3D+1 Transport Processes in Ocean Models

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Outline

Approximating 3D FTLEs using 2D velocities

- FTLE theory
- Error assessment in idealized models: ABC flow and a quadrupole
- Error assessment in an ocean model: GOM HYCOM

Assessment of trajectory and FTLE uncertainties in a 16-member GOM NCOM ensemble

3D ocean process studies

- Loop Current Ring formation in the Gulf of Mexico
- Transport boundaries near a GoM quadrupole

Lagrangian Coherent Structures

Defined by Haller (2000) in terms of FTLE (used here)

Other LCS characterizations

FSLE	Joseph and Legras (2002)
Minimal trajectories	Mancho and Mendoza (2010)
Mesohyperbolicity	Mezic et al. (2010)
Geodesic surfaces	Haller (2011)
Complexity methods	Rypina (2011)

Most GFD studies restricted to 2D velocities

Theory applies to R^n

Are LCS important in GFD?

MODE/POLYMODE (circa 1975): Mesoscale eddies transport heat, salinity, and momentum. Yet.....

- How do eddies form?
- How many eddies are there?
- How do eddies exchange heat, etc. with environment?

Since MODE/POLYMODE

- Growing Lagrangian user community
- Dramatic oil spills
- Increased model resolution exposed energetic submesoscale

Little Compton meeting (circa 1990): DST methods applied to 2D mesoscale and submesoscale transport

To date: Analyses focus only on kinematic descriptions

FTLE Theory

$$\mathbf{S} = \frac{\partial \mathbf{x}}{\partial \mathbf{x}_0}$$

Strain tensor

$$\mathbf{C} = \mathbf{S}^T \mathbf{S}$$

Cauchy-Green tensor

$$\mathrm{FTLE} = \frac{\log \sqrt{\lambda_{max}(\mathbf{C})}}{t - t_0}$$

Finite-time Lyapunov exponent

$$3D : \begin{bmatrix} \frac{\partial x}{\partial x_0} & \frac{\partial x}{\partial y_0} & \frac{\partial x}{\partial z_0} \\ \frac{\partial y}{\partial x_0} & \frac{\partial y}{\partial y_0} & \frac{\partial y}{\partial z_0} \\ \frac{\partial z}{\partial x_0} & \frac{\partial z}{\partial y_0} & \frac{\partial z}{\partial z_0} \end{bmatrix}$$

$$3D2D : \begin{bmatrix} \frac{\partial x}{\partial x_0} & \frac{\partial x}{\partial y_0} & \frac{\partial x}{\partial z_0} \\ \frac{\partial y}{\partial x_0} & \frac{\partial y}{\partial y_0} & \frac{\partial y}{\partial z_0} \\ 0 & 0 & 1 \end{bmatrix}$$

$$2D : \begin{bmatrix} \frac{\partial x}{\partial x_0} & \frac{\partial x}{\partial y_0} \\ \frac{\partial y}{\partial x_0} & \frac{\partial y}{\partial y_0} \end{bmatrix}$$

Three strain tensor forms

FTLE Diagnostics

$$S_w = \sqrt{\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2}$$

Gradients of w
3D only

$$S_v = \sqrt{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2}$$

Vertical gradients of (u,v)
3D and 3D2D

Idealized Models: Approximating 3D FTLEs using 2D Velocities

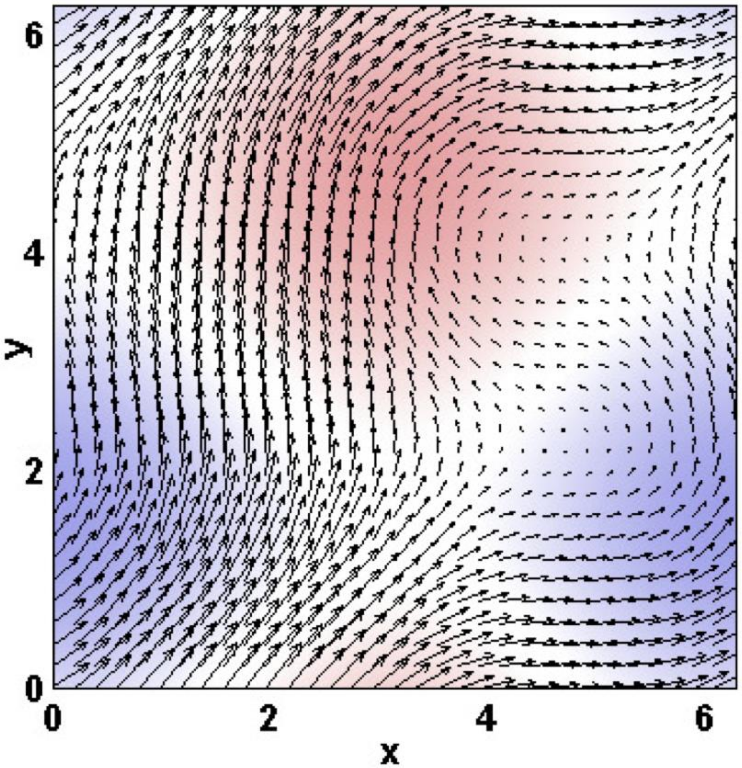
ABC Flow and a Simple Quadrupole

Two approximate FTLEs (3D and 2D) computed from 2D trajectories are compared with full 3D FTLE from 3D trajectories.

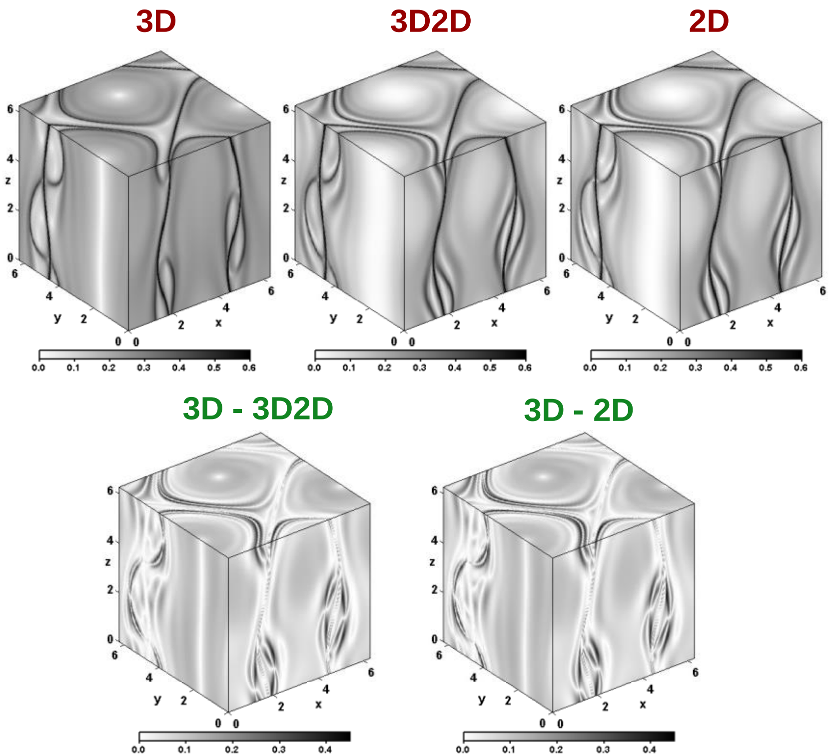
Two diagnostics are developed.

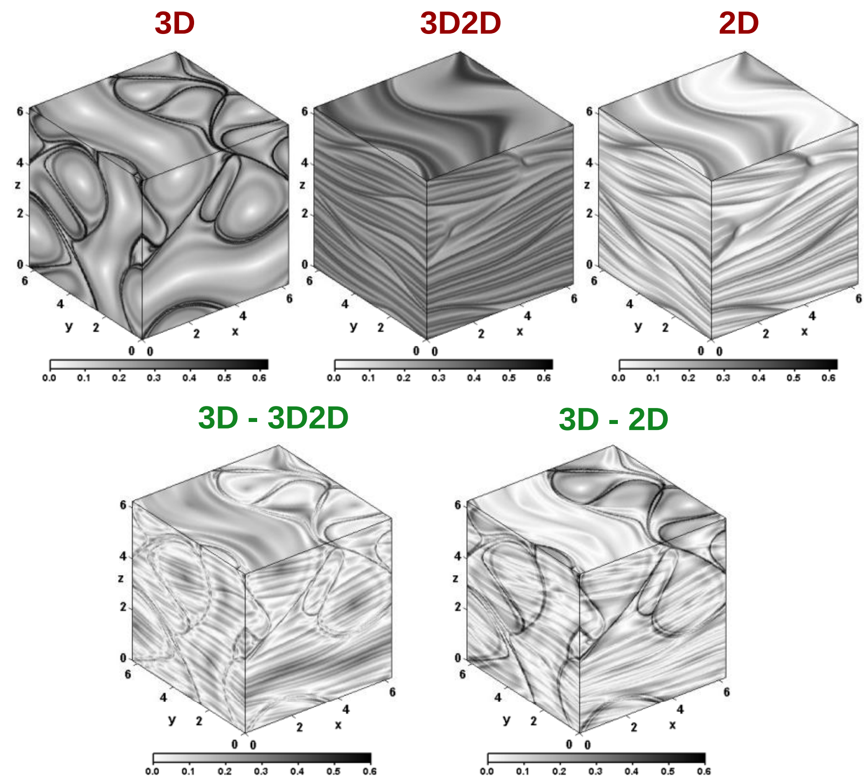
ABC Flow

$$u = A\sin(z) + C\cos(y)$$
$$v = B\sin(x) + A\cos(z)$$
$$w = C\sin(y) + B\cos(x)$$



ABC Flow: FTLE in a 3D Cube



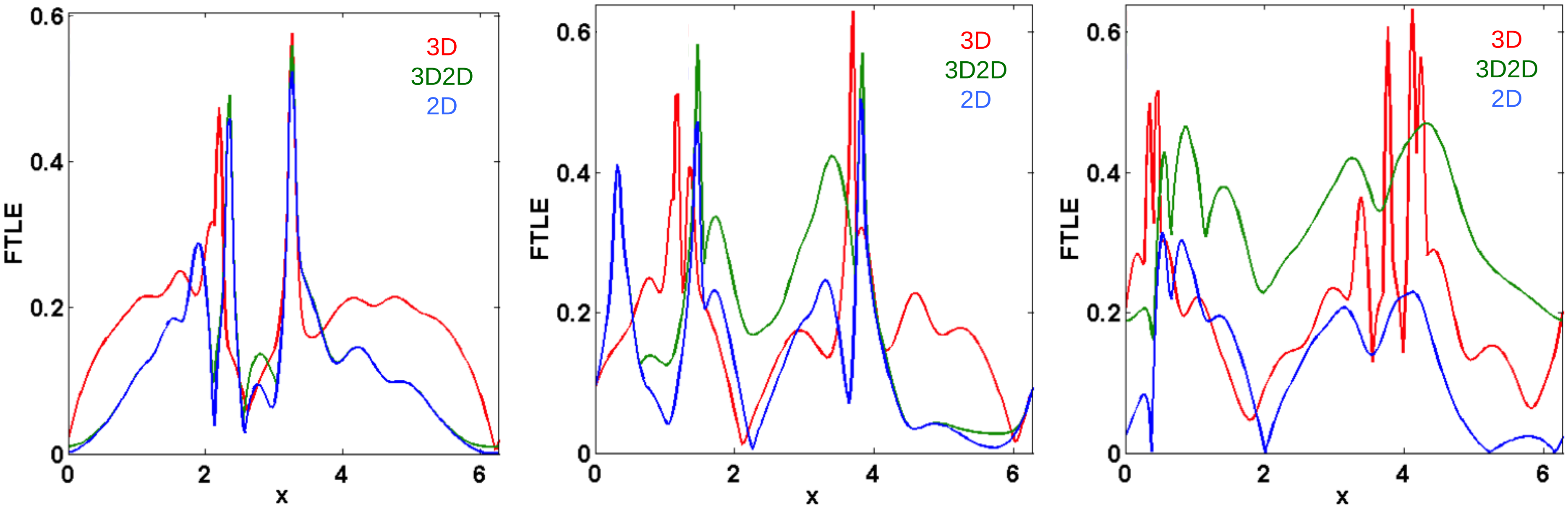


Weak S_v: Small errors

Strong S_v: Large errors

Sulman et al. (2013, under revision)

ABC Flow: FTLE Cross-Sections



$S_v = 0.1$ | $S_v = 0.5$ | $S_v = 1.1$

Ridge position errors increase with increasing S_v

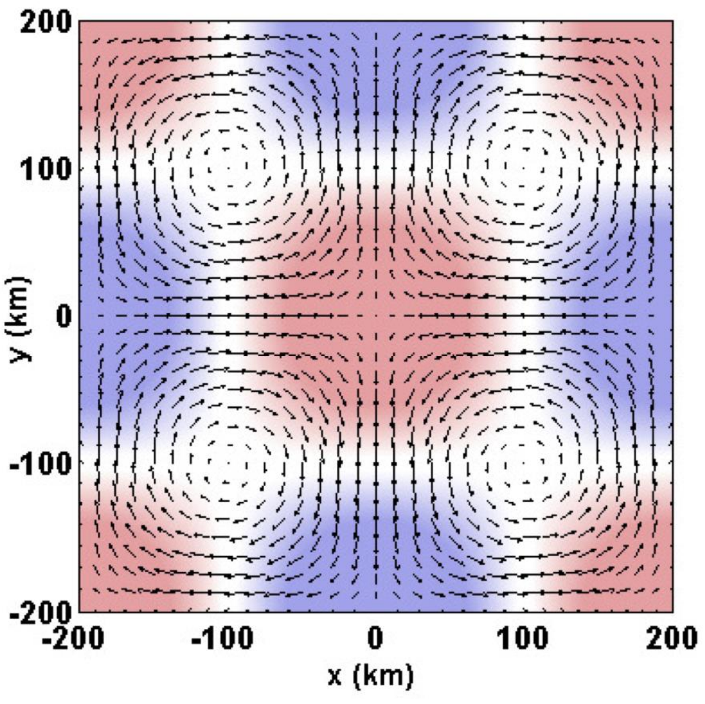
Sulman et al. (2013, under revision)

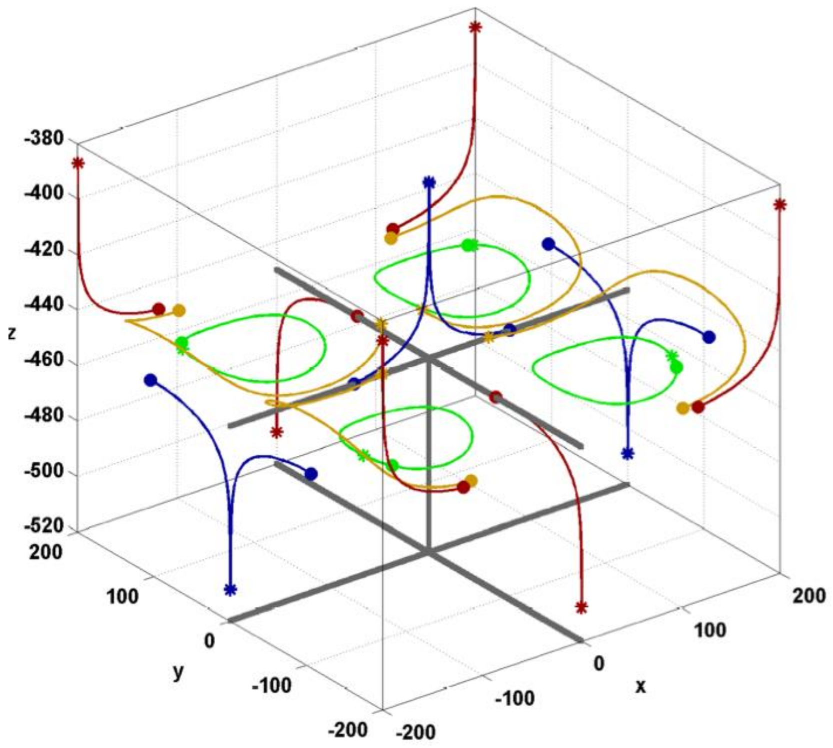
Simple Quadrupole

$$u = -k\,[A(z) + B_z(z)]\,\sin kx \cos ky$$

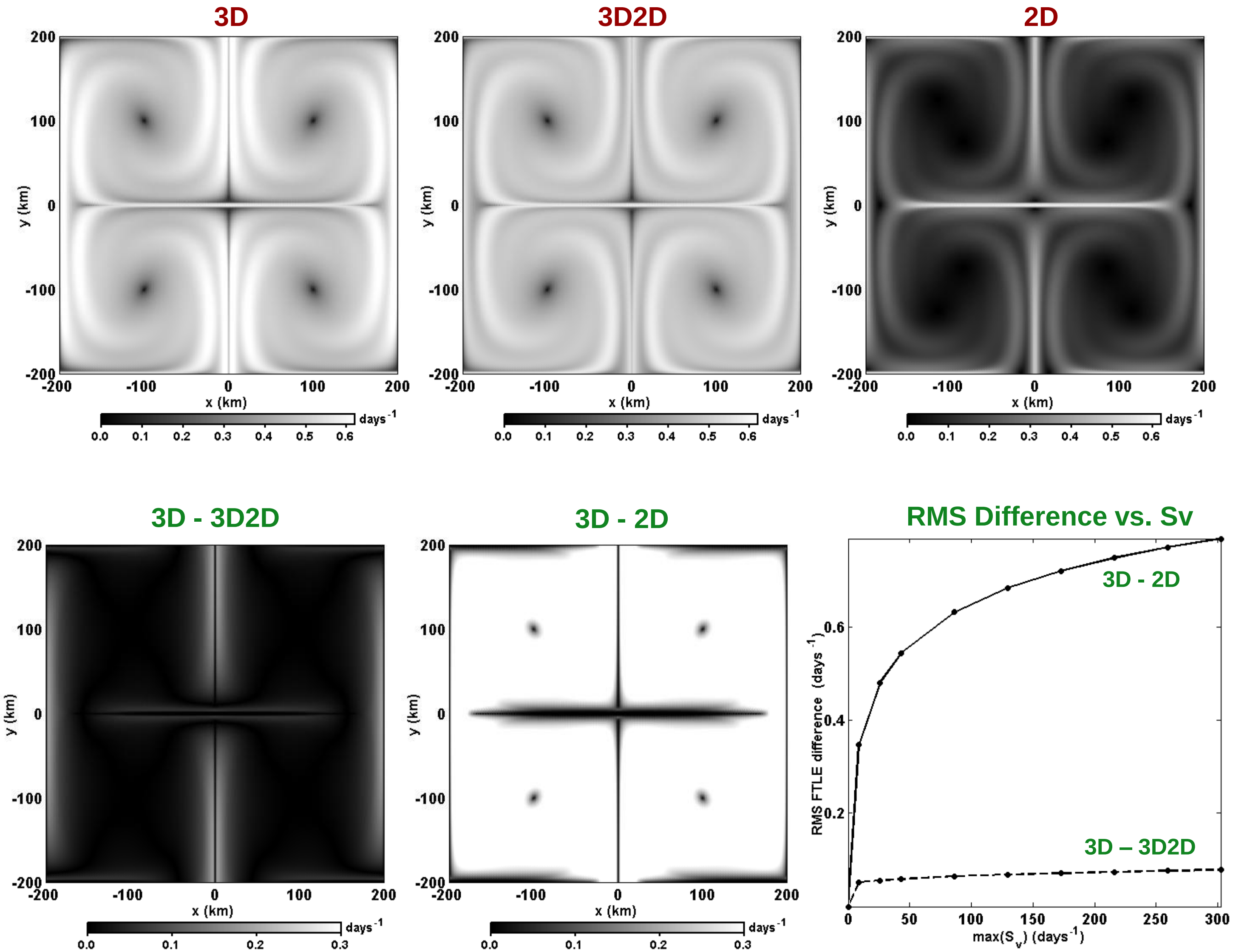
$$v = k\,[A(z) - B_z(z)]\,\cos kx \sin ky$$

$$w = 2k^2 B_z(z) \cos kx \cos ky$$





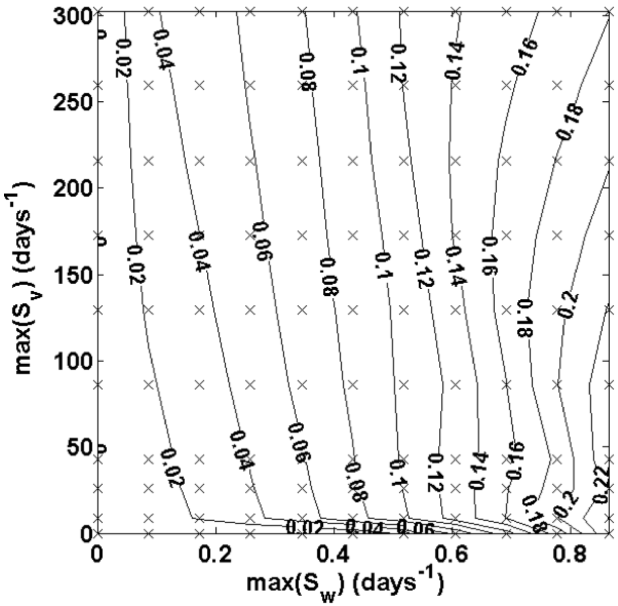
Quadrupole: FTLE at One Depth

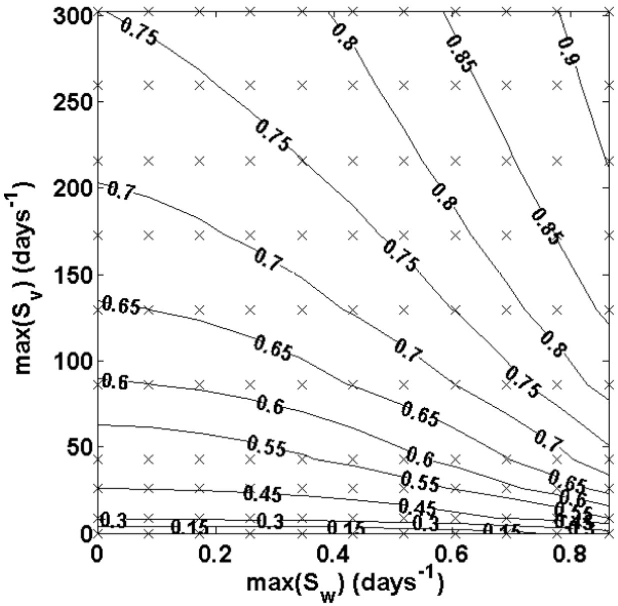


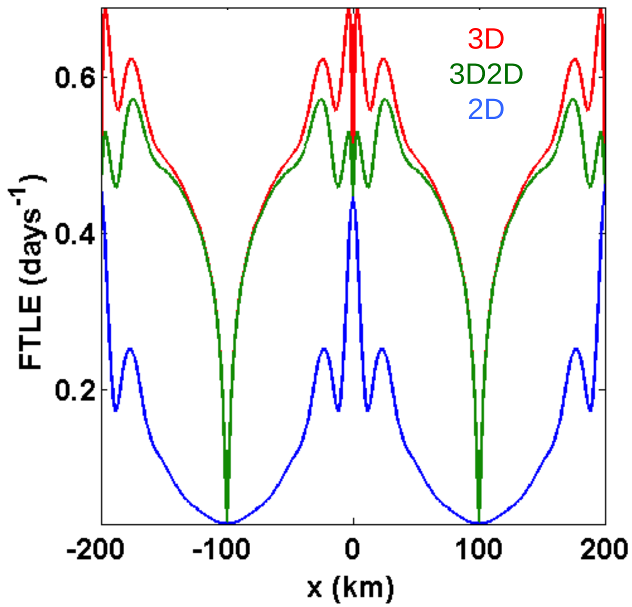
Sulman et al. (2013, under revision)

QP

Quadrupole







$FTLE_{3D} – FTLE_{3D2D}$

Errors increase as Sw increases

Little influence of increasing Sv

$FTLE_{3D} – FTLE_{2D}$

Sv effects dominate

Sw effects negligible for small Sv

FTLE cross sections

Ridge positions are "certain"

Sulman et al. (2013, under revision)

Learned from Idealized Models

- Expanded strain tensor (3D2D) includes dispersion due to vertical (u,v) gradients.
- S_v is an important ocean diagnostic.
- **ABC flow**: Moderate to large S_v prevents accurate location of FTLE ridges.
- **Quadrupole**: Both approximate FTLEs yield LCS consistent with "truth".

For mesoscale ocean flows, vertical shear is constrained by

Richardson criterion: $N^2/S_v^2 > 0.25$

Ocean model FTLE approximations will therefore likely prove useful.

GOM HYCOM: Approximating 3D FTLEs using 2D Velocities

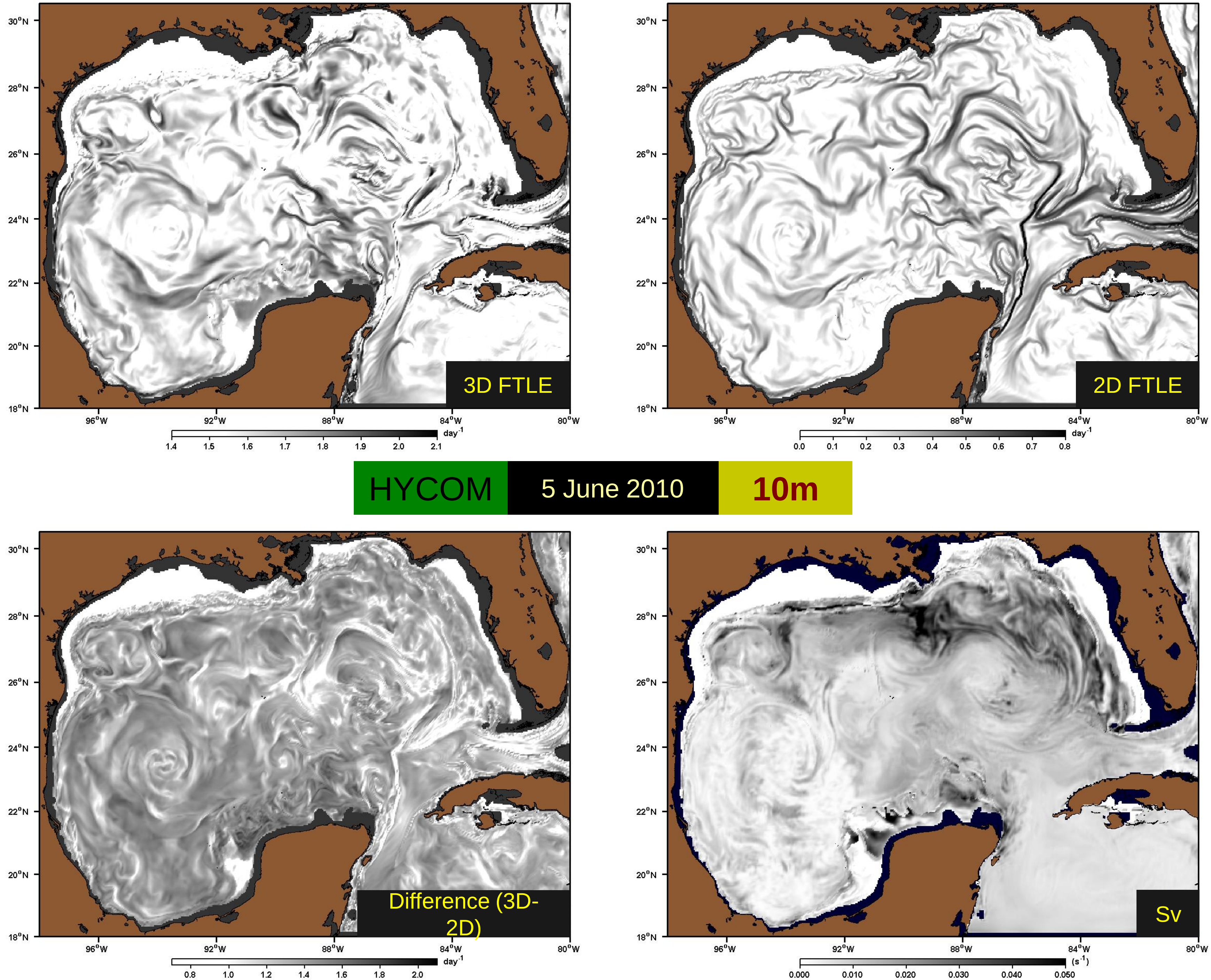
Two FTLE forms (3D and 2D) computed from 2D trajectories

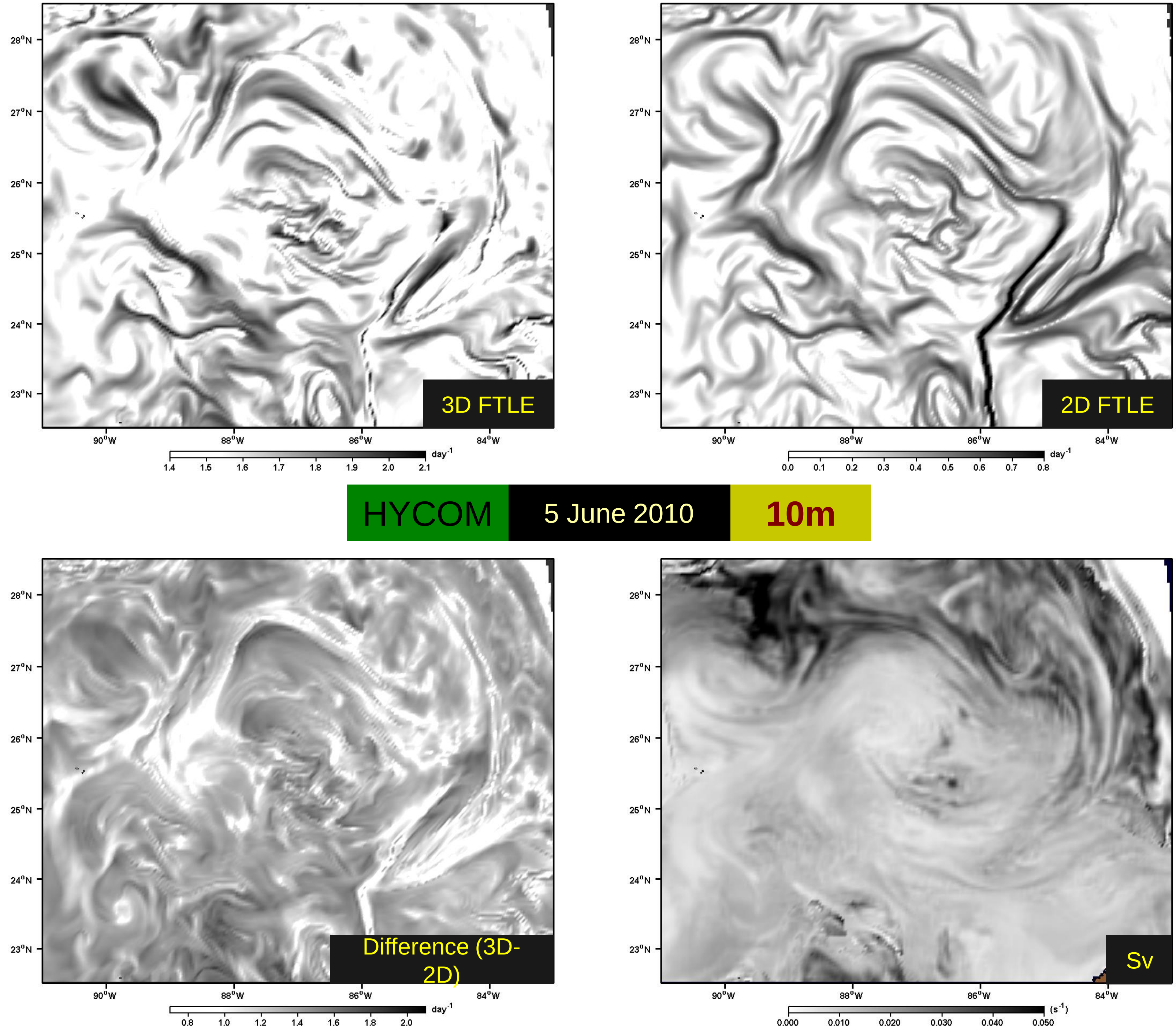
How do 3D and 2D FTLEs differ? How do differences change with depth?

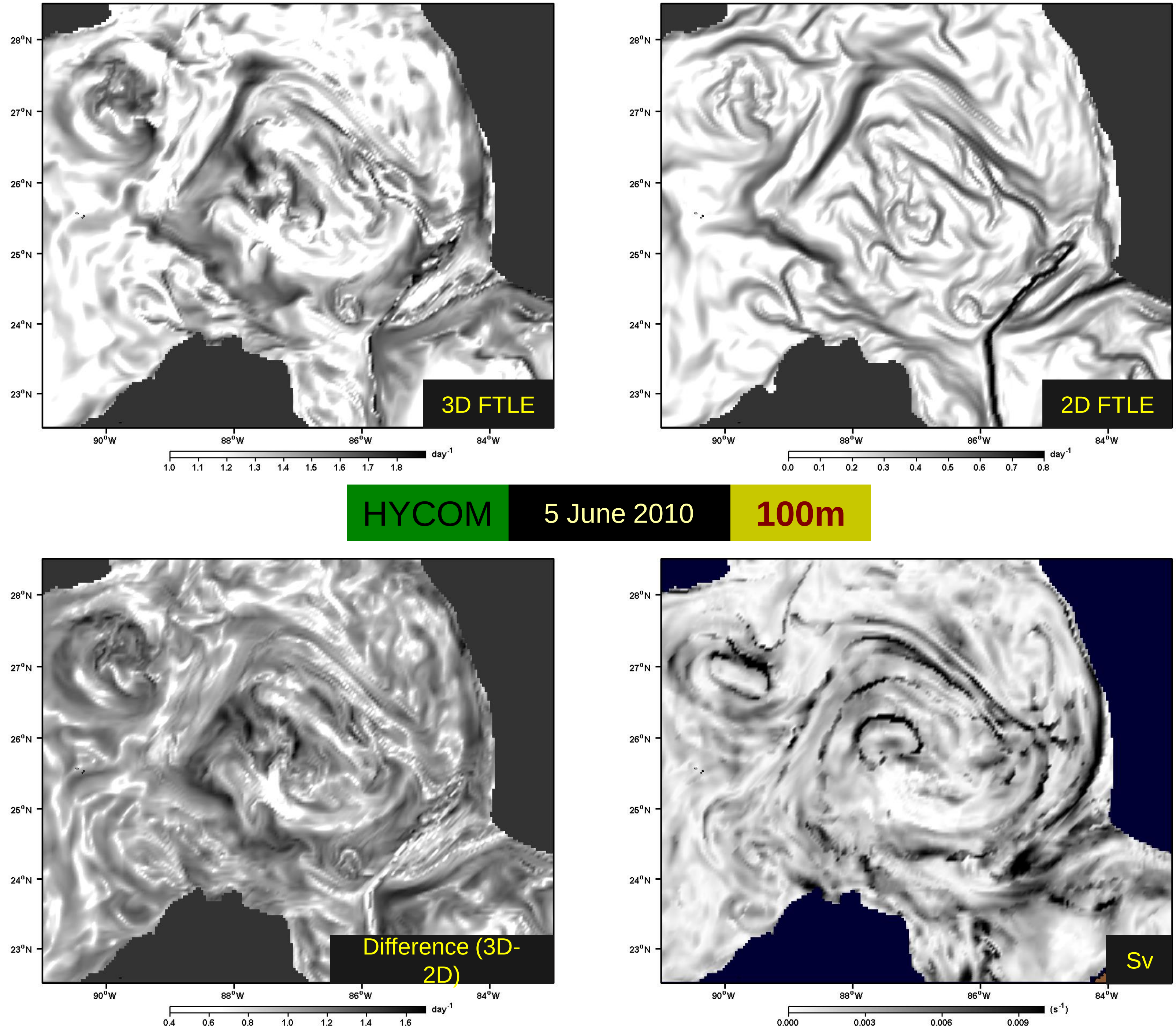
What are typical values for vertical (u,v) gradients (S_v) in an ocean model? How do they change with depth?

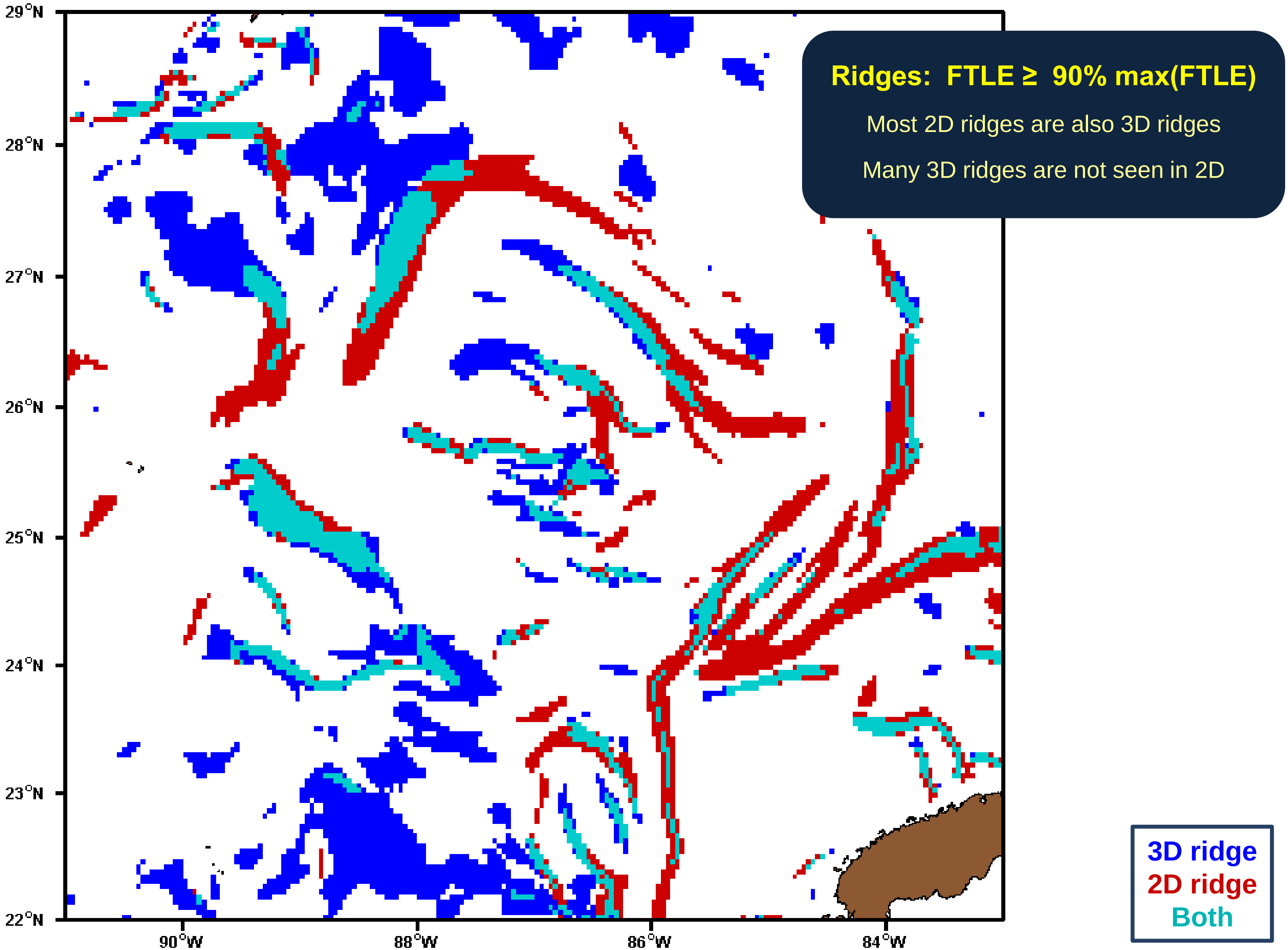
Are (3D-2D) FTLE differences related to S_v?

Do 3D FTLEs provide important new information?


3D FTLE
2D FTLE
HYCOM
5 June 2010
10m
Difference (3D-2D)
Sv
30°N
28°N
26°N
24°N
22°N
20°N
18°N
96°W
92°W
88°W
84°W
80°W
day-1
1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
0.8 1.0 1.2 1.4 1.6 1.8 2.0
(s-1)
0.000 0.010 0.020 0.030 0.040 0.050

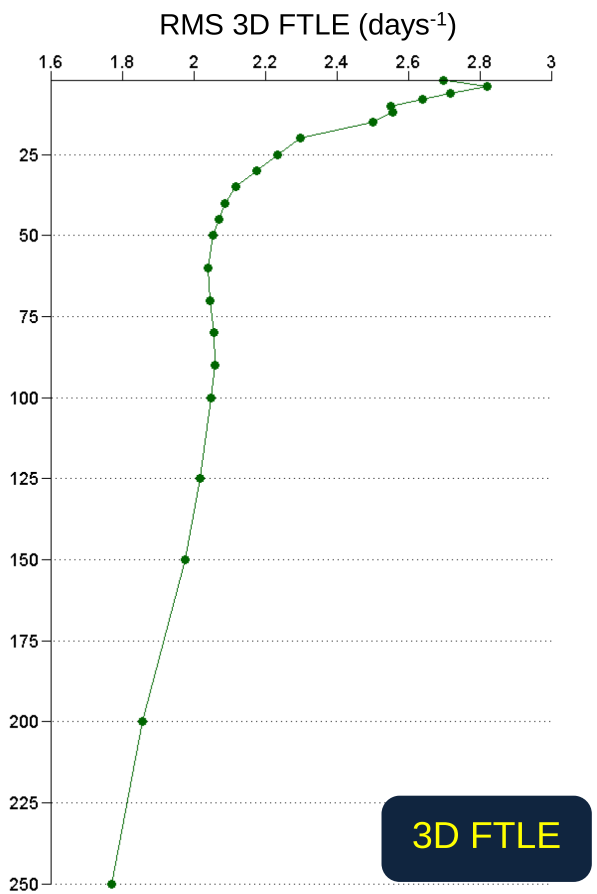

HYCOM
5 June 2010
10m
3D FTLE
2D FTLE
Difference (3D-2D)
Sv
28°N
27°N
26°N
25°N
24°N
23°N
90°W
88°W
86°W
84°W
day-1
1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
0.8 1.0 1.2 1.4 1.6 1.8 2.0
(s-1)
0.000 0.010 0.020 0.030 0.040 0.050

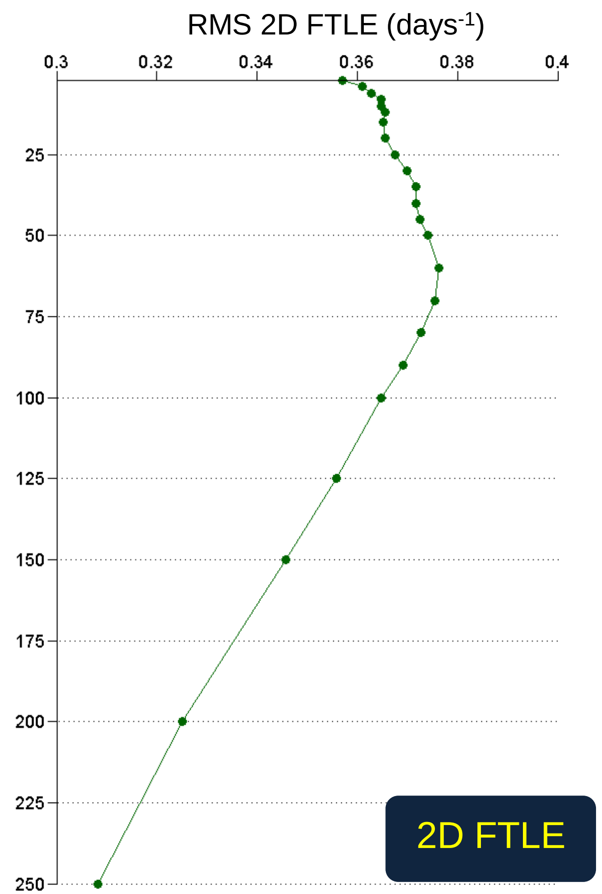

3D FTLE
2D FTLE
HYCOM
5 June 2010
100m
Difference (3D-2D)
Sv
28°N
27°N
26°N
25°N
24°N
23°N
90°W
88°W
86°W
84°W
day-1
1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
0.4 0.6 0.8 1.0 1.2 1.4 1.6
(s-1)
0.000 0.003 0.006 0.009

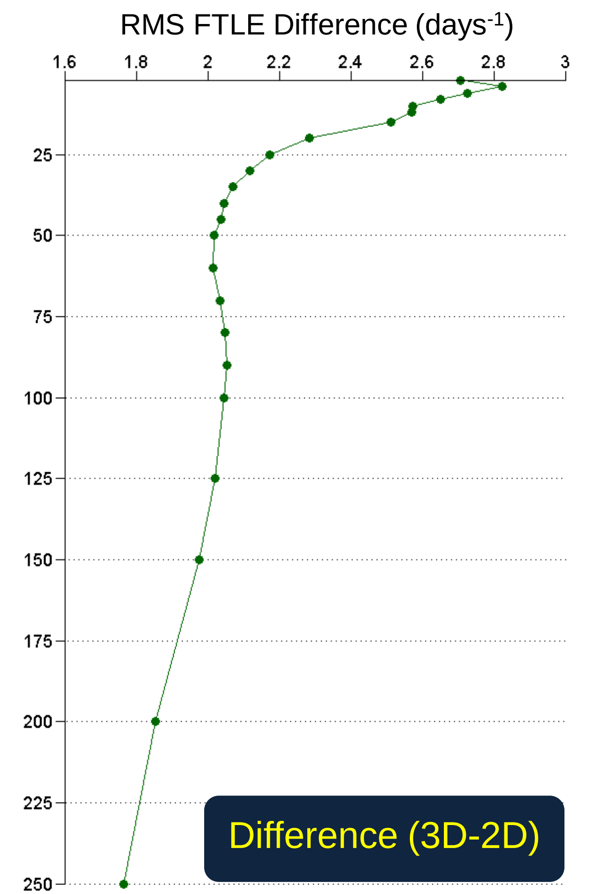


HYCOM 5 June 2010 **10m**

Depth Profiles: RMS FTLE



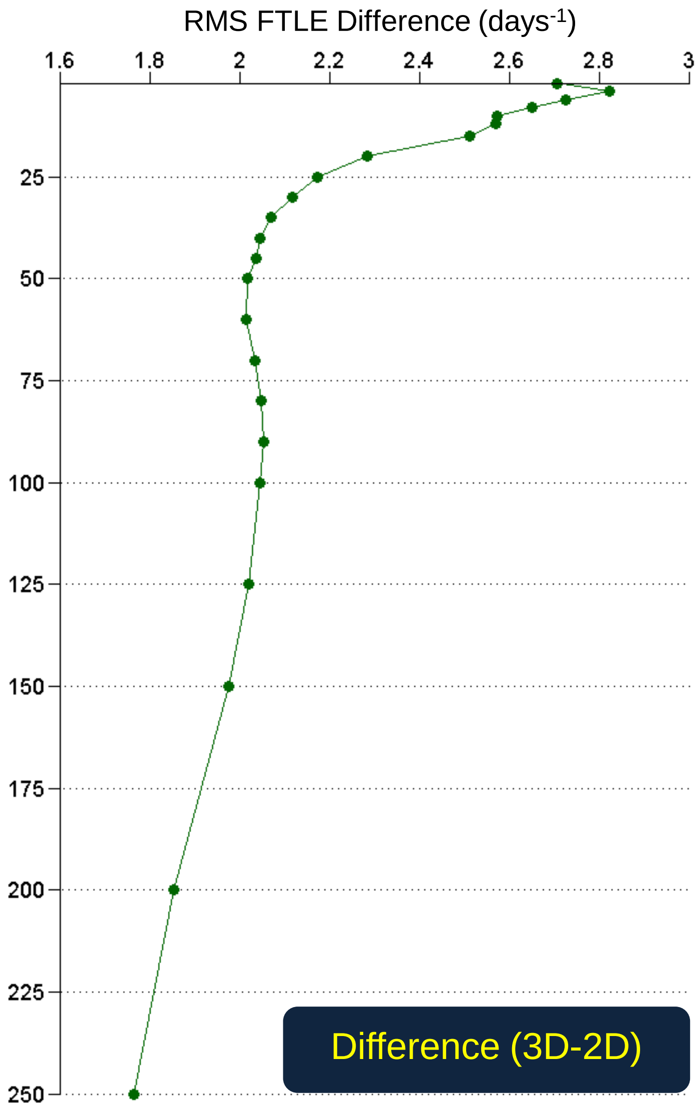


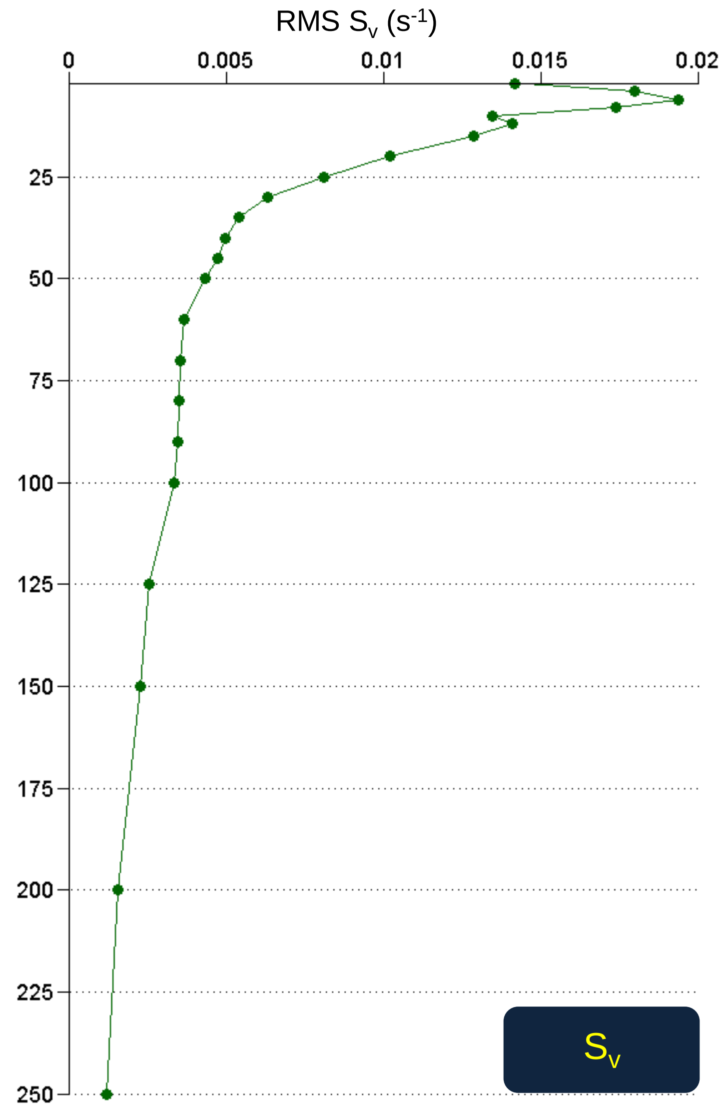


- 3D FTLE **6-7 times larger** than 2D FTLE
- 3D FTLE decreases by **40%** over 250m
- 2D FTLE decreases by **15%** over 250m

HYCOM 5 June 2010

Depth Profiles: RMS FTLE Difference and S_v





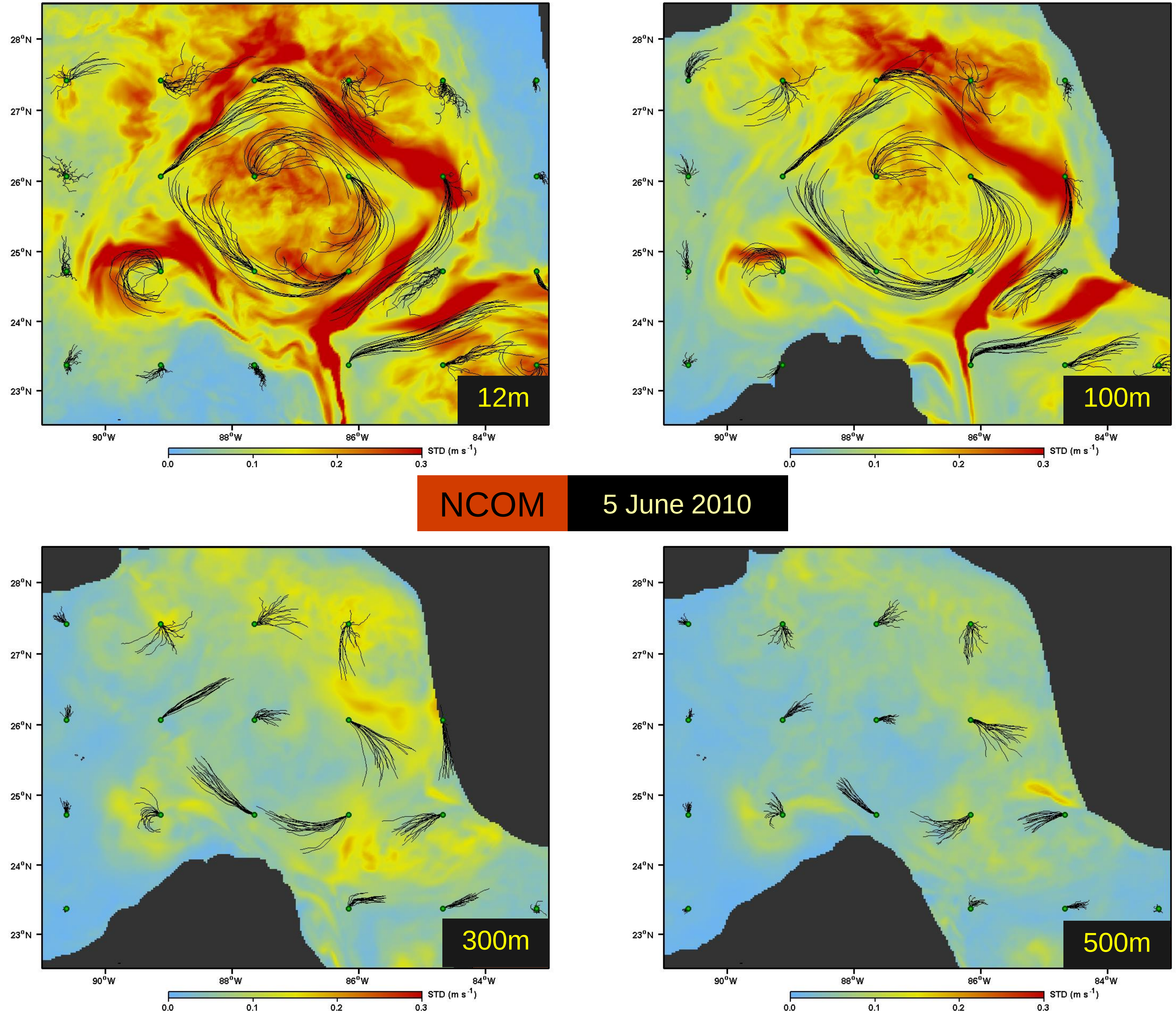
HYCOM 5 June 2010

Assessing Trajectory and FTLE Uncertainties in Ocean Models: GOM NCOM Ensemble

How uncertain are model ensemble trajectory forecasts?

How do trajectory uncertainties relate to FTLE uncertainties in an ensemble?

How do ensemble trajectory and FTLE uncertainties change with depth?


NCOM
5 June 2010
12m
100m
300m
500m
28°N
27°N
26°N
25°N
24°N
23°N
90°W
88°W
86°W
84°W
STD (m s⁻¹)
0.0
0.1
0.2
0.3

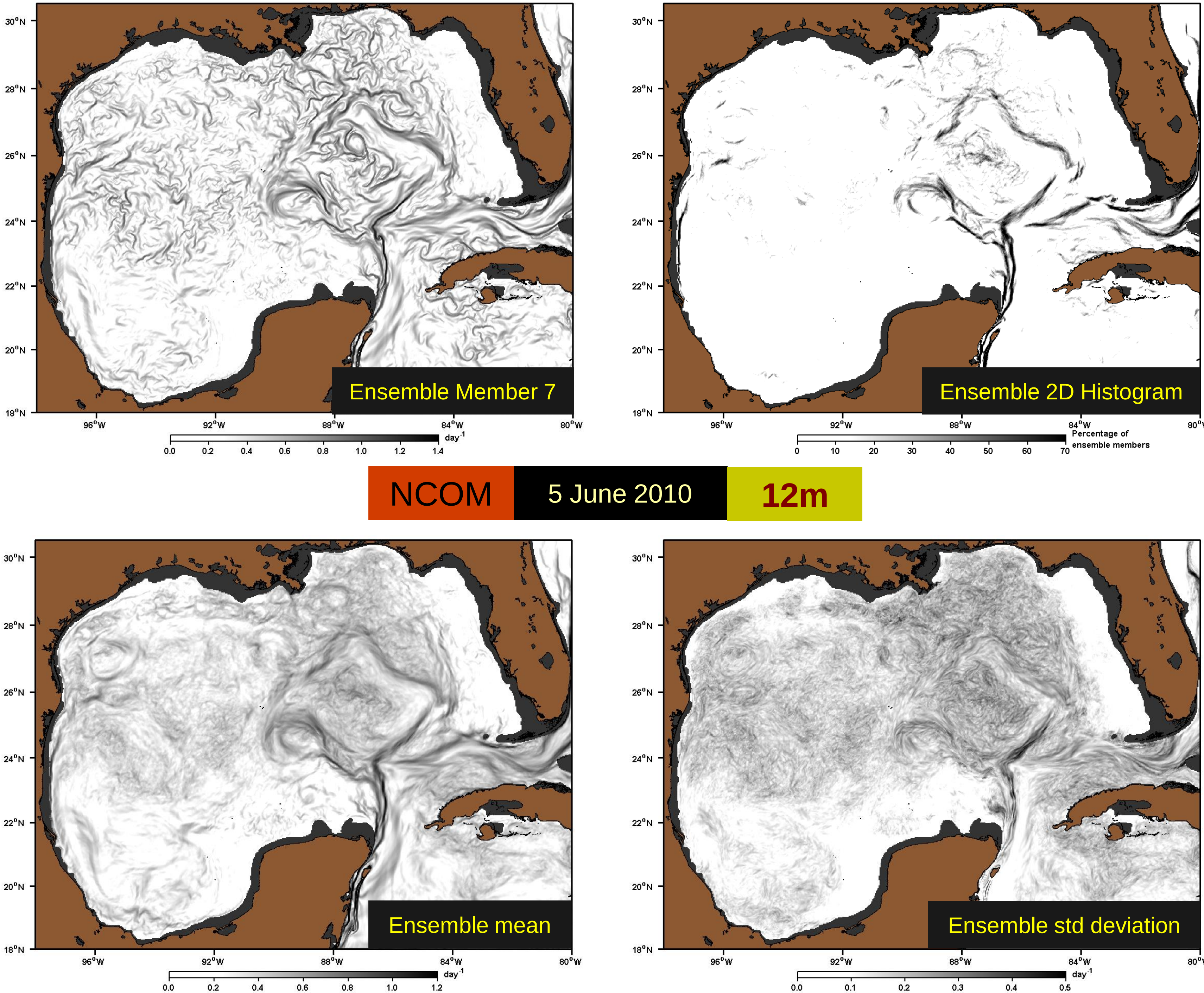
NCOM Lagrangian Predictive Skill Statistics

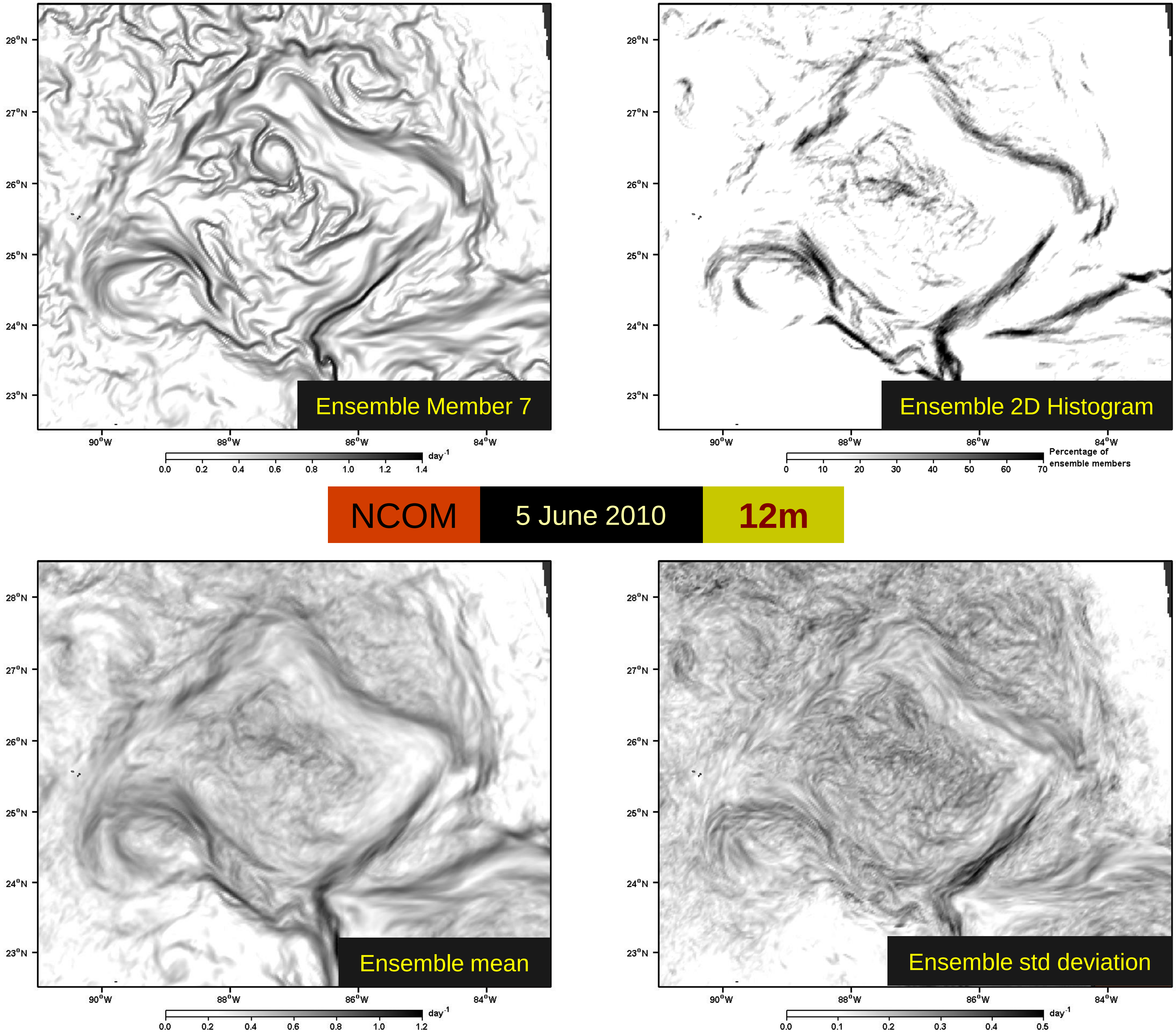
Northern GoM, near Deepwater Horizon (May - July 2010)

- 166 drifters: **1018** independent 3-day trajectory **segments**
- NCOM forecast trajectories and observations **separated by about 1km per hour** on average.

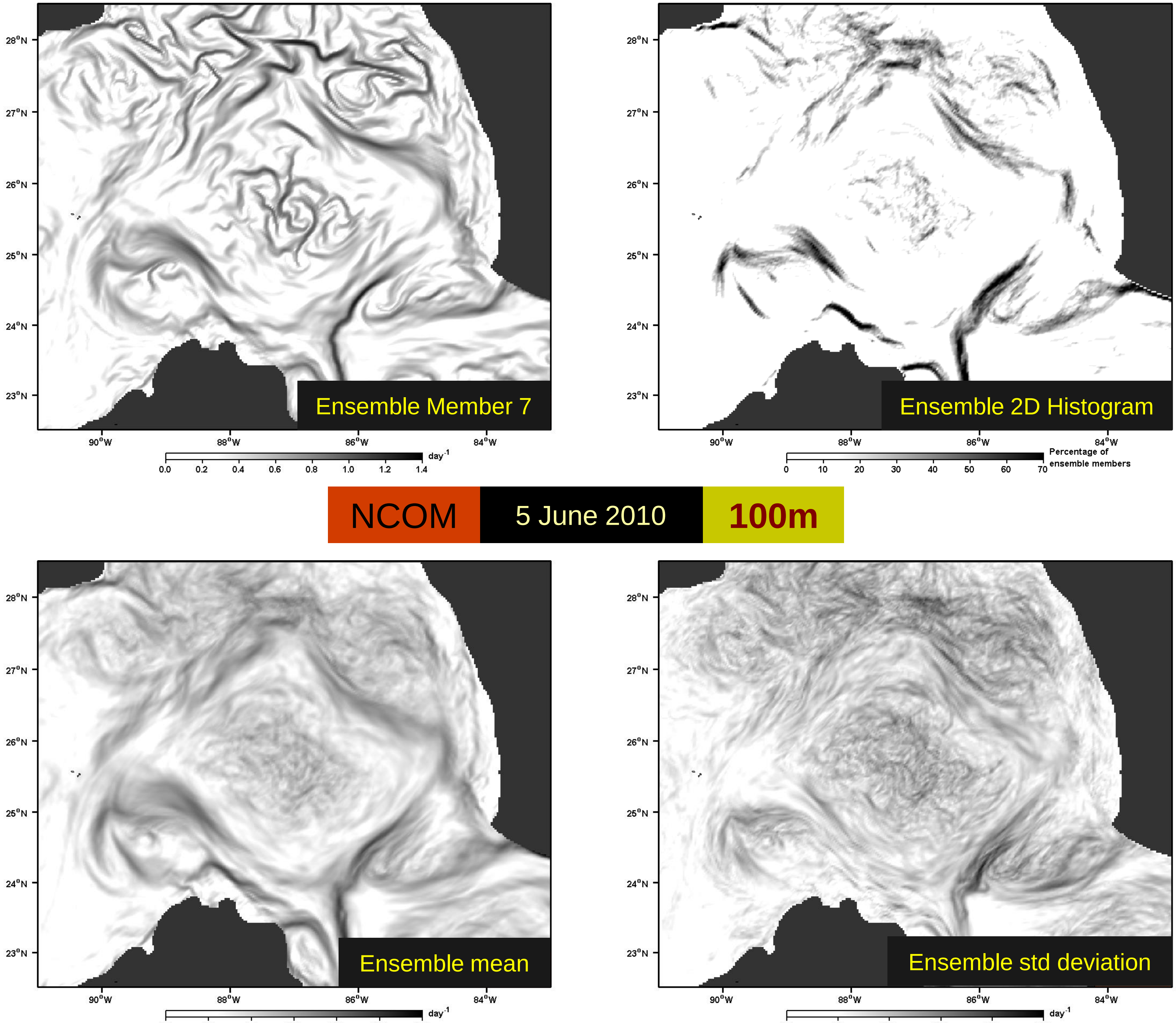
For the 16-member NCOM ensemble:

Fraction of observed drifter end points inside convex hull	
After 1 day	38.0%
After 3 days	36.2%

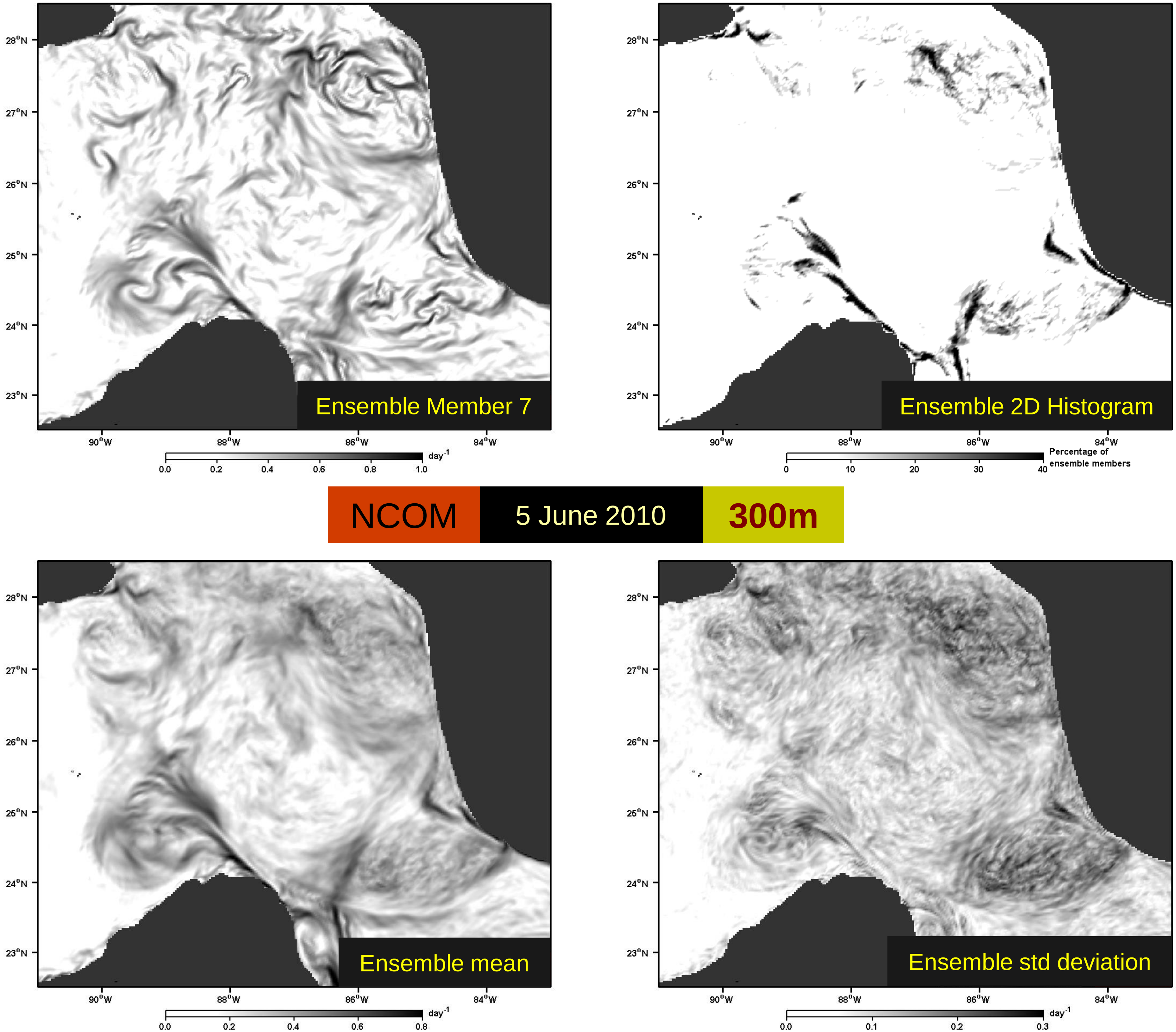

Ensemble Member 7
Ensemble 2D Histogram
NCOM
5 June 2010
12m
Ensemble mean
Ensemble std deviation
30°N
28°N
26°N
24°N
22°N
20°N
18°N
96°W
92°W
88°W
84°W
80°W
day⁻¹
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4
Percentage of ensemble members
0 10 20 30 40 50 60 70
0.0 0.2 0.4 0.6 0.8 1.0 1.2
0.0 0.1 0.2 0.3 0.4 0.5



NCOM
5 June 2010
12m
Ensemble Member 7
Ensemble 2D Histogram
Ensemble mean
Ensemble std deviation
day-1
Percentage of ensemble members
28°N
27°N
26°N
25°N
24°N
23°N
90°W
88°W
86°W
84°W



Ensemble Member 7
Ensemble 2D Histogram
Ensemble mean
Ensemble std deviation
NCOM
5 June 2010
100m
28°N
27°N
26°N
25°N
24°N
23°N
90°W
88°W
86°W
84°W
day-1
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4
Percentage of ensemble members
0 10 20 30 40 50 60 70
0.0 0.2 0.4 0.6 0.8 1.0 1.2
0.0 0.1 0.2 0.3 0.4 0.5



Ensemble Member 7
Ensemble 2D Histogram
NCOM
5 June 2010
300m
Ensemble mean
Ensemble std deviation
28°N
27°N
26°N
25°N
24°N
23°N
90°W
88°W
86°W
84°W
day⁻¹
Percentage of ensemble members

Learned from Ocean Models

FTLE Approximation Errors

Accounting for vertical (u,v) gradients substantially increases FTLE values and reveals new regions of high dispersion. 2D FTLE ridges are preserved.

RMS (3D-2D) FTLE differences closely follow RMS S_v with depth.

Lagrangian Uncertainties

For the NCOM ensemble, even though trajectory forecasts show substantial variability, observations fell within the convex hull of ensemble forecasts only 35-40% of the time after 3 days.

For the NCOM ensemble, probabilistic 2D FTLE maps capture the largest-scale coherent structures. But…..altimetry assimilation is not perturbed….

Loop Current Ring Formation

Ring Formation

Practical and theoretical interest

Diagnostics

- Closed temperature contours (Maul and Vukovich, 1995)
- Subjective assessment of altimeter maps (Sturges and Leben, 2000)
- Subjective assessment of numerical model (Kantha et al., 2005)

What’s wrong with diagnostics?

- Arbitrary and subjective
- Focus on surface layer
- Where’s the physics? – no predictive value

What’s needed?

- Binary criterion based on physical concepts
- Applicable through water column

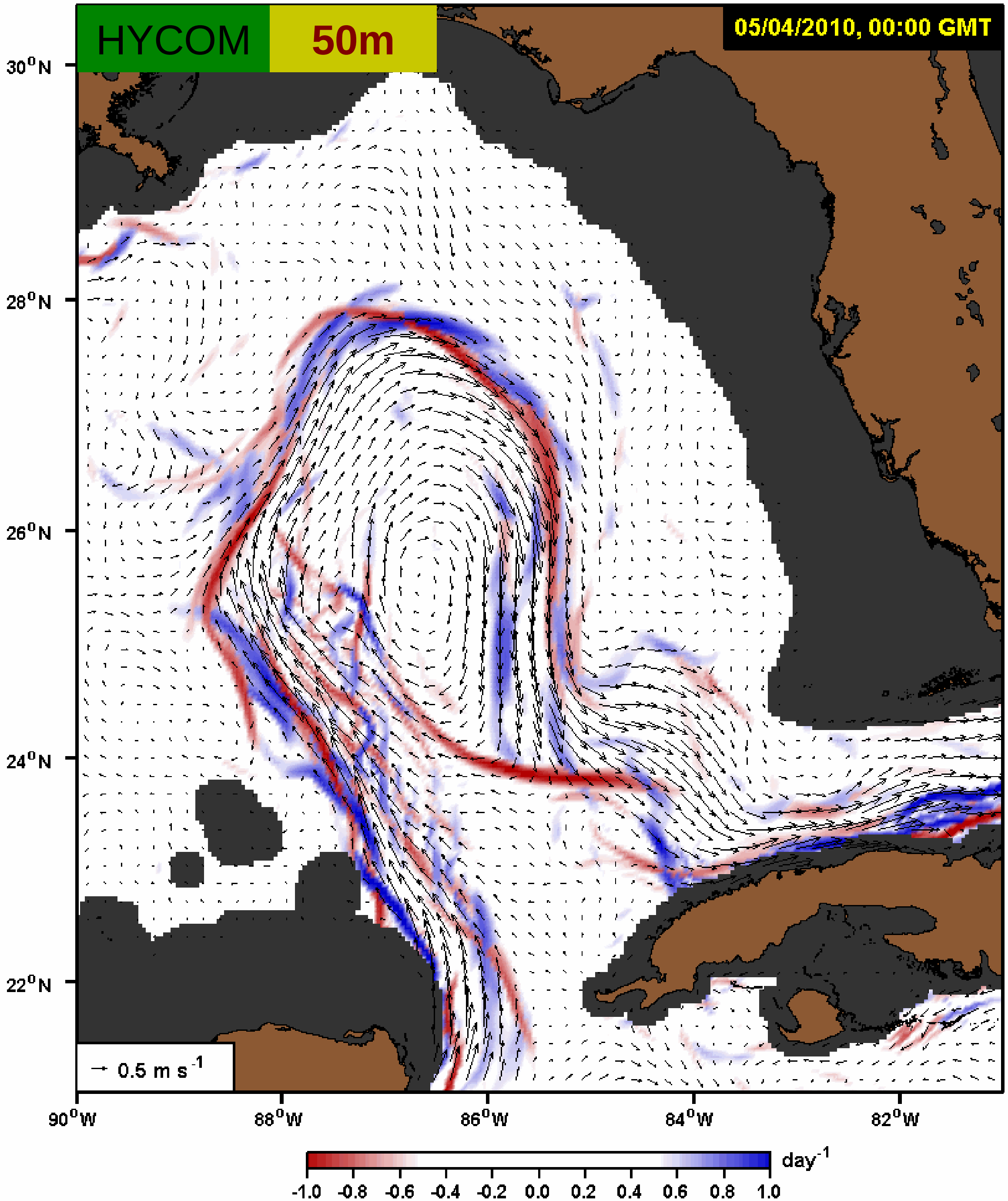
Ocean 3D + 1 Paradigm

Hypothesis: Separation characterized by a robust DHT with transport barriers between the ring and Loop Current

Applicable at any depth

Motivates new questions:

- Is LCR formation baroclinic or barotropic?
- Can we quantify separation/re-attachment?

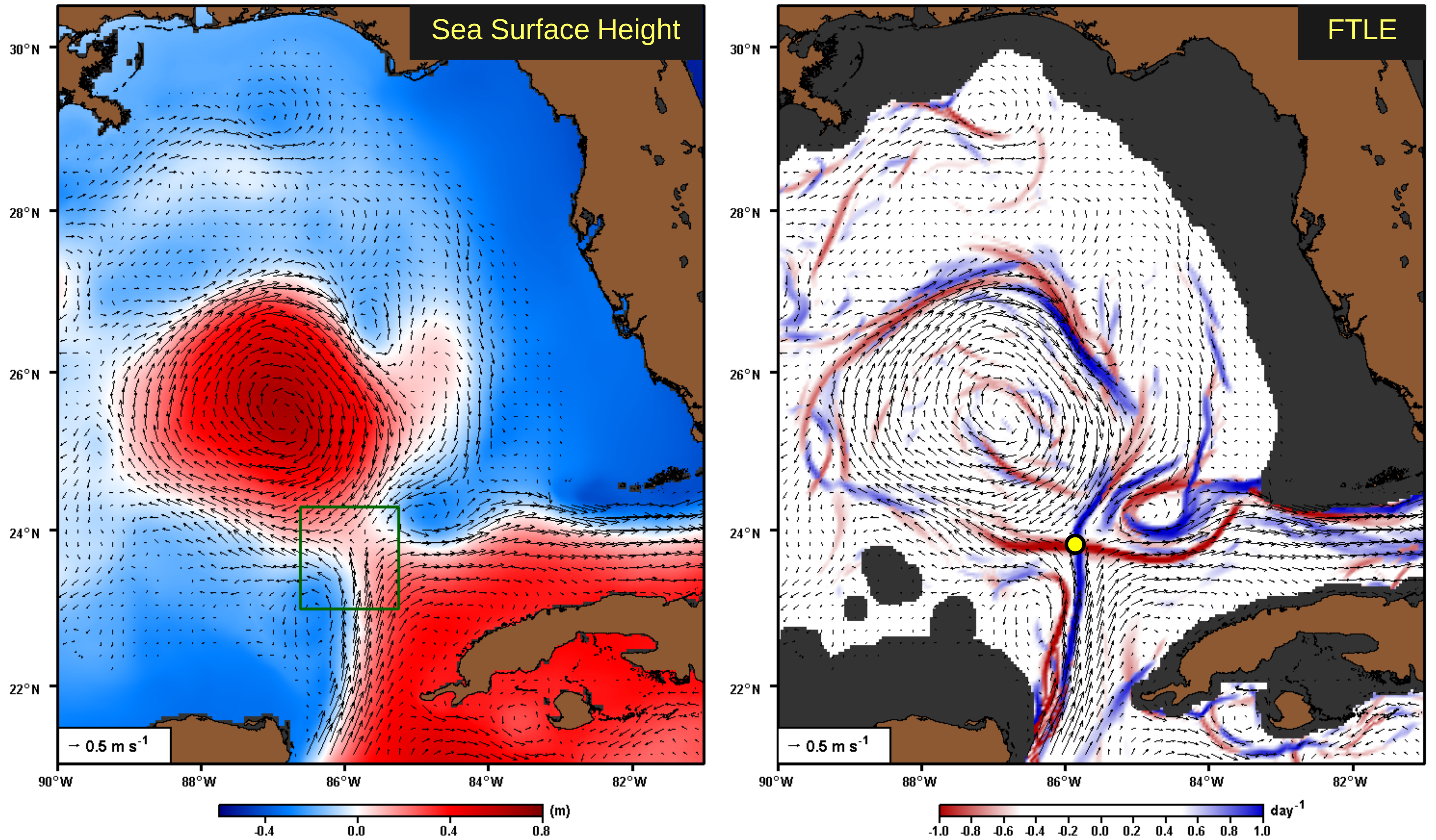


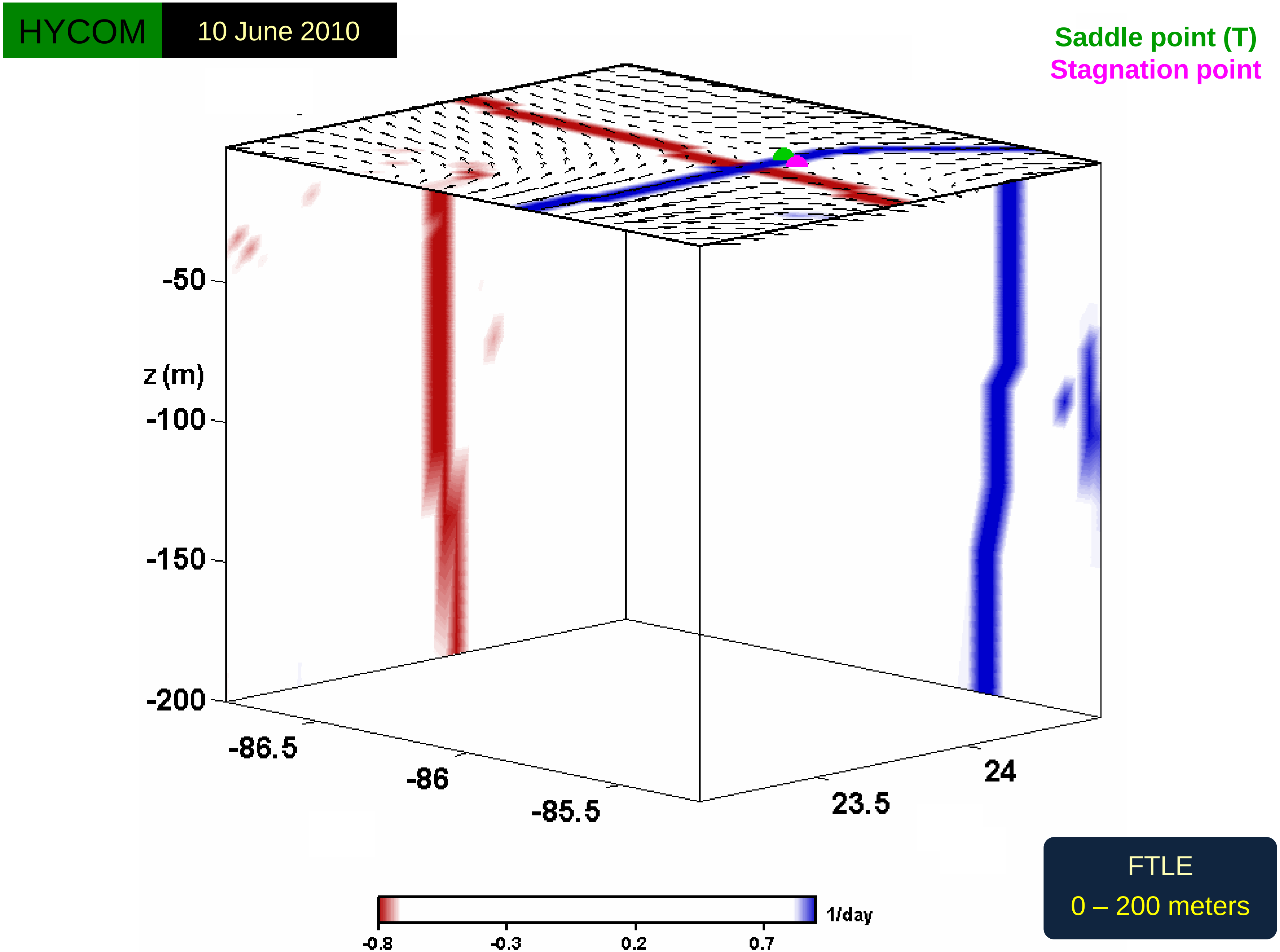
Formation of Eddy Franklin

May – July 2012

- Detachment is characterized by a robust, persistent DHT at depths of 50 to 200m.
- 3D mixing boundaries are vertical “curtains”.
- A stagnation point and a temperature saddle point exist near the DHT at depths of 50 to 200m.

Eddy Franklin: June 10, 2010

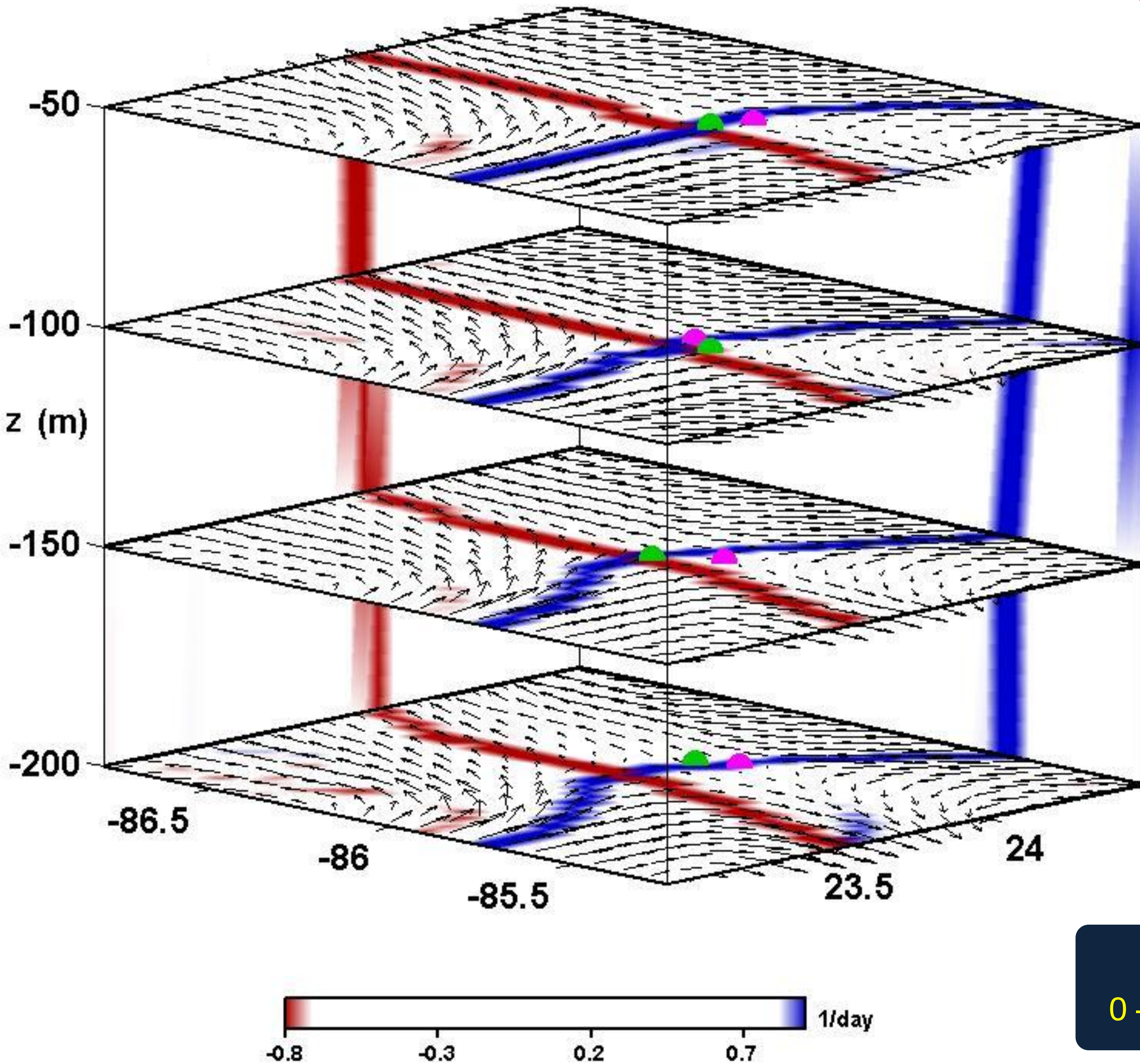




HYCOM
10 June 2010
Saddle point (T)
Stagnation point
-50
z (m)
-100
-150
-200
-86.5
-86
-85.5
23.5
24
-0.8
-0.3
0.2
0.7
1/day
FTLE
0 – 200 meters

HYCOM 10 June 2010

Saddle point (T)
Stagnation point



FTLE
0 – 200 meters

Significant Findings

An idealized quadrupole suggests that **FTLE estimate improve when vertical (u,v) shear effects are included**.

In ocean models, **including effects of vertical (u,v) shear reveals new regions of high dispersion**.

Existing NCOM ensembles include observed 3-day trajectories in their Lagrangian envelope **only 35-40% of the time**.

Probabilistic FTLE maps preserve the largest mesoscale structures for ocean model ensemble forecasts.

Emerging Ocean 3D+1 Hypotheses

Loop Current ring formation is a barotropic process.

(Contrast with the baroclinic eddy structures found by Poje, RSMAS group.)

3D transport barriers are nearly vertical curtains for large mesoscale features.

(Branicki & Kirwan, Bettencourt et al., our recent analyses).

DHTs are linked to temperature/density saddle points during LCR formation.

Characterize Ocean 3D+1 Transport

Challenge: Synoptic Lagrangian observations are not feasible. We must rely on models.

Idealized models

Algorithm development

Application to ocean observations

CUNY Staten Island

LES models

Full 3D dynamics

Submesoscale resolved

Limited domain size

RSMAS

Data-assimilating ocean models

3D process studies

Ring formation/multipoles

U. Delaware

Challenge: Ocean models are imperfect.

Initial condition and forcing uncertainties

How much Lagrangian uncertainty is captured by ensembles?

U. Delaware

Unreliable/missing vertical velocities

How accurate are 3D FTLE approximated with 2D velocities?

U. Delaware

Submesoscale is parameterized

Are submesocales represented accurately?

RSMAS

Next step: Use Lagrangian data assimilation to improve model forecasts.

Publications

Surface drift predictions of the Deepwater Horizon spill: The Lagrangian perspective.
H. S. Huntley, B. L. Lipphardt Jr., and A. D. Kirwan, Jr.
AGU Geophysical Monograph 195: Monitoring and Modeling the Deepwater Oil Spill: A Record-Breaking Enterprise, 179-195, 2011.

Out of Flatland: Three-dimensional aspects of Lagrangian transport in geophysical fluids.
M. H. M. Sulman, H. S. Huntley, B. L. Lipphardt, Jr. and A. D. Kirwan, Jr.
Geophysical Monograph 200: Lagrangian Modeling of the Atmosphere. 77-84, 2012.

Leaving Flatland: Diagnostics for Lagrangian coherent structures in three-dimensional flows.
M. H. M. Sulman, H. S. Huntley, B. L. Lipphardt, Jr. and A. D. Kirwan, Jr.
In revision, Physica D, 2013.

Three-dimensional aspects of Loop Current ring formation.
M. H. M. Sulman, H. S. Huntley, B. L. Lipphardt, Jr., P. Hogan, G. Jacobs, and A. D. Kirwan, Jr.
To be submitted to Nonlinear Processes in Geophysics.