Impact of noise and subsampling on relative dispersion measurements

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GULF OF MEXICO RESEARCH INITIATIVE

Motivation

- SMS flows have an impact on tracer dispersion, oil spills.. etc. and require scale-dependent dispersion measurements (FSLE)

- Issues/constraints on position measurement errors and sampling frequency when the scales of interest are O(1m – 1km).

- Evidence of measurement bias in the latest FSLE measurements of the Gulf Stream surface circulation.

- Raises the questions:
    - Can the noise signal in the FSLE be isolated from the real signal?
    - how does it respond to low-pass filters?

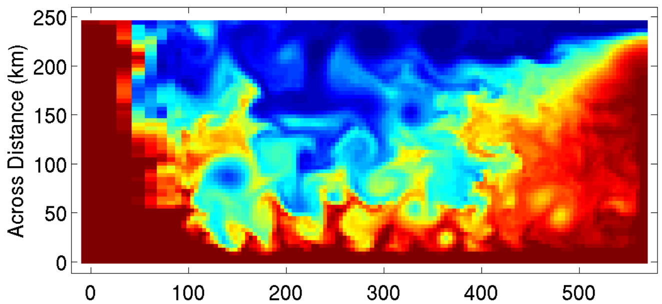
- Simple method: Look at synthetic trajectories and corrupt them with noise.

Relative dispersion from ocean models

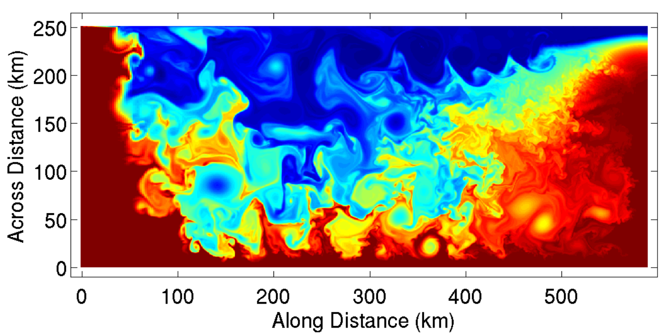
(Poje et al., Ocean Modelling, 31 (2010) 36-50)

Baroclinic jet (ROMS):

res = 4 km

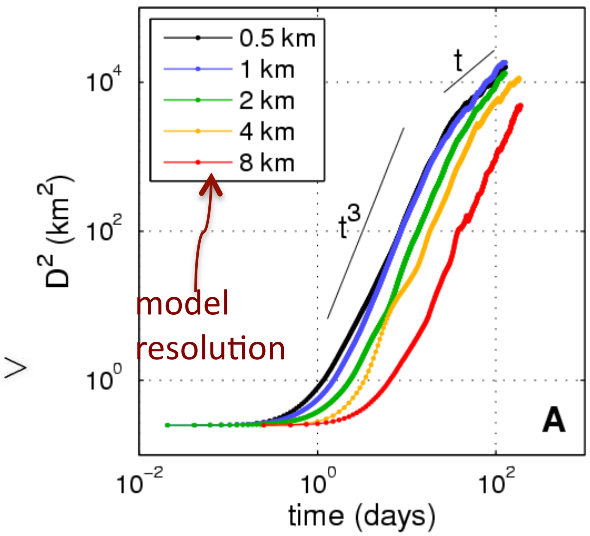


res = 500 m



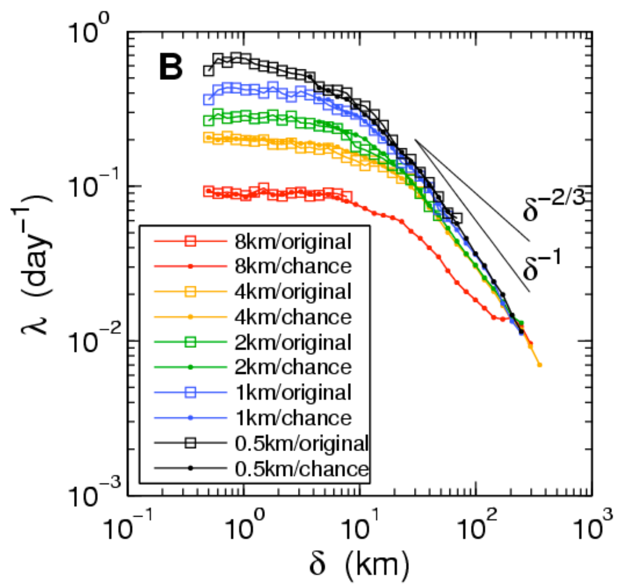
Relative dispersion:

$$D^2(t) = < |\mathbf{x}^{(1)}(t) - \mathbf{x}^{(2)}(t)|^2 >$$



Scale-dependent FSLE:

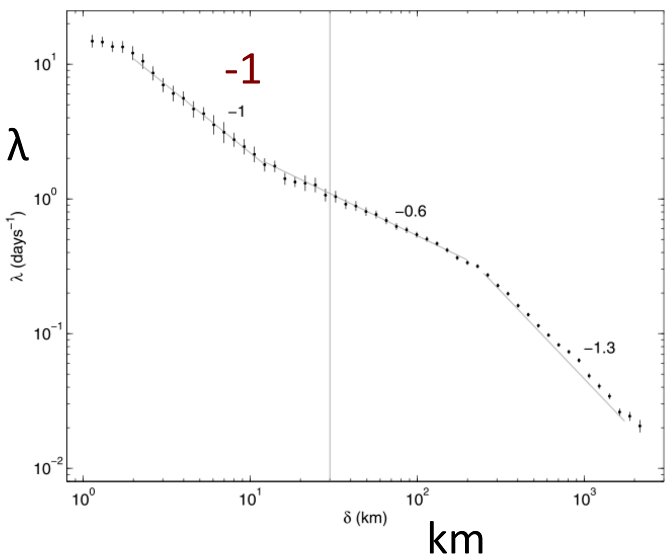
$$\lambda(\delta) = \frac{ln(\alpha)}{< \tau(\delta) >}$$



- Hyperbolicity increases with the horizontal resolution.
- Power-law at large scales is unchanged.

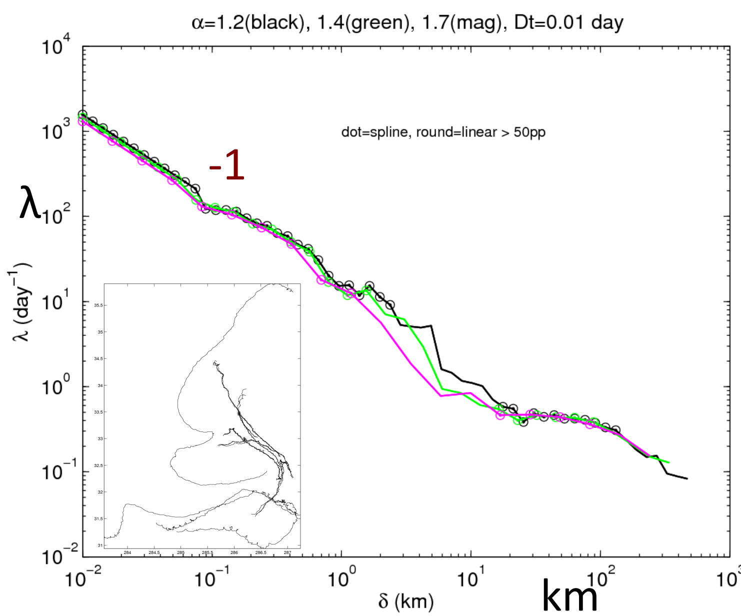
Observed small scale trend (Gulf Stream)

Lumpkin & Elipot, 2010



CLIMODE project (2007)

- 60 satellite tracked drifters launched Feb-Mar 2007.
- Dt = 1-2 hr.
- Error ~ 700m



LAT Mix June 2011 experiment

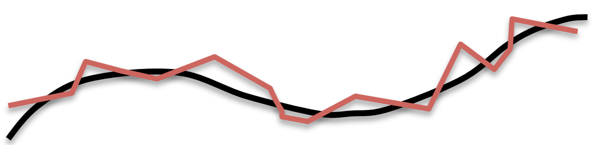
- 20 drifters released in pairs, same type.
- Dt ~ 1.6 hr.
- λ computed from raw data set.

➔ **Recurrent δ^{-1} regime at the smallest measured scales.**

Setting

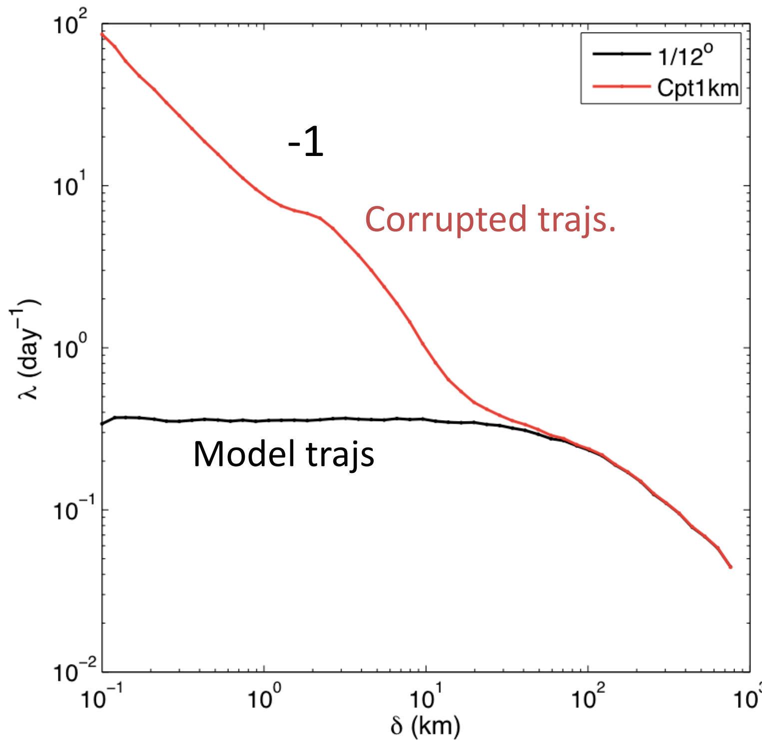
- Synthetic trajectories from Gulf Stream HYCOM 1/12º simulation (weak exponential regime at SMS) assumed to be in-situ drifter trajectories.
- Add noise to each position (time interval Dt).

$$\mathbf{dx} = L_K \, . \, \mathbf{dW_0}$$



L_k = position error std.
dW_0 = random component from normal distribution ($\mu = 0$, $\sigma = 1$)

Example of FSLE for a noise of L_K=1 km, Dt=2 hrs:



Equivalence position error – random walk

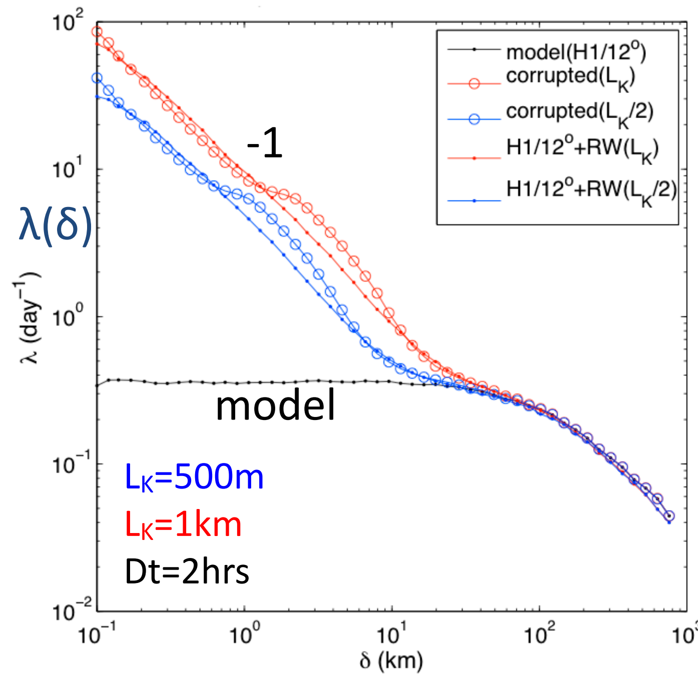
Using the same distribution (LK . dW0(1,0)), compare:

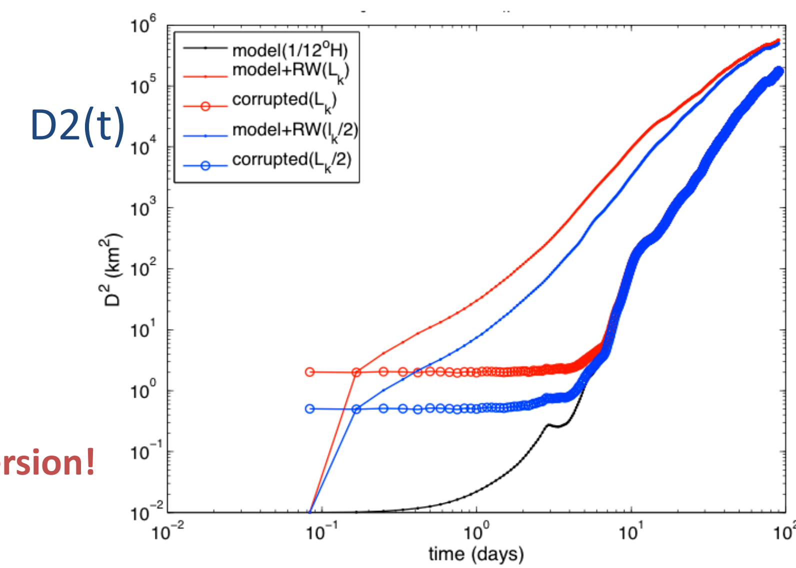
- **Adding a random walk (RW) to particles advected by model velocities.**

- **Corrupting (Cpt) the model trajectories.**

$$\begin{cases} D^2_{model+RW}(t) \gg D^2_{model}(t) \\ D^2_{model+Cpt}(t) = \underbrace{D^2_{model}(t) + 2\,L_K^2}_{\text{no time-cumulative dispersion}} \\ \lambda_{model+Cpt}(\delta) \sim \lambda_{model+RW}(\delta) \end{cases}$$

➔ λ is not directly sensitive to cumulative relative dispersion!





Why do RW and position error yield similar FSLE?

Because of their averaged relative velocities:

$$\Delta V_{RW} \sim \sigma_u = \frac{L_K}{Dt}$$

also:

$$\Delta V_{RW} = \frac{(\alpha - 1)\delta}{\tau(\delta, \alpha)}$$

It follows that:

$$\lambda(\delta) \sim \frac{log(\alpha)}{(\alpha - 1)Dt}\,\frac{L_K}{\delta}$$

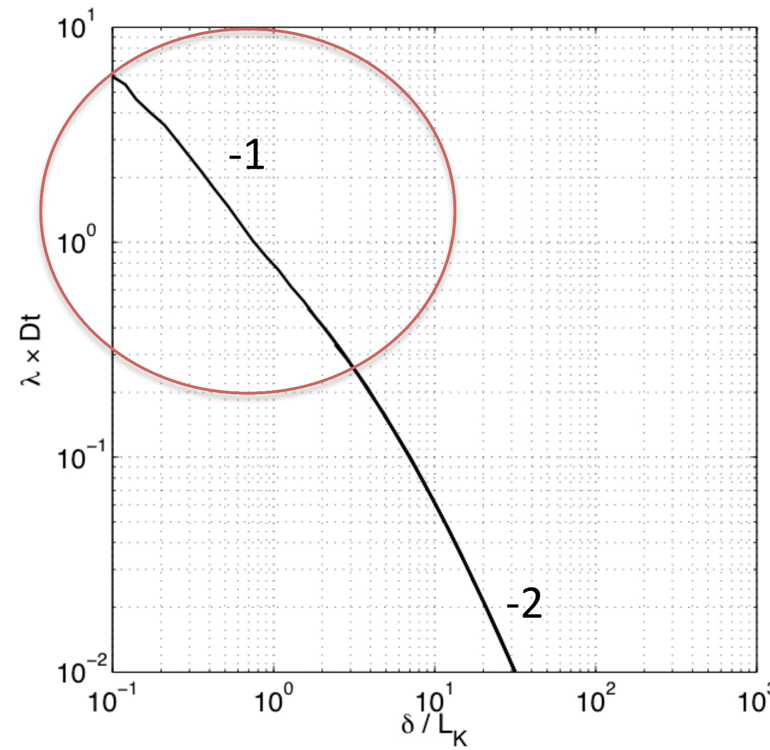
which varies like δ^{-1}.

In the case of noise from position uncertainty:

$$\Delta V_{Cpt} = \frac{L}{T} \sim \frac{NL_K}{NDt} = \frac{L_K}{Dt},$$

and is the same as ΔV_{RW}.

Rescaled FSLE of random walk:

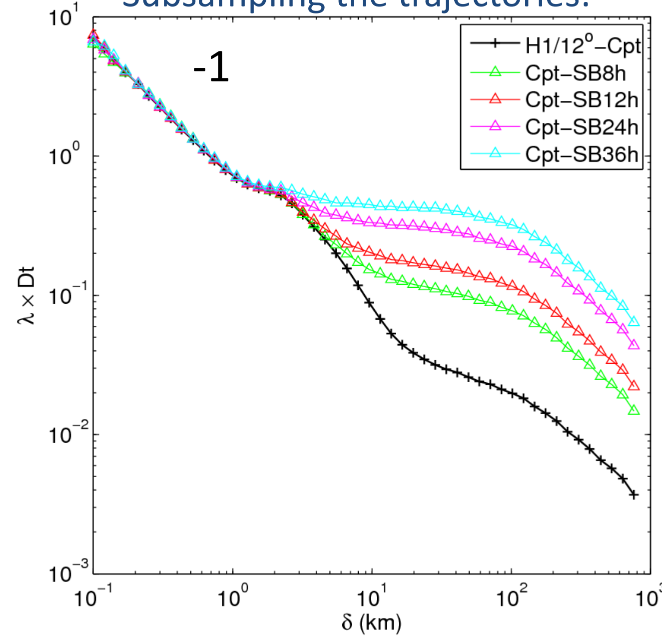


How to evidence pure noise from λ?

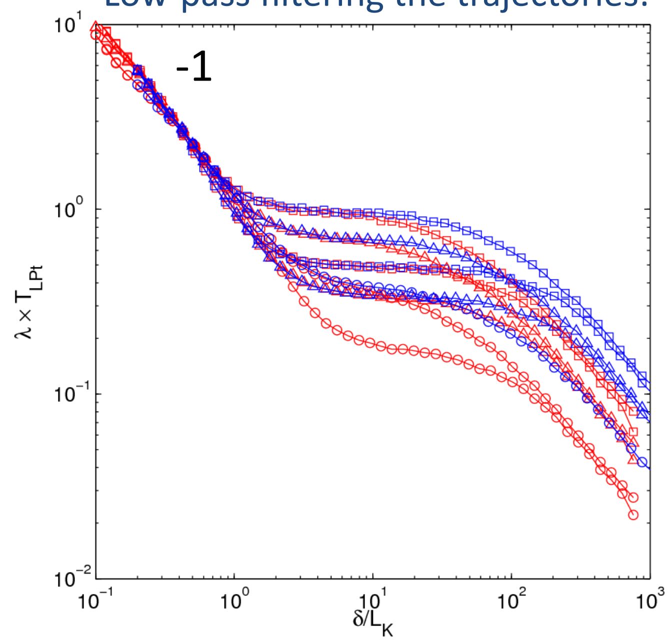
1) ***By sub-sampling*** (SB) the trajectories, i.e. by increasing Dt.

Then re-compute λ and rescale by 1/Dt.
If the curves collapse for $\delta/L_K \le 2$ and $\lambda \sim \delta^{-1}$, then we have pure noise.

Subsampling the trajectories:



Low-pass filtering the trajectories:



2) ***By filtering*** (LPt) the trajectories with a temporal-moving average of window T_{LPt}.

The resulting λ rescaled by $1/T_{LPt}$ also yields a collapse for pure noise.

Low-pass (moving average) filter impact on noise

For each position of indice i (zonal),

$$x^i_{Cpt} = x^i + \frac{L_K}{\sqrt{2}} dWx^i.$$

The low-passed version is :

$$\widetilde{x^i_{Cpt}} \approx x^i + \frac{L_K}{\sqrt{2}} \widetilde{dWx^i}.$$

$\Rightarrow$ new noise: $\widetilde{dW^i}$.

Original noise distribution:

$$f(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} exp \frac{(x-\mu)^2}{2\sigma^2},$$

with $\sigma = \sigma_0 = 1$.

So if $T_{LPt} = N \times Dt$, $\sigma = \sigma_0/\sqrt{N}$,and:

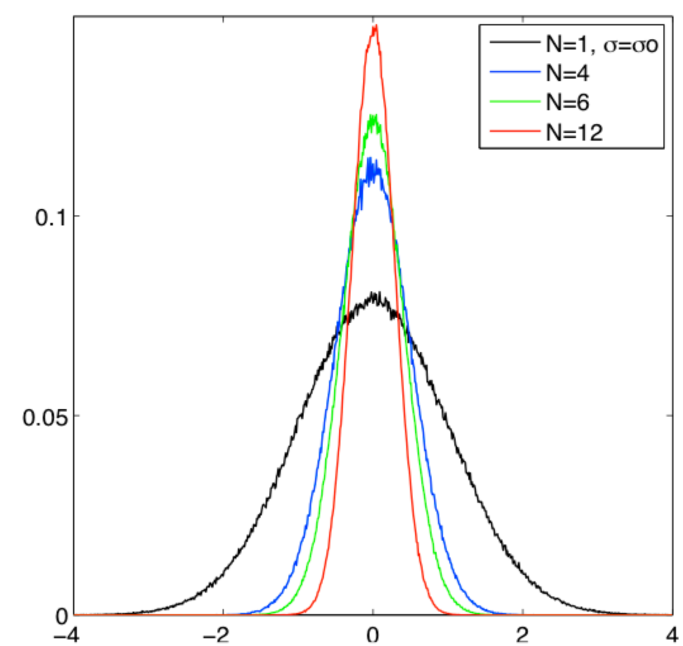
suggested behavior: $\frac{\lambda}{\sigma} \sim \frac{log(\alpha)}{(\alpha-1)Dt} \frac{L_k}{\delta} \times \sigma$

becomes:

$$\lambda \times \frac{N}{\sigma_0^2} \times Dt \sim \frac{log(\alpha)}{(\alpha-1)} \frac{L_k}{\delta}.$$

T_{LPt}

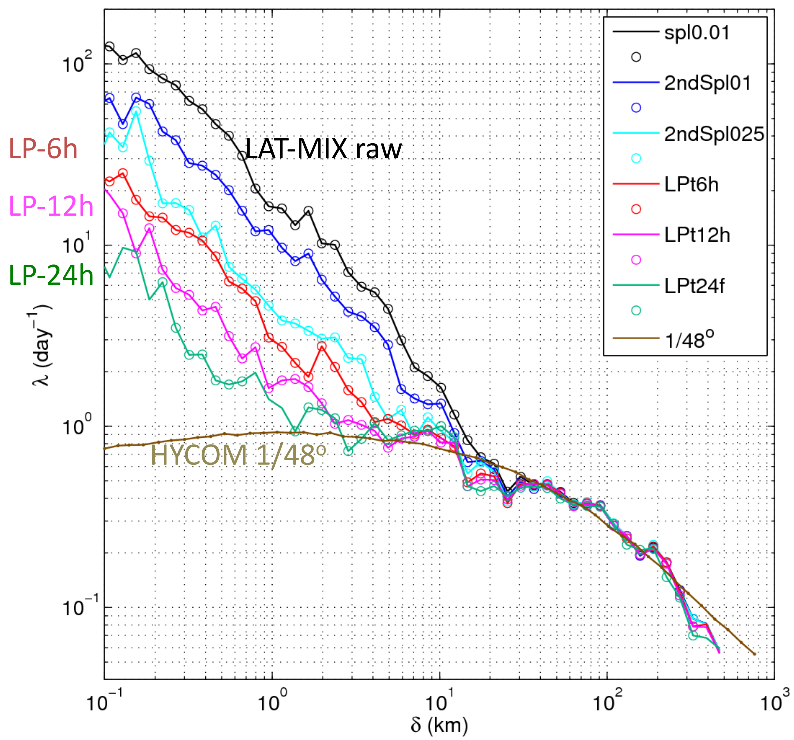
Distribution for filter window T=N.Dt:



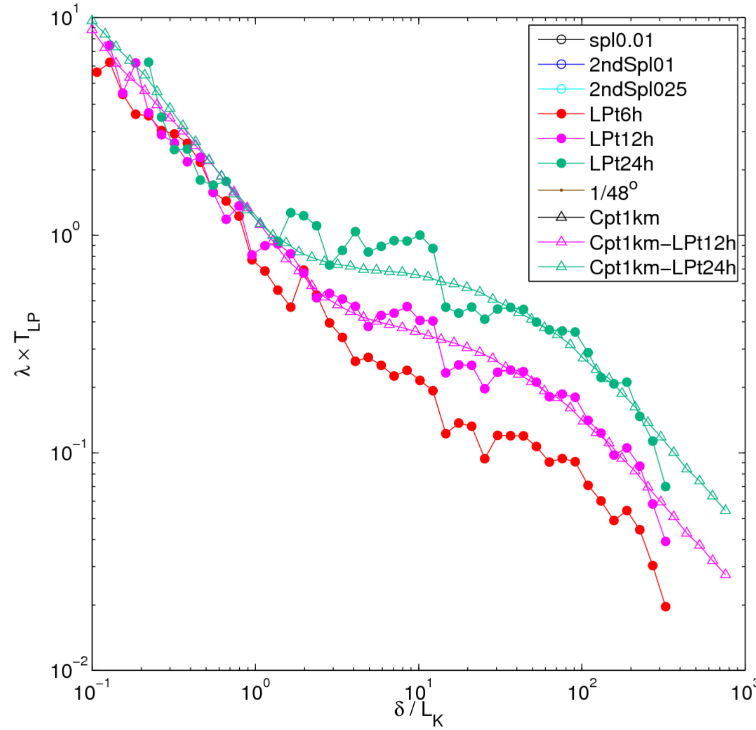
➔ Impact on λ similar to subsampling.

LAT-MIX comparison with HYCOM 1/48°+(1km,2h noise)

λ from low-passed LAT-MIX trajectories:

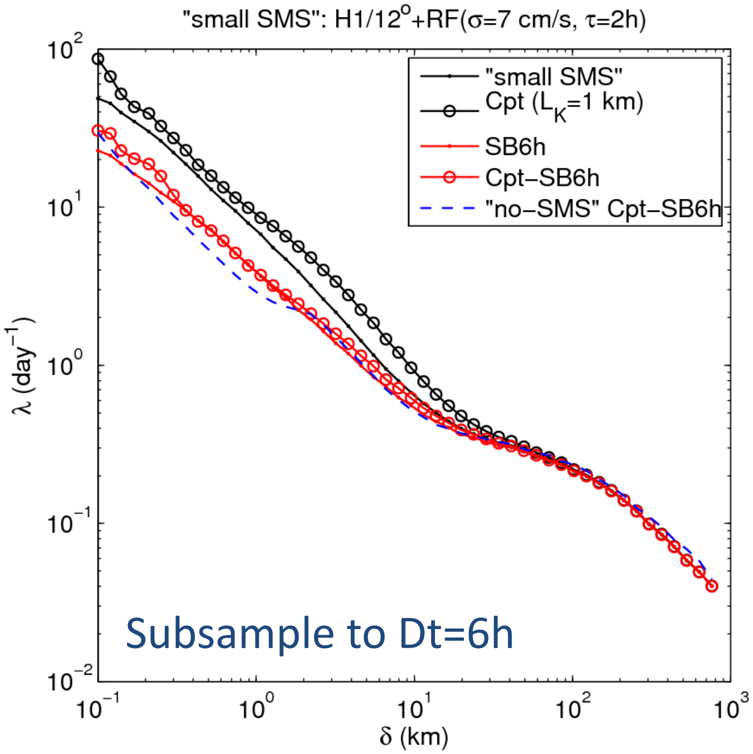


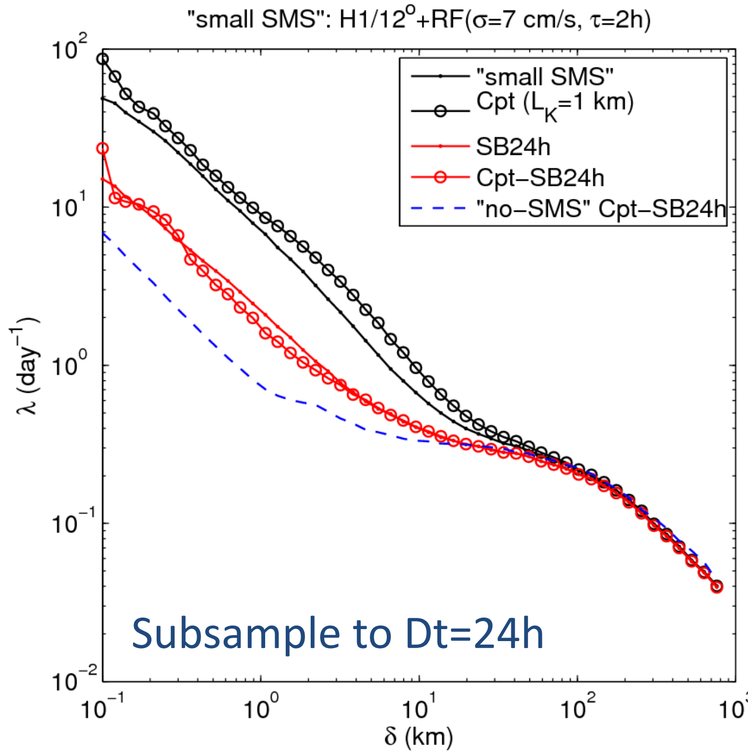
Rescaled λ of LAT-MIX and corrupted H1/48°:



- **LAT-MIX λ can be reproduced with HYCOM1/48°+(1km,2hr) noise.**
- **Indication of pure noise up to δ = 3km.**

What if in-situ SMS and noise have same magnitude λ?





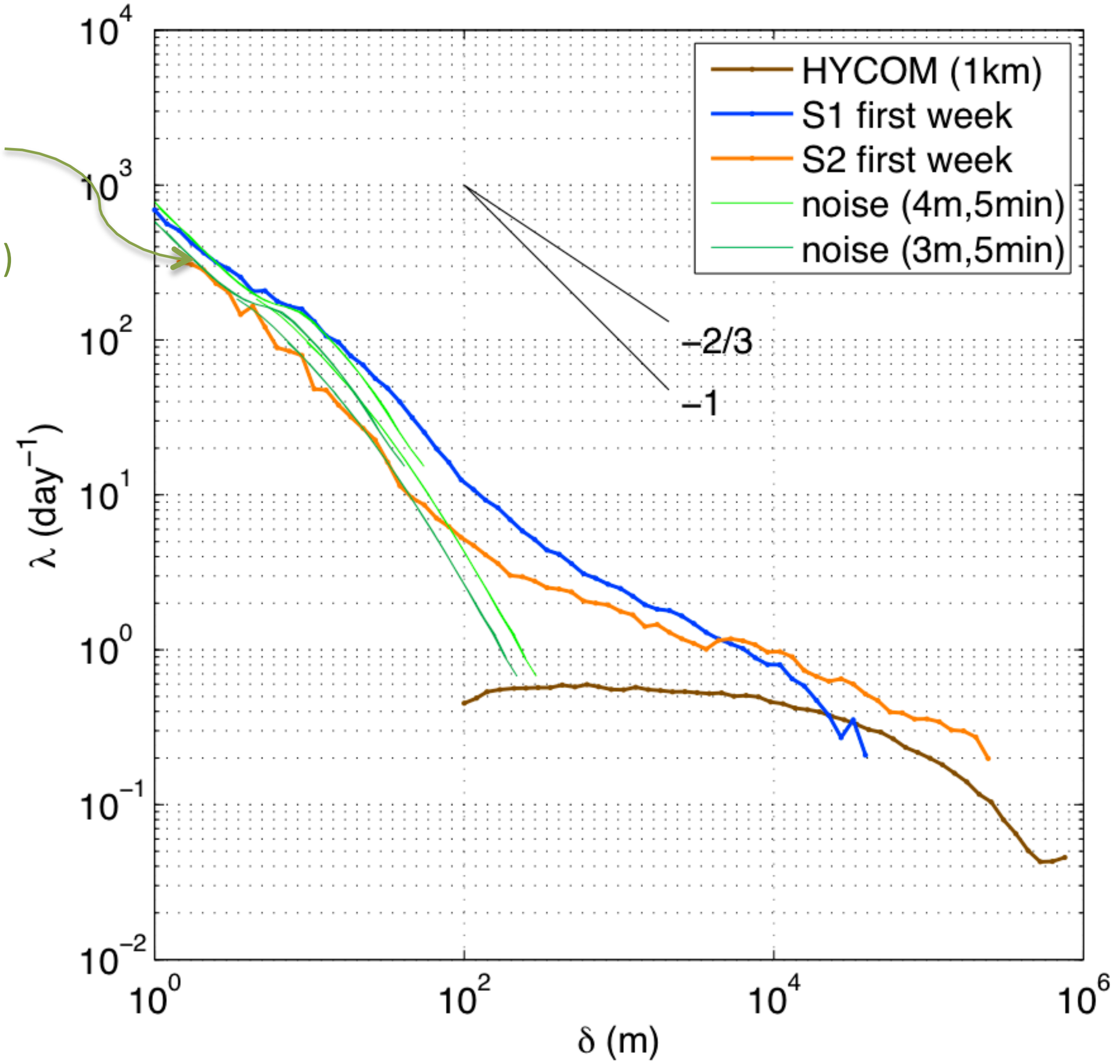
1) Real signal: HYCOM1/12° + Random Flight with small τ. (-)
2) Add noise (1km, 2hrs). (-o-o-o-)
3) subsample both the real (-) and corrupted (-o-o-o-)signals.
4) compare to pure noise. (- - -)

➢ There must be a significant difference between pure noise and filtered signal.

GLAD Noise impact (Near DWH site, Gulf of Mexico)

$$\frac{log(\alpha)}{(\alpha-1)Dt}\frac{L_K}{\delta}$$

(noise contribution)

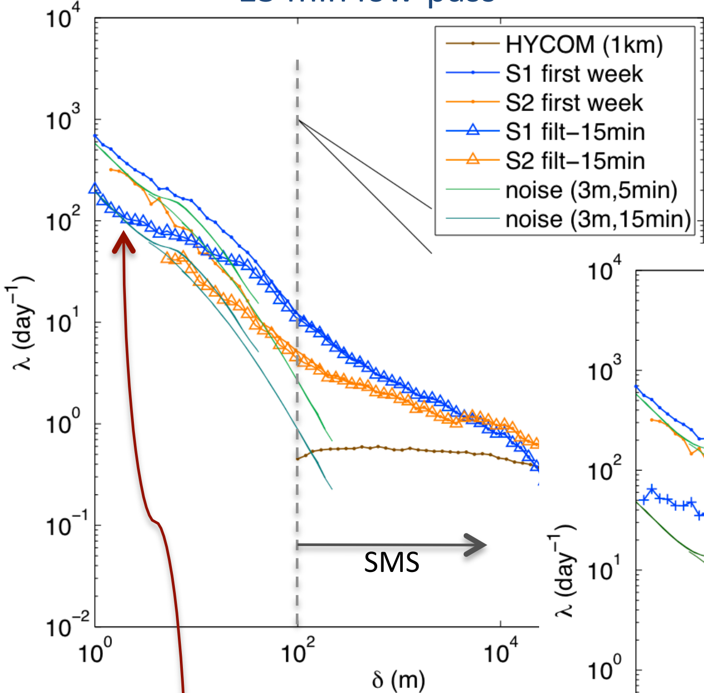


SPOT GPS:
L_K ~ 4 meters
Dt = 5 min

GLAD noise & subsampling impact

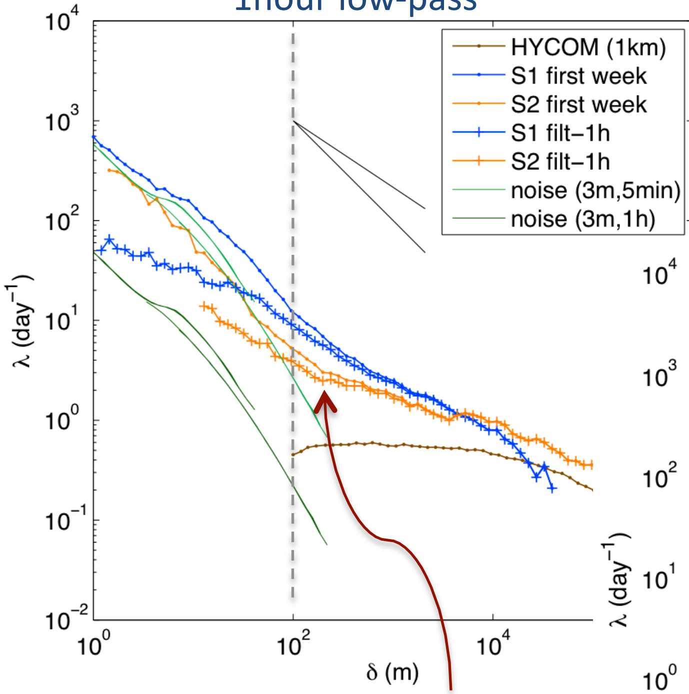
- Increase Dt to reduce the noise impact.

15 min low-pass



pure noise contribution

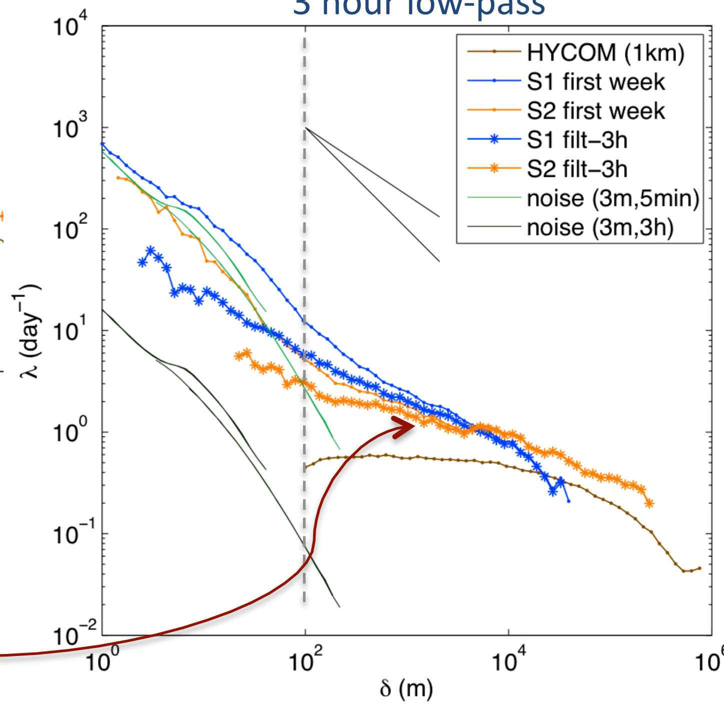
1hour low-pass



λ(δ ≥ 100m) become affected.

➢ SMS features are also contributing to high FSLE at 10-100m besides GPS noise.

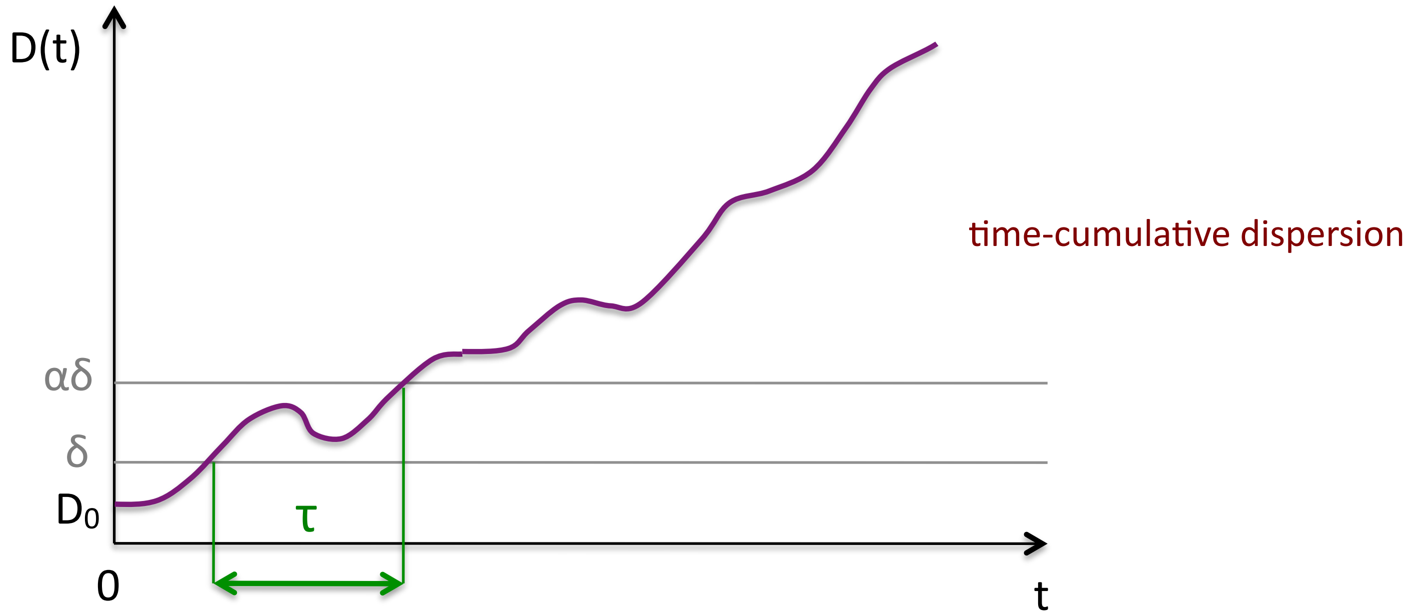
3 hour low-pass



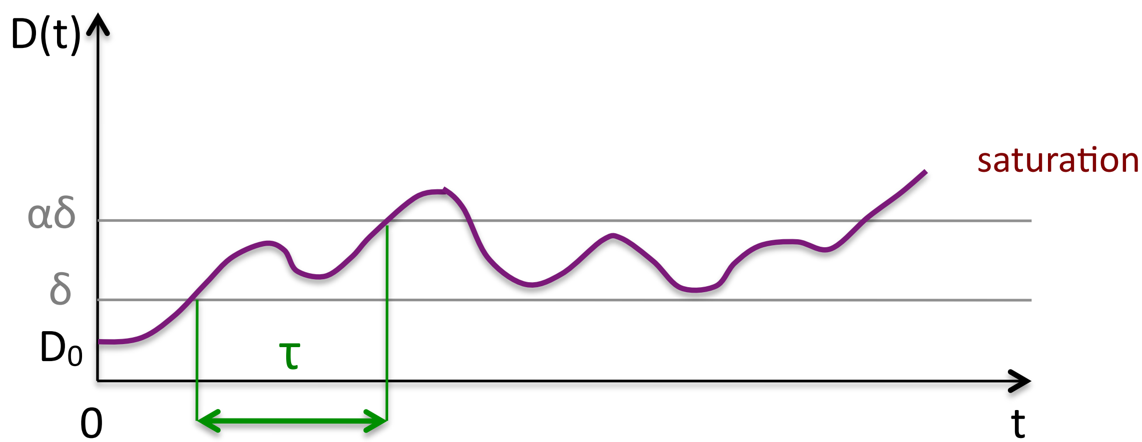
Inertial oscillations affected

Time-cumulative dispersion

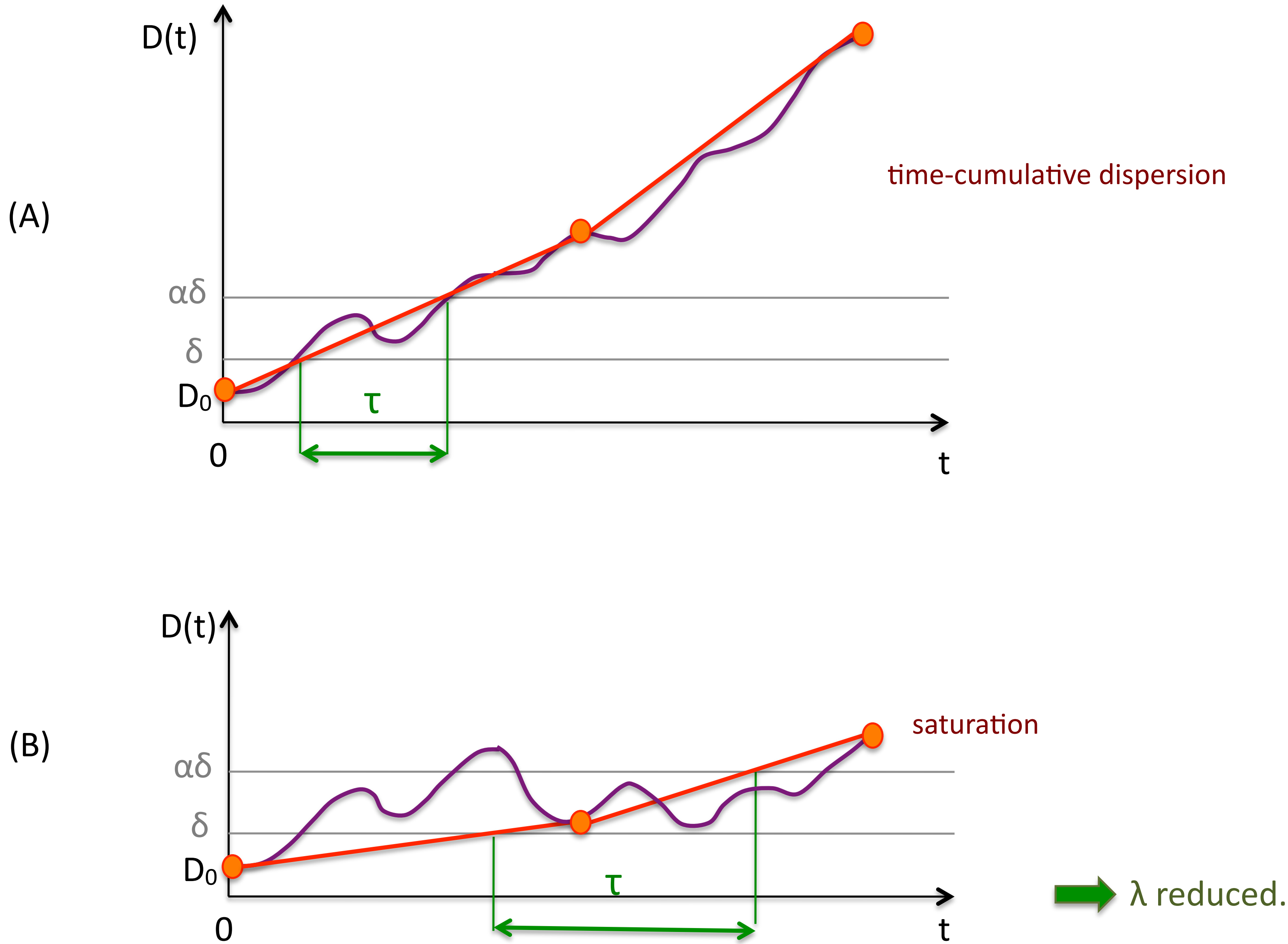
(A)



(B)



Time-cumulative dispersion



Summary

- Measurement noise has a distinct δ^{-1} signature in the scale-dependent FSLE, and is proportional to the position error and sampling frequency.

- It implies a trade-off on the sampling time for designing SMS dispersion experiments.

- Low-passing or subsampling the trajectories reduces the noise signal and can be evidenced by rescaling λ.

- The real SMS signal is less sensitive to the low-pass filters, but it depends on the time-scale of the features controling the relative dispersion at a given scale.

- The recovered signal is obtained from the weakest low-passed trajectories necessary to distinguish noise from the real relative dispersion.