How to Read a Map of Lagrangian Coherent Structures

A Tutorial

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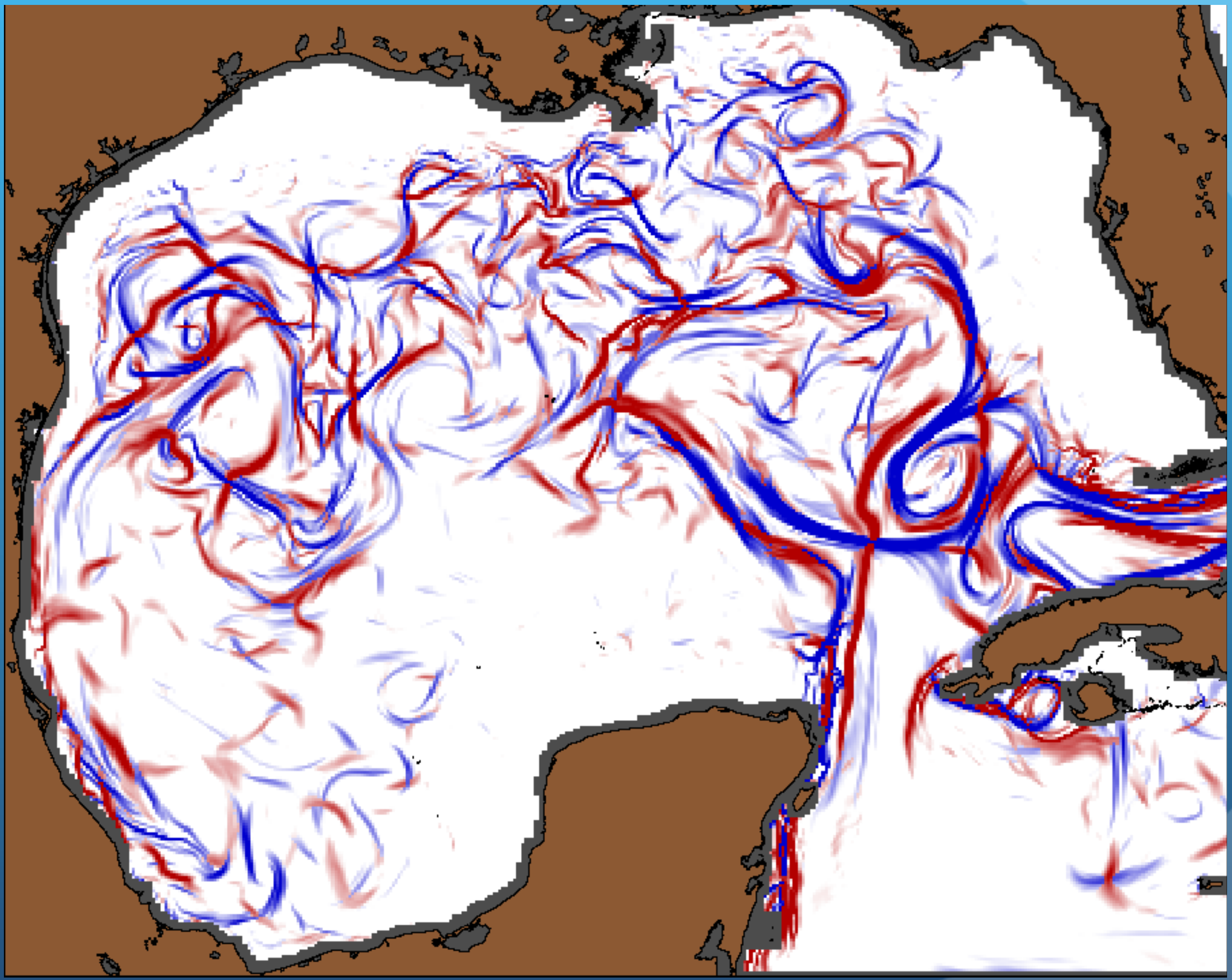
Ocean3D+1

LCS Map: Ocean Model Example

LCS = Lagrangian Coherent Structures

What is this?!?



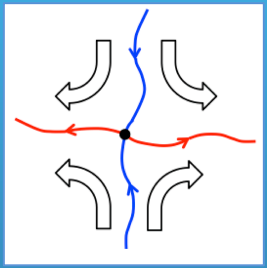
Example based on a Gulf of Mexico implementation of HYCOM, run by NRL-Stennis.

What are LCS?

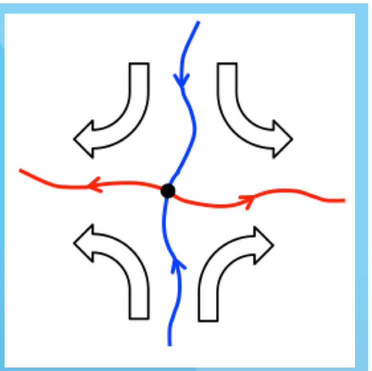
Lagrangian Coherent Structures, or *LCS*, partition a flow field into regions that undergo similar experiences. This may mean any of the following:

- Similar residence time within a region of interest
- Similar origin or fate
- Similar dispersion rates
- Etc.

Manifolds & Critical Trajectories

- Flow regions with high dispersion are often organized like this:  Such regions are termed *hyperbolic*.
- The directions along which the flow converges, marked in blue, are *inflowing manifolds*.
- The directions along which the flow diverges, marked in red, are *outflowing manifolds*.
- The manifolds intersect at a *critical trajectory*.
- Manifolds are material curves: Nothing crosses them.

Manifolds & Dispersion



- Manifolds are difficult to find exactly, even with a perfectly known velocity field.
- Instead they are approximated with diagnostics that are easier to compute.
- Note: Particles that start near the inflowing manifold will separate quickly, while those ending near the outflowing manifold have come from disparate origins.

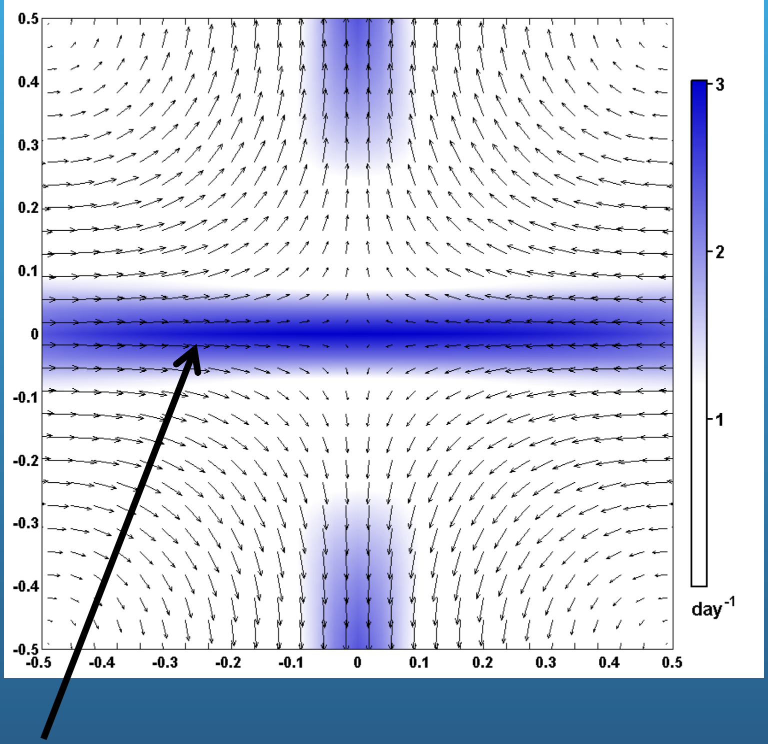
➔ *Areas around the inflowing manifold exhibit high dispersion in forward time; areas around outflowing manifolds exhibit high dispersion in backward time.*

Dispersion & Lyapunov Exponents

- Lyapunov exponents are a mathematical tool to describe dispersion characteristics. They can be used to identify LCS.
- In the context of ocean flows, which are not defined in infinite time or space, the quantities studied are typically *Finite Space Lyapunov Exponents (FSLEs)* or *Finite Time Lyapunov Exponents (FTLEs)*, also called *Direct Lyapunov Exponents (DLEs)*.
- Fundamentally, both FTLEs and FSLEs measure separation time-scales of nearby particles.

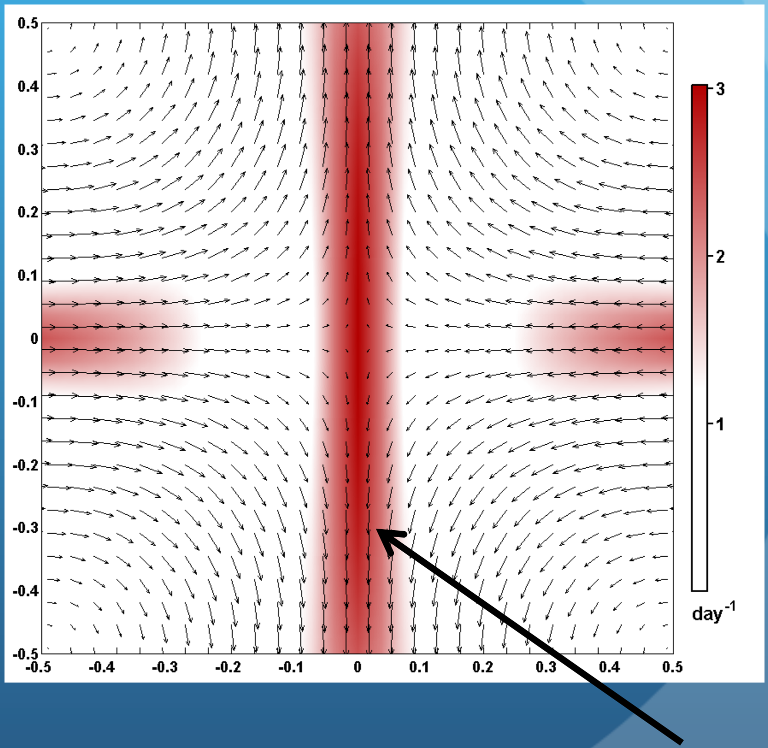
FTLEs in a Simple Stationary Flow

Forward Time FTLE



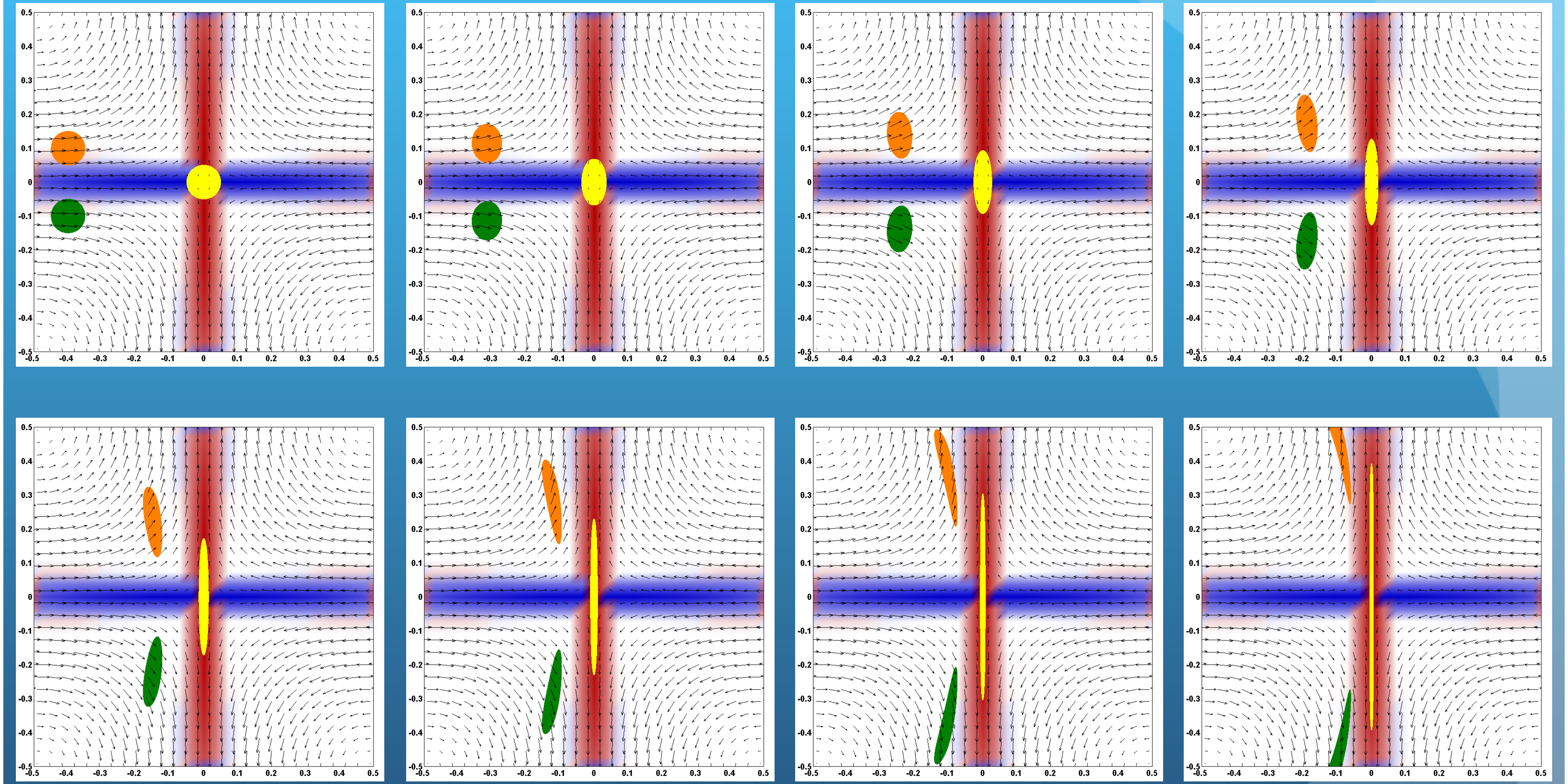
Inflowing manifold

Backward Time FTLE



Outflowing manifold

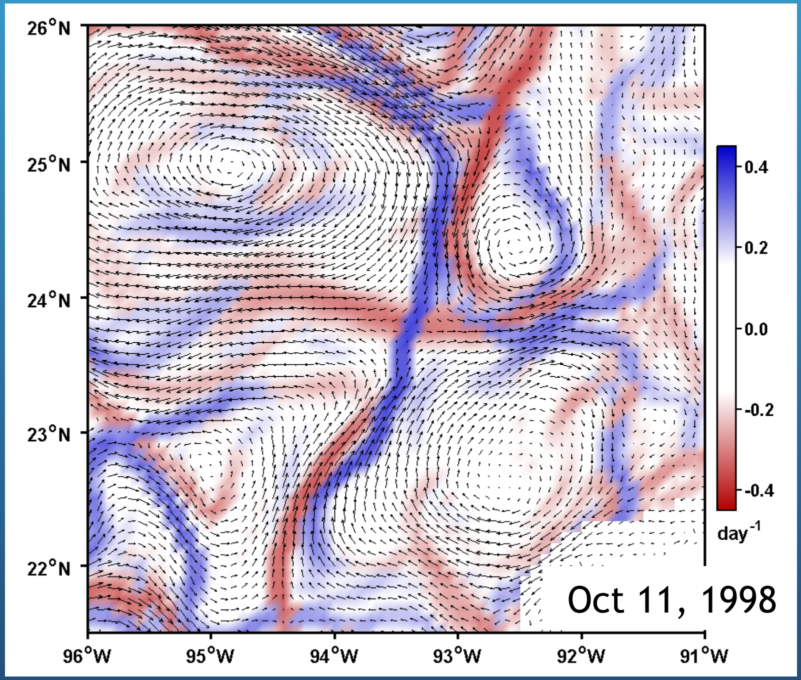
Transport near LCS



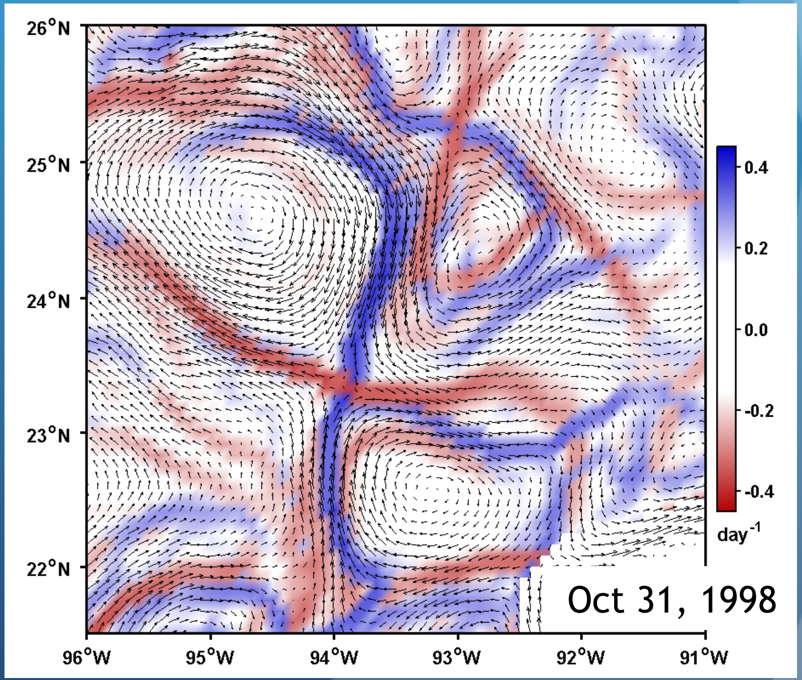
Large stretching along outflowing manifold.
Large differences in fates for nearby initializations near the inflowing manifold.

LCS in Time-Dependent Flows

- LCS given by ridges in FTLE fields still approximate manifolds and show dispersion patterns.
- Material will follow the *evolving* LCS.



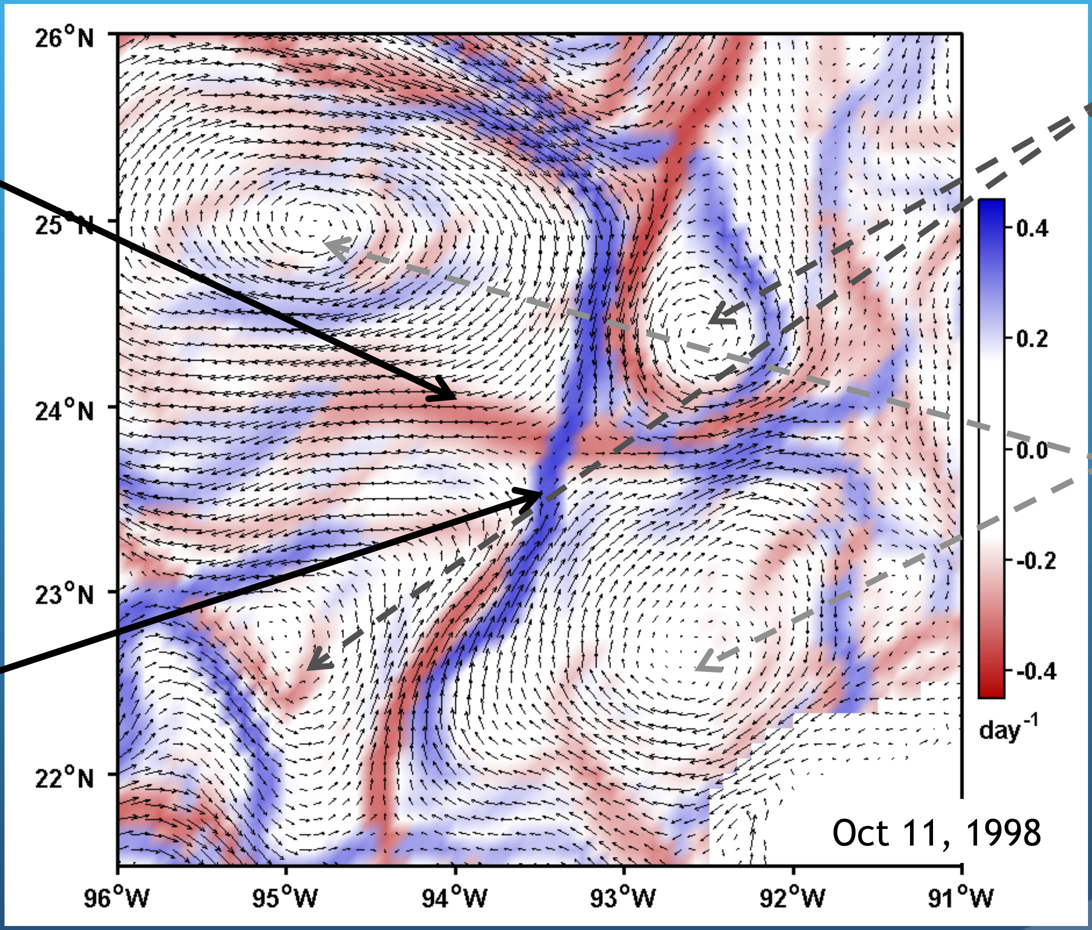
Oct 11, 1998



Oct 31, 1998

Example based on a Gulf of Mexico implementation of CUPOM, run at the University of Colorado.

Example from the Gulf of Mexico



Oct 11, 1998

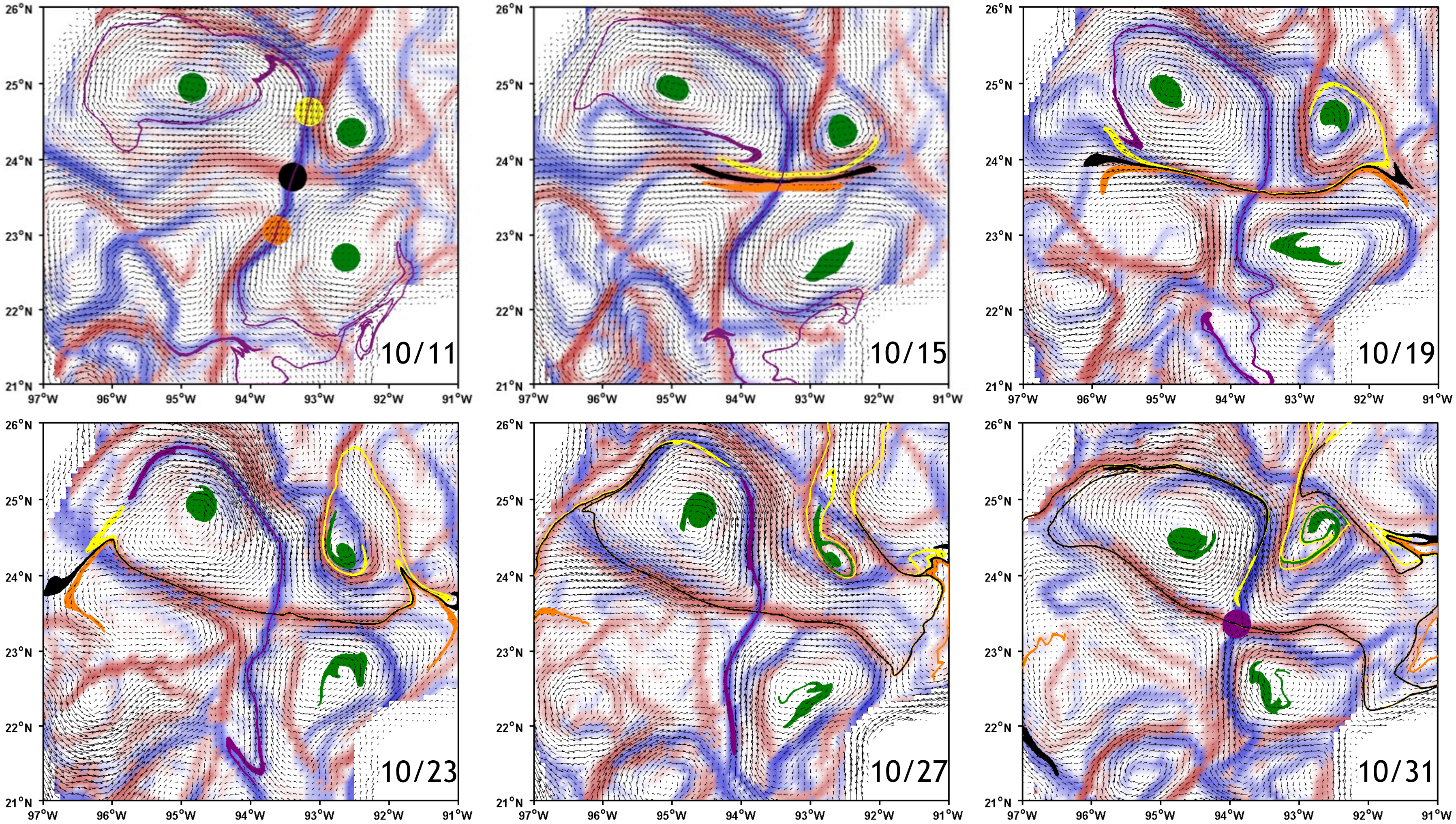
Outflowing manifold

Inflowing manifold

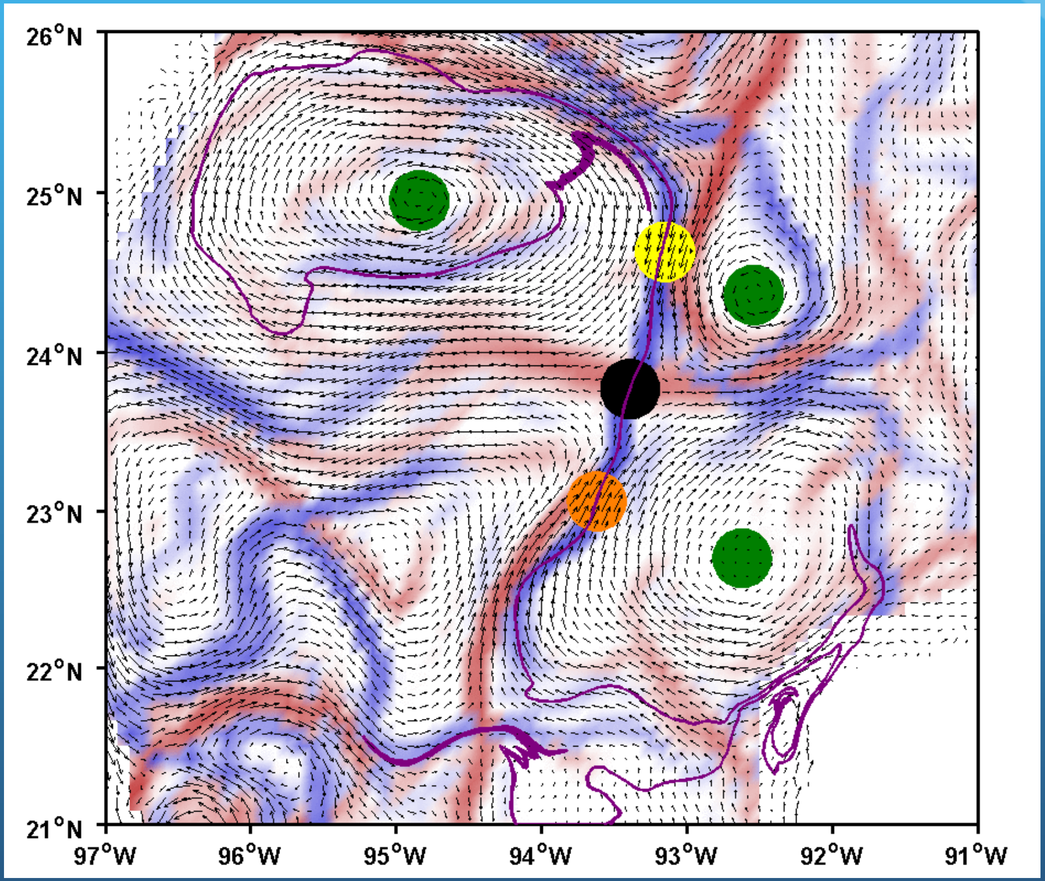
Cyclones

Anti-Cyclones

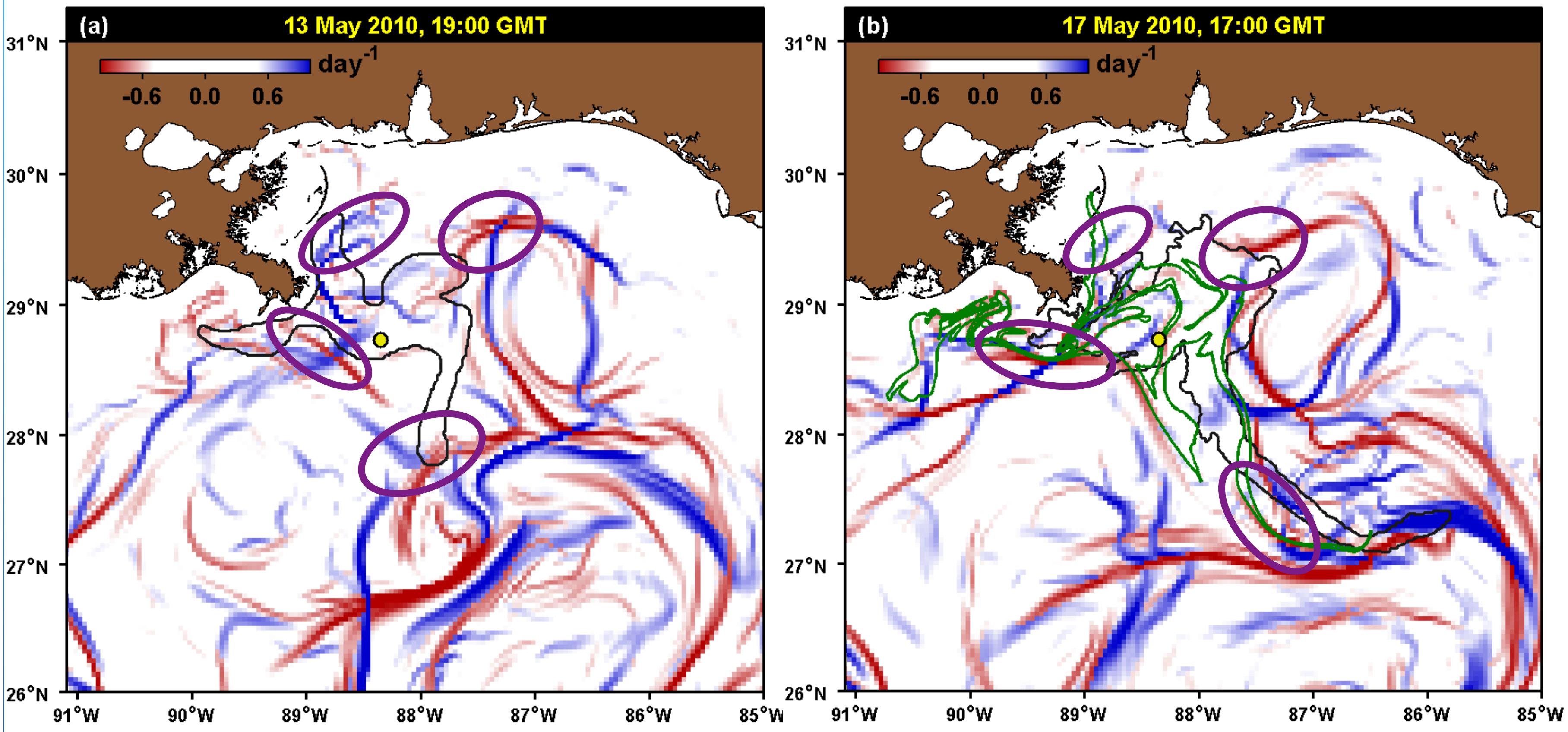
Transport & LCS in an Ocean Flow



Transport & LCS Animation



Practical Application: Oil Spill

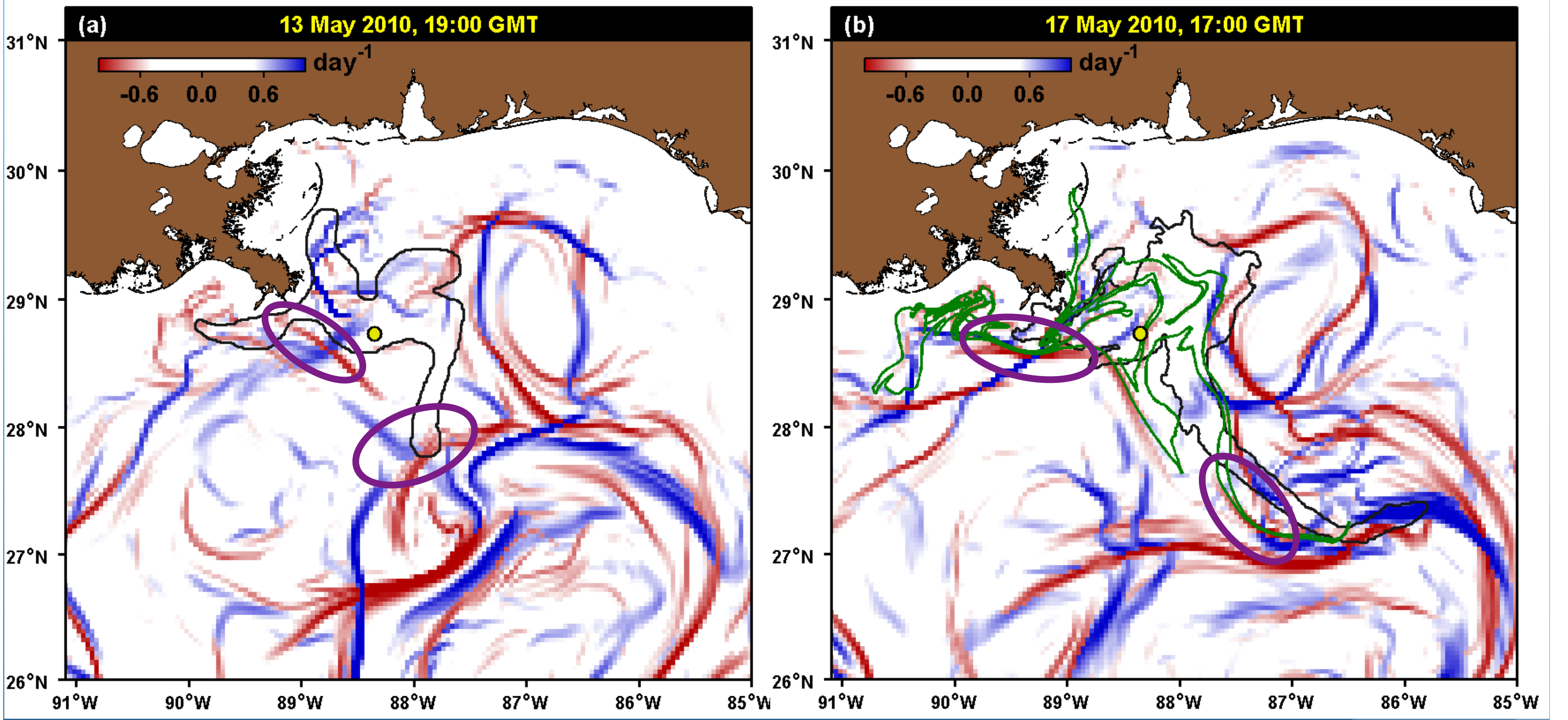


Black: Observed oil slick outline
Green: Modeled oil slick outline

Purple ovals: Hyperbolic regions,
tracked from (a) to (b)

Results from Huntley et al., 2011.

Oil Spill Example: Analysis (I)

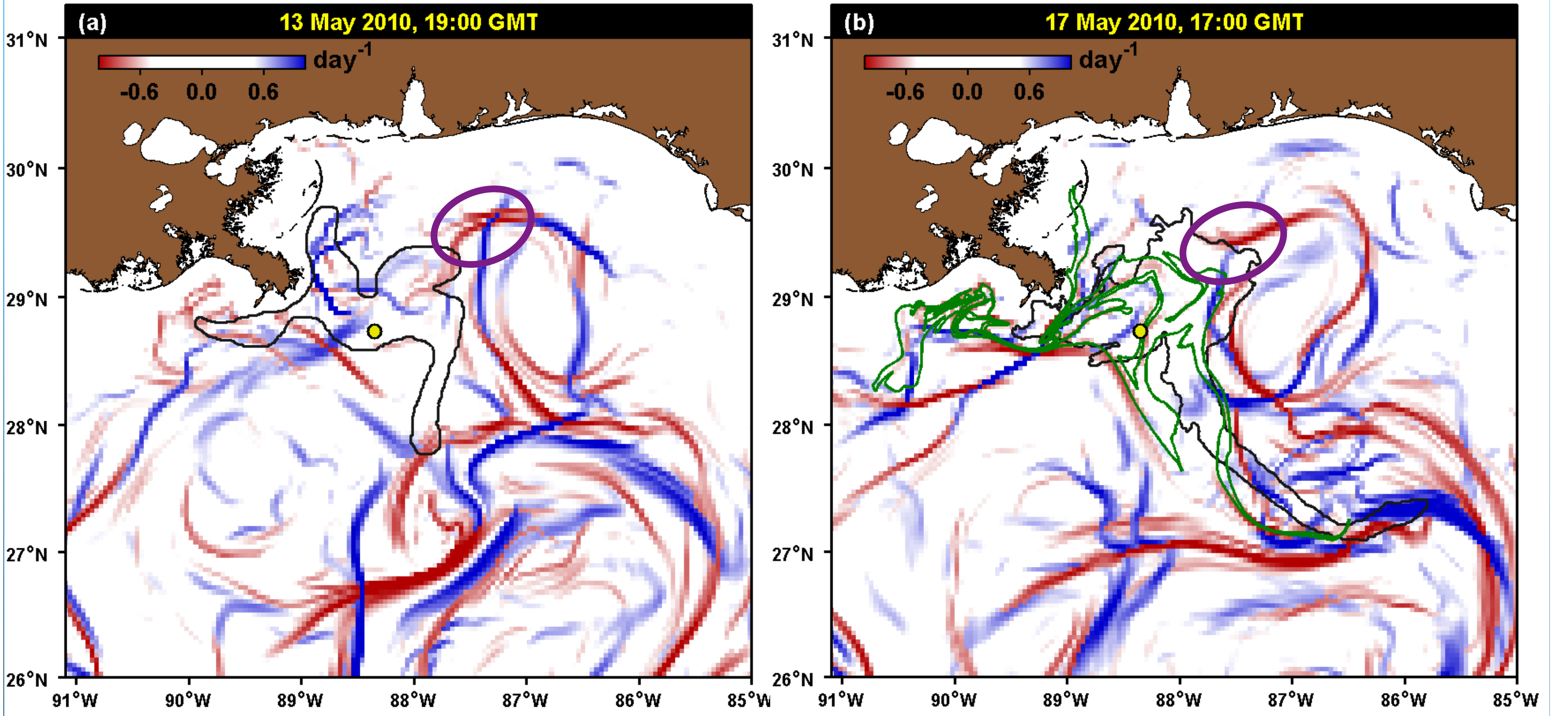


Hyperbolic regions in ovals exhibit:

Stretching along red ridge, away from intersection with blue ridge

Stretching to the SE was also observed;
thin tendrils in the NW may have evaporated or not been visible.

Oil Spill Example: Analysis (II)

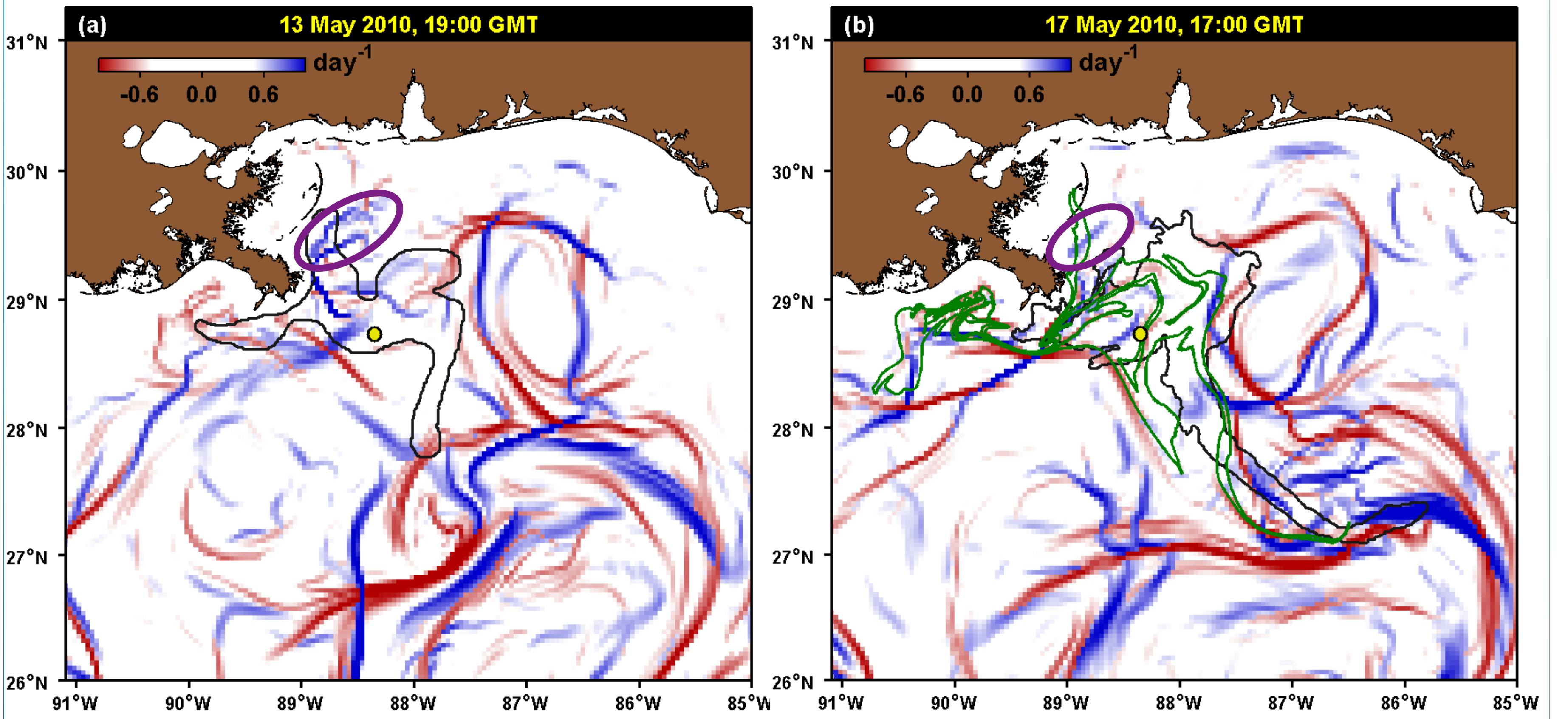


Hyperbolic region in oval exhibits:

Transport along red ridge, away from intersection with blue ridge, leading to consolidation of the oil patch

Observations agree.

Oil Spill Example: Analysis (III)



Initial oil patch crosses blue ridge

- ➔ High sensitivity to initial conditions
- ➔ High forecast uncertainty (& in this case error)

Summary

- Ridges in FTLE fields approximate manifolds.
- Forward-time FTLE ridges (inflowing manifolds) show high initial-condition sensitivity and forecast uncertainty. (Forecast uncertainty is even greater, since the FTLE map itself is based on models with their own uncertainties.)
- Backward-time FTLE ridges (outflowing manifolds) show directions of high stretching.
- Regions away from FTLE ridges are relatively quiescent.
- Edges of strong currents and eddies are typically marked by FTLE ridges.

Some Warnings

- FTLE (and FSLE) ridges are imperfect surrogates for the actual manifolds and, under certain conditions, do not align with the manifold structure and may permit material transport across them.
- For a complete picture, both forward-time and backward-time calculations are needed. Especially the former may be subject to significant model forecast errors.
- Hyperbolic regions may lose their hyperbolicity over time, so that it is not always possible to track a ridge intersection over time.

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