Parametrization of particle transport at submesoscales using Lagrangian subgridscale models

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Motivation

- Transport carried out by submesoscale (SMS) flows is significant at those scales, yet it is challenging to approach deterministically.

- Can we combine transport from underresolved SMS motions computed with statistical Lagrangian subgridscale (LSGS) models, with mesoscale LCS computed from OGCMs?

- Can we adapt an LSGS model to improve the scale dependent relative dispersion statistics, yet maintain the Lagrangian transport barriers of the mesoscale eddy field?

Relative dispersion statistics

Statistics of relative dispersion defined as:

$$\boxed{D^2(t) = < |\mathbf{x}^{(1)}(t) - \mathbf{x}^{(2)}(t)|^2 >}$$

Relative dispersion
(for all particle-pairs)

$$K(t) = \frac{1}{2}\frac{dD^2}{dt} = \langle \mathbf{D}(t, \mathbf{D}_0) \cdot \delta \mathbf{v}(t, \mathbf{D}_0) \rangle$$

Relative diffusion

$$\boxed{\lambda(\delta) = \frac{ln(\alpha)}{< \tau(\delta) >}}$$

FSLE value for the scale δ
(<τ> = averaged time for all particle pairs to separate from distances **δ** to **αδ**)

Typically, 1< α <2

> The impact of the relative dispersion on each scale δ is isolated.

RD vs dual energy cascade

The enstrophy cascade is typically associated with a steep, non-local, slope of the energy spectrum $E(k) \sim k^{-3}$ while Kolmogorov–Kraichnan inertial range arguments in the inverse cascade range imply $E(k) \sim k^{-5/3}$.

$$S(D) = 2\int_0^\infty E(k)(1 - J_0(kD))dk$$

$\langle \delta \mathbf{v}^2(\mathbf{D}) \rangle = \langle \mathbf{u}(x+\mathbf{D},t) - \mathbf{u}(x,t) \rangle^2 = S(\mathbf{D})$ relative velocity term

$K(D) = \langle \mathbf{D} \cdot \delta \mathbf{v} \rangle \sim S^{1/2} D$ Relative diffusivity

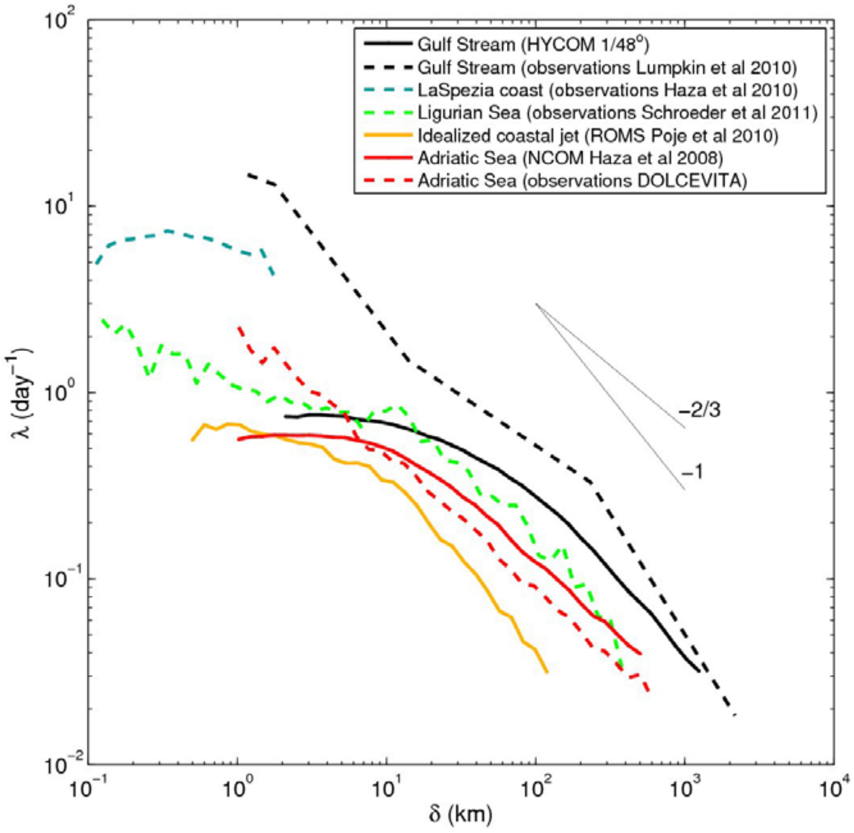
$$E(k) \sim k^{-\beta} \qquad \begin{array}{llll} S(D) \sim D^{\beta-1}, & K(D) \sim D^{(\beta+1)/2}, & D \sim t^{2/(3-\beta)}, & 1 < \beta < 3 \\ S(D) \sim D^2, & K(D) \sim D^2, & D \sim e^{\alpha t}, & \beta > 3. \end{array}$$

Dimensional equivalences for λ:

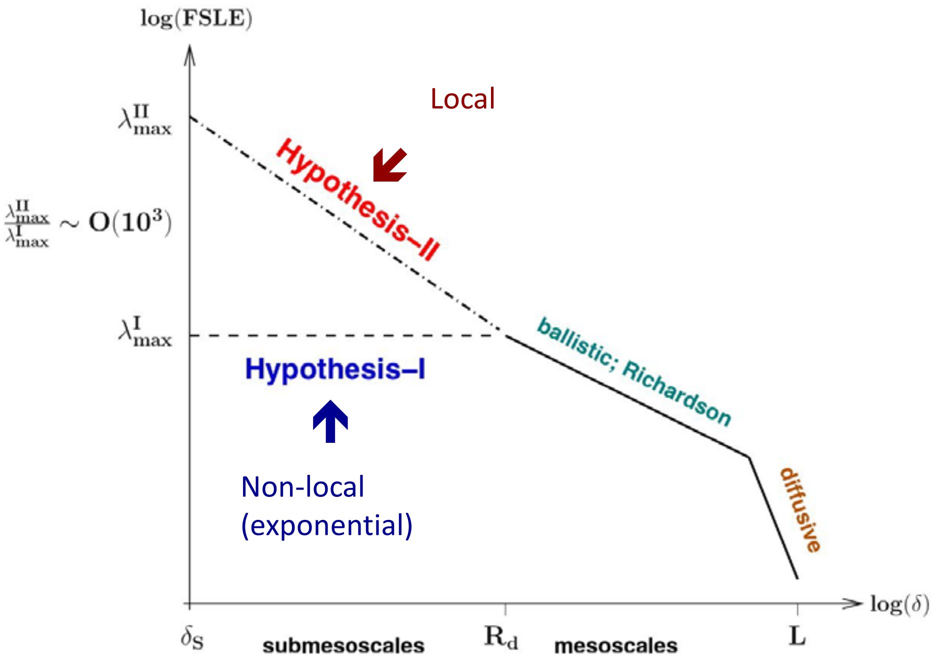
Non-local regime: $D \sim e^{\lambda t} \Leftrightarrow \lambda \sim \lambda_{max}$ (exponential)
Local regimes: $D^2 \sim t^3 \Leftrightarrow \lambda \sim \delta^{-2/3}$ (Richardson)
$D^2 \sim t^2 \Leftrightarrow \lambda \sim \delta^{-1}$ (ballistic, shear)
$D^2 \sim t \Leftrightarrow \lambda \sim \delta^{-2}$ (diffusive)

Submesoscale regimes: possible scenarios

FSLE from models and observations:



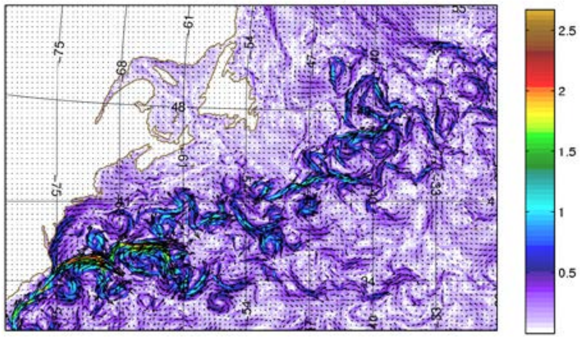
Two possible regimes at the submesoscales:



Relative dispersion from ocean models

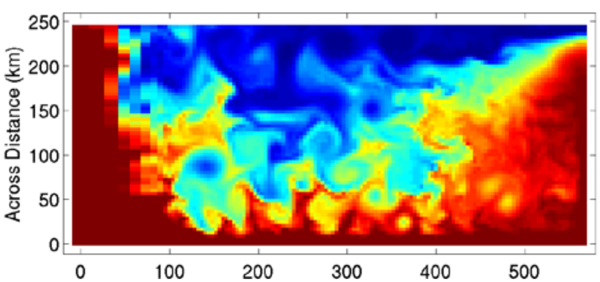
(Poje et al., Ocean Modelling, 31 (2010) 36-50)

Gulf Stream / HYCOM

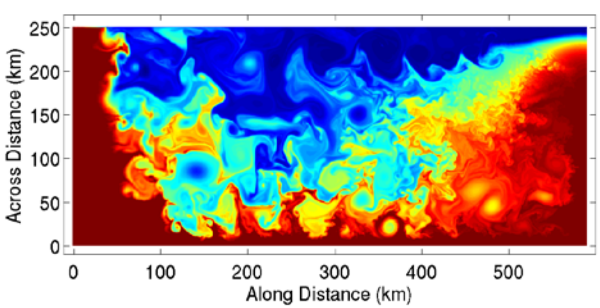


Baroclinic jet / ROMS

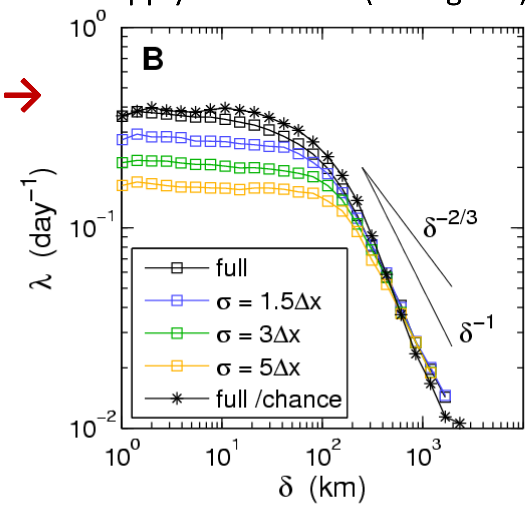
4 km



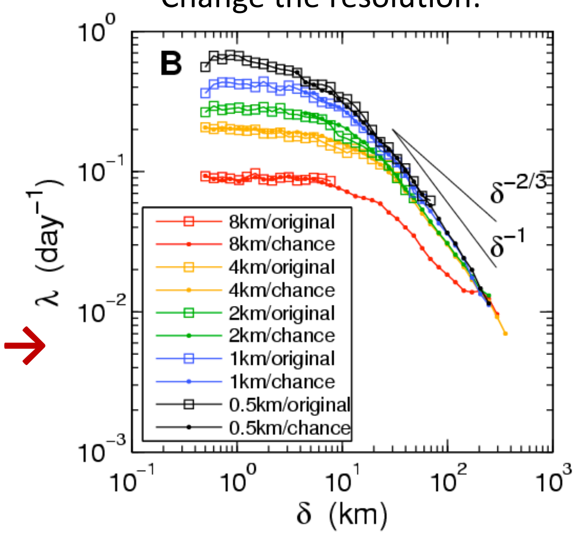
500 m



Apply Gauss filter (strength σ):



Change the resolution:



➢**Exponential regime extended** when resolution (LP smoothing) decreases (increases).

➢**Hyperbolicity ↘** when resolution (smoothing) decreases (increases).

> power-law between Richardson and ballistic at large scales unchanged.

Relative dispersion from ocean models

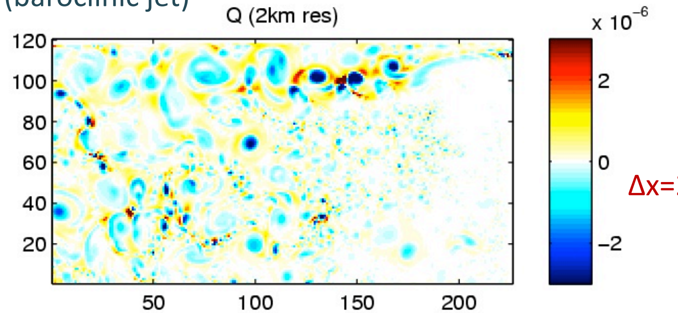
The rate of separation of particle pairs depends on an Eulerian quantity related to the horizontal strain field.

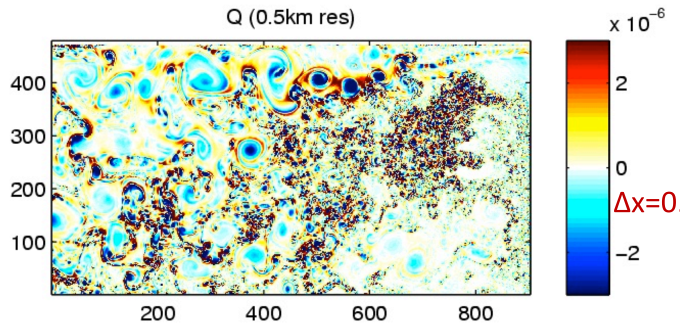
Okubo-Weiss

$$Q^2 = S^2 - \omega^2 .$$

$$S^2 = S_n^2 + S_s^2, \quad S_n = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}, \quad S_s = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y},$$

(baroclinic jet)



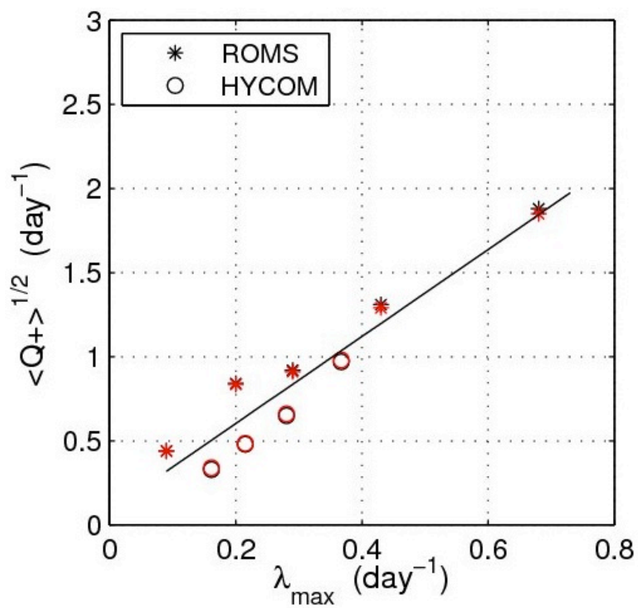


Q² > 0 ⇔ *regions dominated by strain S and deformation* (exponential divergence)

Define the statistical O.W. focusing on the strain ($Q^2 > 0$):

$$\overline{Q} = A^{-1} \int \sqrt{Q^2}\, dA$$

= Eulerian quantity ***similar to the average hyperbolicity*** of the model fields.



Relative dispersion from ocean models

- **Large scale dispersion:** insensitive to resolution.

- **Short scale dispersion:** defined by the spatial resolution of Eulerian measures of velocity gradients.

- **Intermediate scales:** resolution dependent, even at scales much larger than the grid-scale.

Conclusions for LSM applications

- **Submesoscale dispersion** is most often underestimated in coastal and ocean models, although it is unclear which regime dominates at those scales. LSM parametrizations should aim to **enhance the unrealistically low strain-level at the submesoscale range.**

- Due to the increasing complexity in the FSLE barriers, the Lagrangian subgridscale (LSGS) parametrization should be statistical **by incorporating the net effect of turbulent motions on the SMS relative dispersion.**

- The LSM **should not be detrimental to the mesoscale transport barriers!**

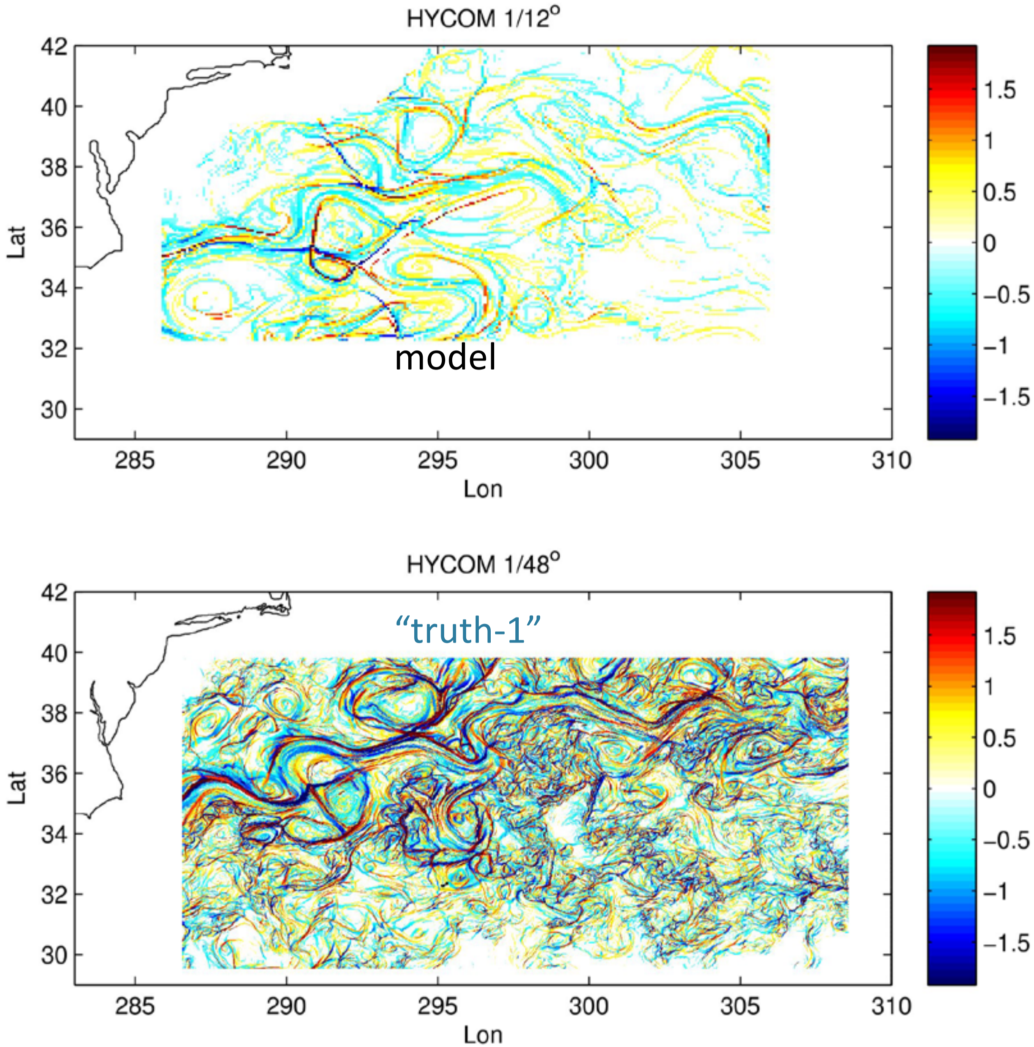
Methodology

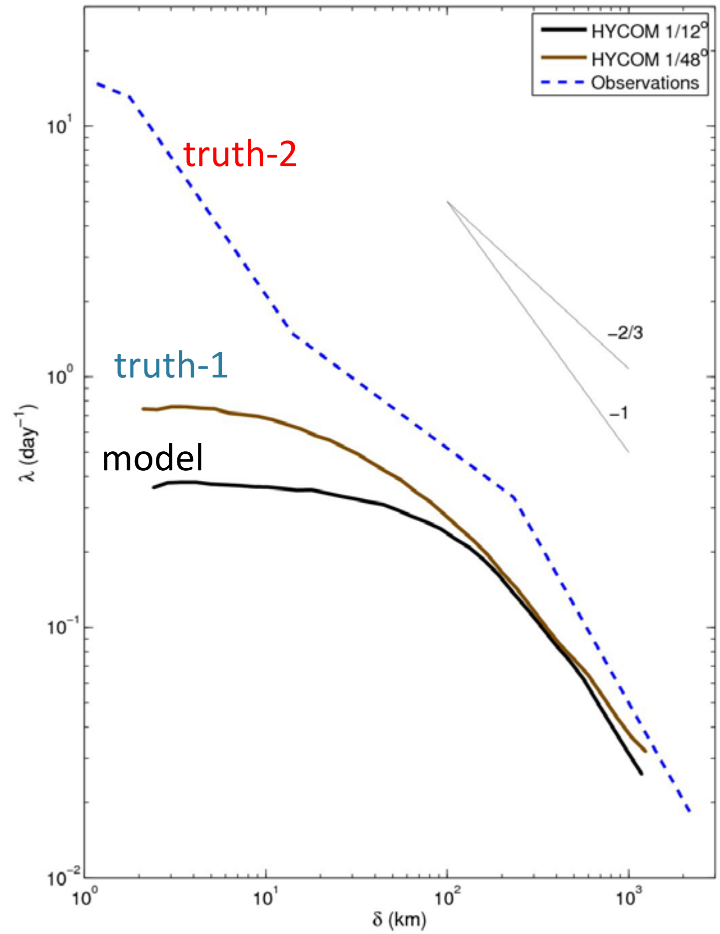
➢ We implement three LSMs of increasing complexity and test their impact on the scale dependent relative dispersion in the Gulf Stream region with HYCOM 1/12^{o}:

1) **Random walk (2 particles).**
2) **Random flight (2 particles).**
3) **IMC Markov-1 model**. *Ocean Modelling, 17 (2007) 68-91.*

➢ Two different types of SMS regimes are used for reference: non-local 1/48^{o} HYCOM simulation, and RD in-situ measurements (*Lumpkin & Ellipot, JMR, 2010*)

LSM setting (Gulf Stream region / HYCOM 1/12deg)





Haza et al., Ocean Modelling, 42 (2012) 31-49.

LSM-1 (random walk with spatial correlation)

- **Zeroth order Markov model** for **2-particle** motion (correlation time-scale τ = dt/2)
- Turbulent component ***entirely parameterized***.
- ***Ocean model velocities = deterministic drift***.

- Given 2 particles δ-apart, their increments in the zonal direction are:

$$\boxed{dx_1 = L_K\, dw_{0A} \quad \text{and} \quad dx_2 = \mathcal{D}\, dx_1 + (1-\mathcal{D}) L_K\, dw_{0B}}$$

With: $\mathbf{L_K}$ = σ.dt, and $\sigma^2 = \langle u'^2 \rangle$ turbulent velocity variance,
dw_{0A}, dw_{0B} = 2 independent random increments.

$$\boxed{\mathcal{D} = \exp\left(-\frac{\delta^2}{2L_\mathcal{D}^2}\right)}$$

≅ effect of submesoscale eddies on particle-pairs

And L_D = space correlation scale.

- If uncorrelated: $L_D = 0$, D ➔0 and

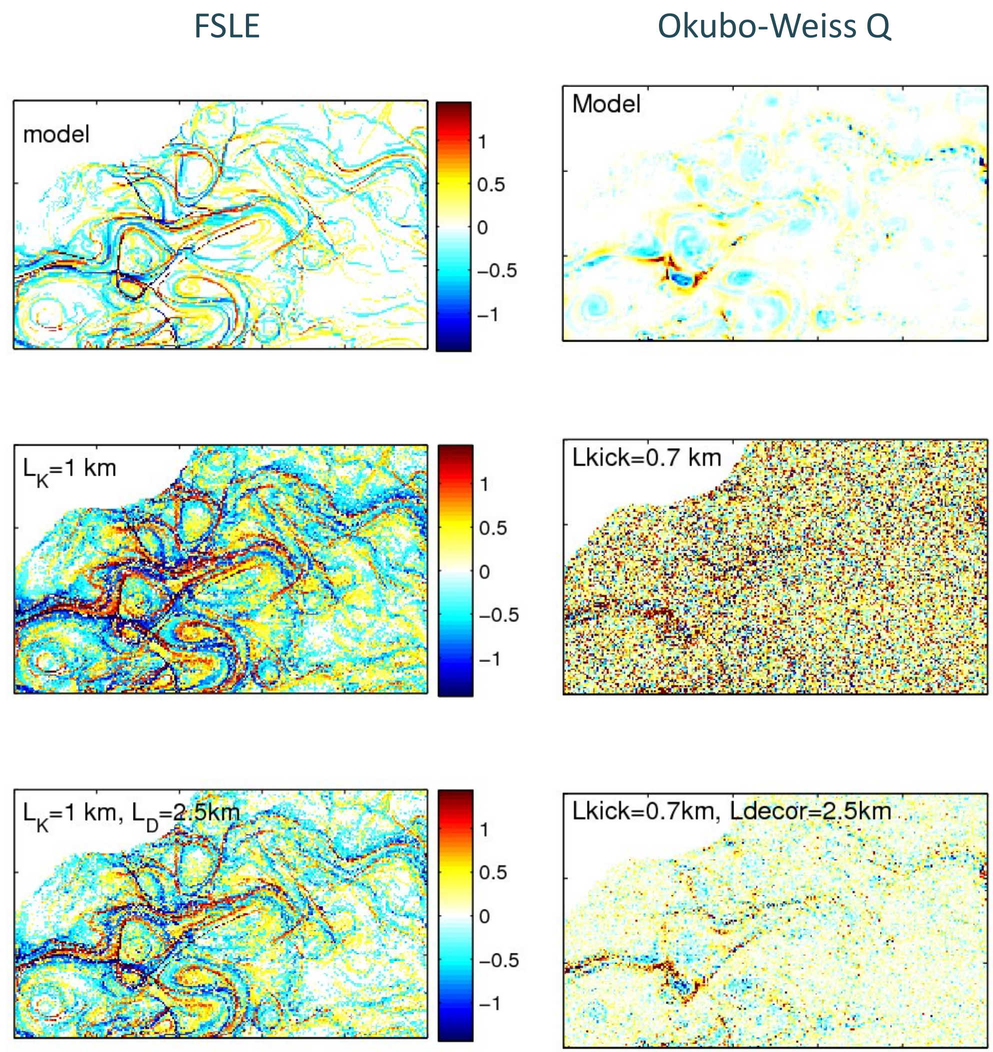
$$dx = \sigma\sqrt{dt/2}\, dw_0, \quad \langle \left(\frac{dx}{dt}\right)^2 \rangle = \sigma^2.$$

LSM-1: impact on the Lagrangian transport barriers

Impact on the mesoscale LCS and strain field:

- **LCS barriers are maintained.**
- **white noise added on the strain field.**

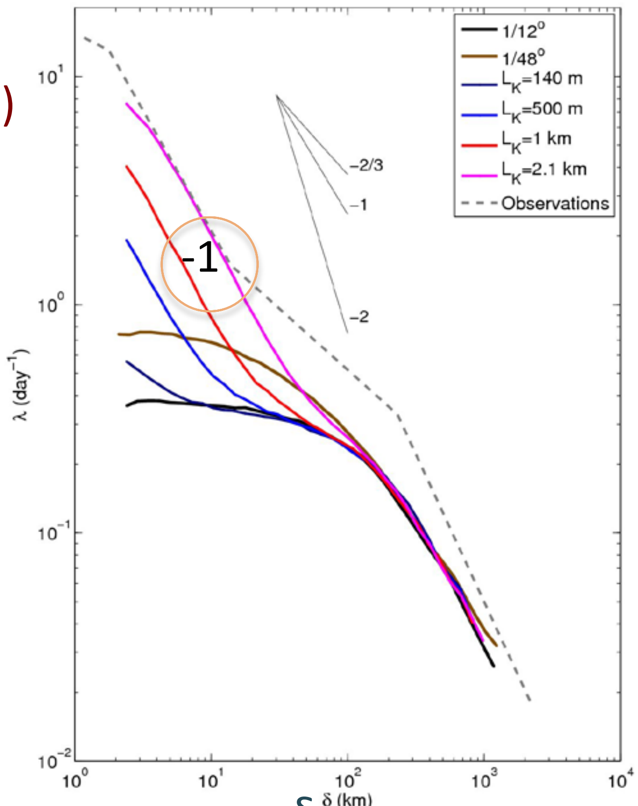
i.e. turbulent fluctuations added everywhere.



LSM-1 scale-dependent relative dispersion

Spatially uncorrelated RW

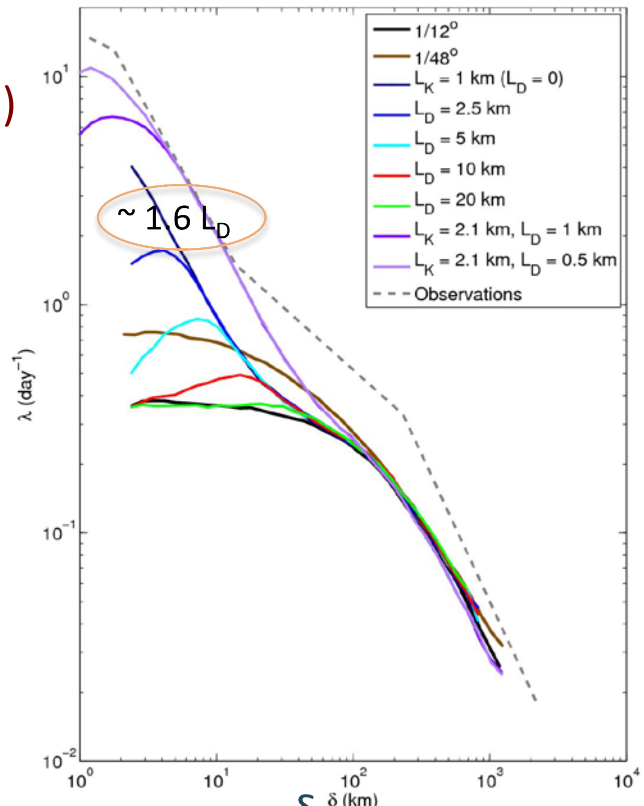
$\lambda(\delta)$



δ

Space correlated RW

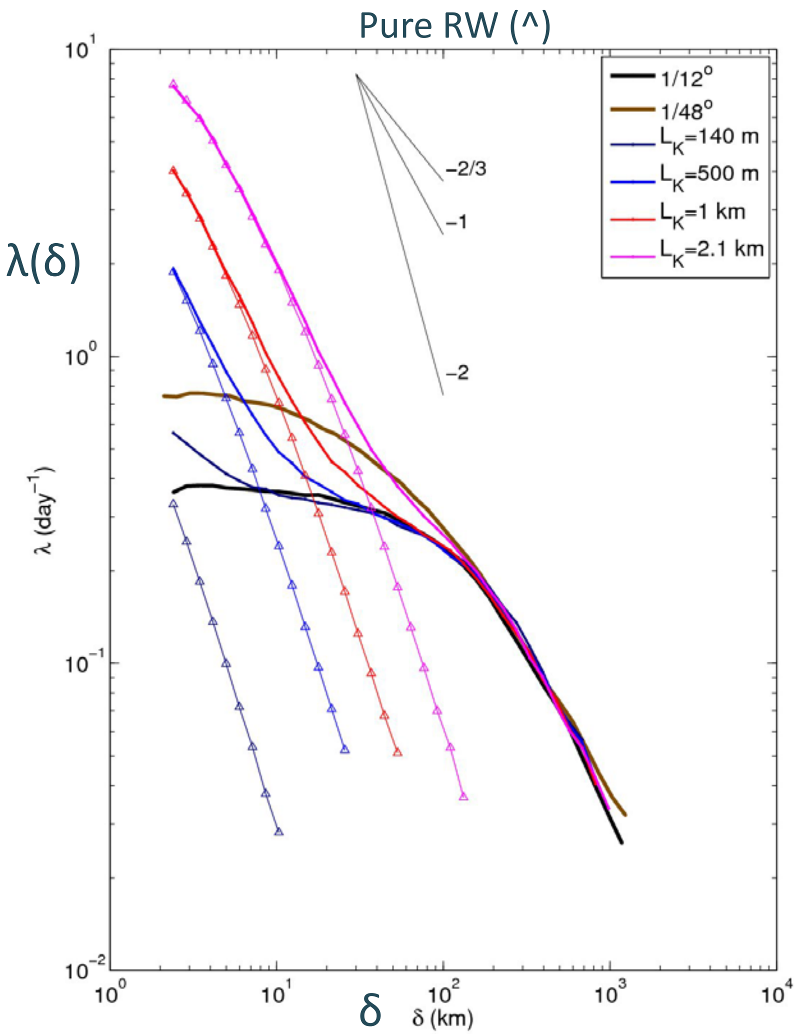
$\lambda(\delta)$



δ

- **δ^{-1} (RW diffusion) in the submesoscales.**
- **local λ_{max} from spatial correlation at about 1.6 L_D.**
- **large scale RD unchanged.**

LSM-1 : uncorrelated random walk contribution



$$V_{RW} \sim \left\langle |\mathbf{v}_2 - \mathbf{v}_1|^2 \right\rangle^{1/12} \sim \left\langle |\mathbf{v}|^2 \right\rangle^{1/12}$$

Therefore,

$$V_{RW} \sim \sigma_{\tilde{u}} \ ,$$

and

$$V_{total} = \frac{(\alpha - 1)\delta}{\tau_\lambda} \sim \ \sigma_{\tilde{u}} \ .$$

This expression leads to:

$$\frac{log(\alpha)}{\tau_\lambda} \sim \frac{log(\alpha)\ \sigma_{\tilde{u}}}{(\alpha - 1)\delta},$$

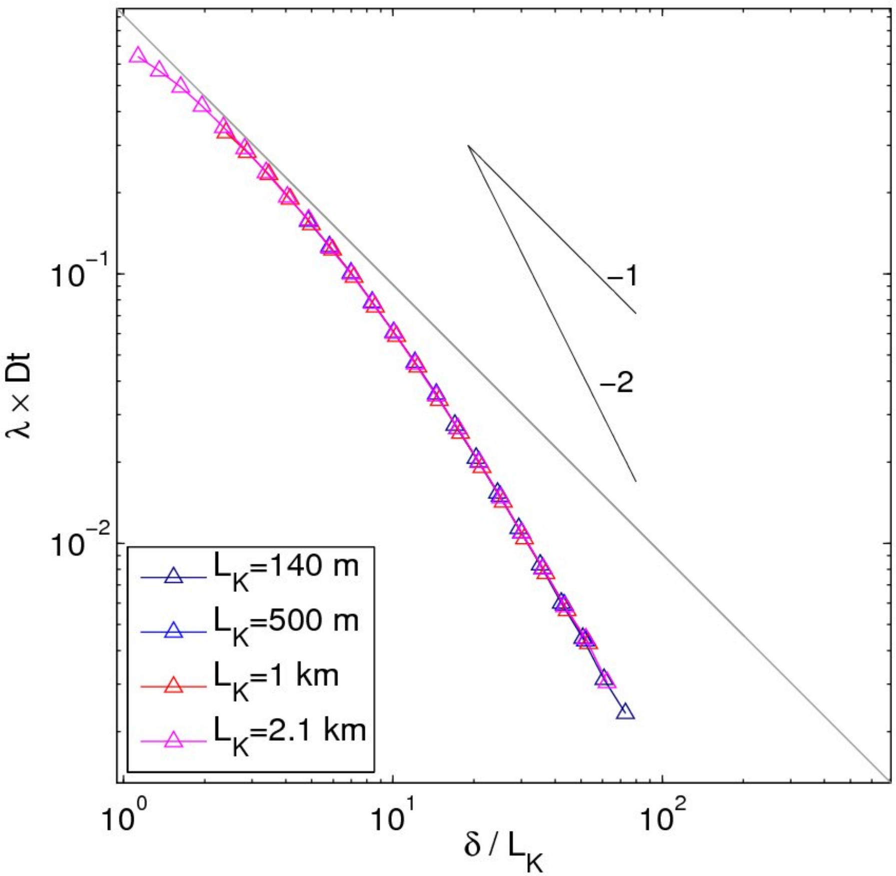
or

$$\boxed{\lambda \sim \frac{log(\alpha)}{(\alpha - 1)dt} \cdot \frac{L_K}{\delta}}$$

NB: δ-1 slope in this range = diffusion signature of a RW with discrete time (finite ΔT) and continuous jump distribution.

LSM-1 : uncorrelated random walk contribution

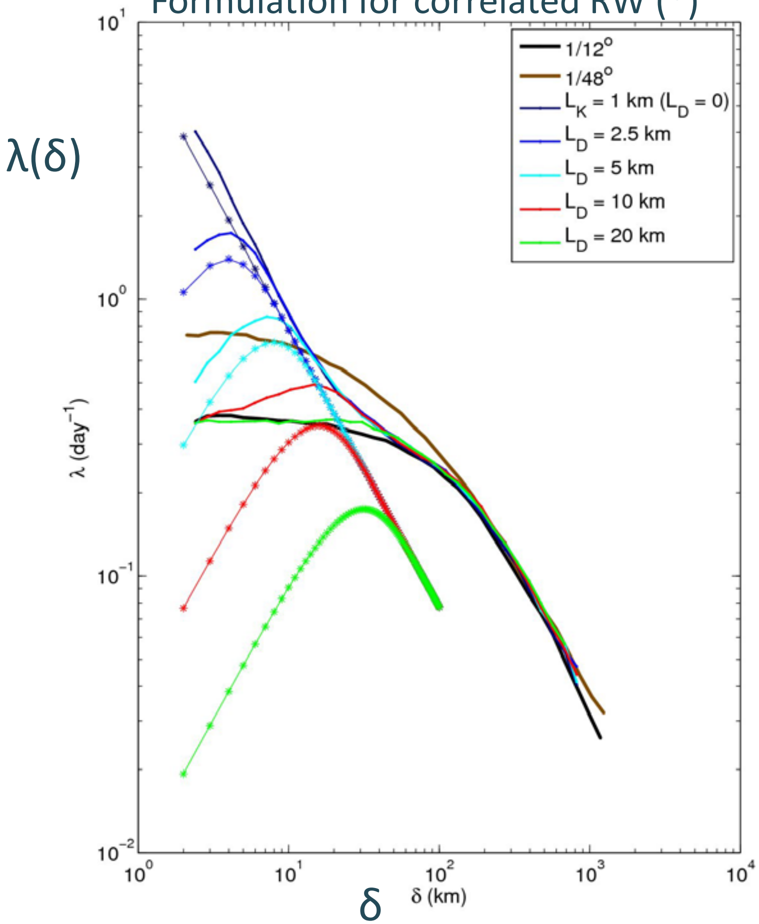
Pure RW rescaled



- $\lambda \sim \delta^{-1}$ regime is possible for **discrete time (Dt), continuous jump distribution**.
- Diffusive $\lambda \sim \delta^{-2}$ regime occurs in the limit Dt goes to zero, or large δs.

LSM-1: correlated random walk contribution

Formulation for correlated RW (*)



$$V_{RW} \sim \left\langle |\mathbf{v}_2 - \mathbf{v}_1|^2 \right\rangle^{1/12}, \qquad with \quad \mathbf{v}_2 = \mathcal{D}\mathbf{v}_1 + (1-\mathcal{D})\,\frac{d\mathbf{w_0}}{dt},$$

so that

$$V_{RW} \sim (1-\mathcal{D})\left\langle |\mathbf{v}^2| \right\rangle^{1/12} \sim (1-\mathcal{D})\;\sigma_{\tilde{u}},$$

and

$$\boxed{\lambda \sim \frac{log(\alpha)}{(\alpha-1)dt}\,.\,\frac{L_K}{\delta}\,.\,\left(1 - e^{-\frac{\delta^2}{2.L_{\mathcal{D}}^2}}\right).}$$

- δ of the local maximum is the solution of:

$$e^u - 2u - 1 = 0, \quad \text{with} \quad u = \frac{\delta^2}{2L_{\mathcal{D}}^2},$$

$$u_0 \approx 1.26, \quad \text{and} \quad \delta_0 \approx 1.59\ L_{\mathcal{D}}.$$

LSM-2: random flight with space correlation

- **First order Markov model**.
- turbulent component ***entirely parametrized***.
- ***ocean model velocities = deterministic drift***.

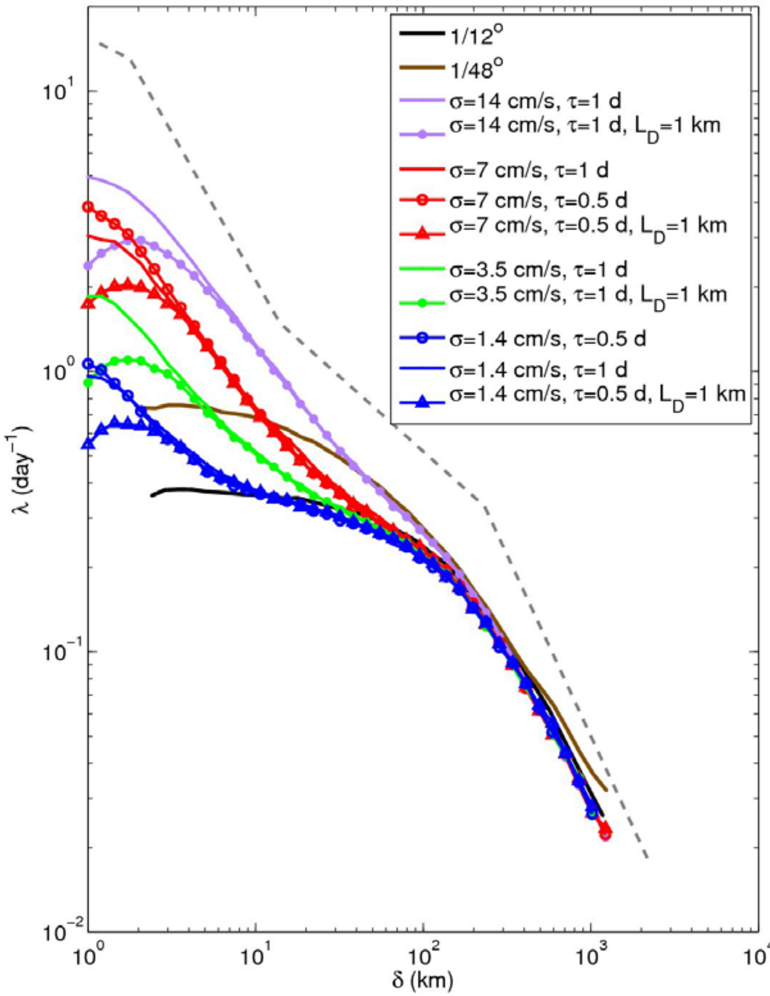
$$du' = -u'\frac{dt}{\tau} + \sigma\sqrt{\frac{2dt}{\tau}}dw_0$$

With σ^2=turbulent velocity variance,
τ = correlation time-scale >> dt/2.

Similar parameter ***D*** can be included for spatial correlation.

For the same diffusivity as LSM1, $K \cong \sigma^2\tau$,

$$\sigma_{RF} \sim \sigma_{RW}\sqrt{\frac{dt}{2\tau}}$$



- ➢ **FSLE in reasonable agreement with K equiv.**
- ➢ **submesoscales slope is about -0.8 (shallower than LSM-1)**
- ➢ **Large-scale RD unchanged.**

LSM-3 formulation

Goal: modify (σ_m,τ_m) to reach real/targeted (σ_r,τ_r) under the Markov-1 assumption.

Model Lagrangian velocity: $\dfrac{d\mathbf{x}_m}{dt} = \mathbf{u}_m(t,\mathbf{x}_m) = \mathbf{U}_m(t,\mathbf{x}_m) + \mathbf{u}'_m(t,\mathbf{x}_m)$

Requires intrinsic flow decomposition

Corrected Lagrangian velocity: $\dfrac{d\mathbf{x}_c}{dt} = \mathbf{U}_m(t,\mathbf{x}_c) + \underbrace{\mathbf{u}'_m(t,\mathbf{x}_c) + \boldsymbol{\eta}(t)}_{u'_c(t,x_c)}$

Missing component:

$$\boxed{\frac{d\eta(t)}{dt} = a\frac{du'_m(t,\mathbf{r}_c(t))}{dt} + bu'_m(t,\mathbf{r}_c(t)) + c\eta(t)}$$

Parameters:

$$\boxed{a = \frac{\sigma_r\sqrt{\tau_m}}{\sigma_m\sqrt{\tau_r}} - 1, \quad b = \frac{\sigma_r}{\sigma_m\sqrt{\tau_r\tau_m}} - \frac{1}{\tau_r}, \quad c = -\frac{1}{\tau_r}}$$

Std (missing component):

$$\overline{\eta^2} = \frac{(\sigma_r\sqrt{\tau_m} - \sigma_m\sqrt{\tau_r})^2 + (\sigma_r\sqrt{\tau_r} - \sigma_m\sqrt{\tau_m})^2}{\tau_r + \tau_m}$$

Turbulent velocity (new/model) Correlation: $r_{mc} = \dfrac{2\sqrt{\gamma}}{1+\gamma}$ > 80% $\qquad \gamma = \tau_m/\tau_r$

LSM-3 without LCS considerations (Adriatic Sea / NCOM)

Haza et al., Ocean Modelling (2007) 17, 68-91.

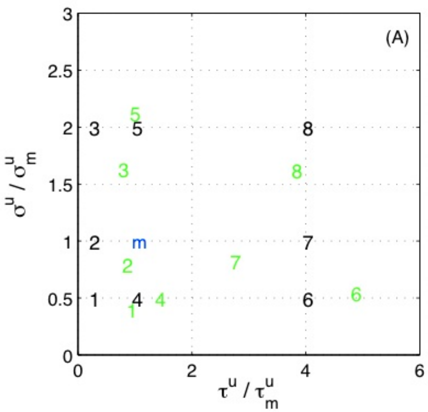
Experimental matrix:

	$\frac{\sigma_r}{\sigma_m} = \frac{1}{2}$	$\frac{\sigma_r}{\sigma_m} = 1$	$\frac{\sigma_r}{\sigma_m} = 2$
$\frac{\tau_r}{\tau_m} = \frac{1}{4}$	Exp-1	Exp-2	Exp-3
$\frac{\tau_r}{\tau_m} = 1$	Exp-4	Model	Exp-5
$\frac{\tau_r}{\tau_m} = 4$	Exp-6	Exp-7	Exp-8

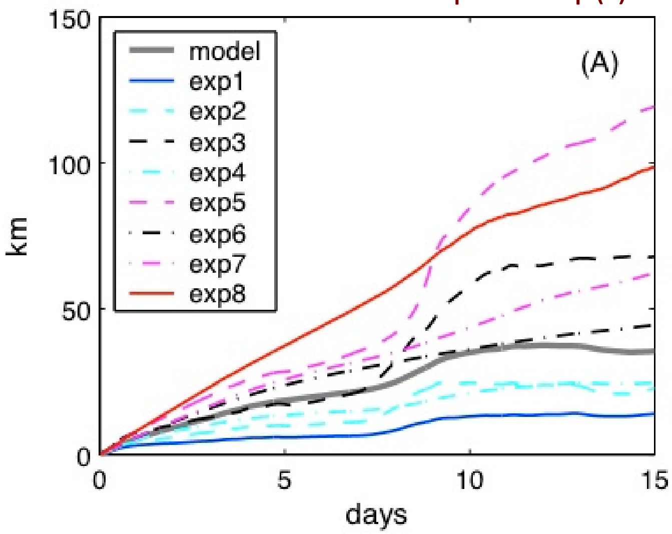
Arranged according to the ratio:

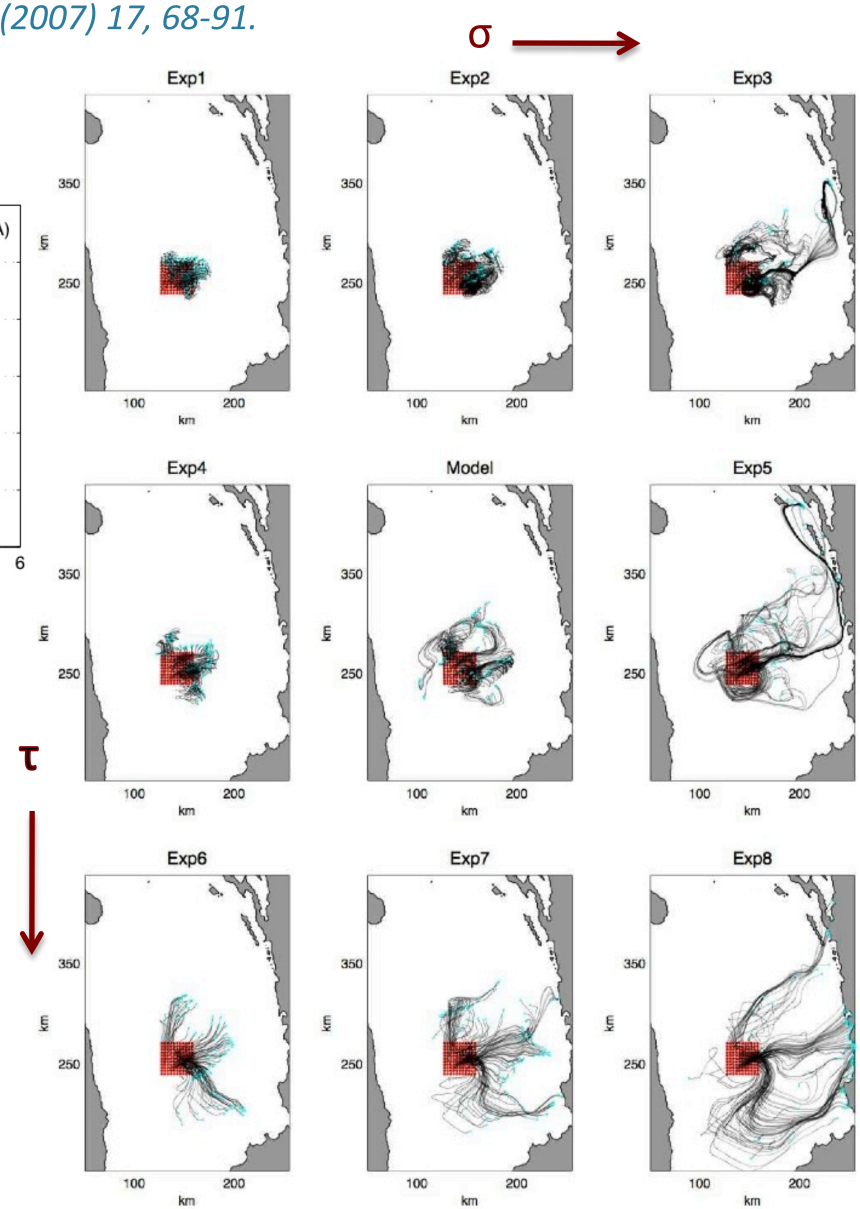
$$\boxed{\frac{\rho_r(t)}{\rho_m(t)} \approx \frac{\sigma_r\sqrt{\tau_r}}{\sigma_m\sqrt{\tau_m}}}$$

Corrected (σ,τ)



Corrected absolute dispersion ρ(t)



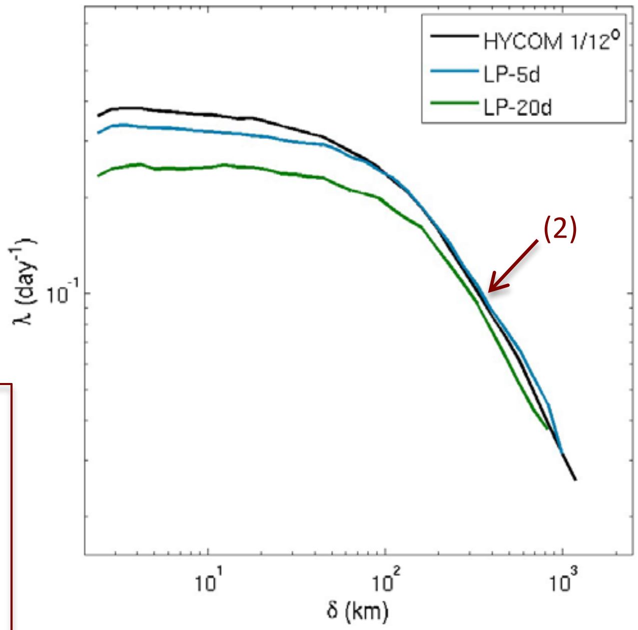


LSM-3 flow decomposition preserving the mesoscale LCS

requirements:

- Enhance turbulence velocity fluctuations.
- Preserve mesoscale transport pathways.

FSLE from low-passed velocities



(1) $\mathbf{u'}_c \ll \mathbf{U}_m$

(2) λ (δ > 2R_d) should remain unchanged.
> Most of the mesoscale flow should be included in $\mathbf{U}_m$.

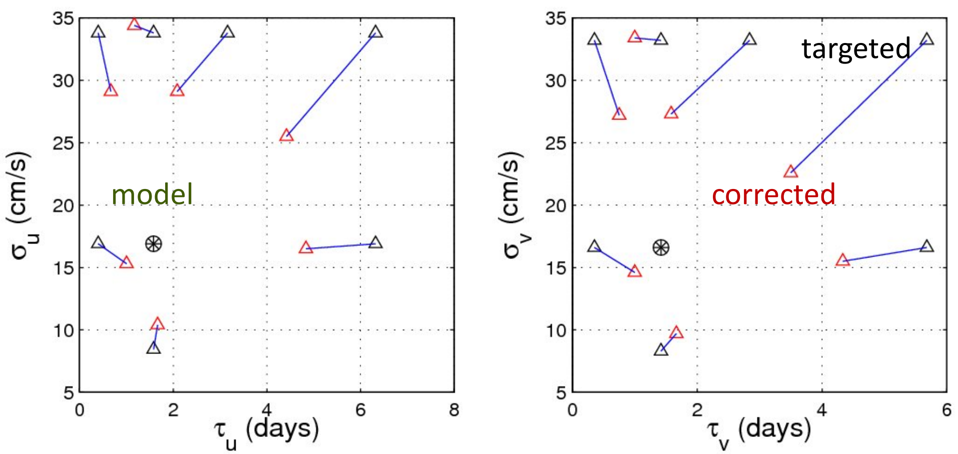
➔A temporal low-pass filter with time-windows of 5-20 days satisfies conditions (1) and (2).

LSM-3 performance (single particle statistics)

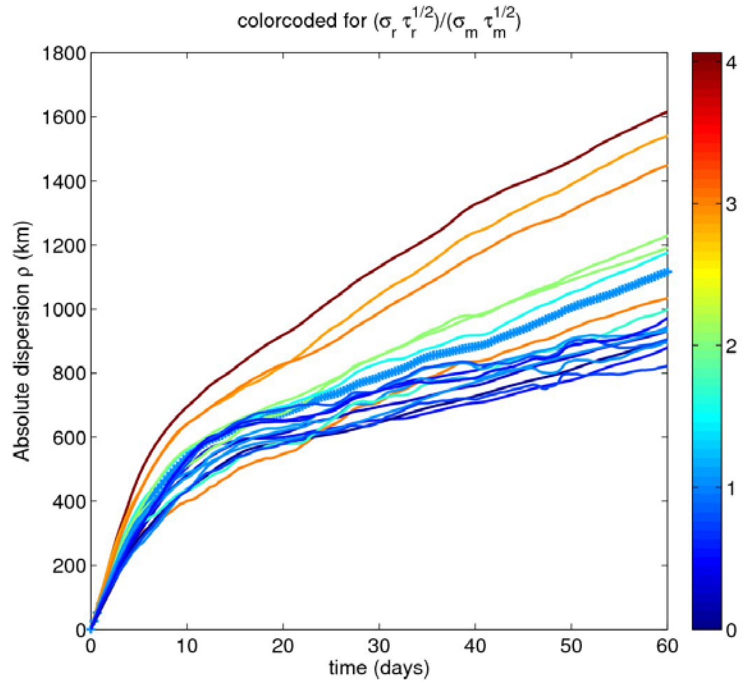
Statistical parameters = f(decomposition)

	τ_m^u (day)	σ_m^u (cm/s)	τ_m^v (day)	σ_m^v (cm/s)
HYCOM 1/12^o (LP20)	1.67	17.9	1.5	17.8
HYCOM 1/12^o (LP5)	1.06	12.25	1.02	10.8
HYCOM 1/48^o (LP20)	1.33	17.6	1.17	17.4
HYCOM 1/48^o (LP5)	0.92	11.7	0.83	11.1

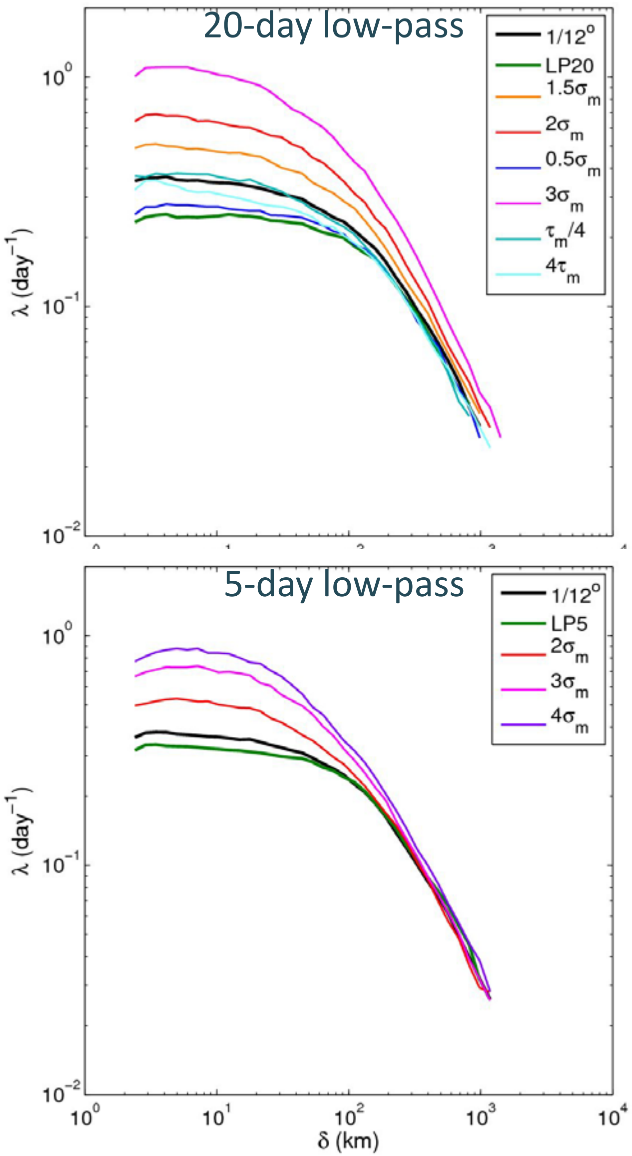
Corrected (σ_r,τ_r)

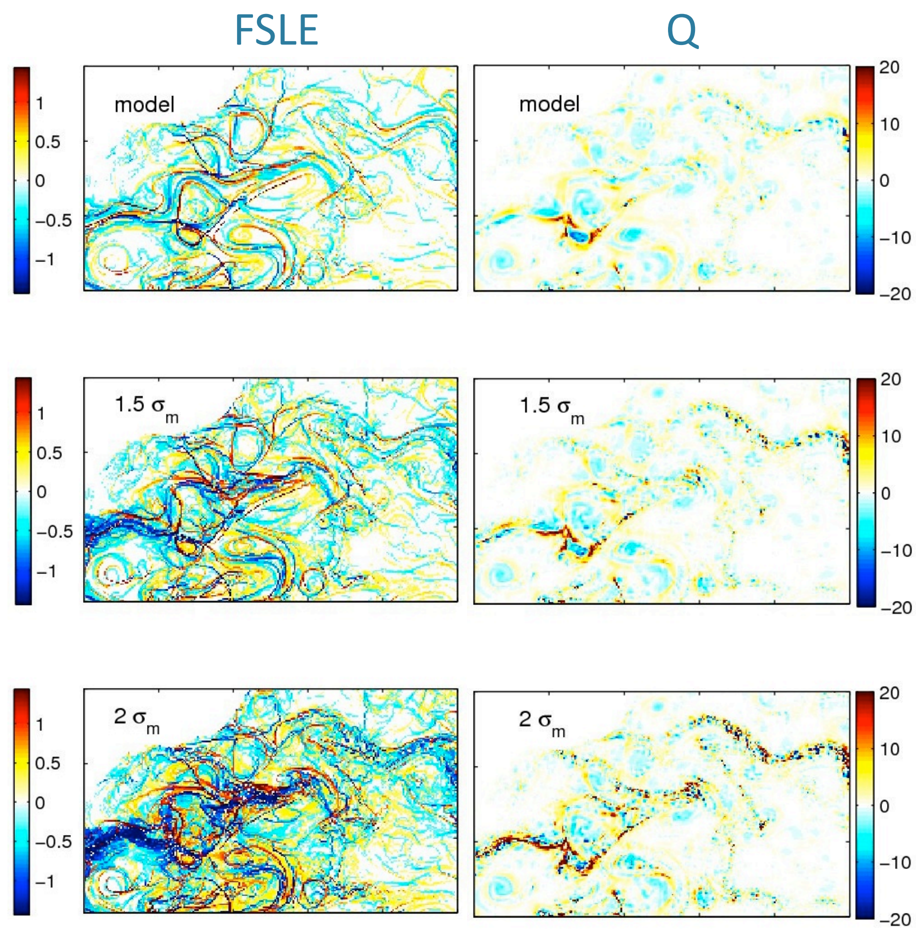


Corrected absolute dispersion ρ(t)



LSM-3 (varying only the σ-ratio)

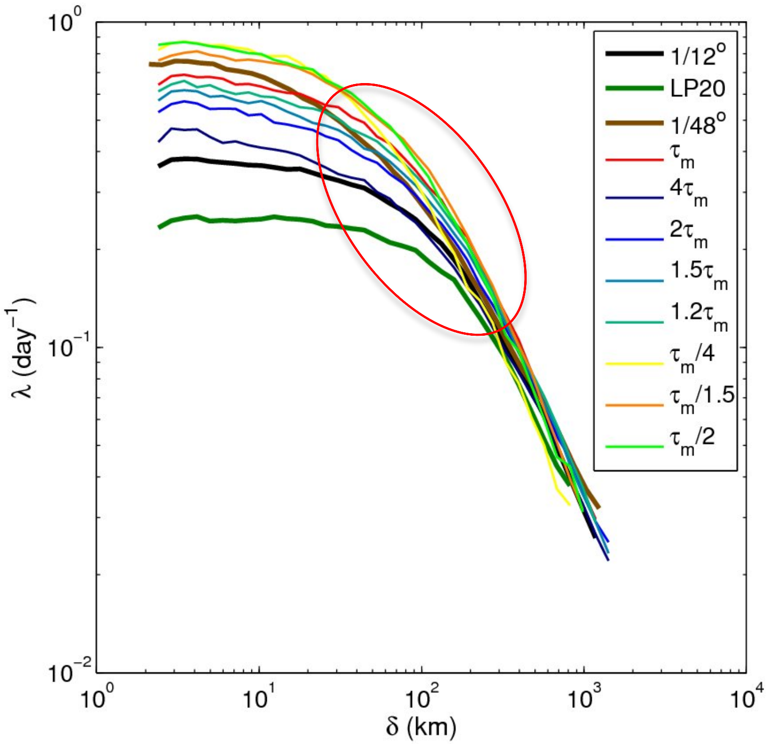


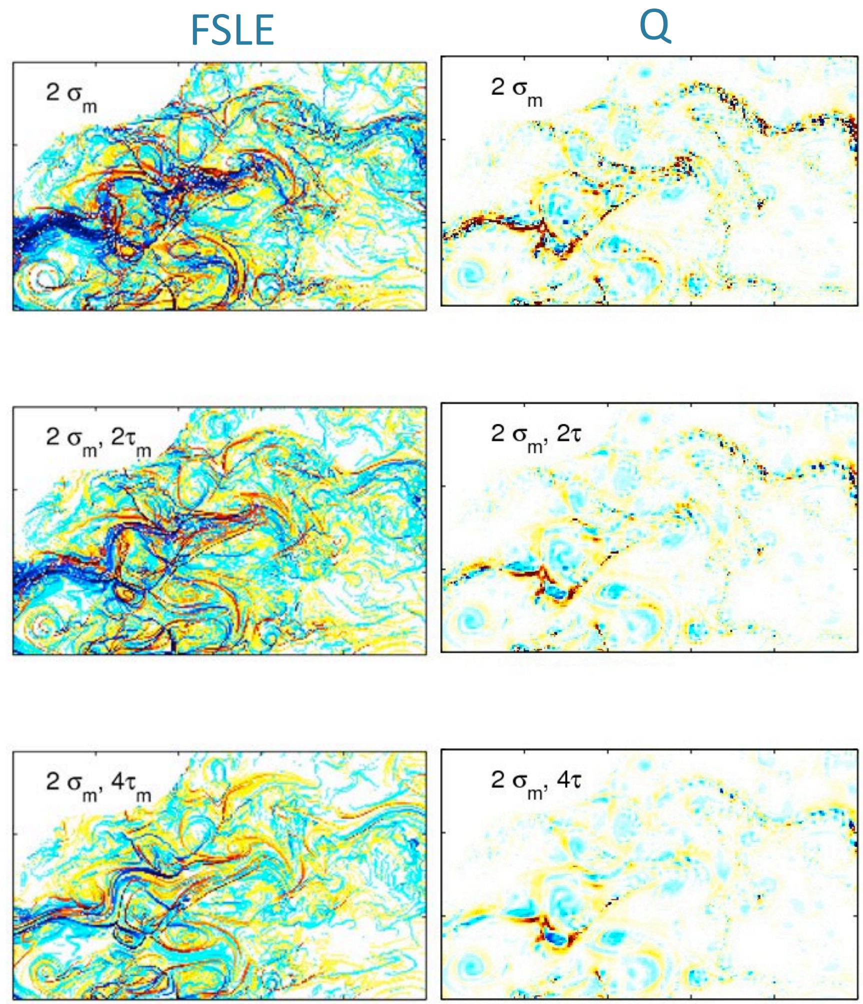


- ➢**Fitting function:** $\lambda_c \sim \lambda_{LP} + \left(\frac{\sigma_r}{\sigma_m}\right)^{\gamma} (\lambda_m - \lambda_{LP})$ γ ~1.9
- ➢**Enhances selectively regions of high stretching rate.**
- ➢**RD at the intermediate scales (2-4 R_d) are still too impacted.**

LSM-3 (varying the τ-ratio for $\sigma_r = 2\sigma_m$)

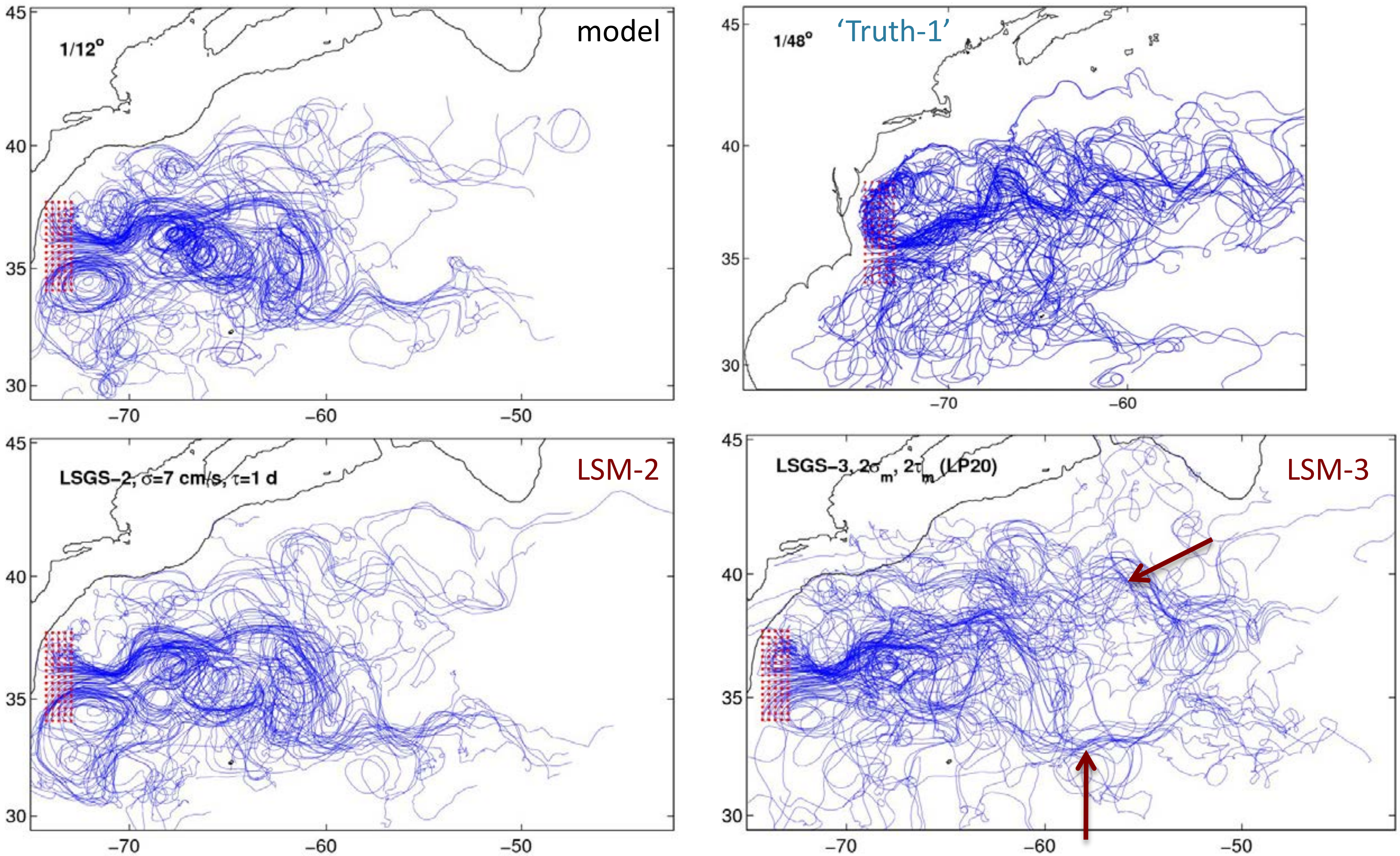
Effect of increasing τ:





- **reduces the rate of dispersion at intermediate scales.**
- **Less invasive on the mesoscale transport barriers.**

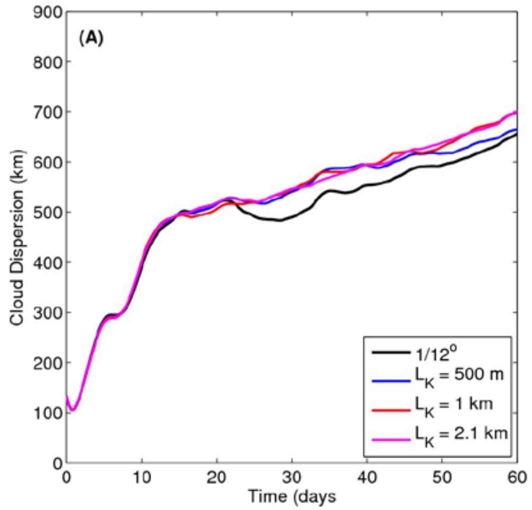
LSM-comparisons



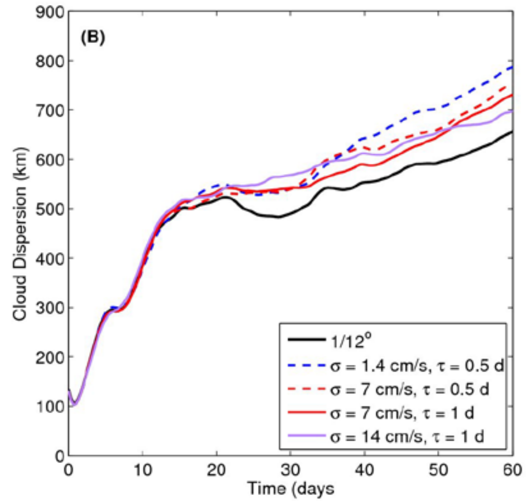
> particles still use the mesoscale transport pathways

LSM-comparisons: cloud dispersion

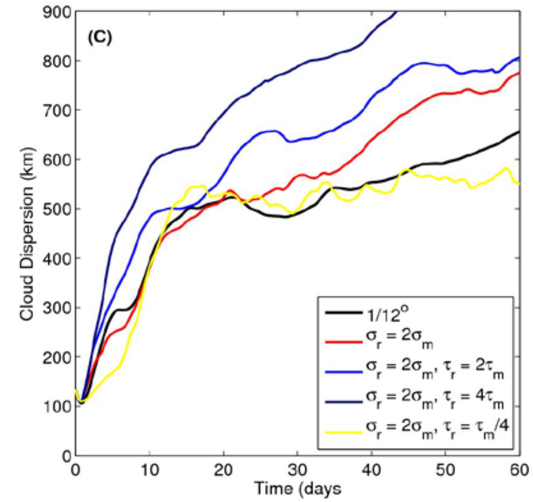
LSM-1



LSM-2

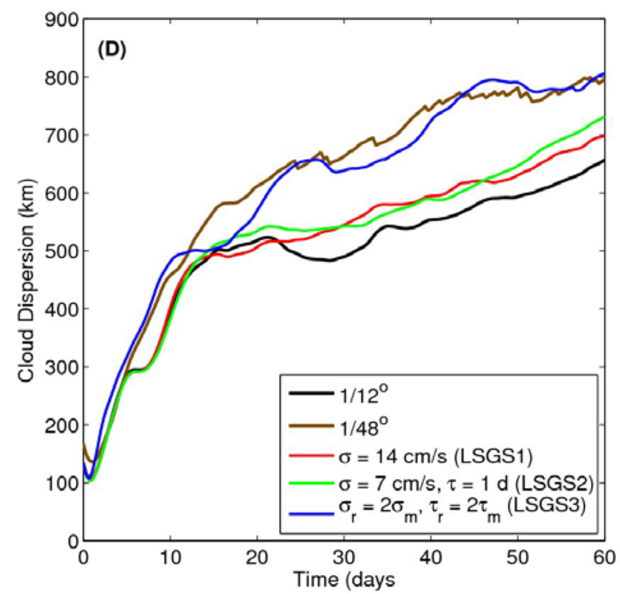


LSM-3



2nd moment of particle displacement (cloud dispersion):

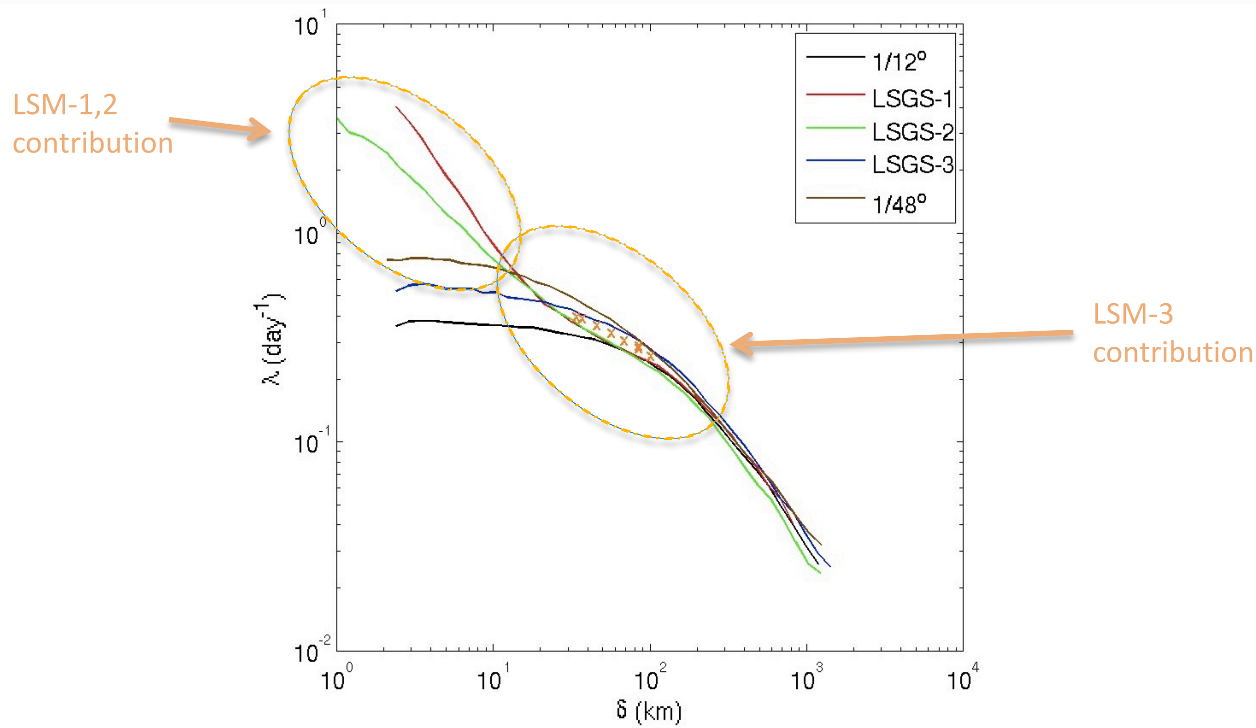
$$\sigma_{CM}(t) = \langle (\mathbf{r}(t) - \langle \mathbf{r}(t) \rangle)^2 \rangle^{1/2}$$



LSM-3 (LP20-2σ,2τ) and Hycom1/48° have similar cloud dispersion.

LSM-conclusion

	LSGS-1	LSGS-2	LSGS-3
	Markov 0	Markov 1	Markov 1
mean flow $\mathbf{V}$	$\mathbf{v}_{HYCOM1/12^o}$	$\mathbf{v}_{HYCOM1/12^o}$	low-passed $\mathbf{v}_{HYCOM1/12^o}$
$\rho(\mathbf{V}, \mathbf{v}')$	0 (uncorrelated)	0(uncorrelated)	$\geq 80\%$
SMS FSLE	$\lambda \sim \delta^{-1}$	$\lambda \sim \delta^{-0.8}$	$\lambda = \lambda_{max}$ exponential
SMS dispersive regime	local	local	non-local
SMS FSLE enhancement	0(10)	0(10)	2-3
spatial impact	uniformly	uniformly	high stretching rate regions



LSM-conclusion

- The **intermediate scales** (around the Rd) play an important role in the cloud dispersion.
  - Still underestimated by the classic LSMs (random walk & random flight).
  - Can be achieved with the LSM-3 (IMC).

- Next:
  - Testing in the GoM.
  - On going work on Lagrangian accelerations, freq. spectra, boundaries ..
  - SMS intermittency.