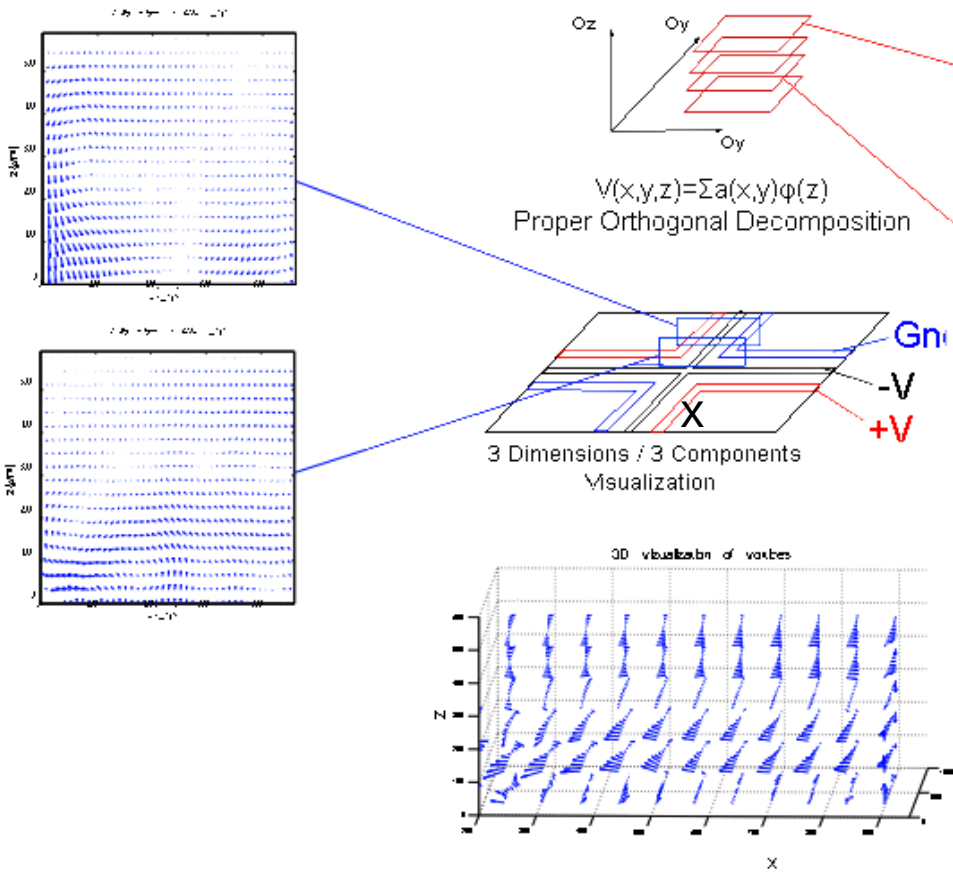
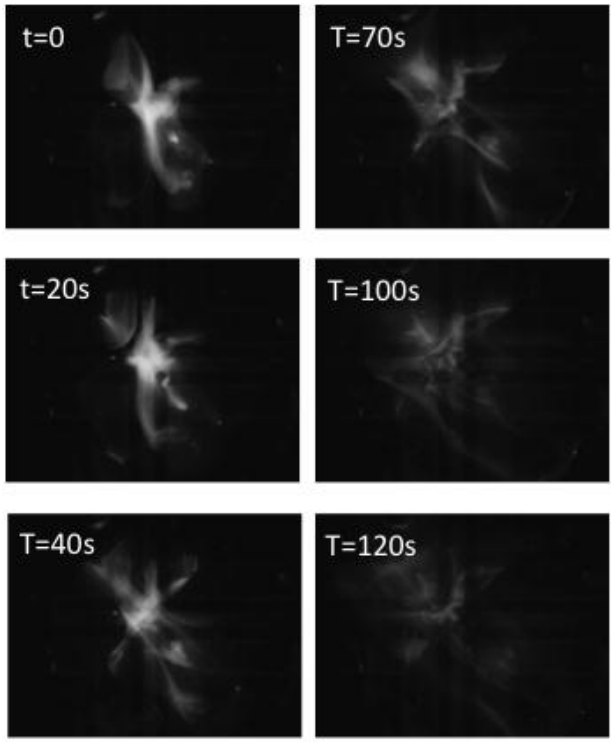
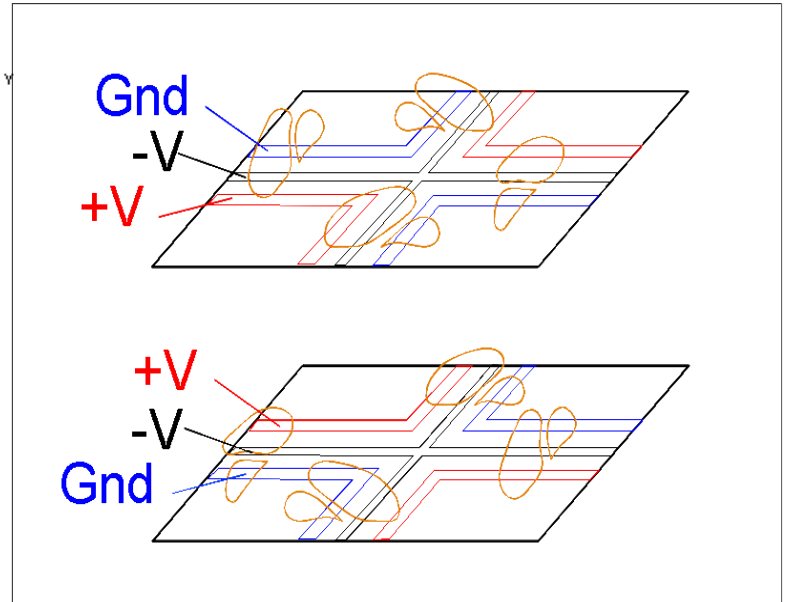
Fundamental advance 1: A versatile experimental testbed for 3D+1 mixing characterization.





Top Views of 1µm bead mixing over time



Properties:

-Ability to emulate physical forces with arbitrary dependence on time.
-Full PIV-based characterization of 3D+1velocity field enable linkage with theoretical progress on visualization of 3-D structures (see next slide).

Fundamental advance 2: An ergodic-theory based theoretical and computational framework for 3D+1 mixing.

UNSTEADY HILL'S VORTEX IN 3D

Example:

$$\begin{bmatrix}\dot{R}\\ \dot{z}\\ \dot{\theta}\end{bmatrix} = \underbrace{\begin{bmatrix}2Rz\\ 1-4R-z^2\\ \frac{c}{2R}\end{bmatrix}}_{\text{Hill's Vortex}} + \underbrace{\varepsilon \begin{bmatrix}\sqrt{2R}\sin\theta\\ \frac{z}{\sqrt{2R}}\sin\theta\\ 2\cos\theta\end{bmatrix}\sin 2\pi t}_{\text{Perturbation div}=0}$$

Swirl

$(R, z, \theta) \in \mathbb{R}^+ \times \mathbb{R} \times \mathbb{T}$

GEOMETRY

Extrinsic Coordinates: Averaged Functions $\tilde{g}(x)$, $\tilde{f}(x)$

Intrinsic Coordinates: Diffusion Modes $\chi_2(x)$, $\chi_1(x)$

Diffusion Maps

- ambient space: negative-index Sobolev space [Mathew '10]
- Diffusion Maps extracts intrinsic coordinates [Coifman '06]

VISUALIZATION

Ergodic Quotient $\chi_2(x)$, $\chi_1(x)$

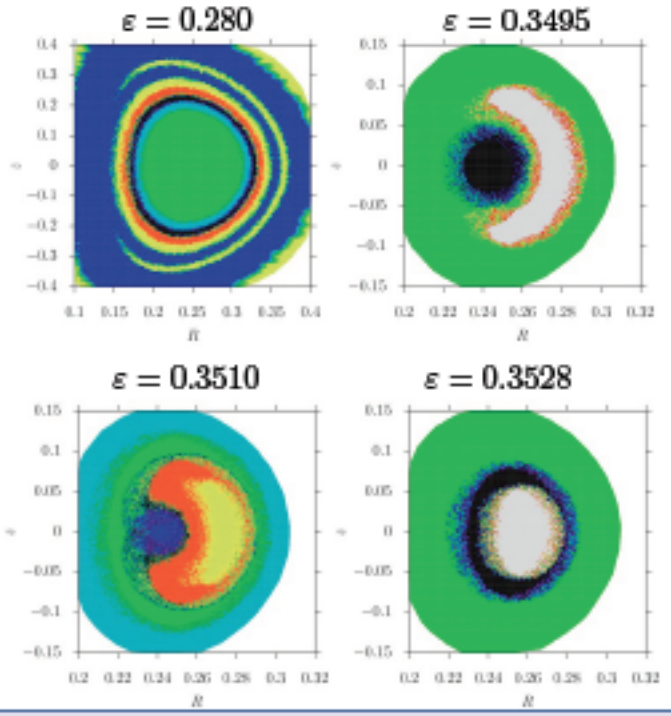
k-means clustering

level set plotting

State Space

- euclidean (chordal) distance in diffusion modes corresponds to intrinsic distance along the EQ
- euclidean (e.g. k-means) clustering is applicable

The spiral island replaces the initial core.



Conclusions:

- Ergodic Quotient is useful for discovering unknown features
- New bifurcation uncovered, seemingly consistent with a saddle-node collision mechanism for periodic sets
- Similar phenomenon seen in 1:2 resonances for Hamiltonian systems, e.g., spring-pendulum oscillator [Broer '03]

Publications

1) M. Budišić, I. Mezić, “Geometry of the ergodic quotient reveals coherent structures in flows” Physica D 241 (2012) 1255–1269

2) S. Loire, P. Kauffmann, I. Mezić and C. D. Meinhart, “A theoretical and experimental study of AC electrothermal flows”, J. Phys. D: Appl. Phys. **45** (2012) 185301.

3) D. L. Valentine, I. Mezić, Senka Maćešić, Nelida Crnjarić-Žic, Stefan Ivić, P. J. Hogand, V. A. Fonoberov, and S. Loire, “Dynamic autoinoculation and the microbial ecology of a deep water hydrocarbon irruption”, Proceedings of teh National Academy of Sciences (2012).