Equations of motion. For example:

$$\left(\partial_t + \mathbf{u}\cdot\nabla\right)\mathbf{u} + 2\Omega\times\mathbf{u} + \nabla p/\rho - \mathbf{g} = \nu\nabla^2\mathbf{u}$$

$$\left(\partial_t + \mathbf{u}\cdot\nabla\right)\theta = \kappa\nabla^2\theta$$

$$\left(\partial_t + \mathbf{u}\cdot\nabla\right)S = \mu\nabla^2 S$$

and bndry / initial conditions. This is *really old stuff*.

Our job is numeric approx solutions ("modeling")

Equations of motion. For example:

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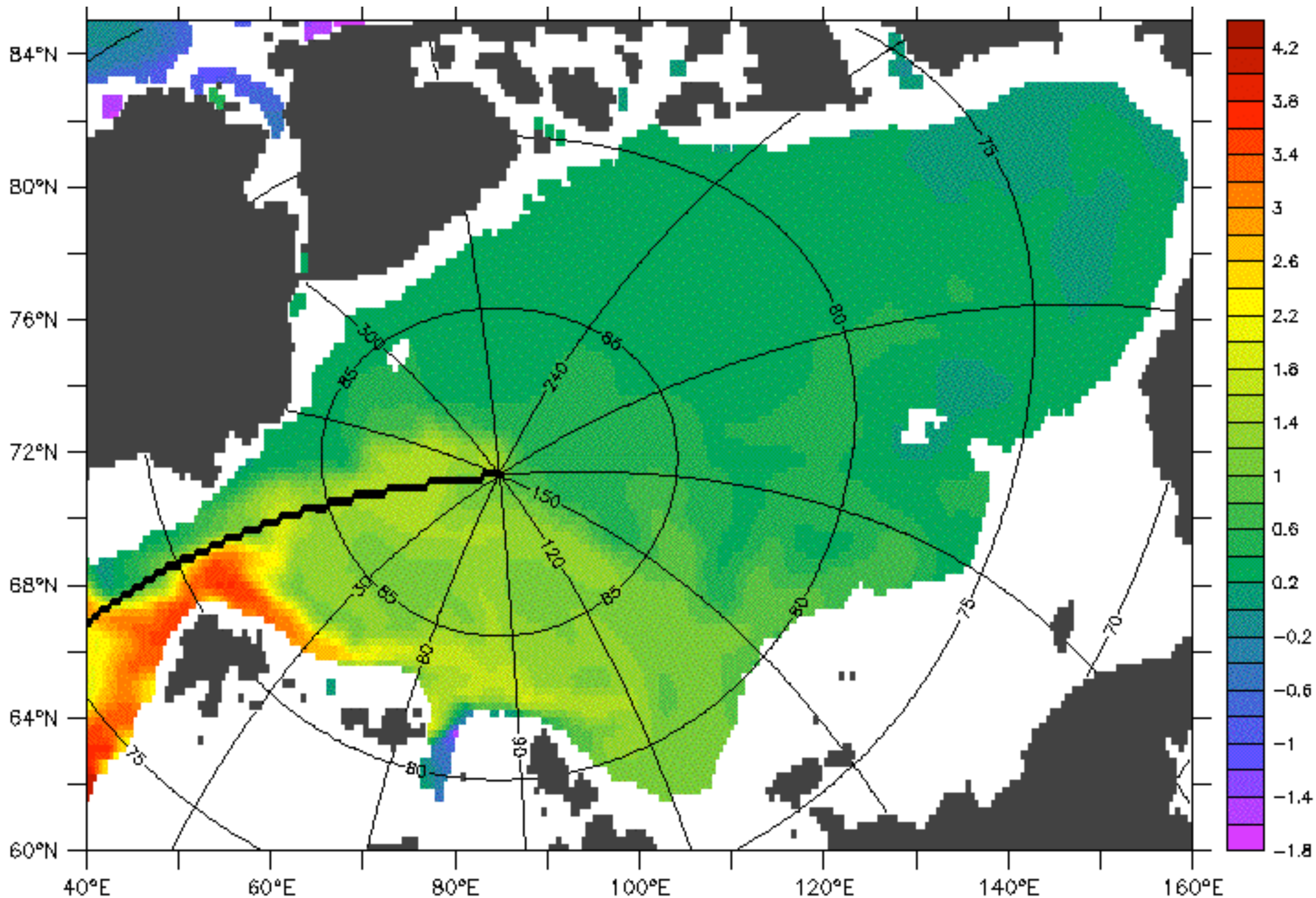
and bndry / initial conditions. This is *really old stuff.*

Our job is numeric approx solutions ("modeling")
-- maybe fudging some coefs ("parameterization").

This talk: Rethink equations of motion. Why?
Consider dependent variables (stuff that "moves")?

Hint: it matters.

What is this stuff? θ at some x,y,z,t? How about $\langle\theta\rangle$?



Rewrite symbolically $\partial \mathbf{Y} / \partial t = \mathbf{F}(\mathbf{Y}) + \mathbf{G}$

where Y collects everything that interests you.

This equation, solved honestly, requires mm grid.
Ignore small scale details => large scale expectations.

Probability! $dP = P(\mathbf{Y})d\mathbf{Y}$ $\langle \mathbf{Y} \rangle \equiv \int \mathbf{Y} dP$

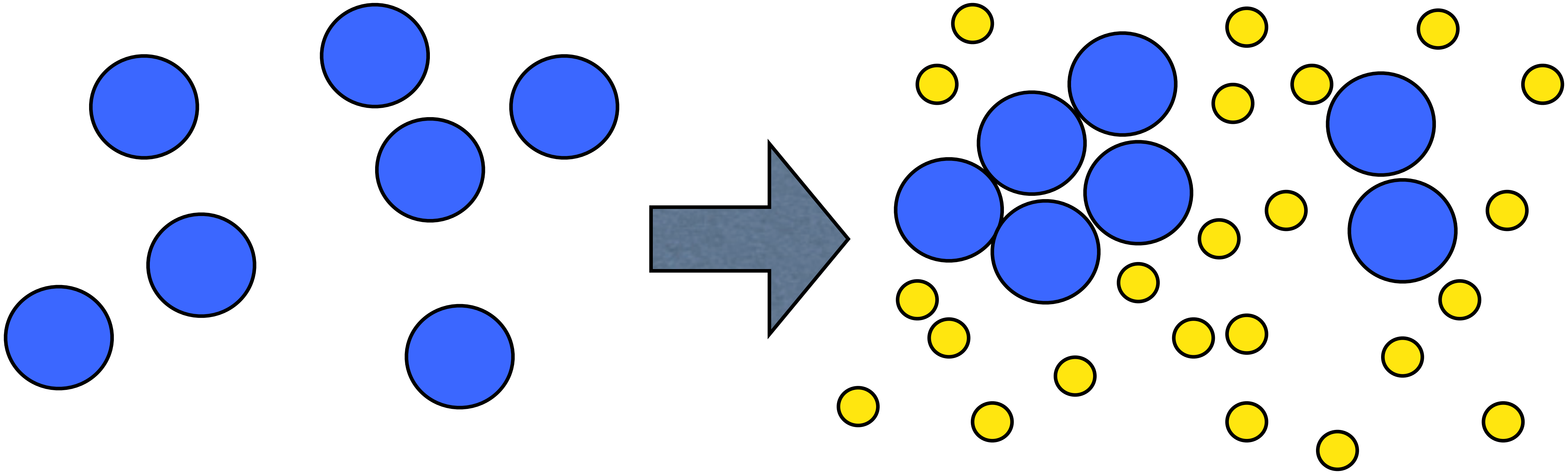
$$\partial \langle \mathbf{Y} \rangle / \partial t = \langle \mathbf{F}(\langle \mathbf{Y} \rangle) \rangle + \langle \mathbf{G} \rangle \ + \ "oops"$$

What to do about "oops"?

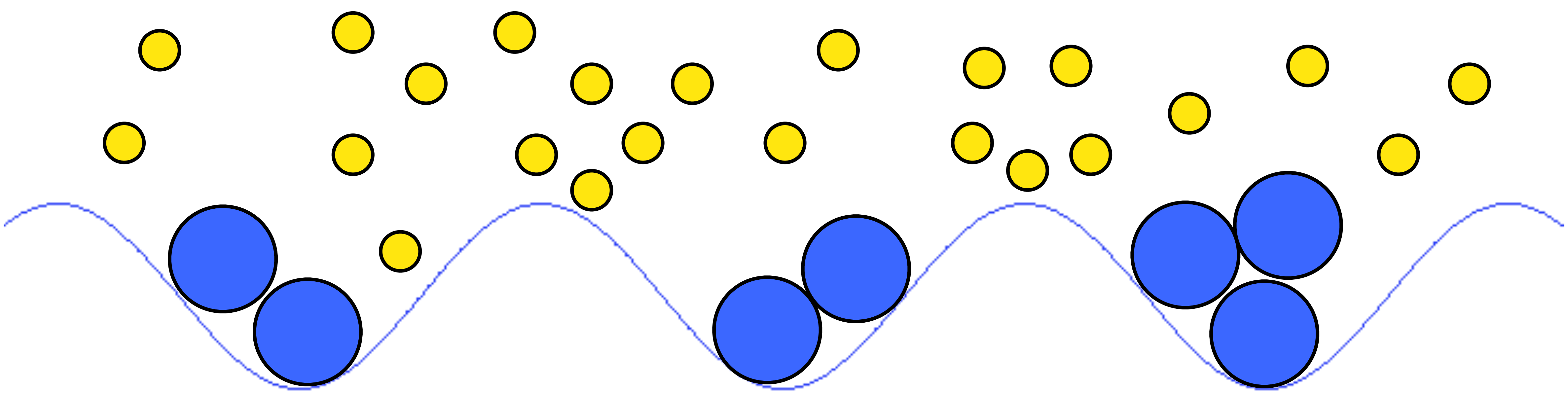
Entropy: $$\boxed{H \equiv -\int dP \log P}$$

then $$\partial \langle \mathbf{Y} \rangle / \partial t \overset{?}{\approx} \langle \mathbf{F}(\langle \mathbf{Y} \rangle) \rangle + \langle \mathbf{G} \rangle \boxed{+ \mathbf{C} \cdot \partial_{\langle \mathbf{Y} \rangle} H}$$

entropic force



Examples from nanoworld (colloids, ‘machines’, microbiol): The only explicit physics is repulsion among balls, and from walls. “See” attraction. “Entropic forcing” in the lab!



Three illustrations: neptune
vertical viscosity
sea ice

neptune (level 1): $$\partial \mathbf{Y} / \partial t = \mathbf{F}(\mathbf{Y}) + \mathbf{G} + \mathbf{C} \cdot \partial_{\mathbf{Y}} H$$

$$\mathbf{C} \cdot \partial_{\mathbf{Y}} H \approx \mathbf{C} \cdot \partial_{\mathbf{Y}} \partial_{\mathbf{Y}} H \cdot (\mathbf{Y}^* - \mathbf{Y}) \equiv \mathbf{K} \cdot (\mathbf{Y}^* - \mathbf{Y})$$

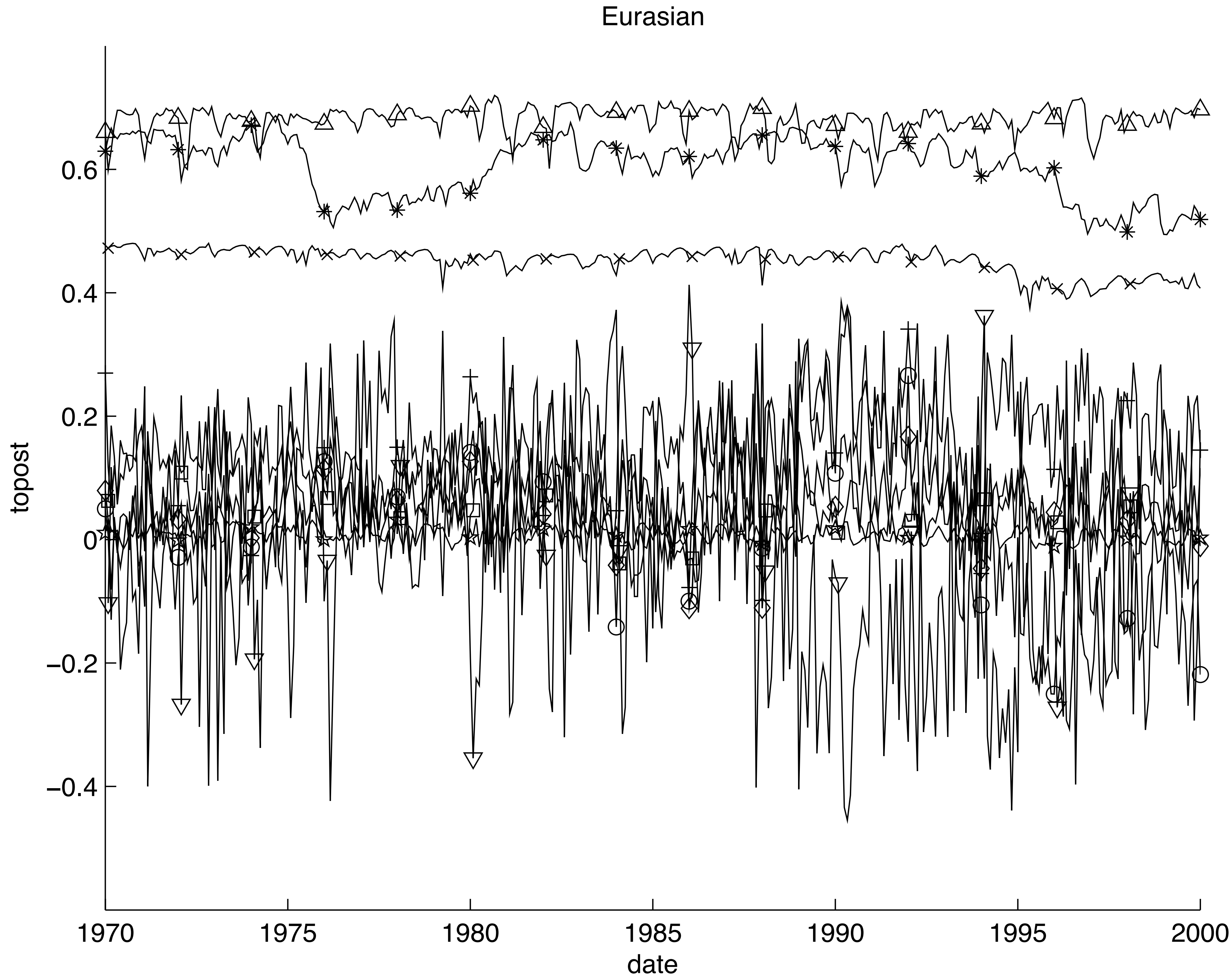
$$\boxed{\mathbf{u}^* = -L^2 \mathbf{f} \times \nabla \log D}$$

(QG energy & enstrophy)

Replace $$\partial_t \mathbf{u} + \ldots = \partial_z A_z \partial_z \mathbf{u} + \nabla_h A_h \nabla_h \mathbf{u} + \ldots$$

with $$\partial_t \mathbf{u} + \ldots = \partial_z A_z \partial_z \mathbf{u} + \underline{\nabla_h A_h \nabla_h (\mathbf{u} - \mathbf{u}^*)} + \ldots$$

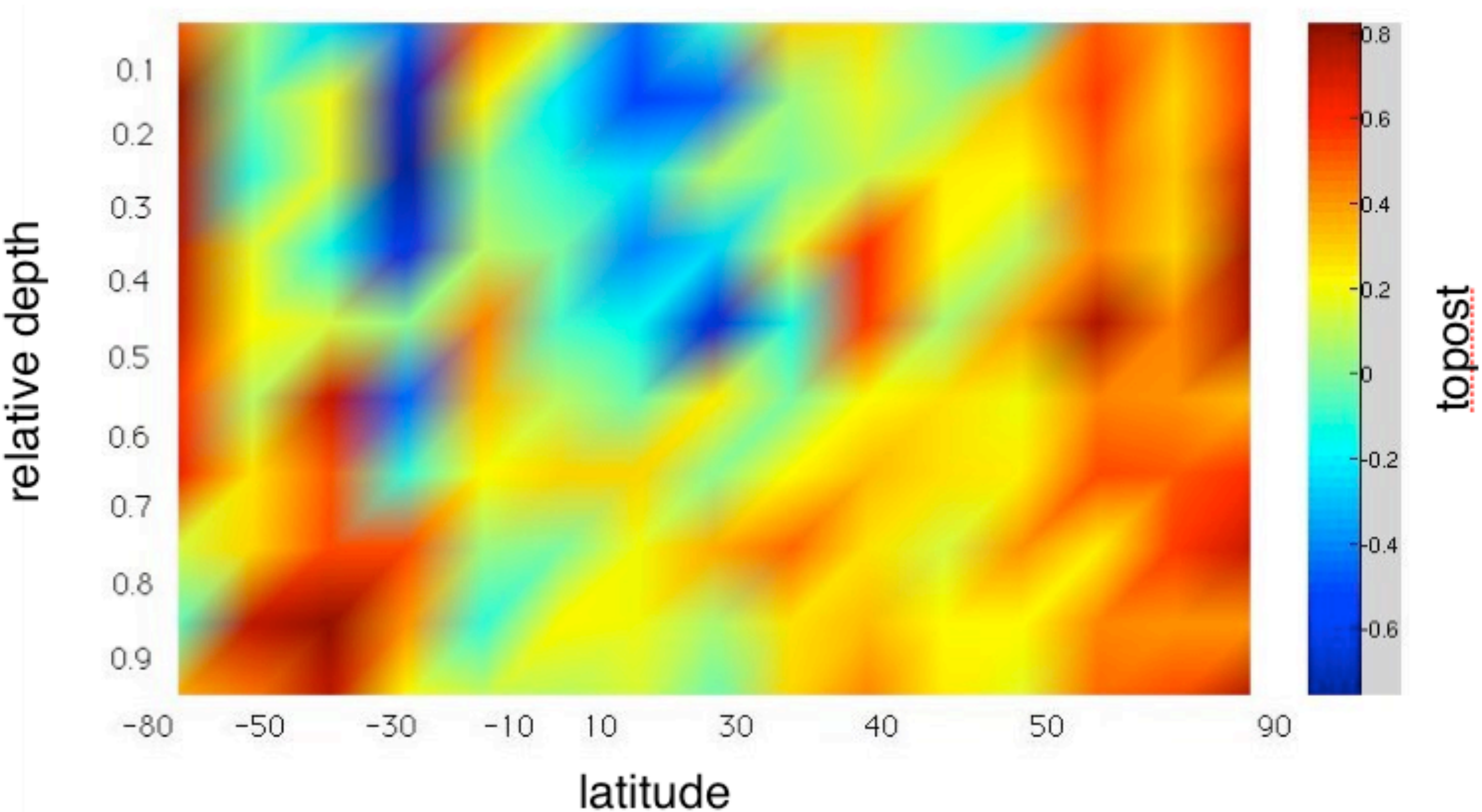
Topostrophy: $\mathbf{f} \times \mathbf{u} \cdot \nabla D / \sqrt{|\mathbf{f} \times \mathbf{u}|^2 |\nabla D|^2}$, basin-averaged from 10 models (JGR, 2007)



Neptune makes big difference but ... better or worse?

Observed from 17120 current meters over 83087 months (JGR, 2008):

Topostrophy vs. latitude and relative depth



Observations are completely outside the range of traditional modeling????

Three illustrations: neptune
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neptune (level 1): $\partial \mathbf{Y} / \partial t = \mathbf{F}(\mathbf{Y}) + \mathbf{G} + \mathbf{C} \cdot \partial_{\mathbf{Y}} H$

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$\mathbf{u}^* = -L^2 \mathbf{f} \times \nabla \log D$ (QG energy & enstrophy)

Replace $\partial_t \mathbf{u} + \ldots = \partial_z A_z \partial_z \mathbf{u} + \nabla_h A_h \nabla_h \mathbf{u} + \ldots$

with $\partial_t \mathbf{u} + \ldots = \partial_z A_z \partial_z \mathbf{u} + \nabla_h A_h \nabla_h (\mathbf{u} - \mathbf{u}^*) + \ldots$

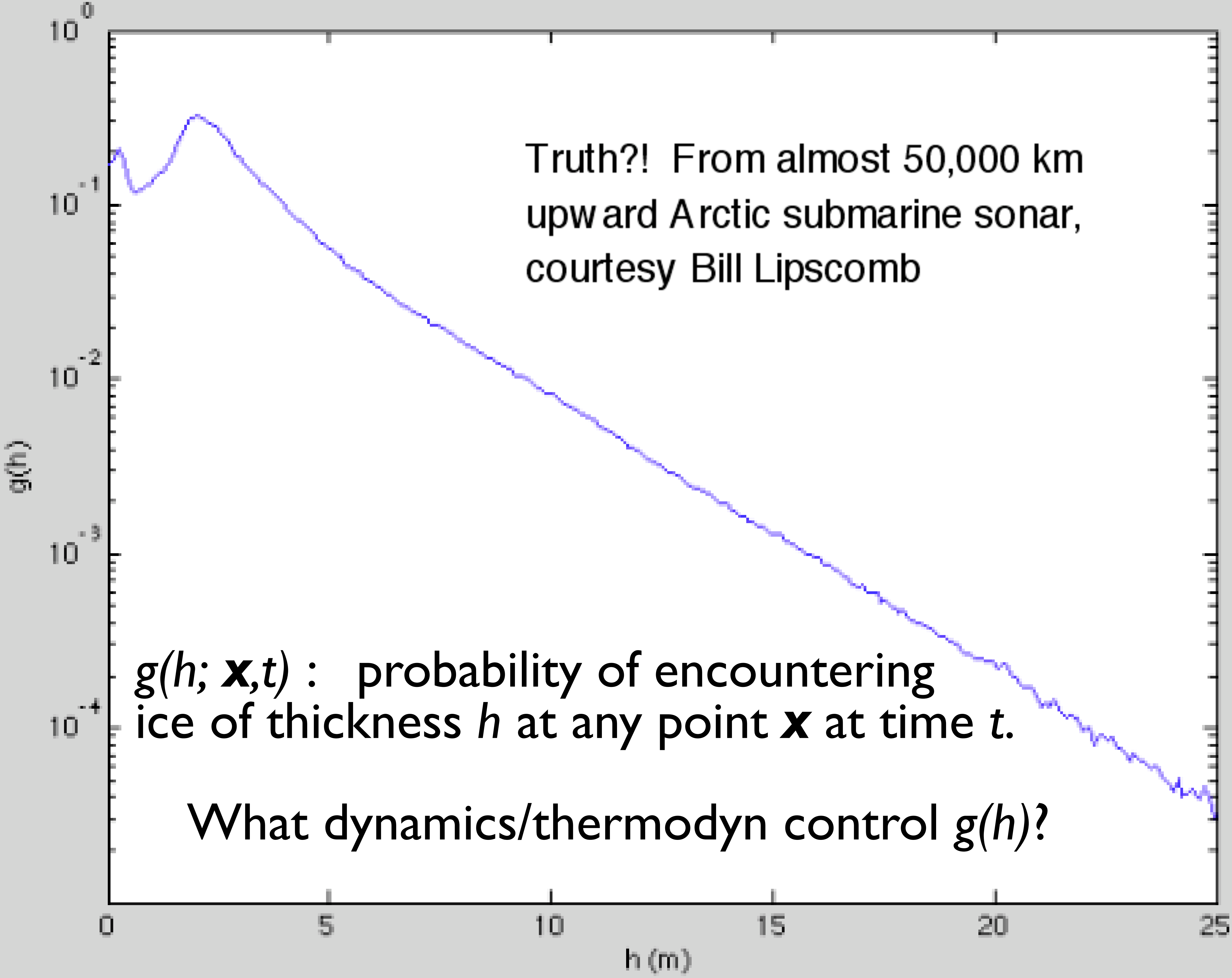
vertical viscosity: $A_z = \left(f^2 / N^2 \right) A_?$

$$f^2 / N^2 \approx 10^{-3}, \quad A_? \approx 10^2, \quad \text{then } A_z \approx 10^{-1} m^2 / s \quad \left(\textit{not } \ 10^{-4} \ \textit{ to } \ 10^{-3} \right)$$

Three illustrations: neptune

vertical viscosity

sea ice



Truth?! From almost 50,000 km upward Arctic submarine sonar, courtesy Bill Lipscomb

g(h; ***x****,t)* : probability of encountering ice of thickness *h* at any point ***x*** at time *t*.

What dynamics/thermodyn control *g(h)*?


santa's
work
shop

A (very!) little math:

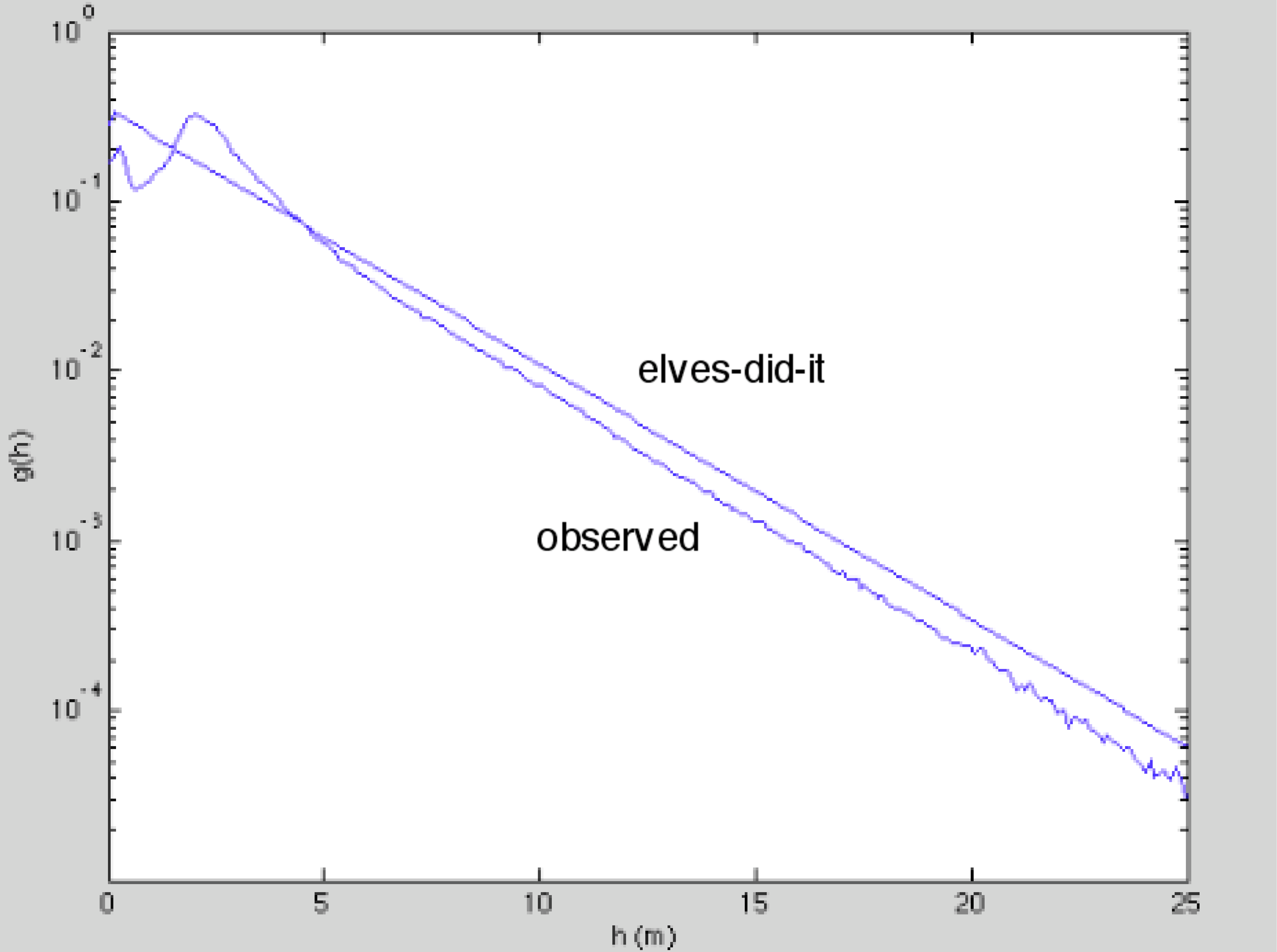
Maximise $S = -\int_0^\infty \log(g) g dh$

subject to $\int_0^\infty g dh = 1$ and $\int_0^\infty g h dh = H$

and *presto!* $g(h) = \exp(-h/H) / H$

Thus, Santa's elves, underoccupied during off-season, randomly toss ice into the Arctic. Is that easy, or what?

Let's see!



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
g(h)
elves-did-it
observed
0
5
10
15
20
25
h (m)

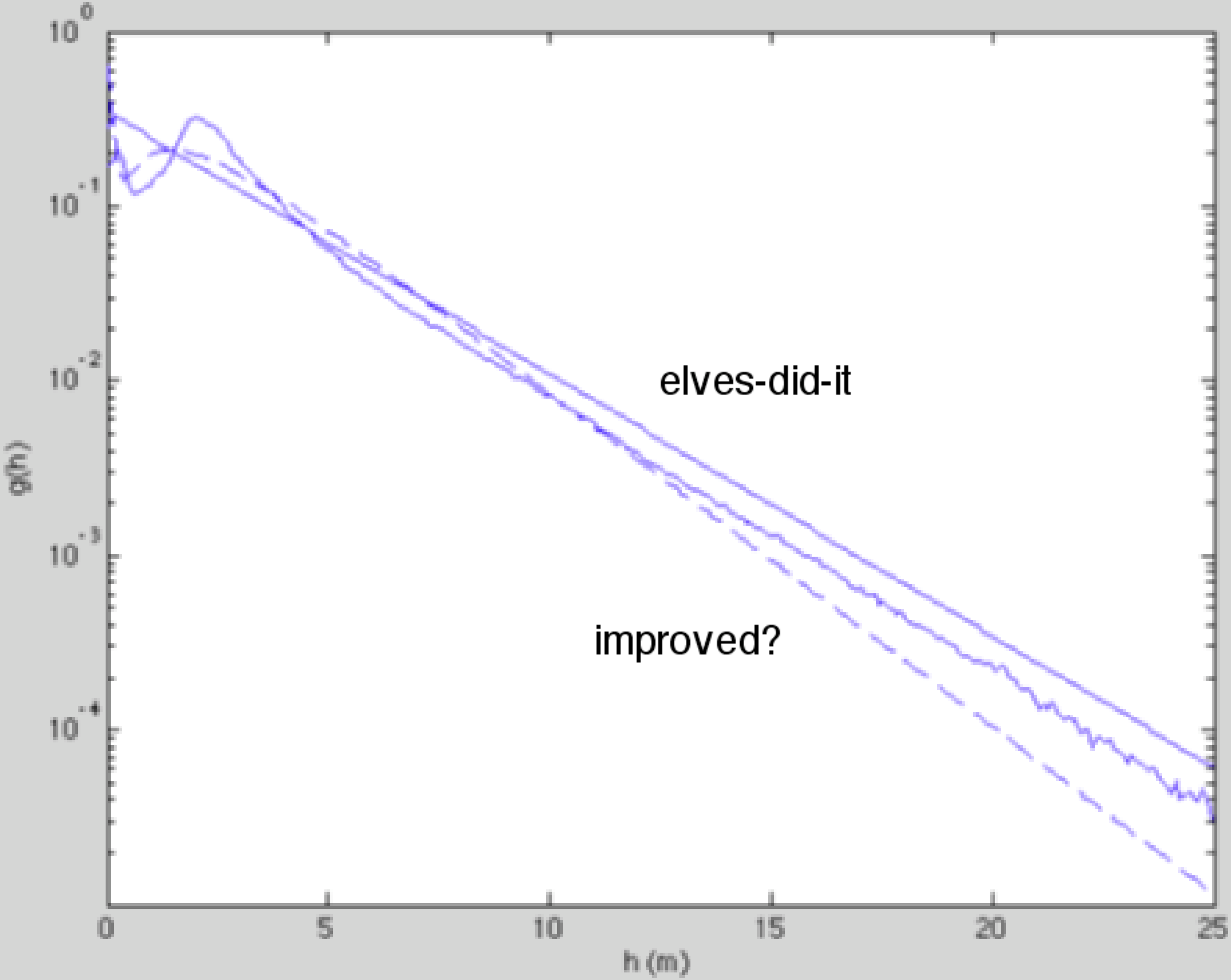
OK, not the greatest ever. But fiddle just a little.

Wind shifts, tides, inertial oscillations, ... open ice.

Recognize an “open water” (thin ice?) fraction 1-*A*. In leads during freezing, thin new ice rapidly forms but is easily smunched into thicker ice. Fix it?

$$D = \frac{A}{(2a-1)H}\left(e^{-\frac{h}{aH}} - e^{-\frac{h}{H-aH}}\right) + \frac{1-A}{bH} e^{-\frac{h}{bH}}$$

Improved? Let’s see.



10^0
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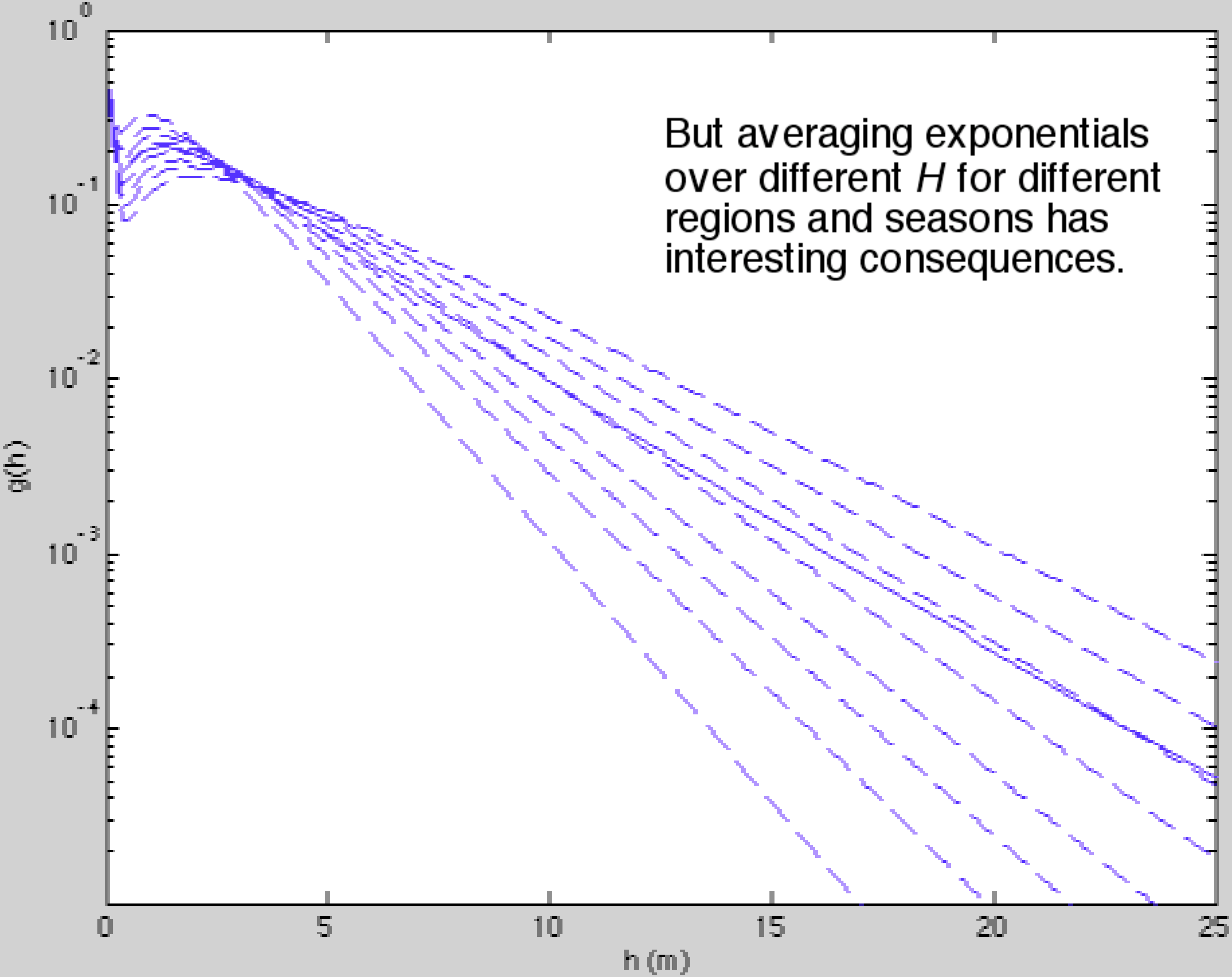


But averaging exponentials over different *H* for different regions and seasons has interesting consequences.

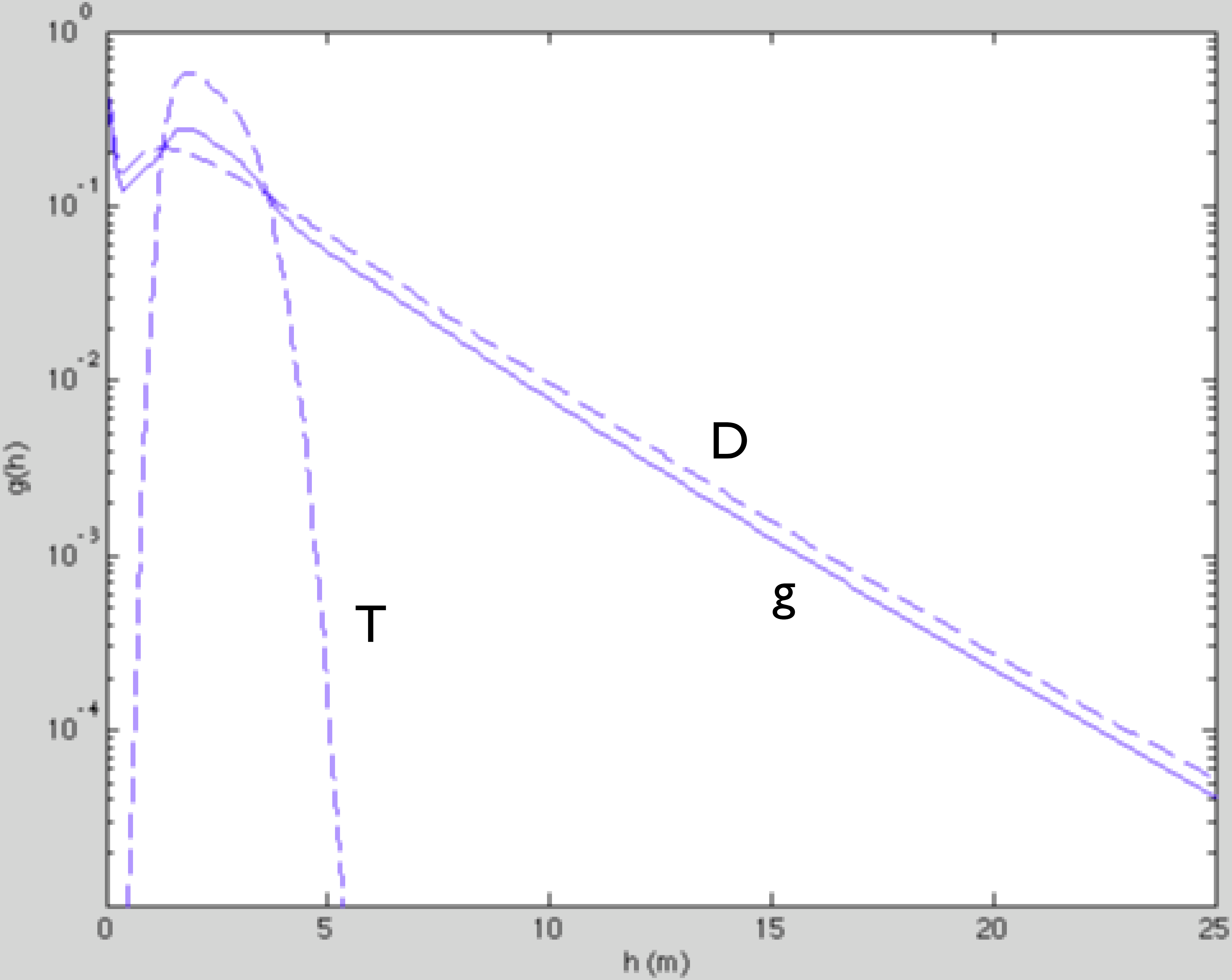
Dynamics -- alone -- spreads out *g(h)*, approaching *D.*

Thermodynamics -- alone -- would focus *g(h)*, e.g. as

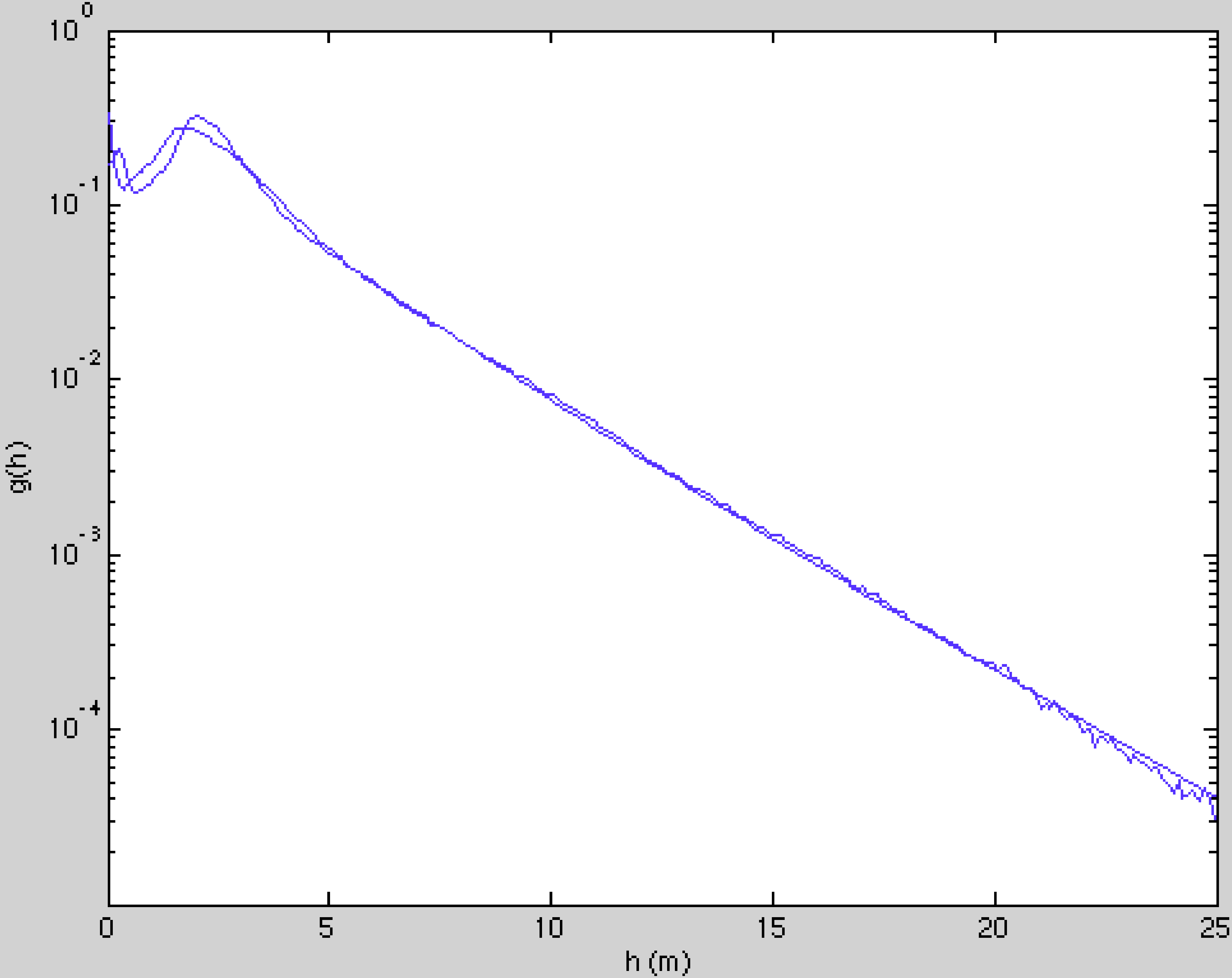
$T = e^{-\frac{(h-aH)^2}{2b^2}} \Big/ \sqrt{2\pi}b$ which also needs be averaged over

various *H*. Combine to $g(h) = cD + (1-c)T$ for some *c.*

See a case $c = 0.8$:



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
g(h)
0
5
10
15
20
25
h (m)
D
g
T



10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
g(h)
0
5
10
15
20
25
h (m)

This is just fudge factors!

Yes. But not so many. *b* only need be small, like *b*~0.1m. *a* is constrained, $0.5 < a < 1$.
I chose $a = 0.7$. That leaves *c*. I chose $c = 0.8$ (more dynamic, not so thermodyn.)

Models solve dA/dt and dH/dt. Fudge something for dc/dt, like integrated deformation? Keep first- and multi-year fractions?

Summary:

Stat mech completes the equations of motion. Includes entropic forcing.

E.g., neptune

E.g., vertical viscosity

Entropy also is about economy (intellectual & computational). When outcomes depend upon many detailed interactions, problems get *easier*. Entropy calculus (a crutch) *avoids* difficult stuff.

E.g., sea ice thickness