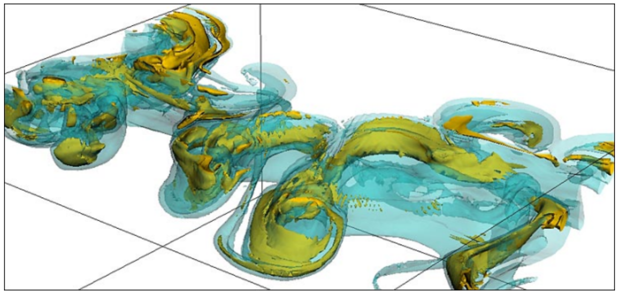
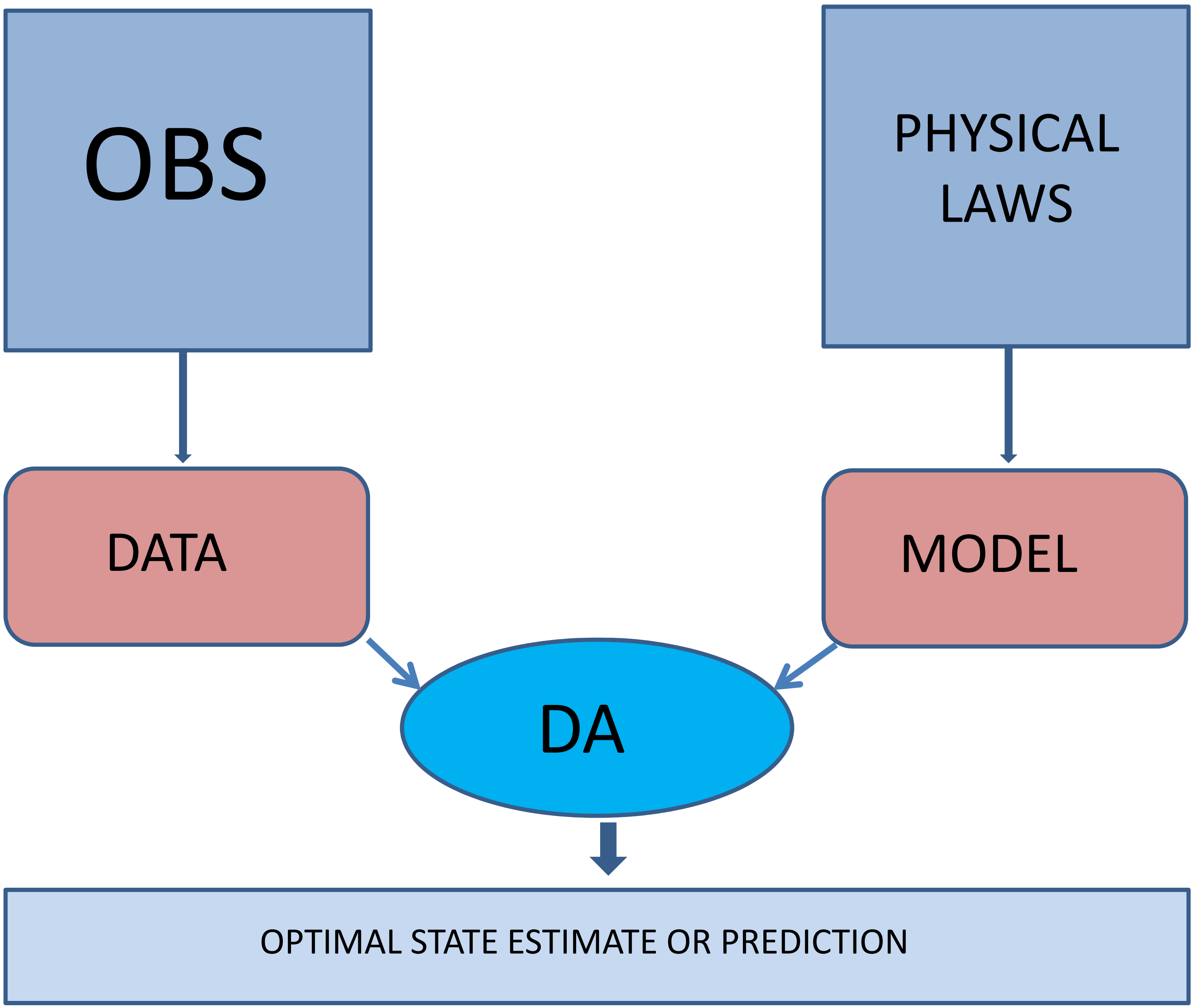
OCEAN 3D+1

Toward 3D Lagrangian DA: Issues and Overview

The DA subgroup based at UNC-CH and Marquette




OBS
PHYSICAL LAWS
DATA
MODEL
DA
OPTIMAL STATE ESTIMATE OR PREDICTION

Typical problems:

1. Estimate state at current time
2. Estimate initial condition
3. Estimate parameters

1. Sequential DA (predictive mode-filtering)

Minimize the cost-function:

$$J(x) = \left\langle x - x_t, (P_t)^{-1}(x - x_t) \right\rangle + \left\langle y - H(x_t), R^{-1}(y - H(x_t)) \right\rangle$$

P_t = background error covariance

R = observational error covariance

2. Variational DA (reanalysis mode-smoothing)

Minimize the cost-function:

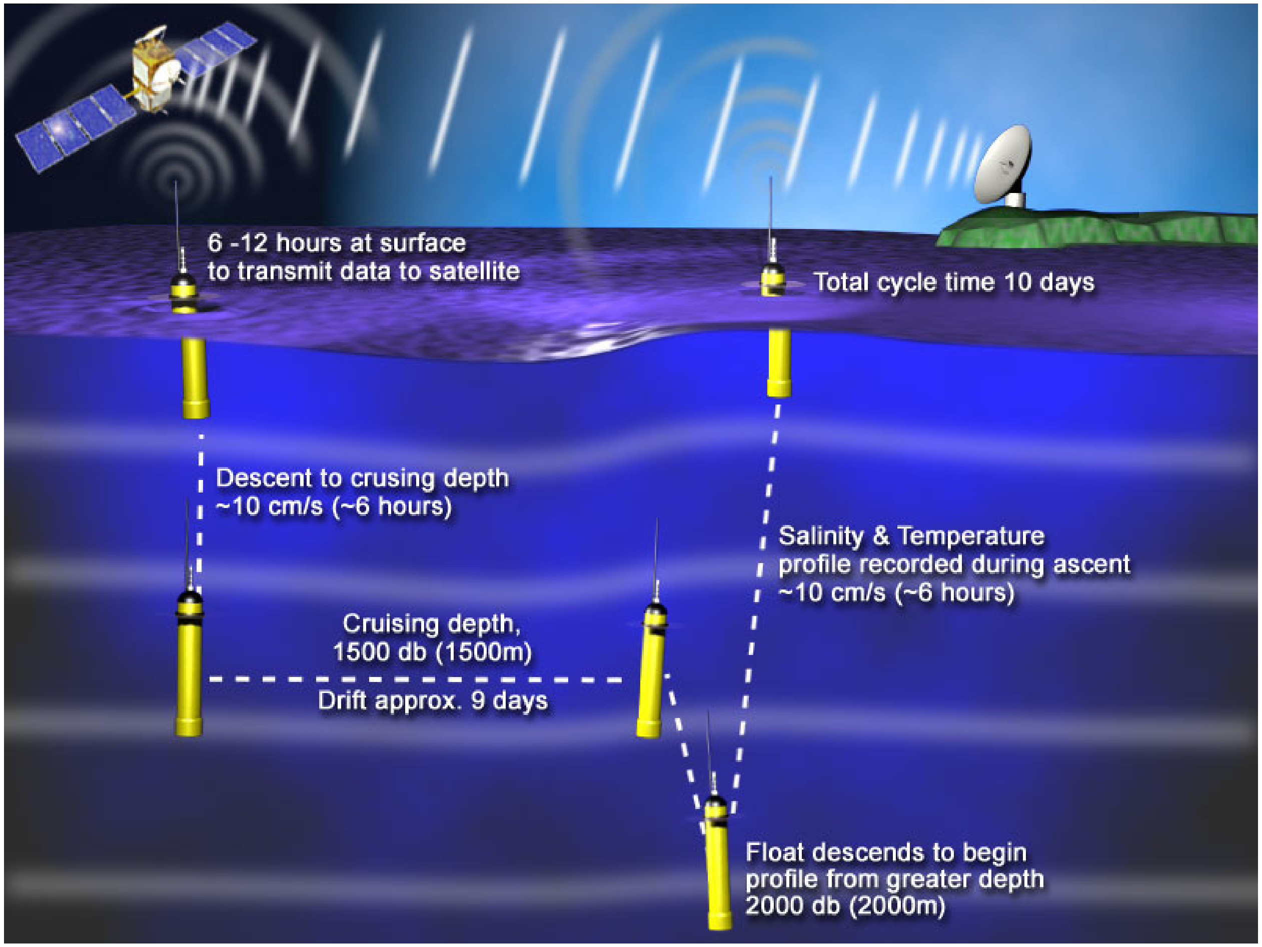
$$J(x) = \left\langle x - x_0^*, (B)^{-1}(x - x_0^*) \right\rangle + \sum_{j=1}^{N} \left\langle y_j - H(x_i), R_j^{-1}(y_j - H(x_j)) \right\rangle$$

B = background error covariance

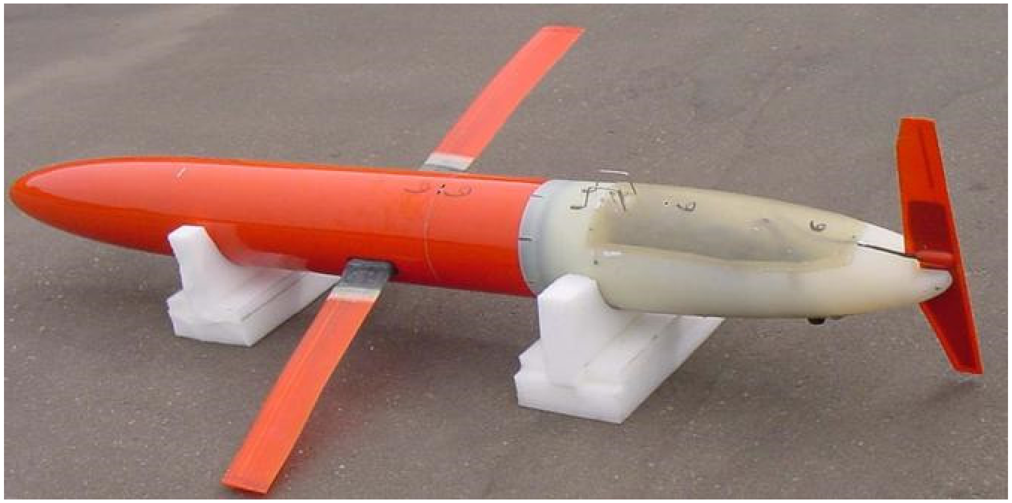
R_j = jth observational error covariance

x_0^* = initial (initial condition) estimate

ARGO Floats

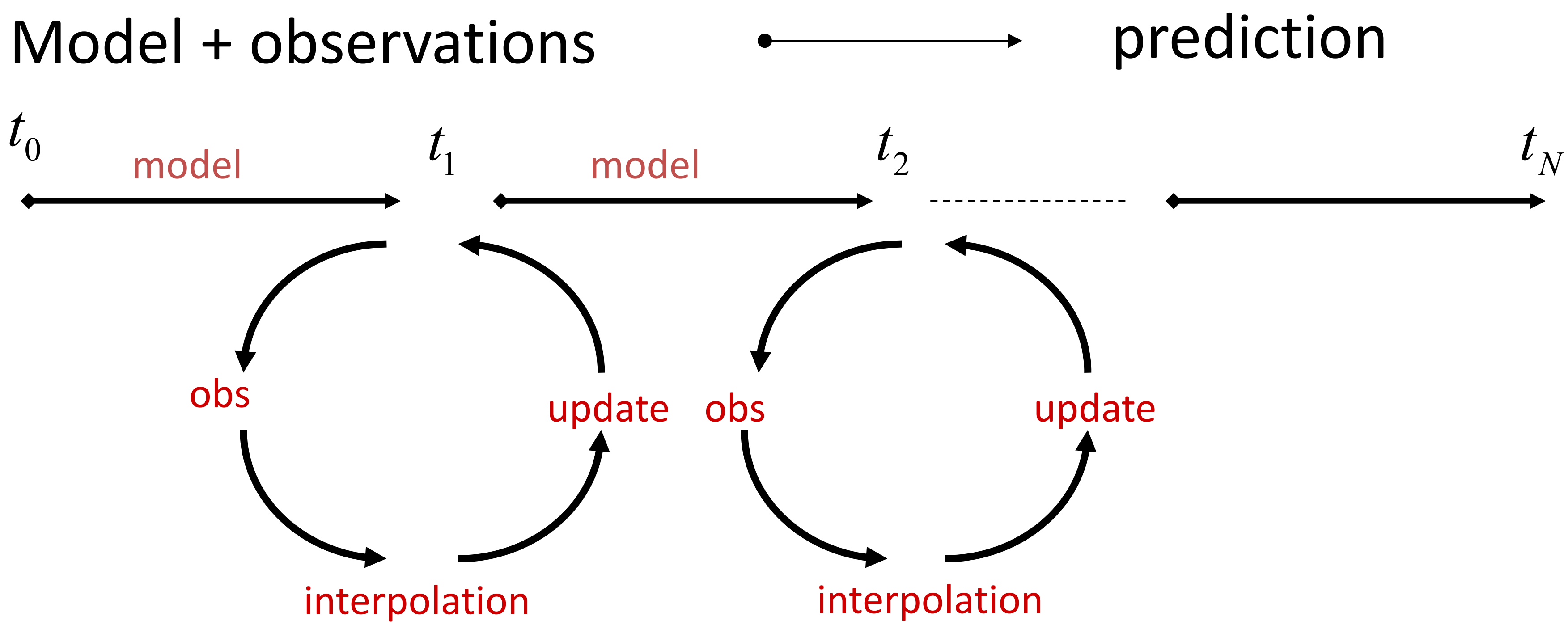

6 -12 hours at surface
to transmit data to satellite
Total cycle time 10 days
Descent to crusing depth
~10 cm/s (~6 hours)
Salinity & Temperature
profile recorded during ascent
~10 cm/s (~6 hours)
Cruising depth,
1500 db (1500m)
Drift approx. 9 days
Float descends to begin
profile from greater depth
2000 db (2000m)

Ocean gliders



://www.locean-ipsl.upmc.fr/gliders/EGO/index.php?html=./srchtml_HowAGliderWorks.html

Data Assimilation in Predictive Mode



Interpolation at $t = t_i$

$$x_i^a = x_i^f + K_i\left(\eta_i - H(x_i^f)\right)$$

Sequential DA/Forward Problem/Filtering

$$x^a = x^f + K\left(\eta - H(x^f)\right)$$

Where the matrix K is determined by the requirement that x^a minimize the cost function:

$$J(x) = \left\langle x - x^f, \left(P^f\right)^{-1}\left(x - x^f\right)\right\rangle + \left\langle \eta - H\left(x^f\right), R^{-1}\left(\eta - H\left(x^f\right)\right)\right\rangle$$

$$K = P^f H^T \left(R + HP^f H^T\right)^{-1}$$

H (linearized) observation operator

R observation error covariance matrix

P^f determined by DA scheme being used!

To Linearize or not to Linearize?

Sequential DA cost-function:

$$J(x) = \left\langle x - x_t, (P_t)^{-1}(x - x_t) \right\rangle + \left\langle y - H(x_t), R^{-1}(y - H(x_t)) \right\rangle$$

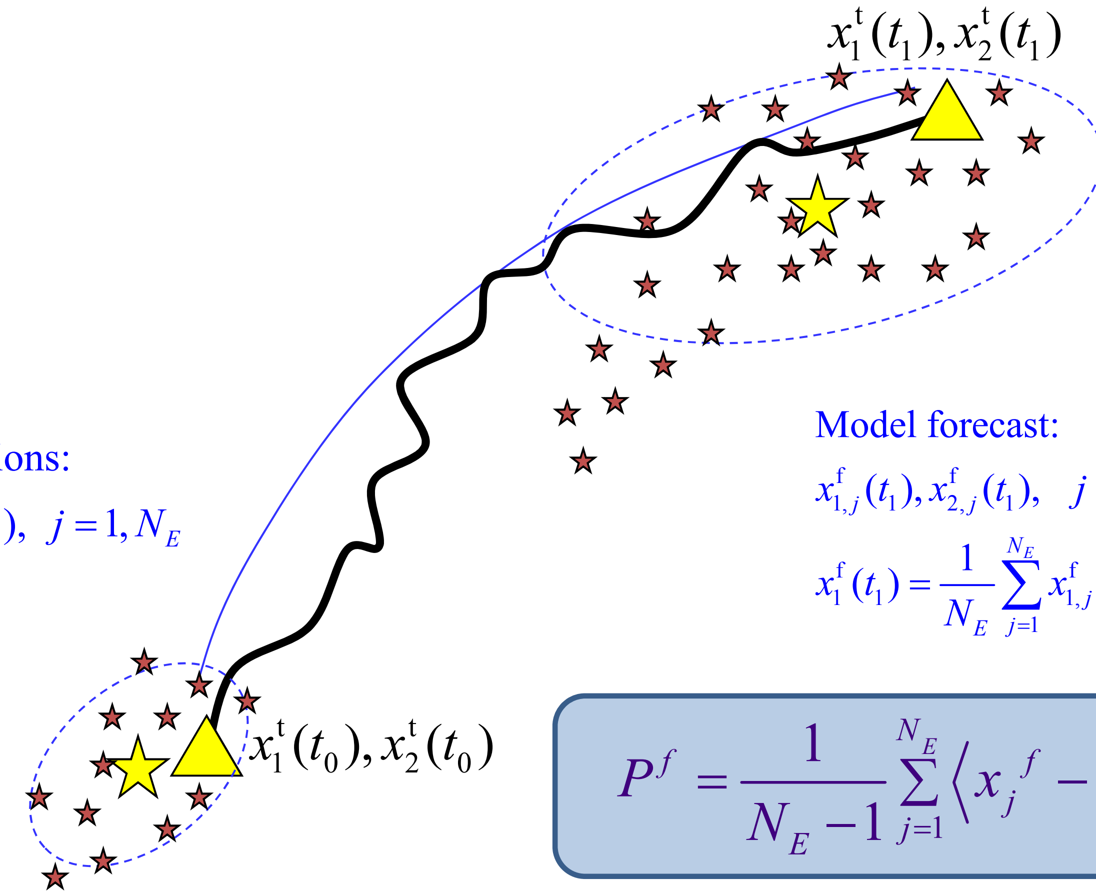
If H is linear (linearized) then cost-function is quadratic

- P^f is constant background error covariance matrix (Optimal Interpolation=OI)
- If model is linear, P^f is evolved under model (Kalman Filter=KF)
- If model is nonlinear, P^f is evolved under tangent linear model (Extended Kalman Filter=EKF)
- P^f is built out of ensembles evolved under full nonlinear model (Ensemble Kalman Filter=EnKF)

For large models, linearization is enacted at some level

Ensemble Kalman Filter (EnKF)

Error covariance is predicted via solution of full nonlinear system for a Monte-Carlo ensemble of states



Initial conditions:

$x_{1,j}^{\mathrm{f}}(t_0), x_{2,j}^{\mathrm{f}}(t_0), \quad j = 1, N_E$

Model forecast:

$x_{1,j}^{\mathrm{f}}(t_1), x_{2,j}^{\mathrm{f}}(t_1), \quad j = 1, N_E$

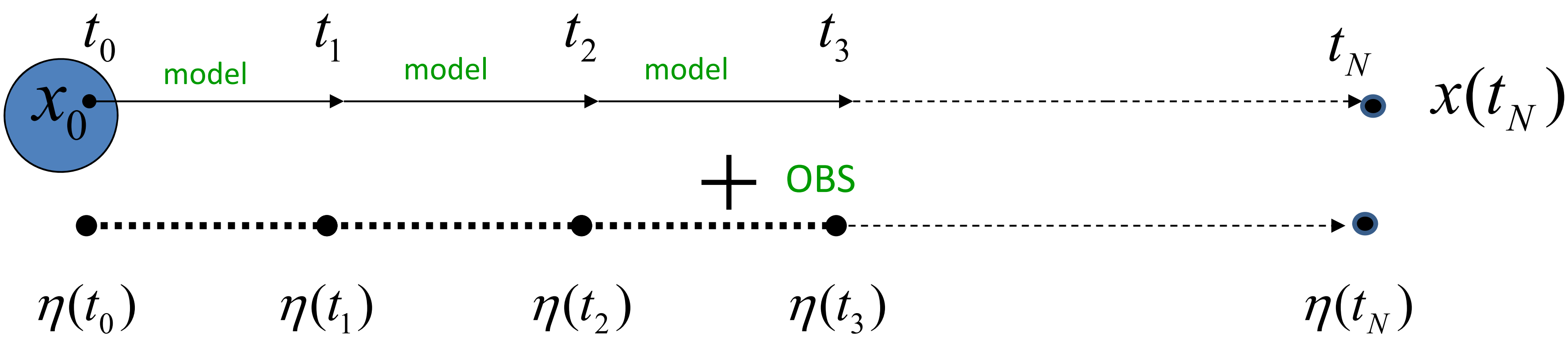
$$x_1^{\mathrm{f}}(t_1) = \frac{1}{N_E}\sum_{j=1}^{N_E} x_{1,j}^{\mathrm{f}}(t_1), \quad x_2^{\mathrm{f}}(t_1) = \frac{1}{N_E}\sum_{j=1}^{N_E} x_{2,j}^{\mathrm{f}}(t_1)$$

$$P^f = \frac{1}{N_E - 1}\sum_{j=1}^{N_E}\left\langle x_j^{\,f} - x^f, x_j^{\,f} - x^f \right\rangle$$

DA in State Estimation Mode

Model run + observations ⟶ state estimate

$t = t_1, \ldots, t_N$



4DVAR: Minimize the cost function:

$$J(x) = \left\langle x_0 - x_0^*, B^{-1}\left(x_0 - x_0^*\right)\right\rangle + \sum_{1}^{N} \left\langle \eta_j - H(x(t_j)), R_j^{-1}\left(\eta_j - H(x(t_j))\right)\right\rangle$$

$x_0^* = \text{estimate}$ $\qquad$ $B = \text{background error covariance matrix}$

Variational DA/Inverse Problem/Smoothing

Bayes Theorem

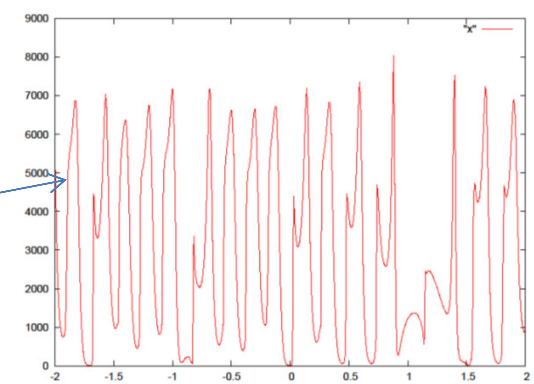
If not linear(ized), there may be multiple minima for the cost-function! Is the global minimizer necessarily the desired answer?

$$P^{\text{posterior}}(x|y) \propto P^{\text{obs}}(y|x)\,P^{\text{prior}}(x)$$

$$P(x\,|\,y) \propto \exp(-J(x))$$

mode ←→ global min

A log posterior of the Lorenz '63 system (courtesy of Jochen Voss)



See: Data Assimilation: Mathematical and Statistical Perspectives, Apte, J, Stuart and Voss, IJNMF 2008

$$P^{\text{posterior}}(x|y) \propto P^{\text{obs}}(y|x)\,P^{\text{prior}}(x)$$

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Sampling strategies:

1. Particle filtering
2. Langevin sampling
3. Metropolis-Hastings
4. Importance sampling

But none are well developed for high-dimensional problems

Augmented system

Append equations for drifters (floats)

$$\mathbf{x} = \begin{pmatrix} \mathbf{x}_F \\ \mathbf{x}_D \end{pmatrix} \qquad \text{-- augmented state vector}$$

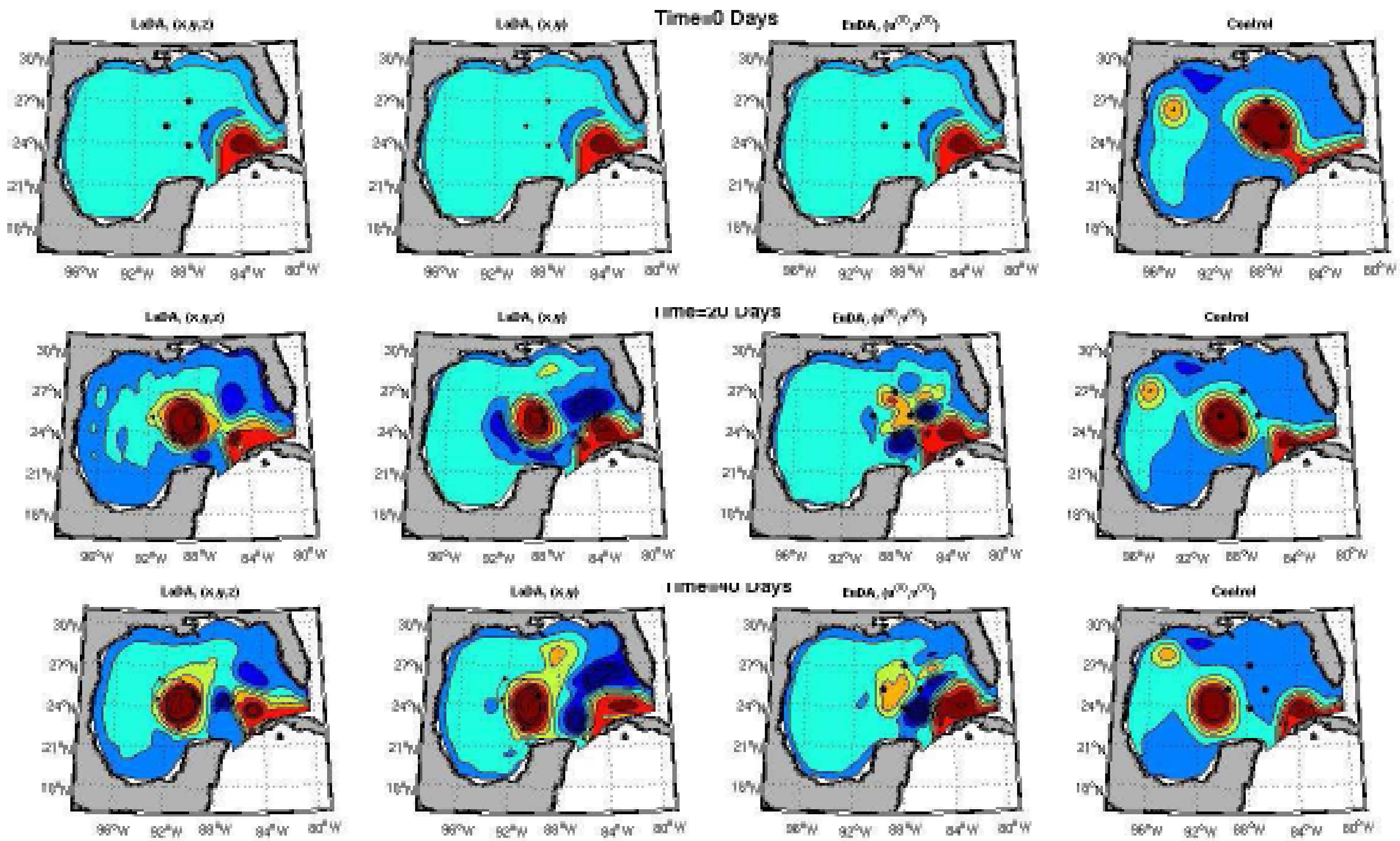
$$\frac{d\mathbf{x}_F^{\mathrm{f}}}{dt} = M_F(\mathbf{x}_F^{\mathrm{f}}, t) \qquad \text{-- flow equations}$$

$$\frac{d\mathbf{x}_D^{\mathrm{f}}}{dt} = M_D(\mathbf{x}_D^{\mathrm{f}}, \mathbf{x}_F^{\mathrm{f}}, t) \qquad \text{-- tracer advection equation}$$

Apply filtering to augmented system

Ide, Jones and Kuznetsov (2002)

Eddies in GoM



Work with Guillaume Vernieres (NASA) and Kayo Ide (MD)

Why does LaDA work so well?

- Lagrangian information content
- Lagrangian structures and their significance
- Low dimensionality of space carrying Lagrangian flow
- Observations in low-dimensional space
- Focusing of nonlinearity into low dimensional subspace

Issues to resolve

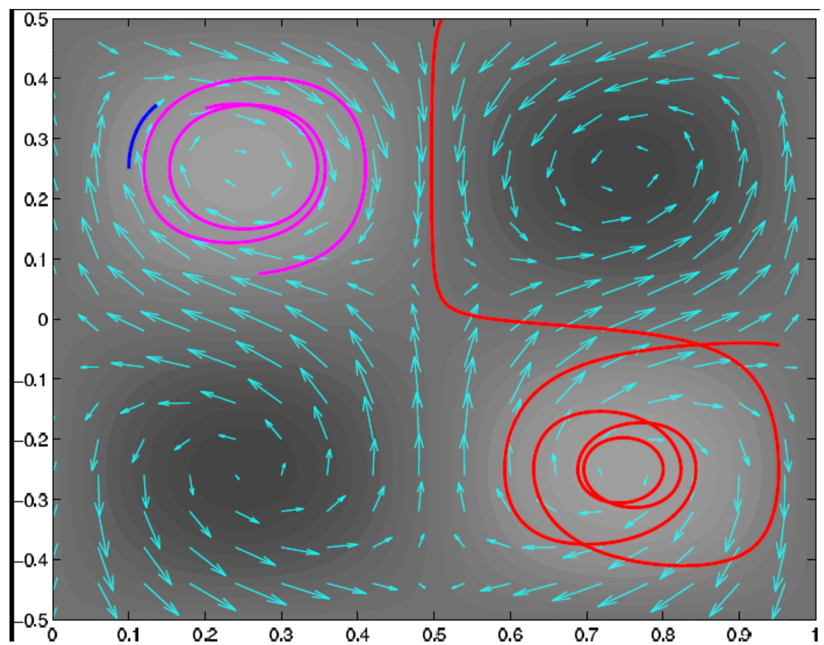
1. Statistical technique that deals with nonlinearity: Amit and C. (Elaine)
2. Estimating position information between surfacings: Elaine, Amit and C.
3. Information transfer between levels: Nara, Elaine, Regis and C. (based on thesis work by Liu @ UNC-CH)
4. Control and DA communication: Damon and C.

Perturbed Cellular Flow Field

$$\frac{\partial u}{\partial t} = v - \frac{\partial h}{\partial x},$$
$$\frac{\partial v}{\partial t} = -u - \frac{\partial h}{\partial y},$$
$$\frac{\partial h}{\partial t} = -\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y},$$

$$u(x,y,t) = -2\pi l \sin(2\pi kx)\cos(2\pi ly)u_0 + \cos(2\pi my)u_1(t),$$
$$v(x,y,t) = 2\pi k \cos(2\pi kx)\sin(2\pi ly)u_0 + \cos(2\pi my)v_1(t),$$
$$h(x,y,t) = \sin(2\pi kx)\sin(2\pi ly)u_0 + \sin(2\pi my)h_1(t),$$

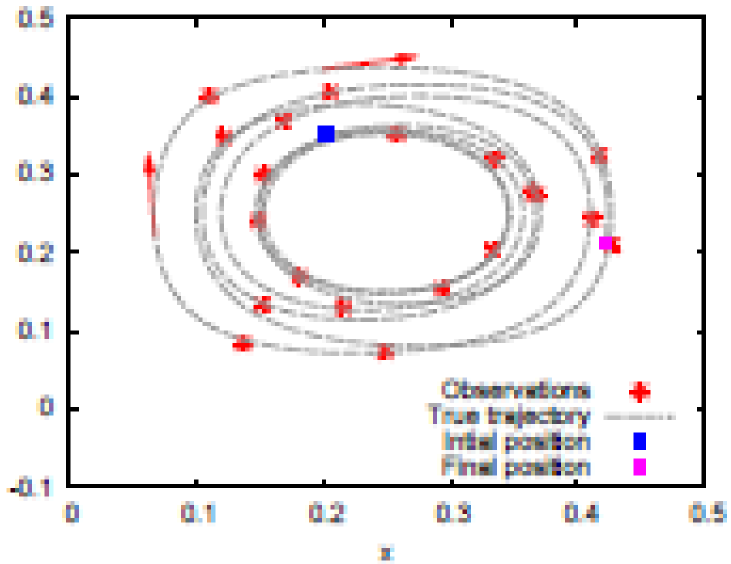
$$\dot{u_0} = 0,$$
$$\dot{u_1} = v_1,$$
$$\dot{v_1} = -u_1 - 2\pi m h_1,$$
$$\dot{h_1} = 2\pi m v_1,$$

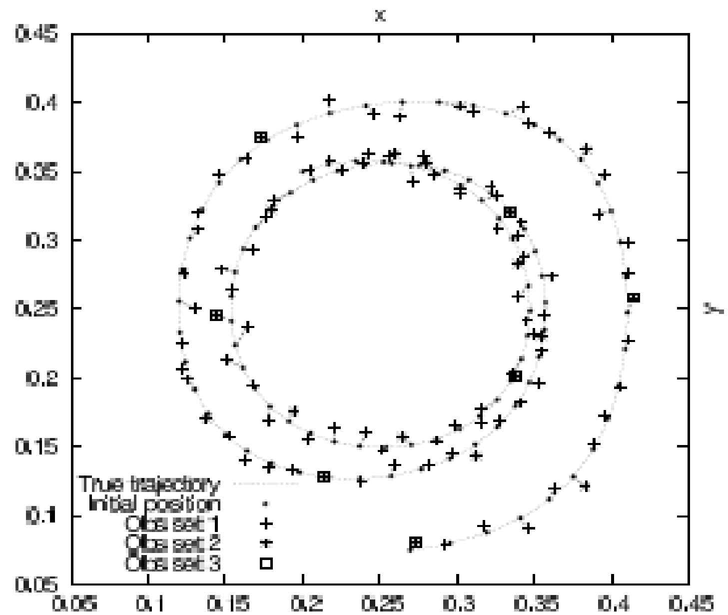


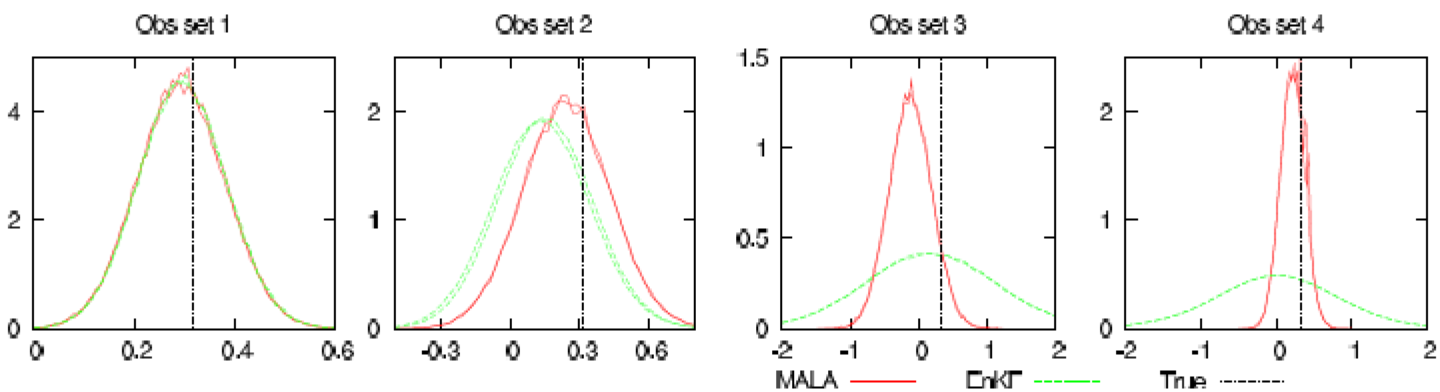
Apte, J and Stuart Tellus A 2008
Apte and J. (in progress)

Assimilating from trajectory staying in one cell

Expt: estimate i.c. from observations of trajectory

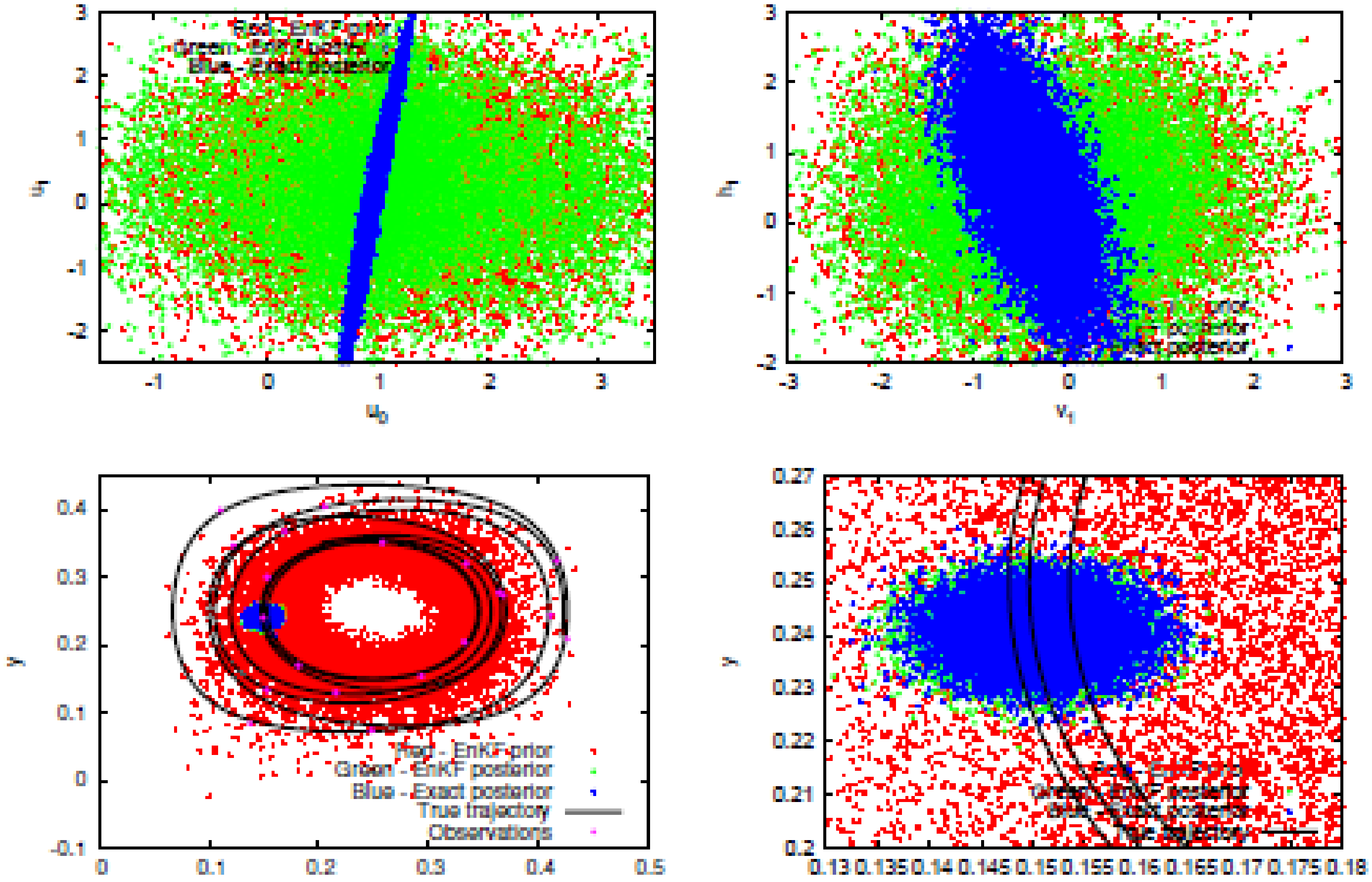






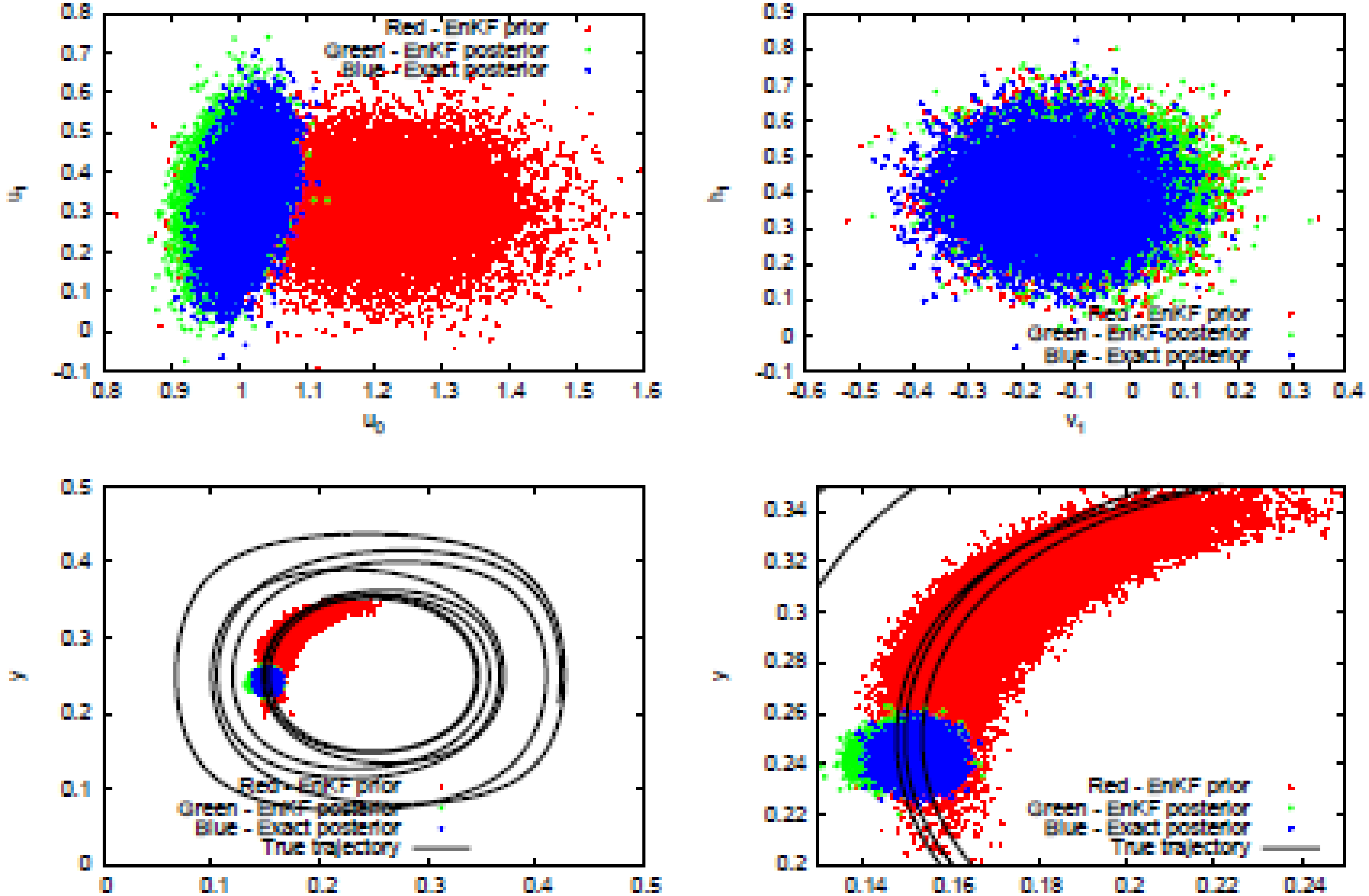
Nonlinear shear around center $\Longrightarrow$ non-Gaussianity

- Distribution functions prior and posterior to 2nd observation



Decreasing prior covariance $\Longrightarrow$ weaker non-Gaussianity

1/10 the prior covariance as compared to previous case



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