Journal of Correct Oceanography

Volume 2, Number 1, December 1983

Queries concerning this journal may be directed to
The Editor, J. Correct Oceanography, P.O. Box 6000,
Sidney, B. C., V8L 4B2 Canada.

The Lemon-and-Lime Line: I. Appeal for Data

Bruce Warren

Abstract. A socio-geographical boundary, the Lemon-and-Lime Line, is defined, and an appeal is made for data with which to map it accurately.

During the course of oceanographic travels, I have been struck by the fact that, while in some parts of the world people garnish gin-and-tonic with a piece of lime, in other parts they choose lemon. It is my impression that these differing tastes are nearly everywhere uncompromising and irreconcilable; and indeed, that there exists a distinct, socio-geographic demarcation between the two regions — more deeply rooted in the Social Order than even the Prime Meridian — which I have tentatively called the "Lemon-and-Lime Line". No doubt there are compelling and intriguing cultural reasons for this division, but it seems prudent to defer theoretical investigation until the Line has been mapped with at least fair accuracy. Observations available to me at present, for example, are inadequate even to tell whether the Lime-Preferential and Lemon-Preferential Regions are simply or multiply connected. Nor am I at all sure that the Line is everywhere a line: somewhere it may be only a smudge.

The purpose of this note is not, therefore, to speculate, but to report observations of preference that colleagues and I have made, and to solicit additional data from other curious travelers. Mapping the Lemon-and-Lime Line accurately over the globe will evidently be a large enough task to require a collective effort. Our existing observations of geographical preference, listed in Table 1, could at the very least stand verification.

Table 1. Observations of Preference for Lemon and Lime

Country	Preferred Garnish	Remarks
Canada	Lime	
United States:		
Contiguous U.S.	Lime	Sampling in more than thirty states.
College Station, Texas	Lemon wedges	Unexplained exception in certain Aggie bars.
Alaska		
Anchorage	Lime	
Adak	Lemon	Usually.
Hawaii	Lemon	
Puerto Rico	Lime	Thorough sampling.
Bermuda	Lime	
Mexico	Lime	Pronounced lemon.
Costa Rica	Lime	
Panama	Lemon	Lime often available on request.
Venezuela	Lemon	Spot-sampling near Hilton Hotels.
Curacao	Lime	Extensive sampling.
Ecuador	—	Beyond the pale: an order for gin-and-tonic brought gin-and-7-Up.
Brazil	Mixed	Only lime in Recife.
Argentina	Lime	Only Buenos Aires investigated.
Chile:		
Major cities	Either	Lemon most common.
Punta Arenas	Occasionally lemon	Usually neither available.
Puerto Williams	Neither	Gin not common (!)
England	Lime	
Switzerland	Non-uniform	Cantonal preferences: lime in Geneva, lemon in Zurich.
Italy	Lemon	Pronounced limone. Lime required by expatriates in La Spezia.

Country	Preferred Garnish	Remarks
Soviet Union:		
Moscow and Leningrad	Usually neither	Gin available only in "hard-currency" tourist bars.
Yalta and environs	Lime	Tourist bars only.
Turkey	Lemon	Studies only in Istanbul.
Kenya	Either	Noisy data.
South Africa	Lime	
South West Africa (Namibia)	Lime	Usually. Lemon also available in Swakopmund, where much that is unexpected is found.
Seychelles	Lime	Exceptionally enjoyable at Northolme Hotel.
Mauritius	Lemon	Except at Le Chaland Hotel, which caters to a British clientele, and serves lime.
Sri Lanka	Lime	Sampling in Colombo only.
Australia	Lime	Repeated sampling.
New Zealand	Lemon	(!) Adamantly.
Fiji	Lime	
New Caledonia	Lime preferred, often lemon	Lime pronounced lemon or citron, lemon pronounced citron.
Japan	Lemon	
South Korea	Lemon	Sampling limited to Seoul.

The geographical distribution of data is obviously fragmentary and uneven. The coverage of the Americas, of the Pacific, and of the Indian Ocean is not bad, but there are only three observations for Africa, and only one for all of the Asian Mainland. It is regrettable, moreover, that Europe, Cradle of Western Civilization, should have received so little attention to date.

Nevertheless, important questions are posed by the data already in hand. Why, for example, is New Zealand so anomalous among its neighbors in the South Pacific? Why are Brazil and Kenya so indecisive in their allegiances? And, while the Line appears to pass between Adak and the Alaskan Mainland, at what longitude do ships plying from California to Hawaii switch from lime to lemon?

Clearly more data are needed, and I am appealing herewith to other oceanographers (and anyone else, for that matter) to send me their own observations of local preferences for lemon vs. lime, in order that a well documented map of the Lemon-and-Lime Line may eventually be prepared. It seems doubtful that the importance of this project would warrant color-printing of the resulting map in electric green and yellow, but even a black-and-white version should prove stimulating to sociologists, theoreticians, and bon vivants.

Acknowledgements: I am indebted for some field data to certain colleagues who probably wish to remain anonymous. This research has not been supported by the Office of Naval Research.

NCAR/0501/82-11

A simple mechanical analogue of two-dimensional flow

Philip Duncan Thompson

National Center for Atmospheric Research[1]

Boulder, Colorado 80307

November 1982

---

1 The National Center for Atmospheric Research is sponsored by the National Science Foundation.

1. Introduction

The purpose of this note is to draw attention to a remarkable parallelism between the dynamical properties of a solid body, rotating around three different axes in the absence of external forces, and those of a low-order spectral model of two-dimensional flow. The rates of rotation around the three principal axes of a solid body correspond to the amplitudes of three interacting modes in a spectral representation of the streamfunction for the two-dimensional flow of a homogeneous inviscid fluid, and are governed by evolution equations that have precisely the same mathematical form as those for the three modal amplitudes. The moments of inertia around the three principal axes correspond to the eigenvalues associated with the three interacting modes.

The exact correspondence between these two systems sheds no new light on their behavior, which is completely known and understood; it does, however, provide us with a simple and accessible way of demonstrating experimentally some of the important aspects of nonlinear transfer of energy between interacting modes. This article contains the material and equipment for a modest parlor, classroom or laboratory demonstration.

2. The spectral form of the vorticity equation

The dynamical principle governing the two-dimensional motion of a homogeneous inviscid fluid is that of conservation of vorticity, following the motion of material elements of the fluid. I.e.,

$$\frac{\partial \zeta}{\partial t} + \underset{\sim}{V} \cdot \nabla \zeta = 0 \tag{1}$$

in which $\underset{\sim}{V}$ is the fluid velocity, ∇ is the two-dimensional vector gradient: ζ is the vorticity, $\underset{\sim}{K} \cdot \nabla \times V$, and $\underset{\sim}{K}$ is a unit vector normal to the plane of motion. In addition, we impose the condition of incompressibility, which in this case reduces to:

$$\nabla \cdot \underset{\sim}{V} = 0 \ .$$

From this equation we infer that

$$\underset{\sim}{V} = \underset{\sim}{K} \times \nabla \psi \qquad\qquad \zeta = \nabla^2 \psi \ . \tag{2}$$

The streamfunction ψ contains an arbitrary additive function of time, which we are free to specify. Since the component of velocity normal to a fixed boundary necessarily vanishes, so also does the derivative of ψ along the boundary--i.e., ψ is constant on a fixed closed boundary. We specify the arbitrary function contained in ψ by setting $\psi = 0$ on the boundary at all times. Substituting from (2) into (1), we arrive at a single equation with only one dependent variable, namely,

$$\frac{\partial}{\partial t} \nabla^2 \psi + \underset{\sim}{K} \cdot \nabla \psi \times \nabla(\nabla^2 \psi) = 0 \ . \tag{3}$$

Taken together with initial and boundary conditions on ψ, the equation above completely determines the evolution of the flow.

For many purposes, it is convenient to replace (3) by an equivalent system of coupled ordinary differential equations. These are obtained by representing ψ as

$$\psi(x,y,t) = \sum_{i=1}^{N} A_i(t)\ \phi_i(x,y) \qquad (4)$$

$$(i = 1,2,3,\ldots N)$$

in which the $A_i(t)$ are amplitude factors that depend only on time. The ϕi are the eigensolutions of

$$\nabla^2\phi_i = -\alpha_i^2\ \phi_i \ , \qquad (5)$$

and the αi are the eigenvalues of (5) for $\phi i = 0$ on the closed boundary of some region A.

The orthogonality of the ϕi is readily shown by applying (5) for two distinct eigenvalues αp and αq. I.e.,

$$\nabla^2\ \phi_p = -\alpha_p^2\ \phi_p$$

$$\nabla^2\ \phi_q = -\alpha_q^2\ \phi_q \ .$$

Multiplying the first of these equations by ϕq, multiplying the second by ϕp, and subtracting the second from the first, we find that

$$\nabla \cdot (\phi_q\ \nabla\phi_p - \phi_p\ \nabla\phi_q) = (\alpha_q^2 - \alpha_p^2)\ \phi_p\ \phi_q \ .$$

Thus by Gauss' theorem

$$\oint_B \left(\phi_q \frac{\partial \phi_p}{\partial n} - \phi_p \frac{\partial \phi_q}{\partial n} \right) dB = \left(\alpha_q^{\,2} - \alpha_p^{\,2} \right) \int_A \phi_p \phi_q \, dA \quad ,$$

where the derivative with respect to n is taken normal to the boundary. The area integral over A is taken over the entire flow. The line integral is taken around the closed boundary B of the region A: it vanishes, since the ϕ's vanish on B. The α's being distinct, we conclude that

$$\int_A \phi_p \phi_q \, dA = 0 \quad .$$

Finally, owing to the homogeneity of (5), the ϕ_i may be normalized in such a way that

$$\int_A \phi_i^{\,2} \, dA = 1 \quad .$$

In other words, the ϕ_i are an orthogonal set of eigenfunctions and are presumably complete.

Substituting from (4) into (3), we obtain the spectral form of the vorticity equation:

$$\sum_{i=1}^{N} \alpha_i^{\,2} \frac{dA_i}{dt} \phi_i + \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_j^{\,2} A_i A_j (\underset{\sim}{k} \cdot \nabla \phi_i \times \nabla \phi_j) = 0 \quad .$$

Let us next multiply the equation above by a particular eigenfunction ϕ_k and integrate each term over the area A. Then, owing to the orthogonality of the ϕ_i,

$$\alpha_k{}^2 \frac{dA_k}{dt} + \sum_{i=1}^{N} \sum_{j=1}^{N} \beta_{ijk}\, \alpha_j{}^2\, A_i A_j = 0 \tag{6}$$

where the nonlinear interaction coefficients β_{ijk} are given by

$$\beta_{ijk} = \int_A \phi_k\, \underset{\sim}{k} \cdot \nabla\phi_i \times \nabla\phi_j\, dA \quad .$$

Integrating by parts and introducing the condition that $\phi_i = 0$ on B, one can establish the following general properties of β_{ijk}:

(1) it reverses sign under any noncyclic permutation of indices,

(2) remains unchanged under any cyclic permutation of indices, and

(3) vanishes if any two indices are equal.

The consequences of these properties of β_{ijk} are that certain quadratic functions of the A_i's are invariant. For example, multiplying (6) by X_k and summing over k, we see that

$$\frac{d}{dt}\left(\frac{1}{2} \sum_{k=1}^{N} \alpha_k{}^2 A_k{}^2\right) = \sum_{k=1}^{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \beta_{ijk} \alpha_j{}^2 A_i A_j A_k$$

$$= \sum_{j=1}^{N} \alpha_j{}^2 A_j \sum_{i=1}^{N} \sum_{k=1}^{N} \beta_{ijk} A_i A_k \quad .$$

The summation over i and k vanishes, owing to properties (1) and (3) of β_{ijk}. We conclude, therefore, that the quantity

$$\frac{1}{2} \sum_{k=1}^{N} \alpha_k{}^2 A_k{}^2$$

is invariant. In interpreting this result, we note that

$$\frac{1}{2}\int_A \underset{\sim}{V}\cdot\underset{\sim}{V}\, dA = \frac{1}{2}\int_A \nabla\psi\cdot\nabla\psi\, dA = -\frac{1}{2}\int_A \psi\nabla^2\psi\, dA$$

whence, from the definition (4) and the orthogonality of the ϕ_i,

$$\frac{1}{2}\int_A \underset{\sim}{V}\cdot\underset{\sim}{V}\, dA = \frac{1}{2}\sum_{k=1}^{N} \alpha_k^{\,2} A_k^{\,2} \quad .$$

Thus, the total kinetic energy of the flow is invariant, a fact that can be derived directly from (3) in three steps. The significance of the present result is that the total kinetic energy of <u>any finite truncation</u> of (4) is also invariant, and that such finite truncations retain important dynamical properties of the <u>complete</u> representation.

Similarly, multiplying (6) by $\alpha_k^{\,2} A_k$ and summing over k, we get

$$\frac{d}{dt}\left(\frac{1}{2}\sum_{k=1}^{N} \alpha_k^{\,4} A_k^{\,2}\right) = \sum_{k=1}^{N}\sum_{i=1}^{N}\sum_{j=1}^{N} \beta_{ijk}\alpha_j^{\,2}\alpha_k^{\,2} A_i A_j A_k$$

$$= \sum_{i=1}^{N} A_i \sum_{j=1}^{N}\sum_{k=1}^{N} \beta_{ijk}\alpha_j^{\,2}\alpha_k^{\,2}\, A_j A_k \quad .$$

In this case, owing to properties (1) and (3) of β_{ijk}, the summation over j and k vanishes. Thus, the truncated system possesses a second quadratic invariant,

$$\frac{1}{2}\sum_{k=1}^{N} \alpha_k^{\,4} A_k^{\,2} \quad .$$

This quantity is readily identified as

$$\frac{1}{2}\int_A (\nabla^2\psi)^2 \, dA = \frac{1}{2}\int_A \zeta^2 \, dA$$

or, in recent terminology, "enstrophy."

3. The 3-mode system and its analogy to the "gyroscopic" equations

Specializing (6) for the case of three interacting modes (the simplest nontrivial case), we generate the system of evolution equations for A_1, A_2 and A_3. They are:

$$\alpha_1{}^2 \frac{dA_1}{dt} = \beta_{123} (\alpha_2{}^2 - \alpha_3{}^2) A_2A_3$$

$$\alpha_2{}^2 \frac{dA_2}{dt} = \beta_{123} (\alpha_3{}^2 - \alpha_1{}^2) A_1A_3 \tag{7}$$

$$\alpha_3{}^2 \frac{dA_3}{dt} = \beta_{123} (\alpha_1{}^2 - \alpha_2{}^2) A_1A_2 \quad .$$

Now, it should be noted that these are just the "gyroscopic" equations of Euler (cf. Joos, 1932), if we take A_i to be the rate of rotation of a solid body about the ith principal axis and identify $\alpha_i{}^2$ with the moment of inertia around that axis. Hence, whatever can be said of the dynamics of a rotating solid body (free of external forces) also applies to the behavior of a 3-mode truncation of two-dimensional flow.

We next examine some properties of solutions of the nonlinear system (7). To simplify matters somewhat, let us define:

$$\alpha_1 A_1 = X \qquad \alpha_2 A_2 = Y \qquad \alpha_3 A_3 = Z \qquad t = \frac{\alpha_1\alpha_2\alpha_3}{\beta}\tau \quad .$$

Under these transformations, (7) becomes

$$\frac{dX}{d\tau} = (\alpha_2{}^2 - \alpha_3{}^2)\ YZ$$

$$\frac{dY}{d\tau} = (\alpha_3{}^2 - \alpha_1{}^2)\ XZ \qquad (8)$$

$$\frac{dZ}{d\tau} = (\alpha_1{}^2 - \alpha_2{}^2)\ XY \quad .$$

With the transformations shown above, the invariance of kinetic energy and enstrophy may be expressed as

$$X^2 + Y^2 + Z^2 = X_0{}^2 + Y_0{}^2 + Z_0{}^2$$

$$\alpha_1{}^2X^2 + \alpha_2{}^2Y^2 + \alpha_3{}^2Z^2 = \alpha_1{}^2X_0{}^2 + \alpha_2{}^2Y_0{}^2 + \alpha_3{}^2Z_0{}^2$$

in which the subscript "zero" denotes the initial value of a variable. Multiplying the first of these equations by $\alpha 1^2$, $\alpha 2^2$ and $\alpha 3^2$ in succession, and subtracting the second equation, we get

$$(\alpha_1{}^2 - \alpha_2{}^2)(Y^2 - Y_0{}^2) + (\alpha_1{}^2 - \alpha_3{}^2)(Z^2 - Z_0{}^2) = 0$$

$$(\alpha_2{}^2 - \alpha_1{}^2)(X^2 - X_0{}^2) + (\alpha_2{}^2 - \alpha_3{}^2)(Z^2 - Z_0{}^2) = 0$$

$$(\alpha_3{}^2 - \alpha_1{}^2)(X^2 - X_0{}^2) + (\alpha_3{}^2 - \alpha_2{}^2)(Y^2 - Y_0{}^2) = 0 \quad .$$

The symmetry of these equations suggests that a further transformation might be in order, namely,

$$X = (\alpha_3^{\,2}-\alpha_2^{\,2})^{1/2}x \qquad Y = (\alpha_3^{\,2}-\alpha_1^{\,2})^{1/2}y \qquad Z = (\alpha_2^{\,2}-\alpha_1^{\,2})^{1/2}z \quad . \tag{9}$$

The relationships above then become:

$$(y^2 - y_0^{\,2}) + (z^2 - z_0^{\,2}) = 0$$

$$(x^2 - x_0^{\,2}) - (z^2 - z_0^{\,2}) = 0$$

$$(x^2 - x_0^{\,2}) + (y^2 - y_0^{\,2}) = 0 \quad .$$

Since each pair of these equations expresses two of the variables in terms of the third, equations (8) may now be written as three uncoupled equations in x, y, and z. With the transformations given in (9),

$$\frac{dx}{d\tau} = k\left[(y_0^{\,2} + x_0^{\,2} - x^2)(z_0^{\,2} - x_0^{\,2} + x^2)\right]^{1/2}$$

$$\frac{dy}{d\tau} = -k\left[(x_0^{\,2} + y_0^{\,2} - y^2)(z_0^{\,2} + y_0^{\,2} - y^2)\right]^{1/2} \tag{10}$$

$$\frac{dz}{d\tau} = k\left[(x_0^{\,2} - z_0^{\,2} + z^2)(y_0^{\,2} + z_0^{\,2} - z^2)\right]^{1/2}$$

in which $k = \left[(\alpha_3^{\,2} - \alpha_2^{\,2})(\alpha_3^{\,2} - \alpha_1^{\,2})(\alpha_2^{\,2} - \alpha_1^{\,2})\right]^{1/2}$. Equations (10) may be solved by quadrature. The solutions are easily shown to be bi-periodic: they are, in fact, expressible in terms of Jacobi elliptic functions.

It is not, however, necessary to find the complete solutions of (10) in order to understand the behavior of the system in certain specific circumstances. Let us suppose, for example, that all of the kinetic energy of the system were concentrated initially in one mode. If that were exactly true, then (8) would imply that none of the amplitudes change with time. But what happens if one or both of the other two modes initially contain some small amount of kinetic energy? Is the large supply of energy in one mode transferred to the two energy-deficient modes, or does it remain in that mode?

To study these questions, it is convenient to put equations (10) in a slightly different form, obtainable by squaring both sides of each equation, differentiating with respect to time and removing factors common to all terms. The result is:

$$\frac{d^2x}{d\tau^2} = k^2x\left[2x_0^2 + y_0^2 - z_0^2 - 2x^2\right] \qquad (11a)$$

$$\frac{d^2y}{d\tau^2} = -k^2y\left[x_0^2 + 2y_0^2 + z_0^2 - 2y^2\right] \qquad (11b)$$

$$\frac{d^2z}{d\tau^2} = k^2z\left[-x_0^2 + y_0^2 + 2z_0^2 - 2z^2\right] \quad . \qquad (11c)$$

For future reference, we also order the eigenvalues, such that $\alpha_1 < \alpha_2 < \alpha_3$. This renders k real and k^2 positive. The amplitudes x and z are then associated with the lowest and highest modes, respectively, and y with the intermediate mode.

Let us suppose that $y_0 \ll x_0$, $z_0 \ll x_0$. Near the initial time, then, (11b) and (11c) are approximately:

$$\frac{d^2y}{d\tau^2} = -k^2x_0^2y$$

$$\frac{d^2z}{d\tau^2} \simeq -k^2x_0{}^2z \quad .$$

Near initial time, therefore, the solutions are:

$$y = y_0 \cos kx_0\tau$$

$$z = z_0 \cos kx_0\tau \quad .$$

That is, the intermediate and highest modes undergo oscillations, with amplitudes equal to their small initial values. The conclusion is that, if the kinetic energy is initially concentrated in the lowest mode, it stays there.

In considering the case when $x_0 \ll z_0$ and $y_0 \ll z_0$, we observe that equations (11) are symmetrical in x and z. Accordingly we conclude that, if the kinetic energy is initially concentrated in the highest mode, it stays there and is not transferred to the other two modes.

Finally, examining the case when $x_0 \ll y_0$ and $z_0 \ll y_0$, we note that (11a) and (11c) are approximately:

$$\frac{d^2x}{d\tau^2} \simeq k^2y_0{}^2x$$

$$\frac{d^2z}{d\tau^2} \simeq k^2y_0{}^2z \quad .$$

Near initial time the solutions are:

$$x = x_0 \cosh ky_0\tau$$

$$z = z_0 \cosh ky_0\tau$$

Thus, even if x_0 and z_0 are initially small, x and z grow exponentially, and energy is rapidly transferred from the intermediate mode to the lowest and highest modes.

4. Interpretation of results as applied to the gyroscope

Let us next see how the results of the preceding section apply to the rotation of a solid ellipsoid with principal axes of different lengths. The amplitude x then corresponds to the rotation rate around the shortest axis, z to the rotation rate around the longest axis, and y to the rotation rate around the axis of intermediate length. In view of this correspondence between variables, we would expect that the ellipsoid, if initially spun around either its shortest or longest axis (but with a small component of rotation around its intermediate axis), would not transfer much rotation to motion around either of its other axes. If, on the other hand, the ellipsoid were initially spun around its intermediate axis (again, with small components of rotation around its shortest and longest axes), we would expect that rotation would be transferred rapidly to motions around the shortest and longest axes. In short, the ellipsoid would immediately "wobble."

Conversely, if the latter conclusions were confirmed experimentally, by the slightly imperfect spinning of an ellipsoid around each of its three principal axes, it would also demonstrate the validity of our earlier conclusions about the nonlinear transfer of kinetic energy between three interacting modes in a highly truncated representation of two-dimensional flows.

One usually does not have a suitable ellipsoid lying right at hand. A fair approximation, however, is a rectangular parallopiped whose length, width and thickness are different. Among the most common objects of this description is a matchbox containing 2" wooden matches.[2] It is approximately 9/16" × 1 7/16" × 2 1/16". These dimensions can be altered slightly to increase the disparity between moments of inertia around the principal axes, and thus to enhance the qualitative differences of behavior. One change is to make the box 2 1/2" long to produce more "wobble" when it is spun around the axis spanning its width. One of the pages of this note is printed on heavy stiff paper, showing the outlines of contiguous faces of such a box. To construct the box, cut along the outer lines, fold upward along the inner lines, and fasten the edges together with adhesive tape (1/2" scotch tape is recommended).

5. Conduct of the experiment

It is important to try to impart spin around only one principal axis of the box. To achieve some degree of exactness, it is suggested that the box be placed on the edge of a table, with the desired axis of rotation parallel to the edge and with slightly less than half of the box extending beyond the edge (see Fig. 1.). Then, place the nail of an index finger against the table and beneath the box, with the thumbnail

---

[2] Another common object of this kind, pointed out to me by Dr. Peter Rhines, is the standard American chalkboard eraser.

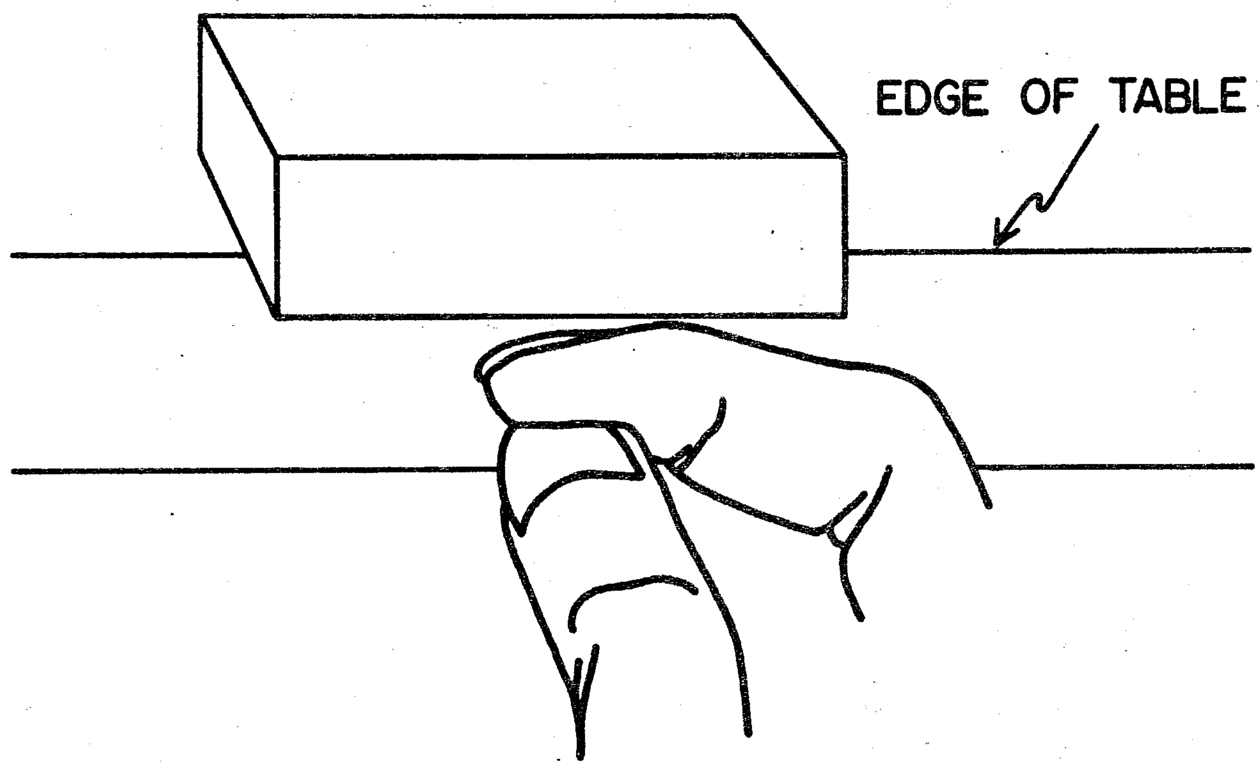

Figure 1

pressing upward on the underside of the index finger and directly beneath the center of the protruding edge of the box (see Fig. 1). Then release the thumb, so that the nail strikes the lower edge of the box forcibly. The box will fly up and slightly toward the table. Its spin will be primarily around the axis parallel to the edge of the table; some slight components of spin around the other two axes will inevitably be imparted, due to small asymmetries of the box, its misalignment or by striking it off center. Nevertheless, with some care, it is not difficult to demonstrate the qualitative differences of behavior described in Section 4.

I have no doubt that there may be considerable improvements on the design of this experiment. In its crudest form, however, it demonstrates some important nonlinear effects in two-dimensional flows.

Acknowledgement

My vague recollection is that the gyroscopic analogy was first suggested to me by Professor Arnt Eliassen about 1970. More recently (in fact, shortly after I discussed this analogy with Dr. C.E. Leith and Dr. Greg Holloway), I discovered that the phenomenon of "wobbling" is part of the folklore of fluid dynamics. The fact that I have never seen it written out is my excuse for setting it down here. I hope it is useful for pedagogical purposes, if nothing else.

Reference

Joos, G., 1932: Theoretical Physics. Hafner Publishing Company, Inc., New York. 748 pp.

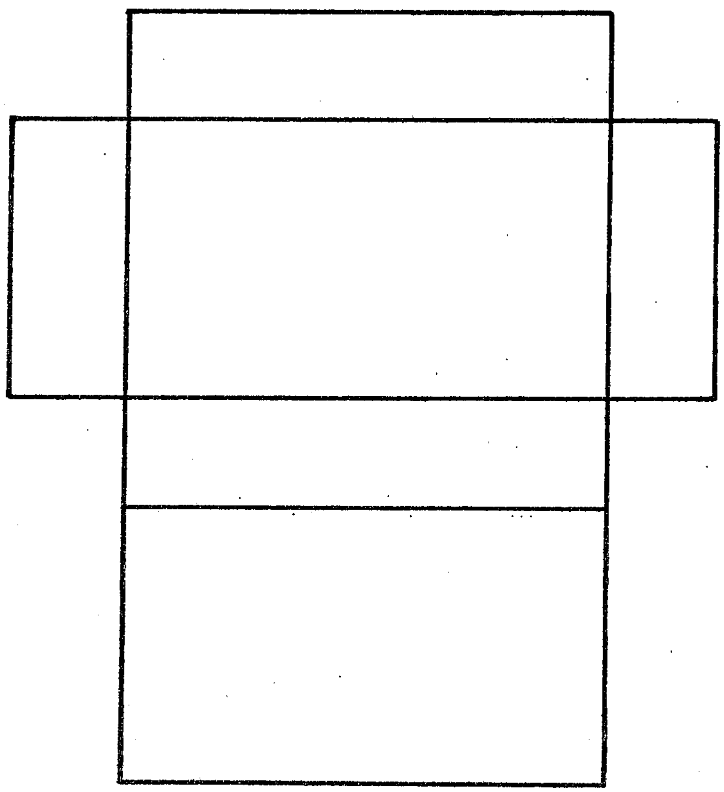

Figure 2