How to Read a Map of Lagrangian Coherent Structures

A Tutorial

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LCS Map: Ocean Model Example

LCS = Lagrangian Coherent Structures

What is this?!?

Example based on a Gulf of Mexico implementation of HYCOM, run by NRL-Stennis.
What are LCS?

*Lagrangian Coherent Structures*, or LCS, partition a flow field into regions that undergo similar experiences. This may mean any of the following:

- Similar residence time within a region of interest
- Similar origin or fate
- Similar dispersion rates
- Etc.
Manifolds & Critical Trajectories

- Flow regions with high dispersion are often organized like this: Such regions are termed *hyperbolic*.

- The directions along which the flow converges, marked in **blue**, are *inflowing manifolds*.

- The directions along which the flow diverges, marked in **red**, are *outflowing manifolds*.

- The manifolds intersect at a *critical trajectory*.

- Manifolds are material curves: Nothing crosses them.
Manifolds & Dispersion

- Manifolds are difficult to find exactly, even with a perfectly known velocity field.

- Instead they are approximated with diagnostics that are easier to compute.

- Note: Particles that start near the inflowing manifold will separate quickly, while those ending near the outflowing manifold have come from disparate origins.

  ➔ Areas around the inflowing manifold exhibit high dispersion in forward time; areas around outflowing manifolds exhibit high dispersion in backward time.
Dispersion & Lyapunov Exponents

- Lyapunov exponents are a mathematical tool to describe dispersion characteristics. They can be used to identify LCS.

- In the context of ocean flows, which are not defined in infinite time or space, the quantities studied are typically *Finite Space Lyapunov Exponents (FSLEs)* or *Finite Time Lyapunov Exponents (FTLEs)*, also called *Direct Lyapunov Exponents (DLEs)*.

- Fundamentally, both FTLEs and FSLEs measure separation time-scales of nearby particles.
FTLEs in a Simple Stationary Flow

Forward Time FTLE

Backward Time FTLE

Inflowing manifold

Outflowing manifold
Transport near LCS

Large stretching along **outflowing** manifold.

Large differences in fates for nearby initializations near the **inflowing** manifold.
LCS in Time-Dependent Flows

- LCS given by ridges in FTLE fields still approximate manifolds and show dispersion patterns.
- Material will follow the evolving LCS.

Example based on a Gulf of Mexico implementation of CUPOM, run at the University of Colorado.
Example from the Gulf of Mexico

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Inflowing manifold

Outflowing manifold

Cyclones

Anti-Cyclones
Transport & LCS in an Ocean Flow
Transport & LCS Animation
Practical Application: Oil Spill

Black: Observed oil slick outline
Green: Modeled oil slick outline
Purple ovals: Hyperbolic regions, tracked from (a) to (b)

Results from Huntley et al., 2011.
Hyperbolic regions in ovals exhibit:

- Stretching along red ridge, away from intersection with blue ridge.
- Stretching to the SE was also observed;
- Thin tendrils in the NW may have evaporated or not been visible.
Oil Spill Example: Analysis (II)

Hyperbolic region in oval exhibits:

Transport along red ridge, away from intersection with blue ridge, leading to consolidation of the oil patch.

Observations agree.
Oil Spill Example: Analysis (III)

Initial oil patch crosses blue ridge

- High sensitivity to initial conditions
- High forecast uncertainty (& in this case error)
Summary

- Ridges in FTLE fields approximate manifolds.

- **Forward-time** FTLE ridges (inflowing manifolds) show high initial-condition sensitivity and forecast uncertainty. (Forecast uncertainty is even greater, since the FTLE map itself is based on models with their own uncertainties.)

- **Backward-time** FTLE ridges (outflowing manifolds) show directions of high stretching.

- Regions away from FTLE ridges are relatively quiescent.

- Edges of strong currents and eddies are typically marked by FTLE ridges.
Some Warnings

- FTLE (and FSLE) ridges are imperfect surrogates for the actual manifolds and, under certain conditions, do not align with the manifold structure and may permit material transport across them.

- For a complete picture, both forward-time and backward-time calculations are needed. Especially the former may be subject to significant model forecast errors.

- Hyperbolic regions may lose their hyperbolicity over time, so that it is not always possible to track a ridge intersection over time.
Some References


Some References (cont’d)


