Ergodicity Defect

Ergodicity Defect (General Idea)

Think of ergodicity in terms of

"time average of observables = space average of observables"

Ergodicity defect evaluates difference between time average and space average for a collection of observables (analyzing functions)

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An alternative metric for distinguishing trajecs & identifying LCS

Ergodicity defect evaluates difference between time average and space average for a collection of observables (analyzing functions)

Ergodicity Defect (ED) on unit interval

Wavelet basis

Analyzing functions are Haar father wavelets – i.e. translations & dilations of the indicator function

$$\phi(x) = \chi_{[0,1)}(x) = \begin{cases} 1, & x \in [0,1) \\ 0, & else \end{cases}$$

Haar father wavelets at scale s are

$$\phi_j^{(s)}(x) = \phi(2^s x - (j-1)), \qquad j = 1, \ldots, 2^s$$

Corresponds to

Partition of [0,1) into 2^s intervals of length $\frac{1}{2^s}$

Ergodicity Defect (ED) on unit square

Analyzing functions are 2 dimensional Haar father wavelets

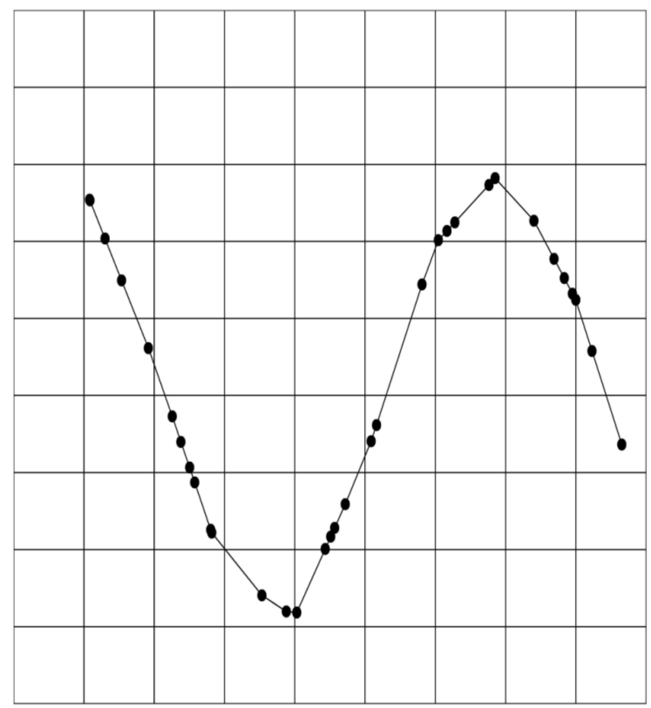
$$\phi_{i_1 i_2}^{(s)}(x,y) = \phi_{i_1}^{(s)}(x)\phi_{i_2}^{(s)}(y) \quad i_1, i_2 = 1,\ldots 2^s$$

Corresponds to

Partition of unit square into 2^{2s} squares each of area $\frac{1}{2^{2s}}$ (where s is the spatial scale)

ED in 2 dimensions – Numerical Algorithm

- Take mapped trajectory in unit square
- Partition the unit square into squares of length s and equal area s^2
- Space average = s^2
- Use number of trajectory points N_j inside jth square to estimate the average time spent in each square (time average)



ED in 2 dimensions – Numerical Algorithm

Time average

For a trajectory with initial conditions $\vec{x}_0, t_0$

Samples best

$$d(s;\vec{x}_0,t_0) = \sum_{j=1}^{s^{-2}} \left(\frac{N_j(s)}{N} - s^2\right)^2$$

Space average

"Ergodic" (most complex) trajectory: $d = 0$

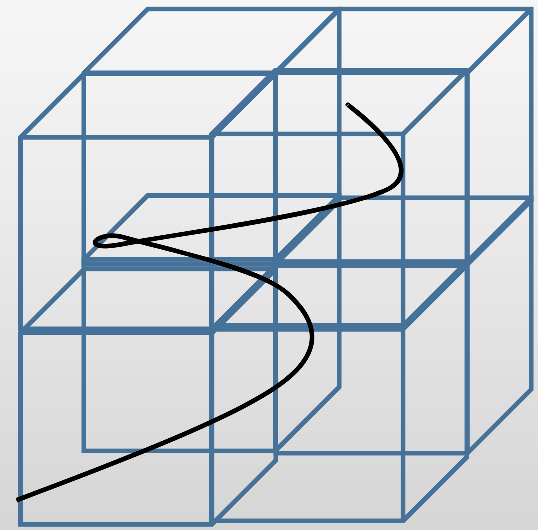
Stationary (least complex) trajectory:

$$d = 1 - s^2 \to 1 \quad \text{as} \quad s \to 0$$

ED in 3 dimensions – Numerical Algorithm

- Take trajectory mapped into unit cube
- Partition the unit cube into smaller cubes of length s and equal volume s^3
- Space average = s^3
- Use number of trajectory points $N_j(s)$

inside jth cube to estimate the average time spent in each cube (time average)



Partition of cube for s=1/2

For a trajectory with initial conditions $\vec{x}_0, t_0$

$$d(s;\vec{x}_0,t_0) = \sum_{j=1}^{s^{-3}} \left(\frac{N_j(s)}{N} - s^3\right)^2$$

ED 3 dimensions + time – Numerical Algorithm

For different fixed initial depth (z) levels,

- Generate trajectory from (time) snapshots
- Take mapped trajectory in unit cube
- Partition the unit cube into smaller cubes with sides of length s
- Space average = s^3
- Use number of trajectory points $N_j(s)$ inside each cube to estimate the average time spent in each cube (time average)
- Combine info from all depth levels

ED & Lagrangian Coherent Structures (LCSs)

Compute the ergodicity defect of individual fluid particle trajectories

Take the mean over scales of interest - d_{mean}

↓

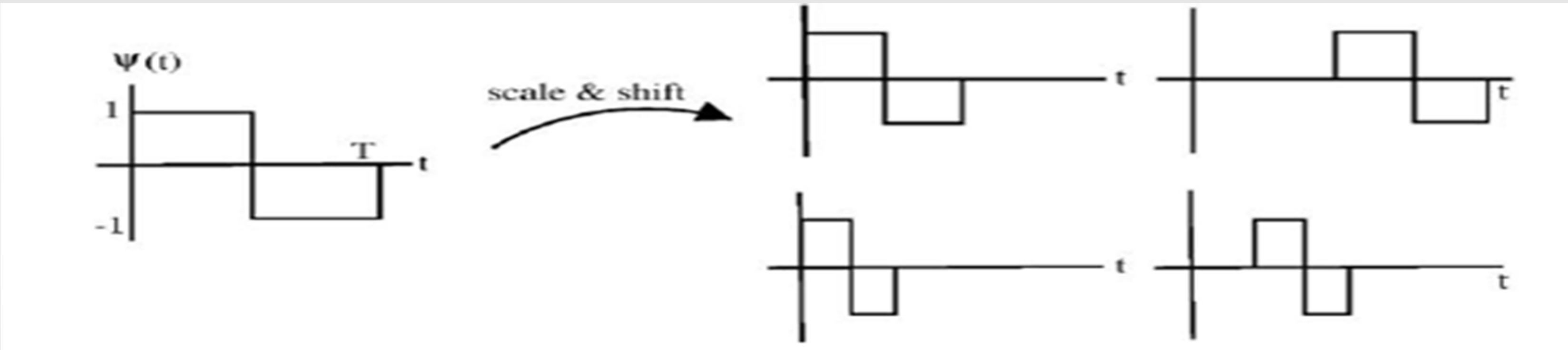
Distinguish each trajectory by the manner in which it samples the space (i.e., by its complexity)

Ergodicity Defect (ED) on unit interval

Can also use Haar mother wavelets -i.e. translations & dilations of the Haar mother wavelet

$$\psi(x) = \chi_{[0,1/2)}(x) - \chi_{[1/2,1)}(x)$$

$$\psi_j^{(s)}(x) = \psi(2^s x - (j-1)), \qquad j = 1,\dots,2^s$$



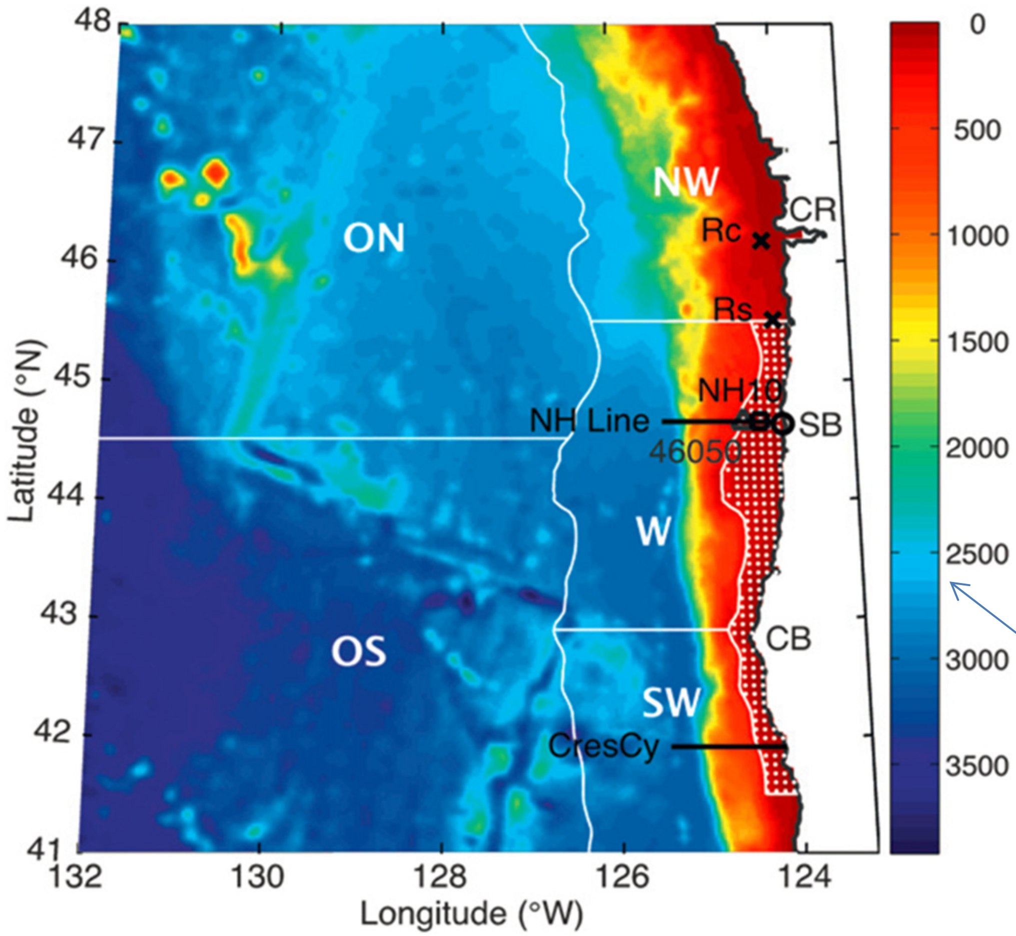
$$d(s) = \frac{2^{s-1}-1}{2^s-1} d(s-1) + \frac{2^{s-1}}{2^s-1} \sum_{j=1}^{2^{s-1}} \| \psi_j^{(s-1),*} \|^2$$

Time average

Better for scaling analysis

ED & an Upwelling flow (Rivas & Samelson) (3D + time example)

Strong Vertical Velocity in Ocean?



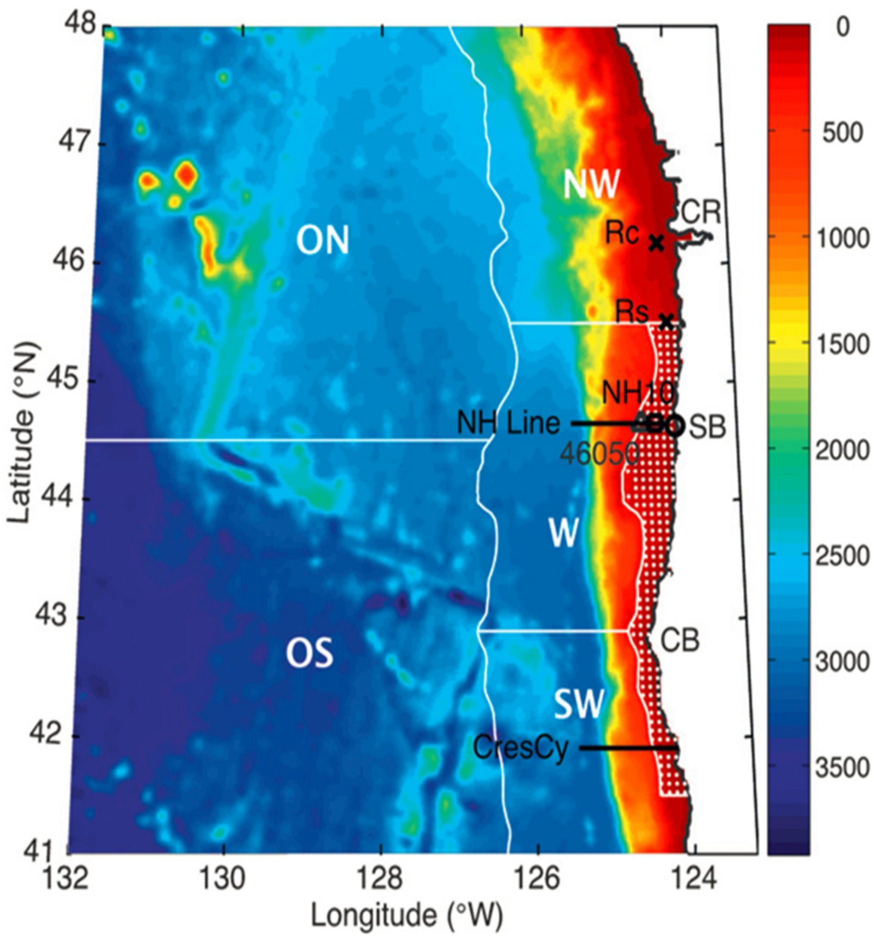
Numerical model off Oregon coast in 2005

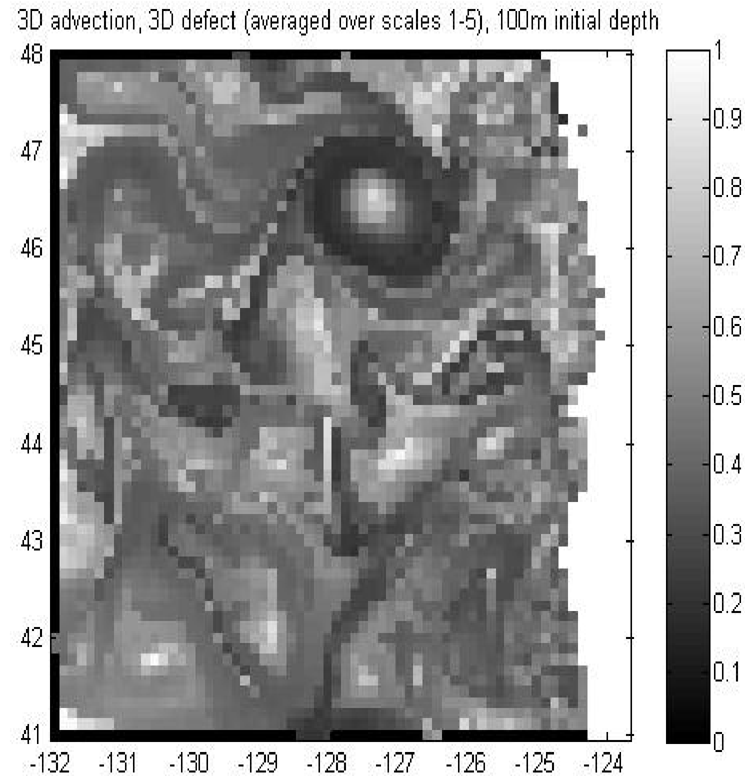
Color=bathymetry

Use Ergodicity Defect to Identify Vertical LCS?

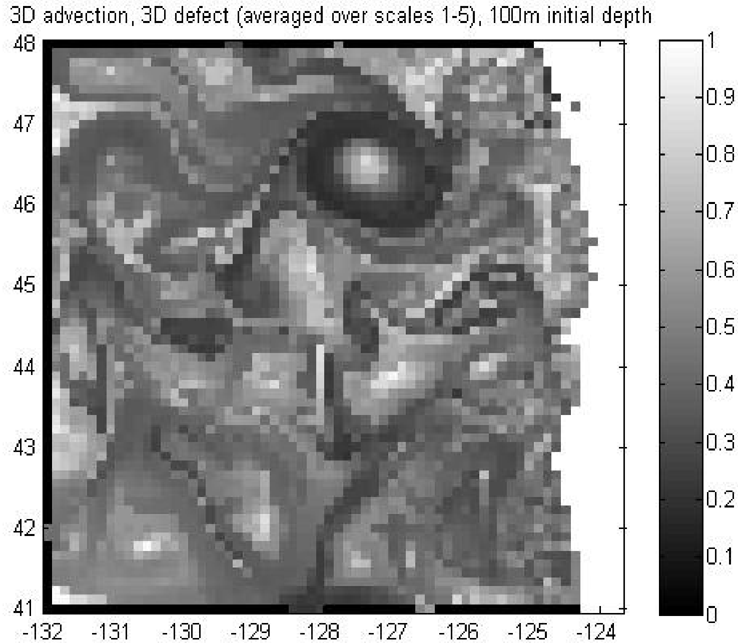
Does 3D Defect (sampling in x,y, & z) give more/different info than just 2D?

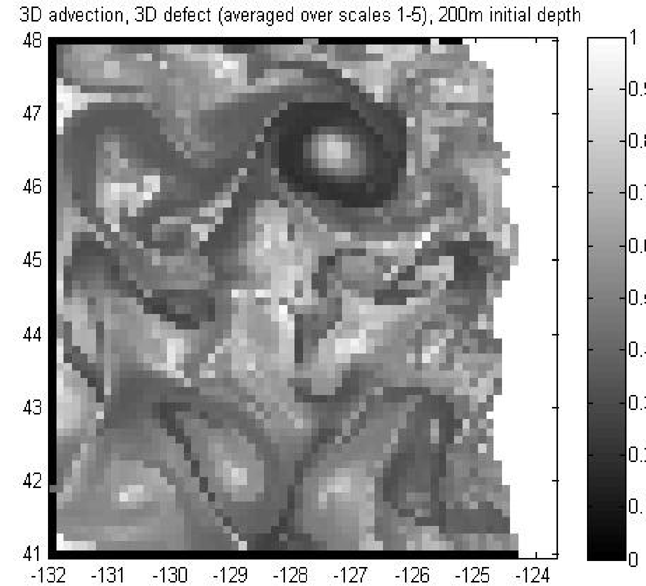
ED & an Upwelling flow (Rivas & Samelson) (3D + time example)

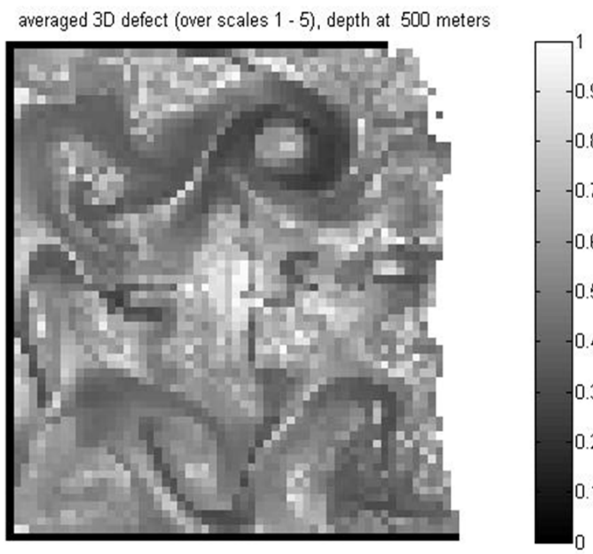


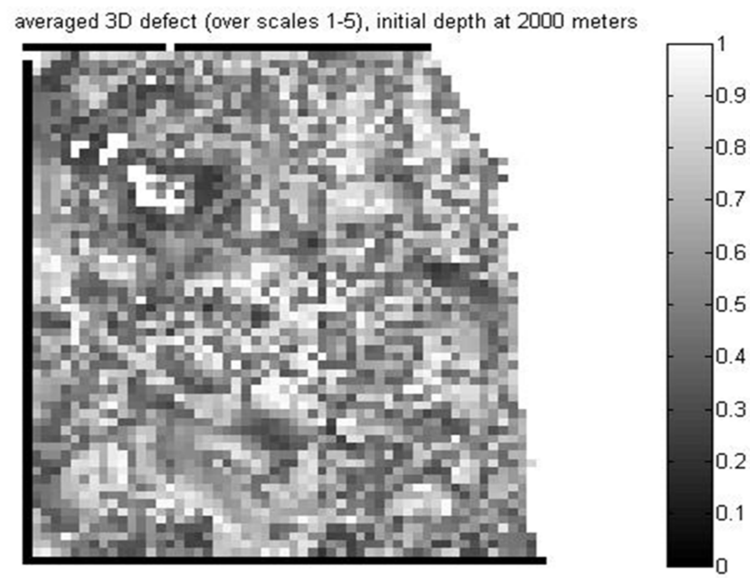


3D ED & Upwelling flow at different depths

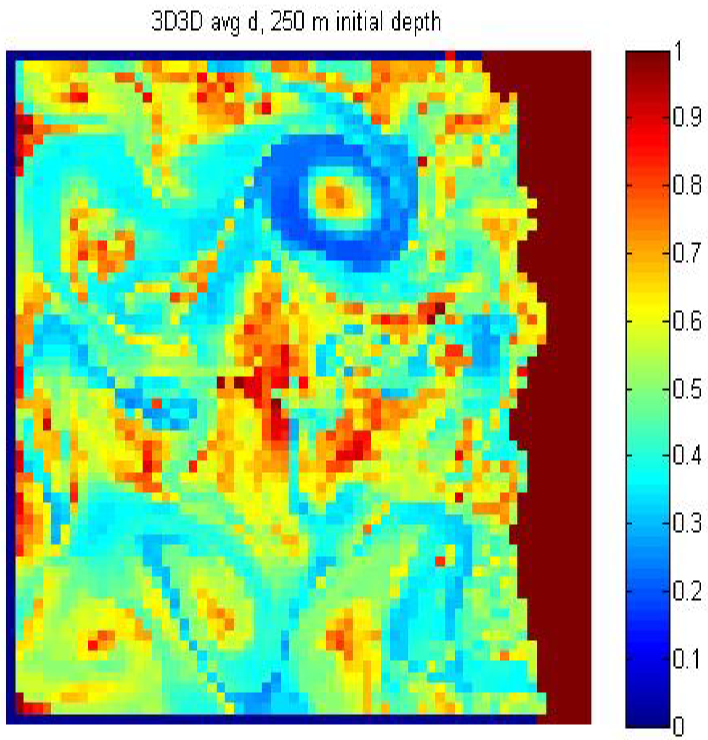


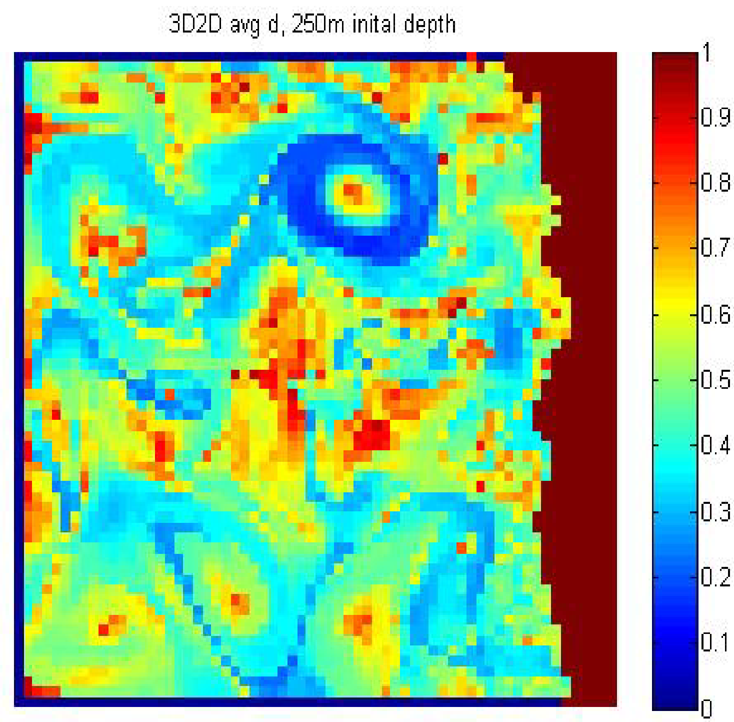






ED & an Upwelling flow full domain, 3D advection





3D defect grayscale
x, y & z sampling

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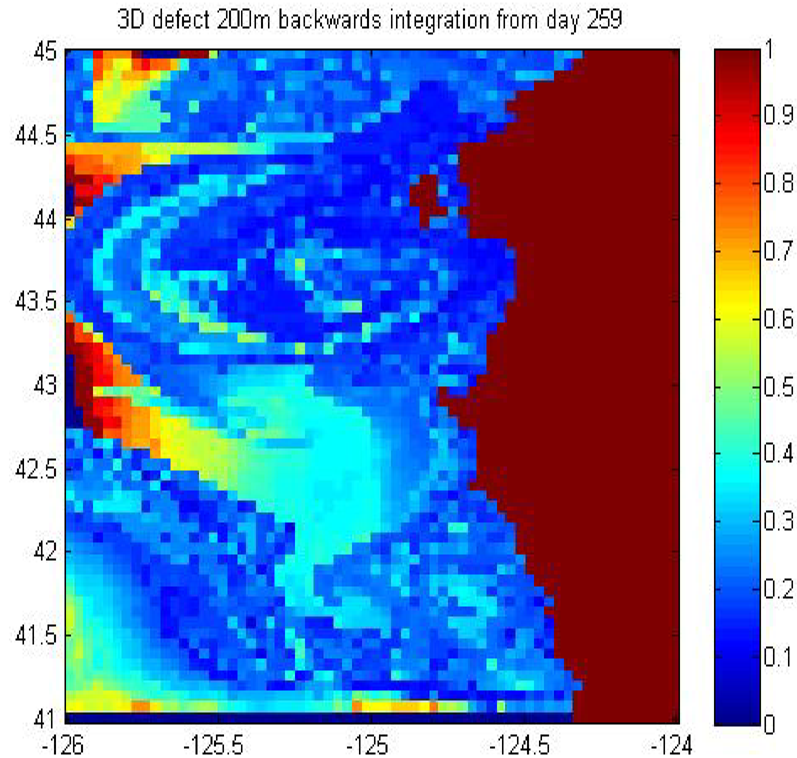
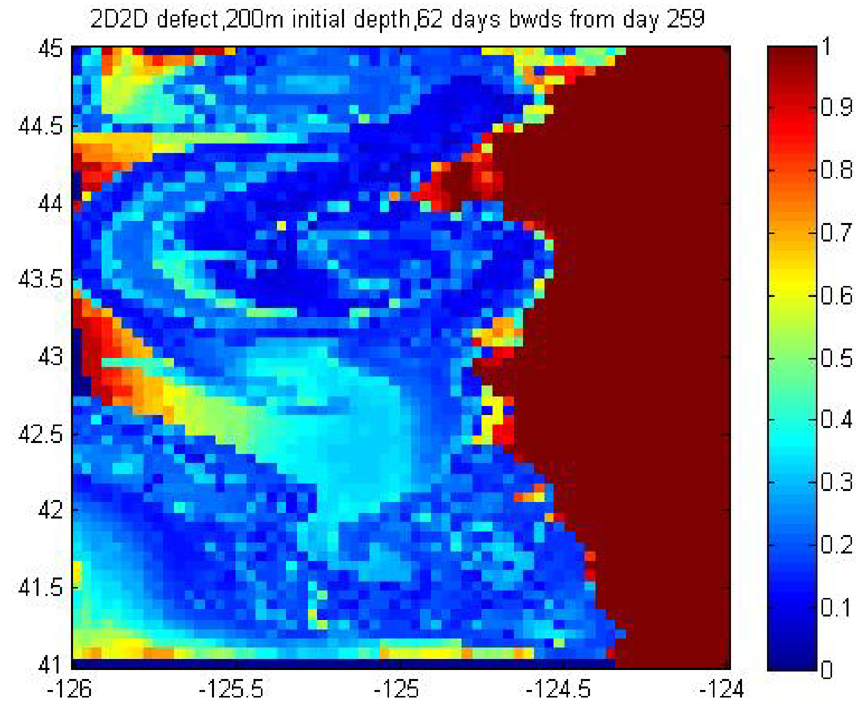
2D defect grayscale
x,y sampling

Upwelling flow
on smaller domain (closer to shore)



3D advection, 3D defect



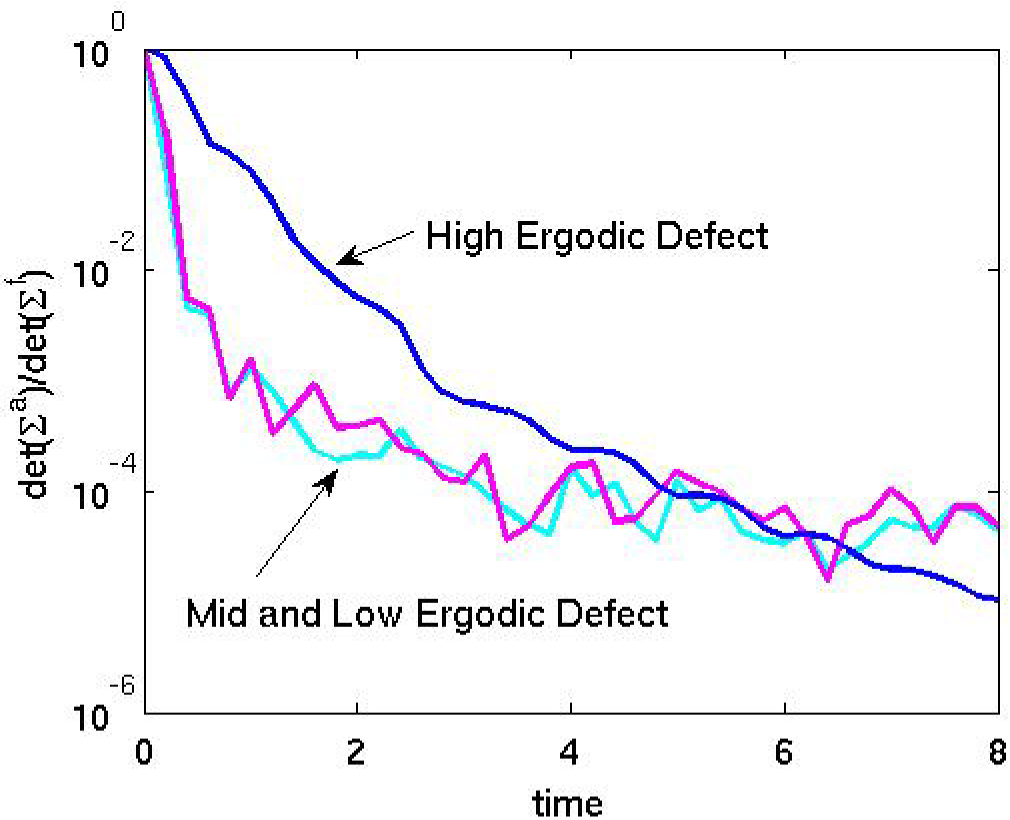
2D advection, 2D defect

Still 3D defect grayscale pic similar 2D defect

Rerunning with better resolution

Ergodicity Defect & LaDA (Linearized Shallow Water & Particle Filter (E. Spiller))

Which trajectory? – Lower defect better



How long?

Doing with rotating can

Summary

Ergodicity Defect (ED) captures trajectory/flow complexity for identifying Lagrangian Coherent Structures

- Understanding barriers to transport
- Understanding/Determining transport of material/flow properties by coherent structures

Advantages of ED

- Distribution of trajectory can be non-uniform/sparse
- Works in both 2 and 3 dimensions
- Scaling analysis component

Other aspects/ideas

Use Ergodicity Defect (ED) to distinguish optimal trajectories/initial conditions

- Use SCALING ANALYSIS – detect fast, small-scale
- Sampling device deployment strategy/path design?
- Quantify transport of materials?
- For estimating fluid flow properties?
  - e.g avg temp over floats/ avg temp over different realizations

NEXT? BIG PIC & CONNECTIONS?

- ABC flow?
- Quadrapole?
- Point Vortices?
- HYCOM?
- NCOM?
- LES?