Los Alamos
NATIONAL LABORATORY

Climate, Ocean and Sea Ice Modeling Program

GM vs biharmonic ocean mixing in the Arctic

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POP:

Bryan-Cox z-coordinate ocean model

hydrostatic, Boussinesq primitive equations for ocean temperature, salinity, momentum

implicit free surface

implicit barotropic fast gravity wave mode; else explicit 3D

KPP vertical mixing parameterization

GM or biharmonic horizontal mixing (on tracers)

biharmonic horizontal friction (on momentum)

CICE:

energy conserving thermodynamics

energy-based ridging and ice strength

elastic-viscous-plastic dynamics

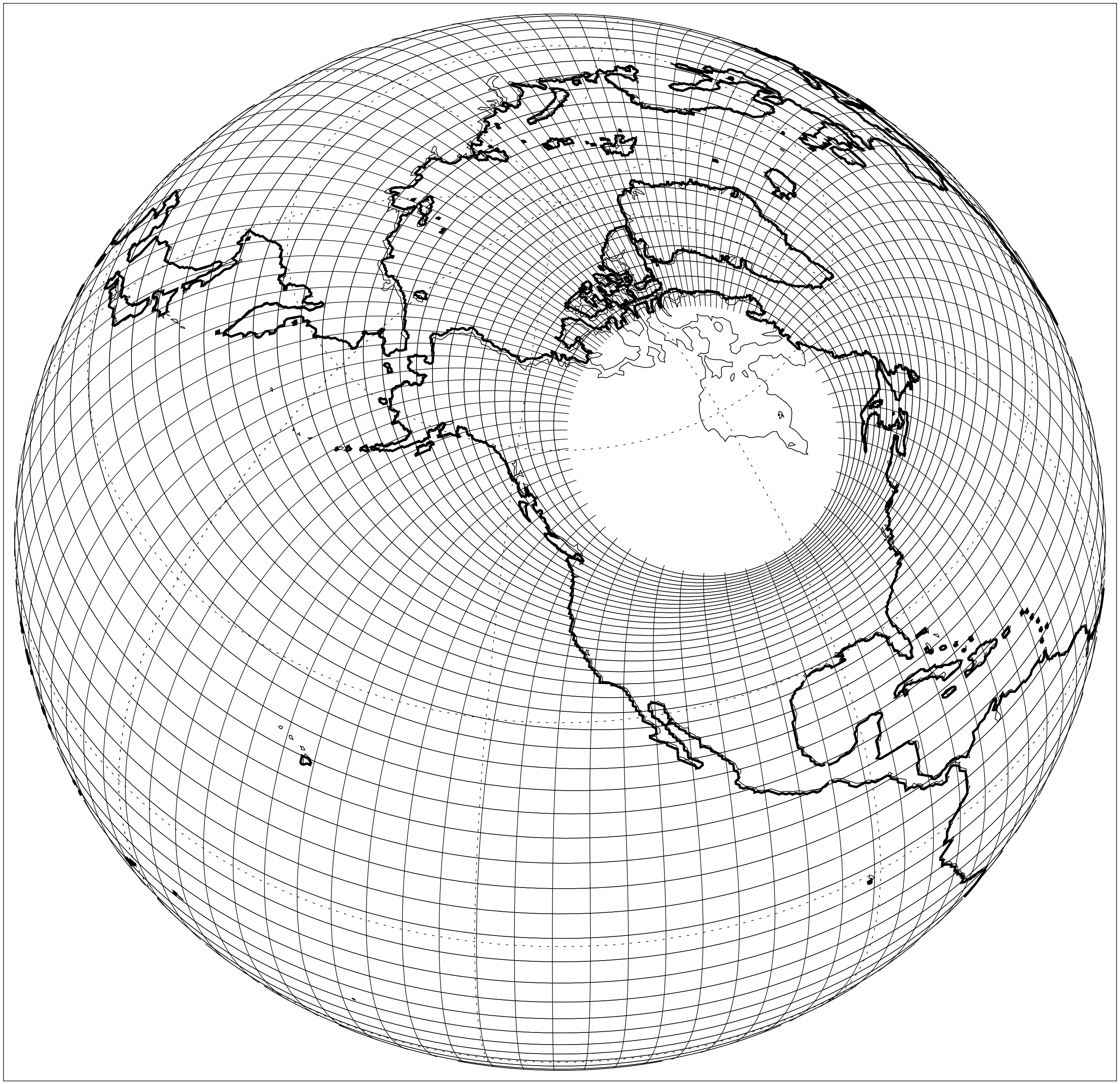
incremental remapping advection

5 thickness categories, 4 layers of ice + 1 layer of snow

variables/tracers (for each thickness category):

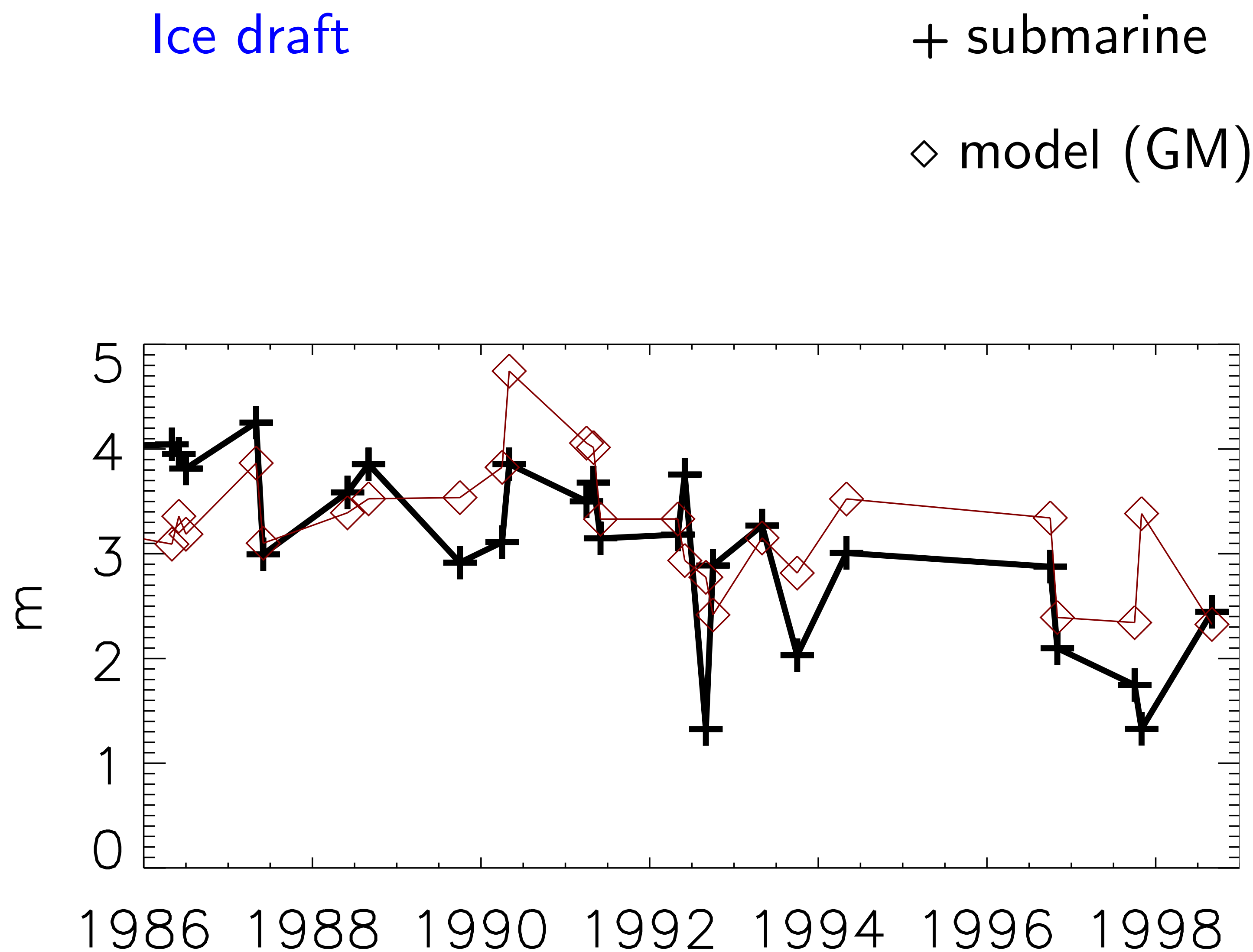
- ice area fraction
- ice/snow volume
- ice/snow energy in each vertical layer
- surface temperature

0.4°: 900x600x40

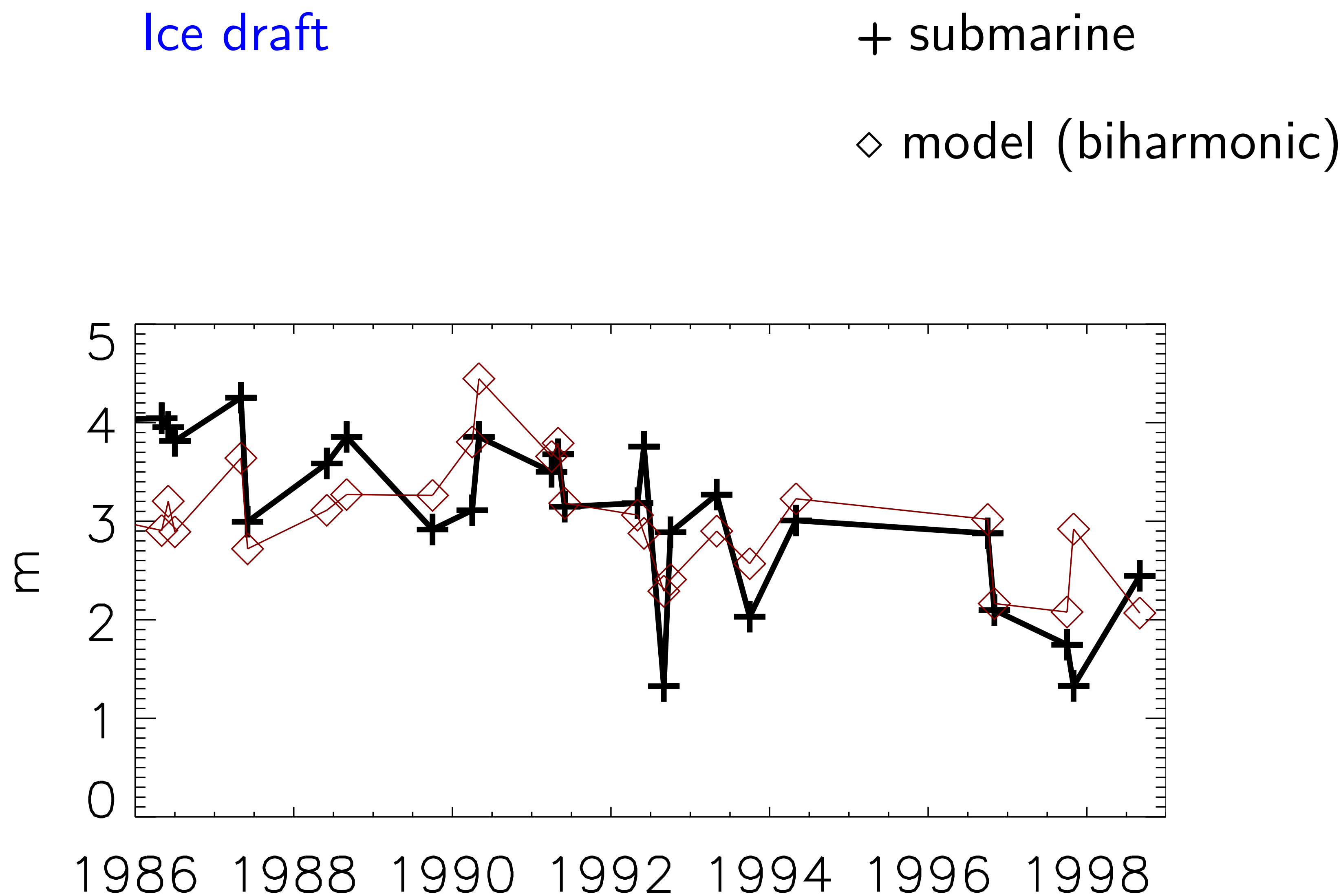


1 of every 100 mesh nodes shown

Ice draft



+ submarine
◇ model (GM)
m
0
1
2
3
4
5
1986
1988
1990
1992
1994
1996
1998



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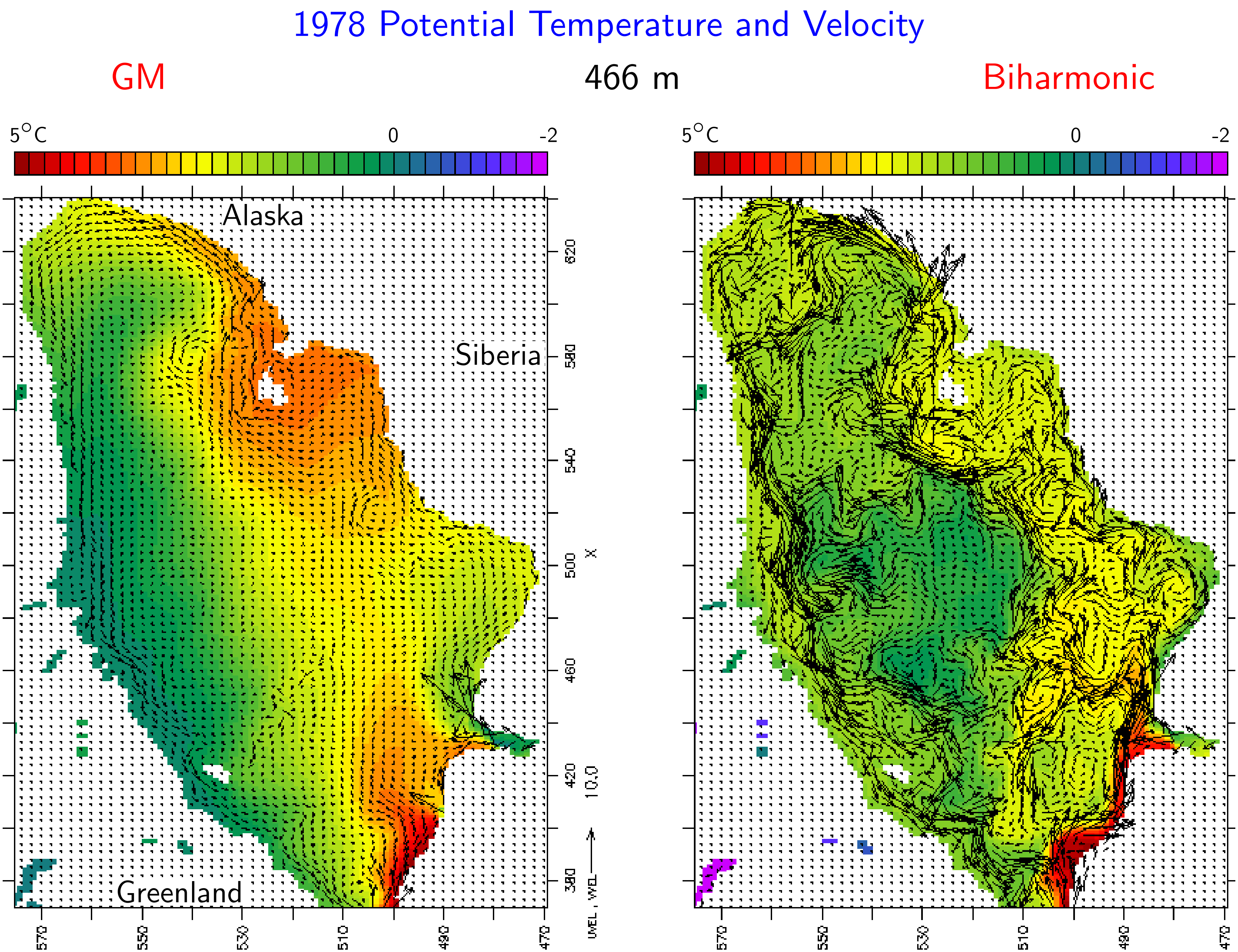
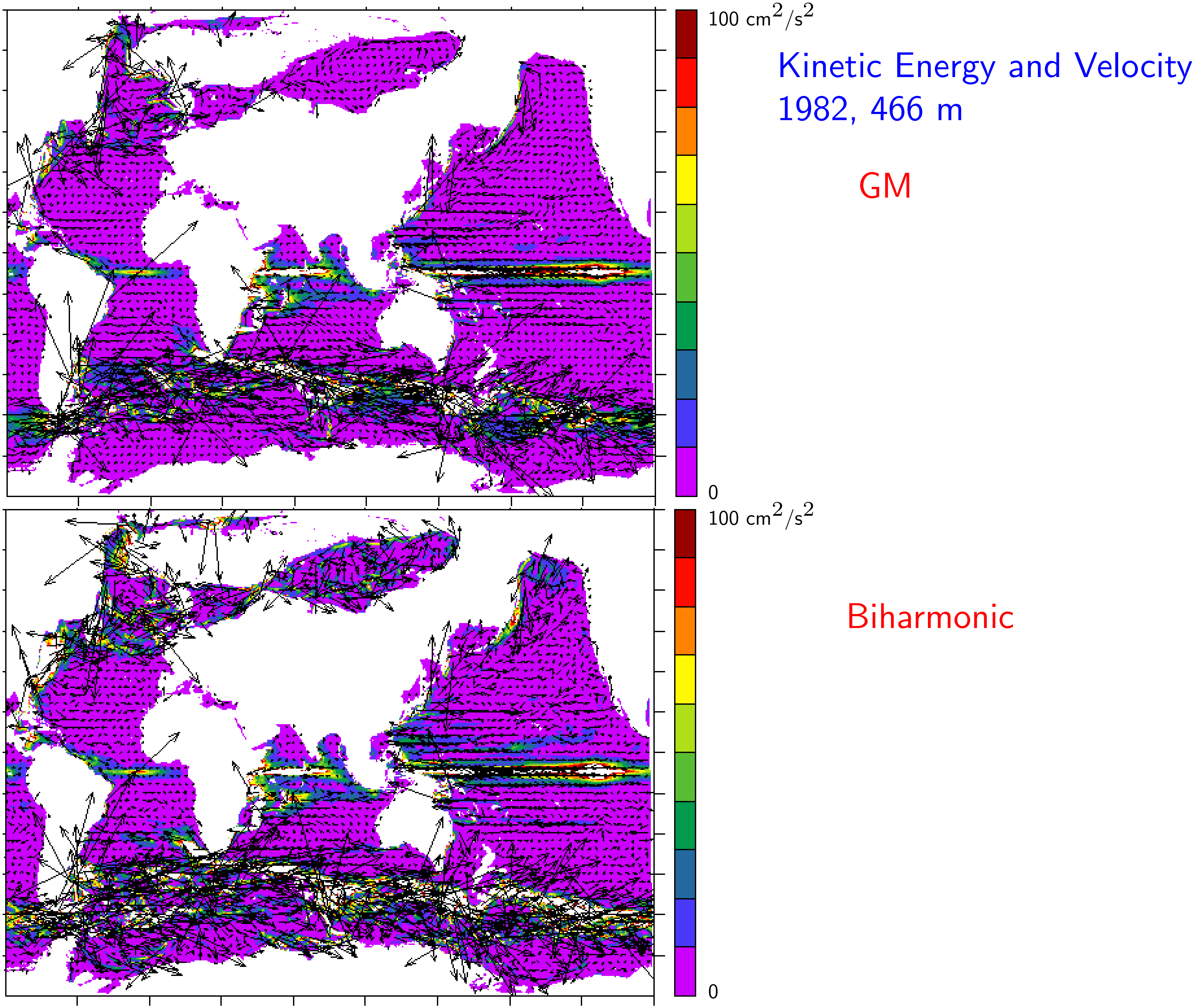

1978 Potential Temperature and Velocity
GM
466 m
Biharmonic
5°C
0
-2
Alaska
Siberia
Greenland
620
580
540
500
460
420
380
X
10.0
UVEL , WVEL
570
550
530
510
490
470


100 cm^2/s^2
0
Kinetic Energy and Velocity
1982, 466 m
GM
100 cm^2/s^2
0
Biharmonic

Tracer transport

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla_2 T + w \frac{\partial T}{\partial z} = D_H(T) + D_V(T)$$

biharmonic mixing

$$D_H = \nabla_2^2 (\kappa_\circ A^{\frac{3}{2}} \nabla_2^2 T)$$

Gent-McWilliams mixing

$$D_H = \nabla_3 \cdot (\mathbf{K} + \mathbf{B}) \cdot \nabla_3 T$$

GM: $$D_H = \nabla_3 \cdot (\mathbf{K} + \mathbf{B}) \cdot \nabla_3 T$$

$$\mathbf{K} = \kappa_I f_1 f_2 \begin{pmatrix} 1 & 0 & -L_x \\ 0 & 1 & -L_y \\ -L_x & -L_y & \frac{|\nabla_2 \rho|^2}{(\partial \rho / \partial z)^2} \end{pmatrix} \qquad \mathbf{B} = \nu f_1 f_2 \begin{pmatrix} 0 & 0 & L_x \\ 0 & 0 & L_y \\ -L_x & -L_y & 0 \end{pmatrix}$$

$$f_1 = \frac{1}{2} + 2\left[\min\left(1, \frac{-z}{||\mathbf{L}||\,\mathrm{R}}\right) - \frac{1}{2}\right]\left[1 - \left|\min\left(1, \frac{-z}{||\mathbf{L}||\,\mathrm{R}}\right) - \frac{1}{2}\right|\right]$$

$$f_2 = \begin{cases} 1 & \text{if } ||\mathbf{L}|| \leq \frac{1}{5} S_M^{(\kappa_I,\nu)} \\ \frac{1}{2} - 2\left(\frac{5||\mathbf{L}||}{2S_M^{(\kappa_I,\nu)}} - 1\right)\left(1 - \left|\frac{5||\mathbf{L}||}{2S_M^{(\kappa_I,\nu)}} - 1\right|\right) & \text{if } \frac{1}{5} S_M^{(\kappa_I,\nu)} < ||\mathbf{L}|| < \frac{3}{5} S_M^{(\kappa_I,\nu)} \\ 0 & \text{if } ||\mathbf{L}|| \geq \frac{3}{5} S_M^{(\kappa_I,\nu)} \end{cases}$$

$$L_x = -\frac{\frac{\partial \rho_p}{\partial \Theta}\frac{\partial \Theta}{\partial x} - \frac{\partial \rho_p}{\partial S}\frac{\partial S}{\partial x}}{\frac{\partial \rho_p}{\partial \Theta}\frac{\partial \Theta}{\partial z} - \frac{\partial \rho_p}{\partial S}\frac{\partial S}{\partial z}} \qquad L_y = -\frac{\frac{\partial \rho_p}{\partial \Theta}\frac{\partial \Theta}{\partial y} - \frac{\partial \rho_p}{\partial S}\frac{\partial S}{\partial y}}{\frac{\partial \rho_p}{\partial \Theta}\frac{\partial \Theta}{\partial z} - \frac{\partial \rho_p}{\partial S}\frac{\partial S}{\partial z}}$$

Theory for simple systems

—mix and match—

Laplacian $T_t = C\,T_{xx}$	biharmonic $T_t = B\,T_{xxxx}$
Scale selectivity Suppose $T = \gamma(t)e^{ikx_n}$, $x_n = n\Delta$ $\Rightarrow$ time scale	Grid dependence Balance advection, diffusion scales $\Rightarrow$ variable coefficients

I. Damping time scales

for simple systems

following Griffies and Hallberg, MWR 2000

Suppose $T = \gamma(t)e^{ikx_n}$, where $x_n = n\Delta$. Then

(1) for the Laplacian case, $T_t = C\,T_{xx}$,

$$\gamma(t) = \exp\left[-C\left(\frac{2}{\Delta}\sin\frac{k\Delta}{2}\right)^2 t\right]$$

$$\text{Damping time scale} = \tau_C = \frac{1}{C\left(\frac{2}{\Delta}\sin\frac{k\Delta}{2}\right)^2}$$

(2) for the biharmonic case, $T_t = B\,T_{xxxx}$,

$$\text{Damping time scale} = \tau_B = \frac{1}{B\left(\frac{2}{\Delta}\sin\frac{k\Delta}{2}\right)^4}$$

$$\Rightarrow \frac{\tau_B}{\tau_C} = \frac{C}{B}\left(\frac{2}{\Delta}\sin\frac{k\Delta}{2}\right)^{-2} \sim \frac{C}{Bk^2} \quad \text{if } \frac{k\Delta}{2} \text{ is small.}$$

Tracer transport

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla_2 T + w \frac{\partial T}{\partial z} = D_H(T) + D_V(T)$$

biharmonic mixing

$$D_H = \nabla_2^2 (\kappa_\circ A^{\frac{3}{2}} \nabla_2^2 T)$$

Gent-McWilliams mixing

$$D_H = \nabla_3 \cdot (\mathbf{K} + \mathbf{B}) \cdot \nabla_3 T$$

II. Scaling argument for grid dependence

for simple systems

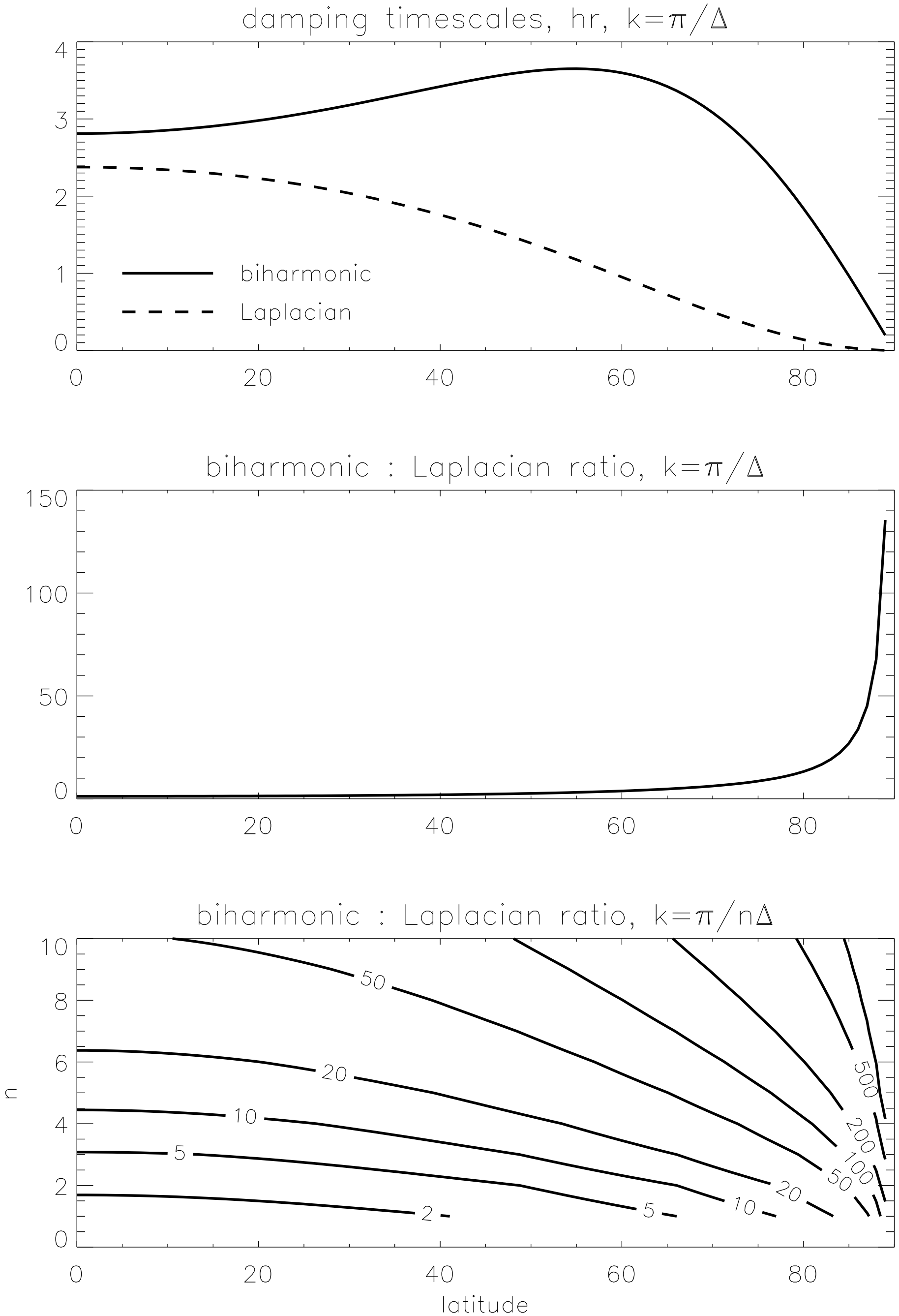
following Maltrud et al., JGR 1998

Balance advective and diffusive terms. Then

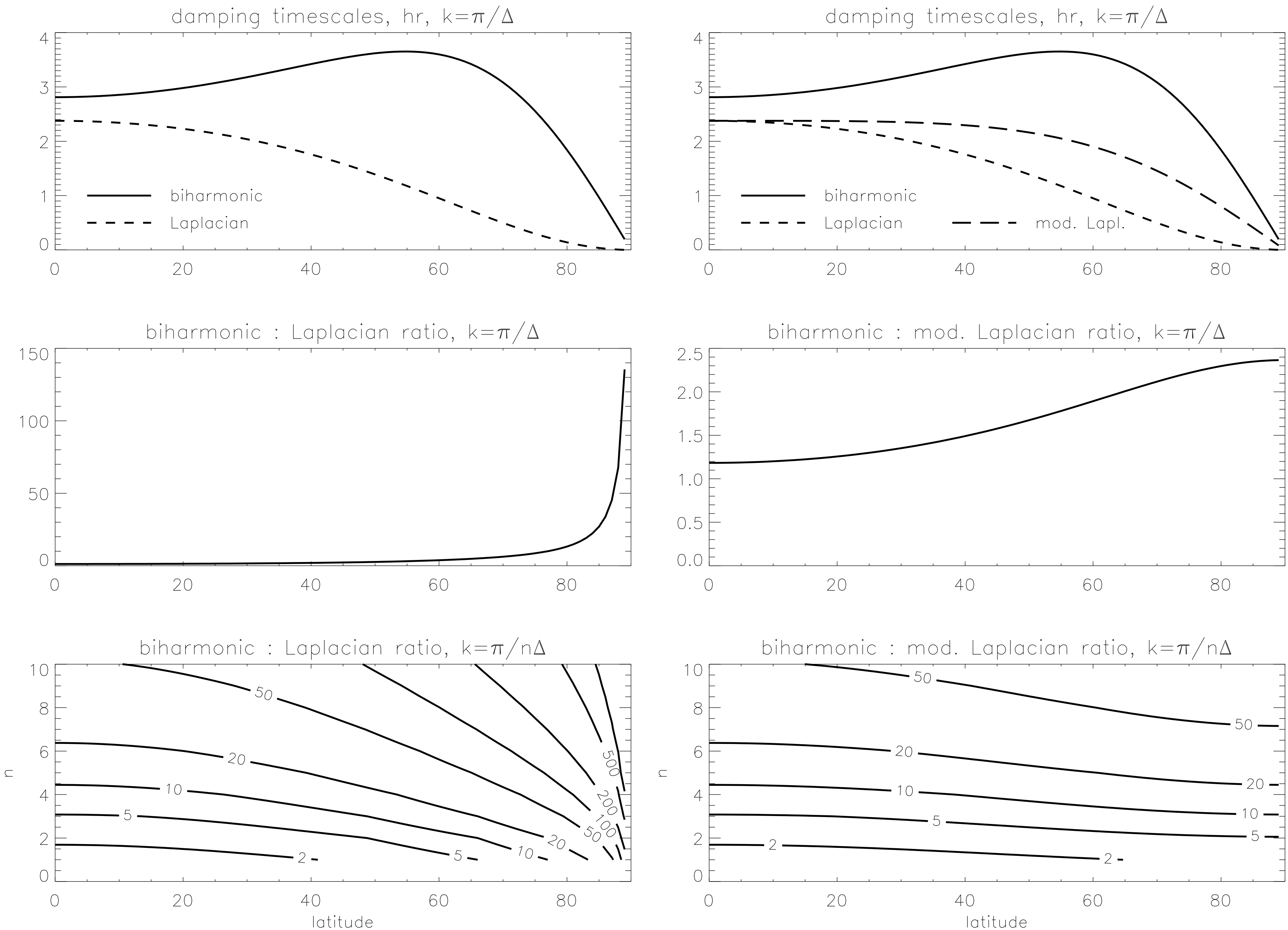
(1) for the Laplacian case, $U\,T_x = C\,T_{xx}$, and C scales with $\Delta \sim dx$,

(2) for the biharmonic case, $U\,T_x = B\,T_{xxxx}$, and B scales with Δ^3.

In 2D, grid cell area $A \sim \Delta^2$, so $C \propto A^{1/2}$ and $B \propto A^{3/2}$.



damping timescales, hr, k=π/Δ
biharmonic
Laplacian
biharmonic : Laplacian ratio, k=π/Δ
biharmonic : Laplacian ratio, k=π/nΔ
n
latitude

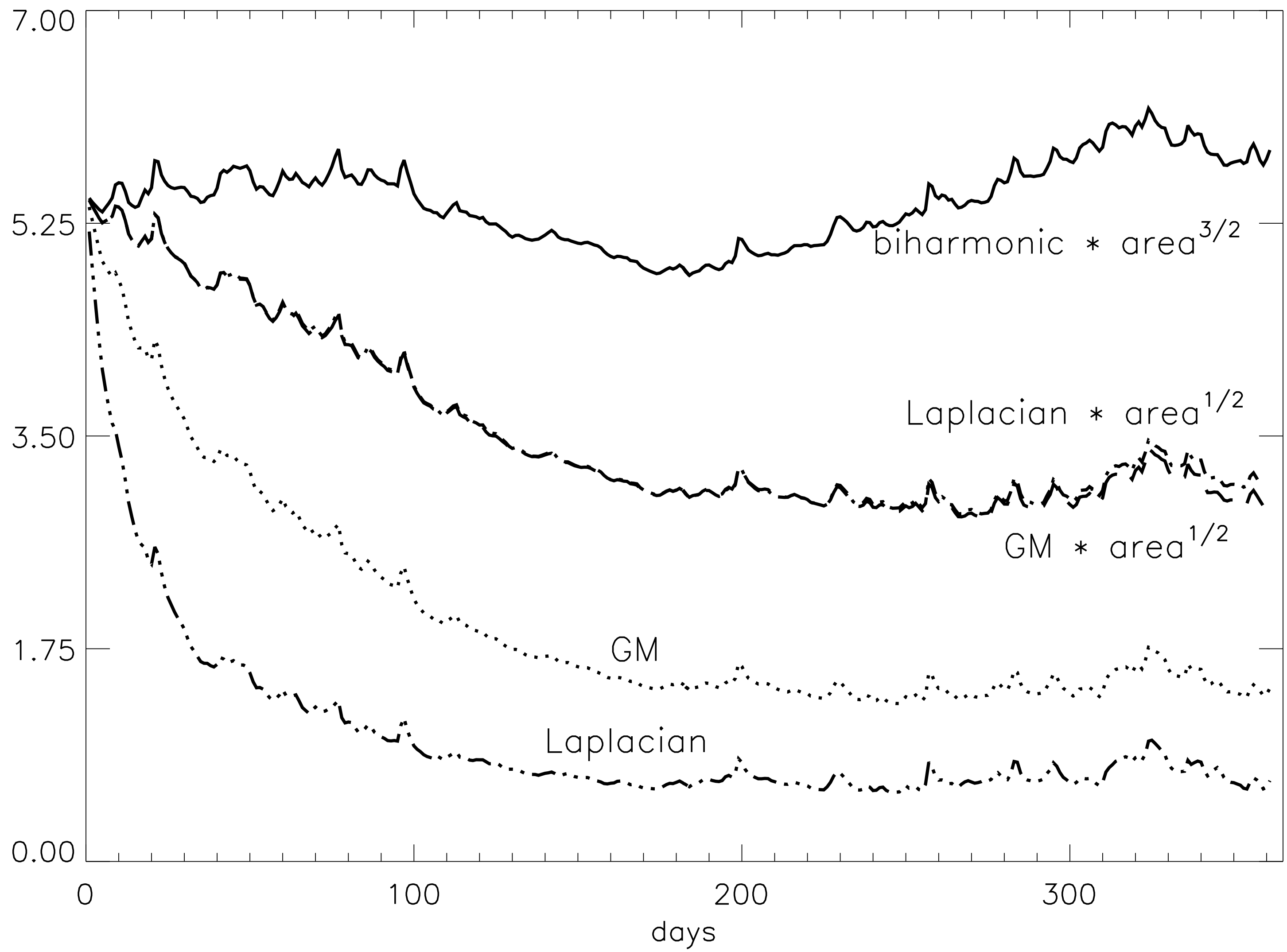


damping timescales, hr, k=π/Δ
biharmonic
Laplacian
damping timescales, hr, k=π/Δ
biharmonic
Laplacian
mod. Lapl.
biharmonic : Laplacian ratio, k=π/Δ
biharmonic : mod. Laplacian ratio, k=π/Δ
biharmonic : Laplacian ratio, k=π/nΔ
n
latitude
biharmonic : mod. Laplacian ratio, k=π/nΔ
n
latitude

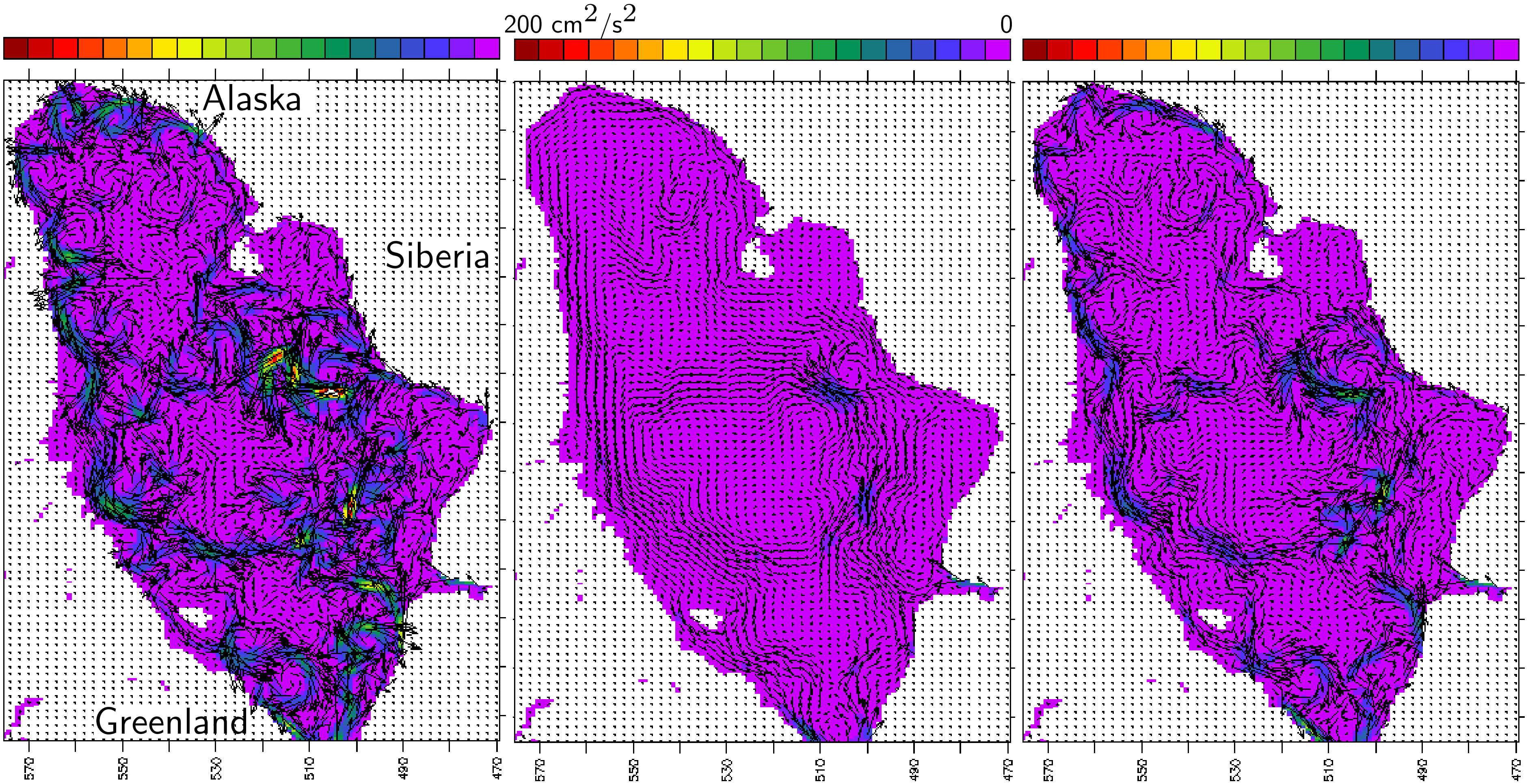
1-year sensitivity runs

Mean kinetic energy

averaged over the ocean volume north of 80°N
initialized from biharmonic AOMIP run, 1 JAN 1982



Kinetic energy and velocity



Biharmonic * area$^{3/2}$

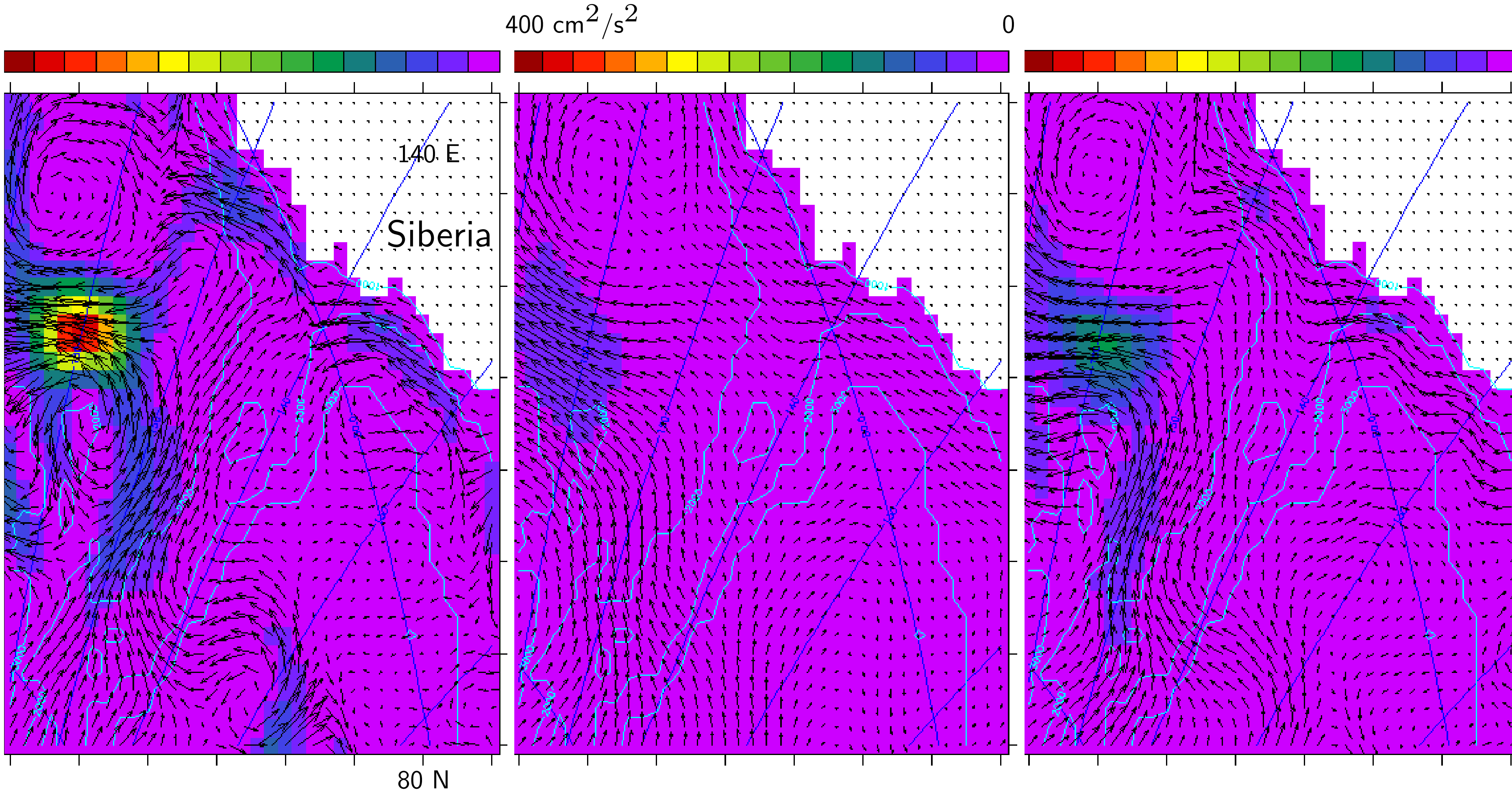
GM

GM * area$^{1/2}$

December 1982, 466 m

1-year sensitivity runs

Kinetic energy and velocity

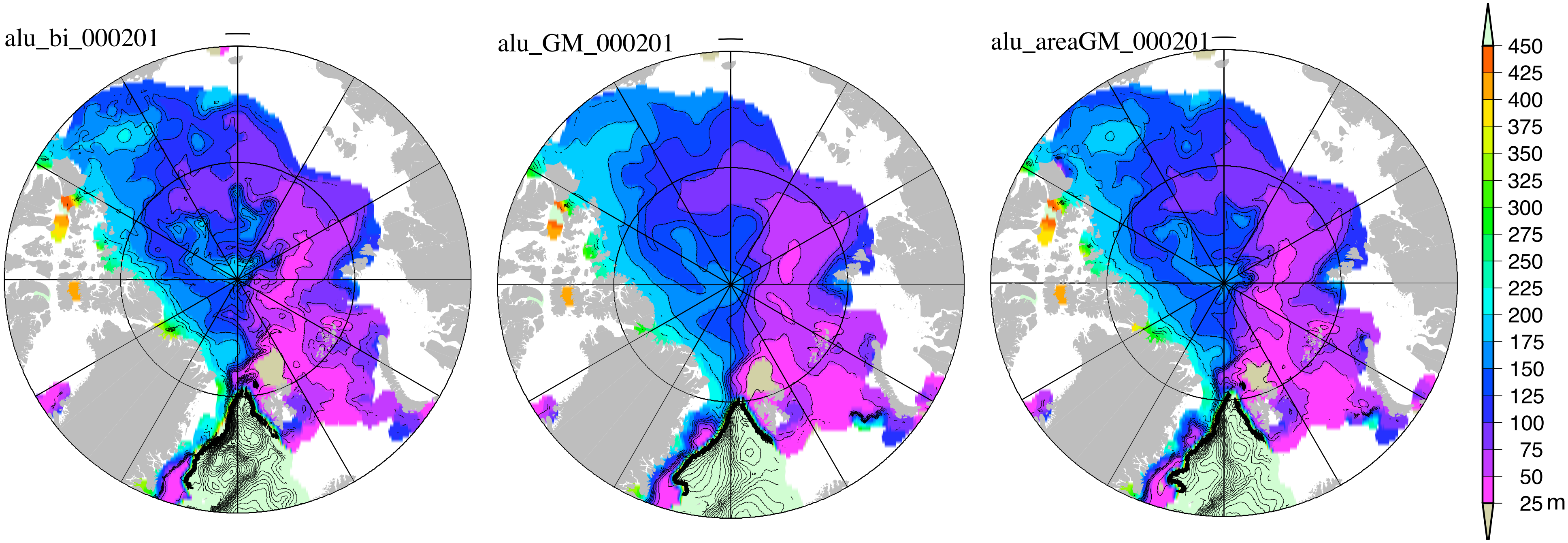


Biharmonic GM area GM

December 1982, 318 m

1-year sensitivity runs

Depth of 0°C isotherm



Biharmonic * area$^{3/2}$

GM

GM * area$^{1/2}$

December 1982

1-year sensitivity runs

Summary

- Scale selectivity and grid-dependent diffusivities both play important roles in high-latitude ocean mixing parameterizations.

- Future global simulations using the GM parameterization should include a diffusivity scaling factor given by the square root of the grid cell area, to prevent diffusion from dominating advection in the evolution of high latitude tracers and circulation.

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