2D Transport Barriers from 2D Velocity Fields

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Support from Office of Naval Research through
MURI *OCEAN 3D+1 grant N00014-11-1-0087*

Ocean 3D + 1 Question:

“How do we characterize all possible Lagrangian advective boundaries in 3D+1 ocean flows?”

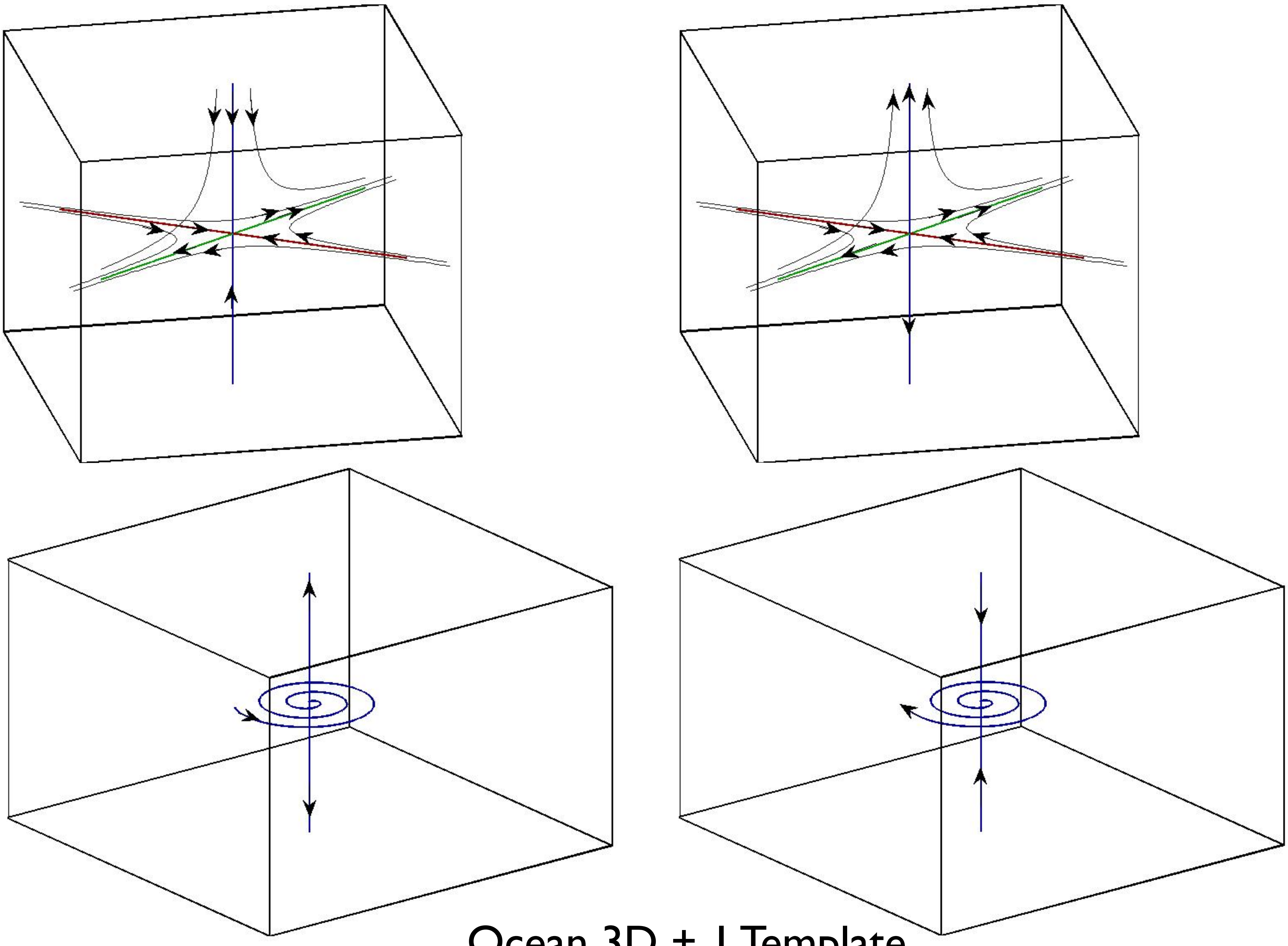
Nota Bene:

Operational data assimilating OGCMs produce 2D velocities along surfaces

Can such 2D velocities characterize 2D transport barriers embedded in a 3D+1 Ocean?

Outline

- Brief review of mesoscale dynamics/LCS
- Summarize Physica D submission
  - ABC results
  - QP scaled to oceanic conditions – mesoscale constraint on vertical shear of horizontal velocity
- Applications – ring separation GoM & QP (Work in Progress)
- Synopsis of 2D transport barriers: Branicki&Kirwan, Bettencourt, GoM
- Now what?



Ocean 3D + 1 Template

Lagrangian Coherent Structures

- Defined by Haller (2000) in terms of FTLE (default metric used here)
- Other LCS characterizations
  - Joseph & Legras (2002) - FSLE
  - Mancho & Mendoza (2010) - minimal trajectories
  - Mezic et al (2010) - mesohyperbolicity
  - Haller (2011) - geodesic material surfaces
  - Rypina – Complexity Method (2011)
- Most studies in GFD confined to 2D velocities
- Yet theory applies to $\mathbf{R}^n$

Are LCS important in GFD?

- MODE/POLYMODE (circa 1975) - Mesoscale eddies transport heat, salinity, and momentum
  But
  - How do eddies form?
  - How many eddies are there?
  - How do eddies exchange heat, etc with environment?
- Since MODE/POLYMODE
  - Growing Lagrangian user community
  - Dramatic oil spills
- Circa 1990 – Little Compton meeting. DST methods applied to 2D mesoscale and submesoscale transport
- To date analyses focused on kinematic descriptions

FTLE Recap

$$\mathbf{C} = \mathbf{A}^{\mathbf{T}}\mathbf{A}$$

$$\mathbf{A} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}$$

$$\mathbf{A}_{3D} = \begin{bmatrix} \partial x/\partial X & \partial x/\partial Y & \partial x/\partial Z \\ \partial y/\partial X & \partial y/\partial Y & \partial y/\partial Z \\ \partial z/\partial X & \partial z/\partial Y & \partial z/\partial Z \end{bmatrix} \qquad \mathbf{A}_{2D} = \begin{bmatrix} \partial x/\partial X & \partial x/\partial Y & 0 \\ \partial y/\partial X & \partial y/\partial Y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_{3D2D} = \begin{bmatrix} \partial x/\partial X & \partial x/\partial Y & \partial x/\partial Z \\ \partial y/\partial X & \partial y/\partial Y & \partial y/\partial Z \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Lambda(t; t_0; \mathbf{X}) = \frac{\log \sqrt{\lambda_{max}(\mathbf{C})}}{t - t_0}$$

FTLE Diagnostics

$$S_v = \sqrt{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2}$$

$$S_w = \sqrt{\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2}$$

Approach*

- Options
  - Follow B & K (2010) paradigm
  - Extend FTLE calculations to include vertical shear of horizontal velocities
  - Calculate 3D trajectories using diagnostic vertical velocity
- Strategy
  - Test options with toy models to control vertical velocity and vertical gradients
  - Apply to data-assimilating OGCMs

*Sulman, et al (submitted, 2012)

ABC Flow Recap

$$
\begin{aligned}
u &= A\sin z + C\cos y \\
v &= B\sin x + A\cos z \\
w &= C\sin y + B\cos x
\end{aligned}
$$

$$S_v = A$$

$$S_w = \sqrt{(B\sin x)^2 + (C\cos y)^2}$$

ABC Flow*

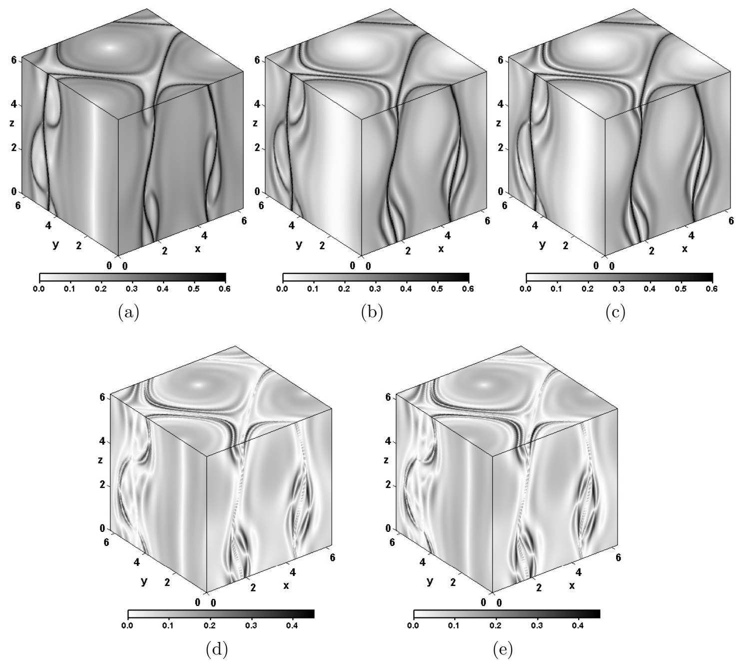


Figure 3: LCS in the steady ABC flow with $A = 0.1$, $B = C = 0.8$, and $t_f - t_0 = 10$. For this case, $S_v = 0.1$ and $\max(S_w) = 1.13$. Each panel shows results on three domain boundaries: $x = 0$, $y = 0$, and $z = 2\pi$. (a) The benchmark FTLE_{3d3d}. (b) Approximation neglecting vertical motion FTLE_{3d2d}. (c) Approximation neglecting the vertical dimension completely FTLE_{2d2d}. (d) $\Delta_2 = |\mathrm{FTLE}_{3d3d} - \mathrm{FTLE}_{3d2d}|$. (e) $\Delta_1 = |\mathrm{FTLE}_{3d3d} - \mathrm{FTLE}_{2d2d}|$.

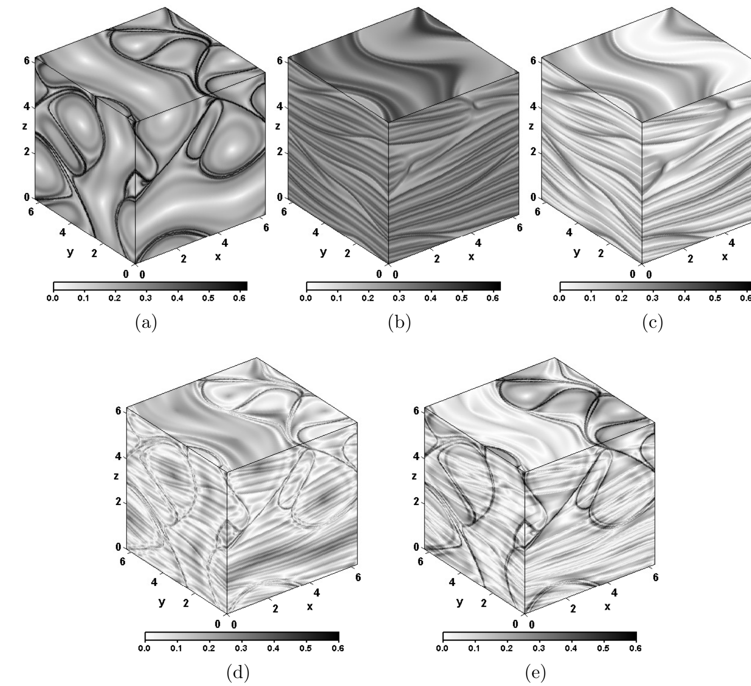


Figure 4: Same as Figure 3, but for $A = 1.1$ and $B = C = 0.8$. For this case, $S_v = 1.1$ and $\max(S_w) = 1.13$.

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16

*Sulman et al (submitted 2012)

ABC Flow*

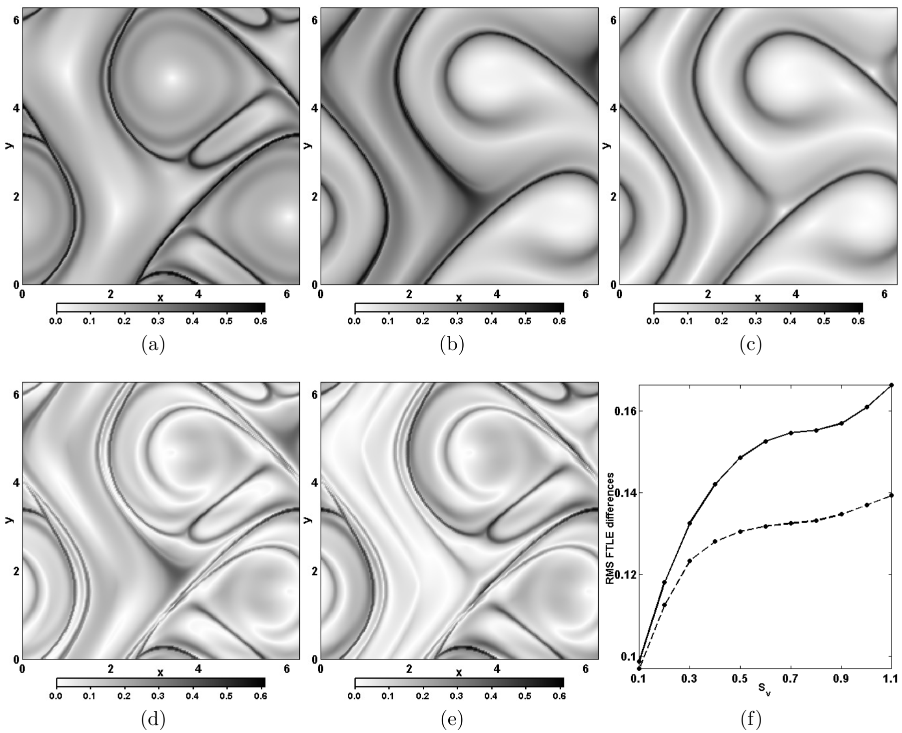


Figure 5: LCS in the steady ABC flow with $A = 0.5$, $B = C = 0.8$, and $t_f - t_0 = 10$ for the cross-section at $z = 2\pi$. For this case, $S_v = 0.5$ and $\max(S_w) = 1.13$. (a) FTLE_{3d3d}. (b) FTLE_{3d2d}. (c) FTLE_{2d2d}. (d) $\Delta_2 = |\mathrm{FTLE}_{3d3d} - \mathrm{FTLE}_{3d2d}|$. (e) $\Delta_1 = |\mathrm{FTLE}_{3d3d} - \mathrm{FTLE}_{2d2d}|$. (f) RMS FTLE differences for the entire domain as a function of S_v. Solid: $\mathrm{RMS}(\Delta_1)$; dashed: $\mathrm{RMS}(\Delta_2)$.

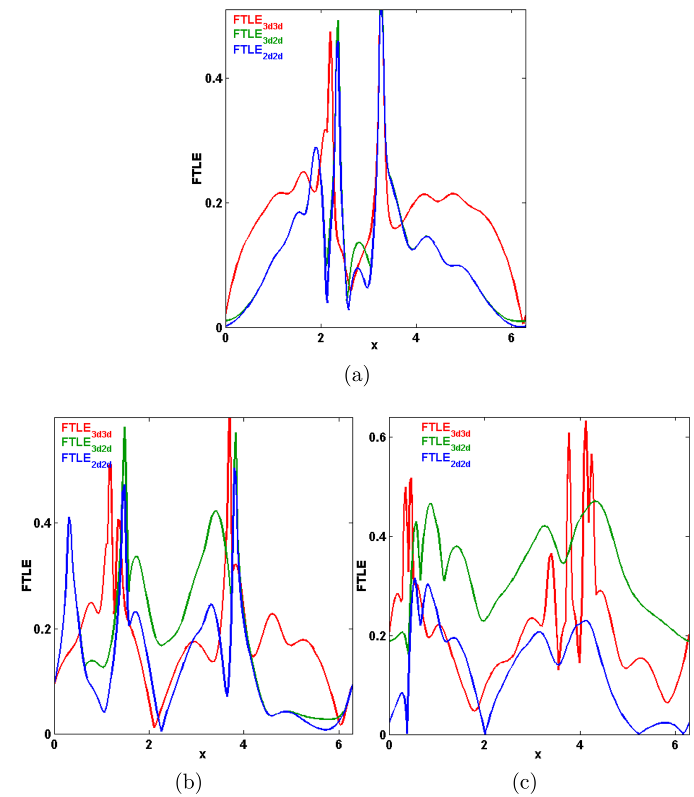


Figure 6: Cross-sections of LCS in the steady ABC flow along the x-direction at $z = 2\pi$, $y = \pi/2$ with $B = C = 0.8$ and $t_f - t_0 = 10$. (a) $A = 0.1$. (b) $A = 0.5$. (c) $A = 1.1$. For all panels, red shows FTLE_{3d3d}, green shows FTLE_{3d2d}, and blue shows FTLE_{2d2d}.

*Sulman et al (submitted, 2012)

ABC Lessons Learned

- Approximations more sensitive to S_v than S_w
- But analysis restricted to $S_w \geq S_v$
- Modified Cauchy Green tensor that includes

$$\partial x/\partial Z, \partial y/\partial Z$$

- In mesoscale ocean flows vertical shear constrained by Richardson criterion $N^2/S_v^2 \geq .25$
- Test with QP model where $S_v \, and \, S_w$ controlled independently

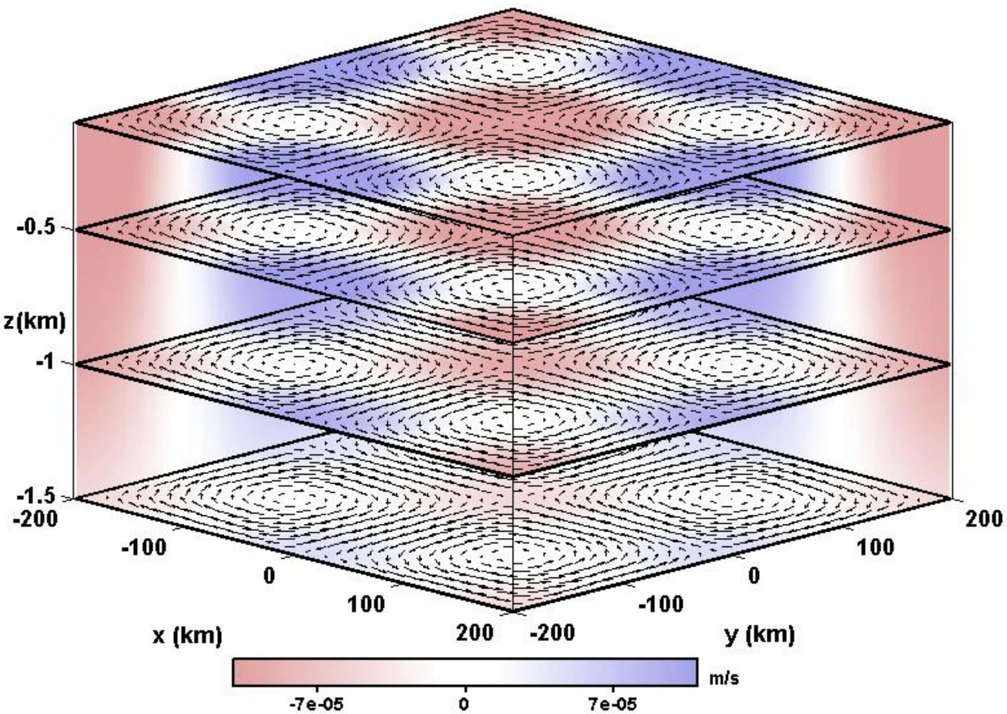
QP Recap

$$
\begin{aligned}
u &= -k\left[A(z) + B_z(z)\right]\sin kx \cos ky \\
v &= k\left[A(z) - B_z(z)\right]\cos kx \sin ky \\
w &= 2k^2 B(z)\cos kx \cos ky
\end{aligned}
$$

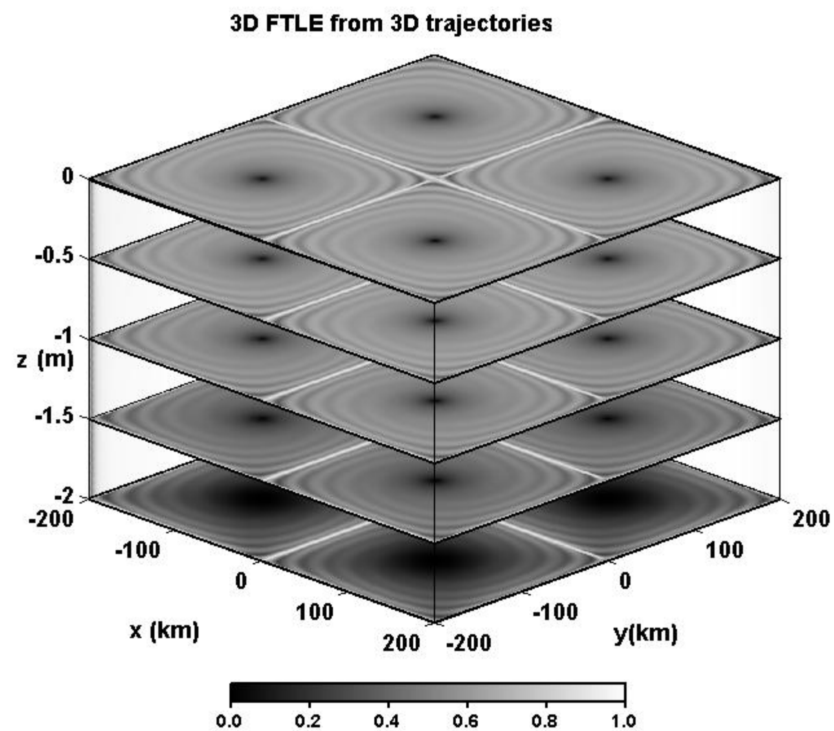
$$S_v = k\sqrt{\left(A_z^2 + B_{zz}^2\right)\left(\sin^2 kx + \cos^2 ky\right) + 2A_z B_{zz}\left(\sin^2 kx - \cos^2 ky\right)}$$

$$S_w = k\sqrt{k^2\left(\sin^2 kx \cos^2 ky + \cos^2 kx \sin^2 ky\right) + B_z^2 \cos^2 kx + \sin^2 ky}$$

QP Cube



-0.5
z(km)
-1
-1.5
-200
-100
0
100
200
x (km)
-200
-100
0
100
200
y (km)
m/s
-7e-05
0
7e-05



3D FTLE from 3D trajectories
0
-0.5
-1
z (m)
-1.5
-2
-200
-100
0
100
200
x (km)
-200
-100
0
100
200
y(km)
0.0
0.2
0.4
0.6
0.8
1.0

Quadrupole*

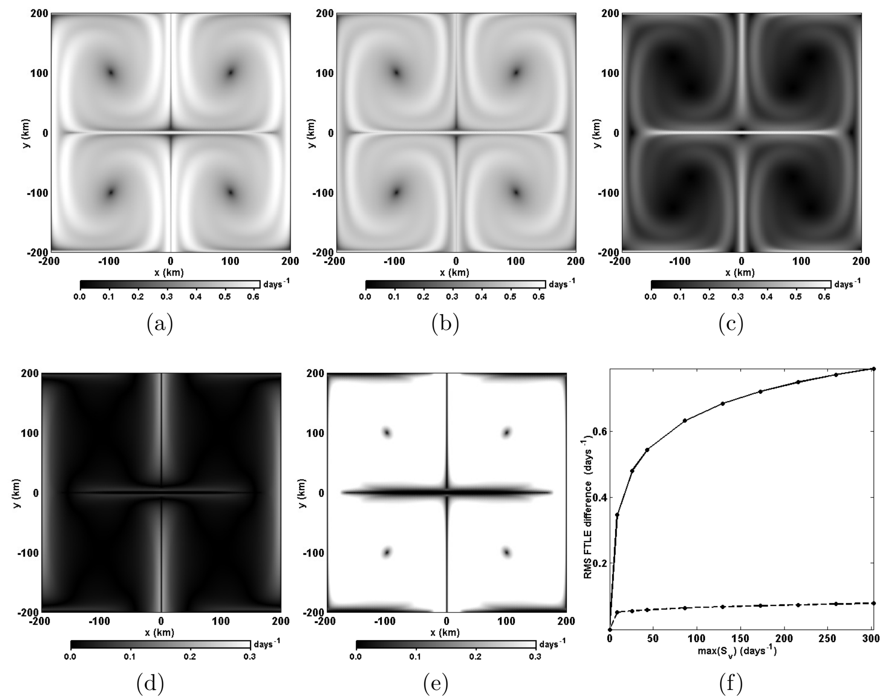


Figure 13: LCS for the quadrupole with nonzero vertical (u, v) shear and nonzero vertical velocity gradient. FTLE at $z = -0.3$ km, with $A_0 = 0.53$ m s^{-1}, $A_1 = 0.10$ m s^{-1} km^{-1}, $B_0 = -1.1 \times 10^{-3}$ m s^{-1}, $B_1 = -4 \times 10^{-3}$ m s^{-1} km^{-1}, and $t_f - t_0 = 8$ days. (a) FTLE_{3d3d}, (b) FTLE_{3d2d}, (c) FTLE_{2d2d}, (d) $\Delta_2 = |\mathrm{FTLE}_{3d3d} - \mathrm{FTLE}_{3d2d}|$, (e) $\Delta_1 = |\mathrm{FTLE}_{3d3d} - \mathrm{FTLE}_{2d2d}|$, and (f) RMS FTLE differences as a function of $\max(S_v)$ with $\max(S_w) = 0.3456$ days^{-1}. Solid: RMS(Δ_1); dashed: RMS(Δ_2).

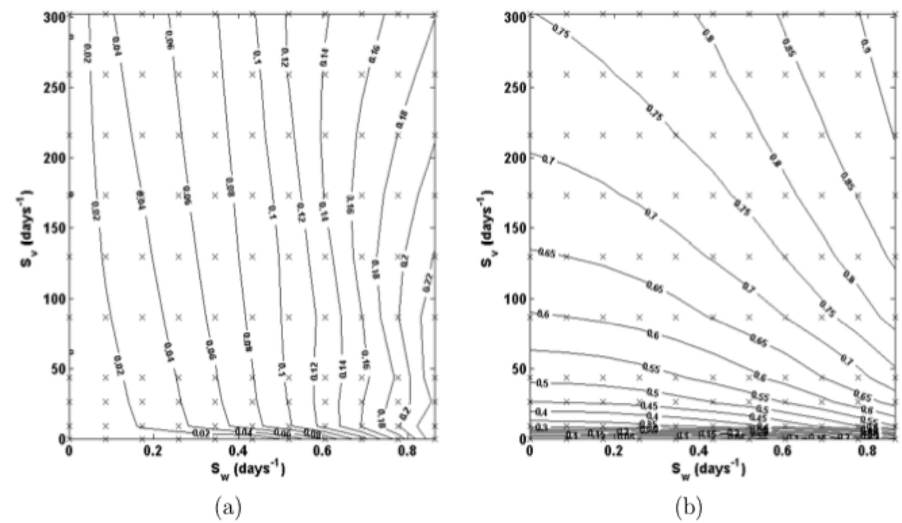


Figure 14: Contour plots of RMS FTLE differences for the quadrupole with nonzero vertical (u, v) gradient and nonzero vertical velocity at $z = -0.3$ km. (a) RMS(Δ_2). (b) RMS(Δ_1). Crosses indicate data points used to construct the contours, whose parameter values are given in (29), (30), (31), and (32).

*Sulman et al (submitted, 2012)

QP Manifold location*

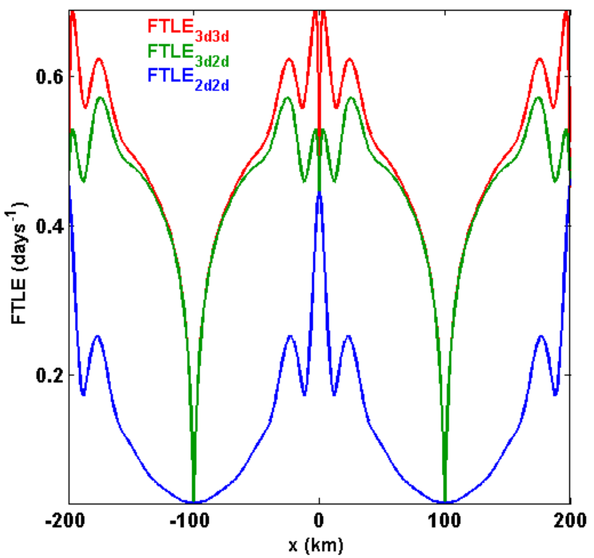


Figure 15: Cross-section at $z = -0.3$ km along $y = 100$ km of the LCS in the quadrupole with $A_0 = 0.53$ m s^{-1}, $A_1 = 0.10$ m s^{-1} km^{-1}, $B_0 = -1.1 \times 10^{-3}$ m s^{-1}, and $B_1 = -4 \times 10^{-3}$ m s^{-1} km^{-1}, giving $S_v = 8.6400$ days^{-1} and $S_w = 0.3456$ days^{-1}. For all panels, red shows FTLE_{3d3d}, green shows FTLE_{3d2d}, and blue shows FTLE_{2d2d}.

*Sulman et al (submitted, 2012)

Lessons Learned from ABC & QP

- For QP flow both 2D FTLE approximations yielded LCS consistent with “truth”
- For ABC flow 2D FTLE poor approximations
- Improved Cauchy Green accounts for vertical shear
- S_v important ocean diagnostic

Ring Formation

- Practical and Theoretical Interest
- Diagnostics
  - Closed contours of 22° isotherm (Sturges circa 1990)
  - Subjective assessment of altimeter maps (Sturges & Leben 2000)
  - Subjective assessment of numerical model (Kantha et al 2005)

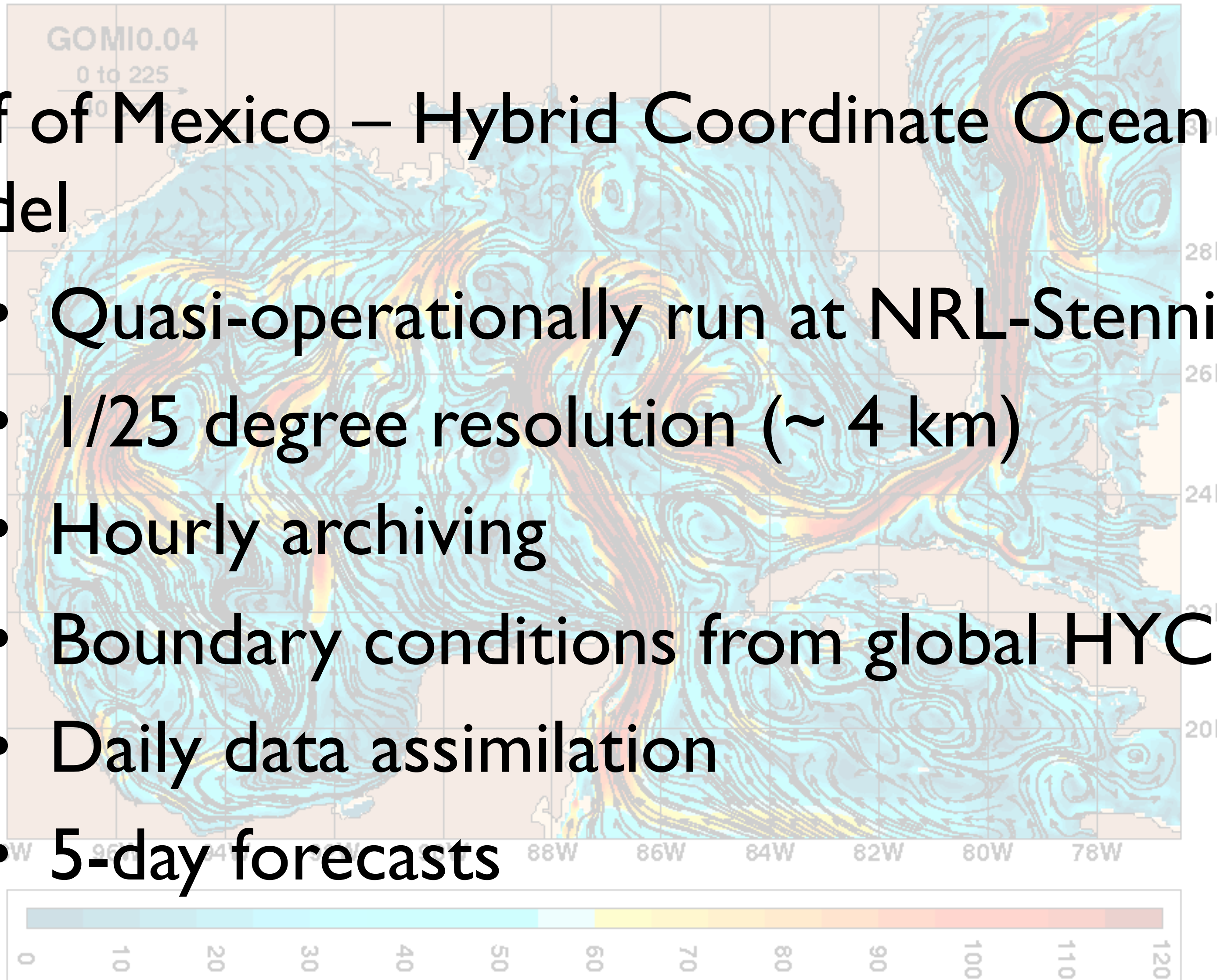
Assessment

- What's wrong with diagnostics?
  - Arbitrary and subjective
  - Focus on surface layer
  - Where's the physics? – no predictive value
- What's needed?
  - Binary criteria based on physical concepts
  - Applicable through water column

Ocean 3D + 1 Paradigm

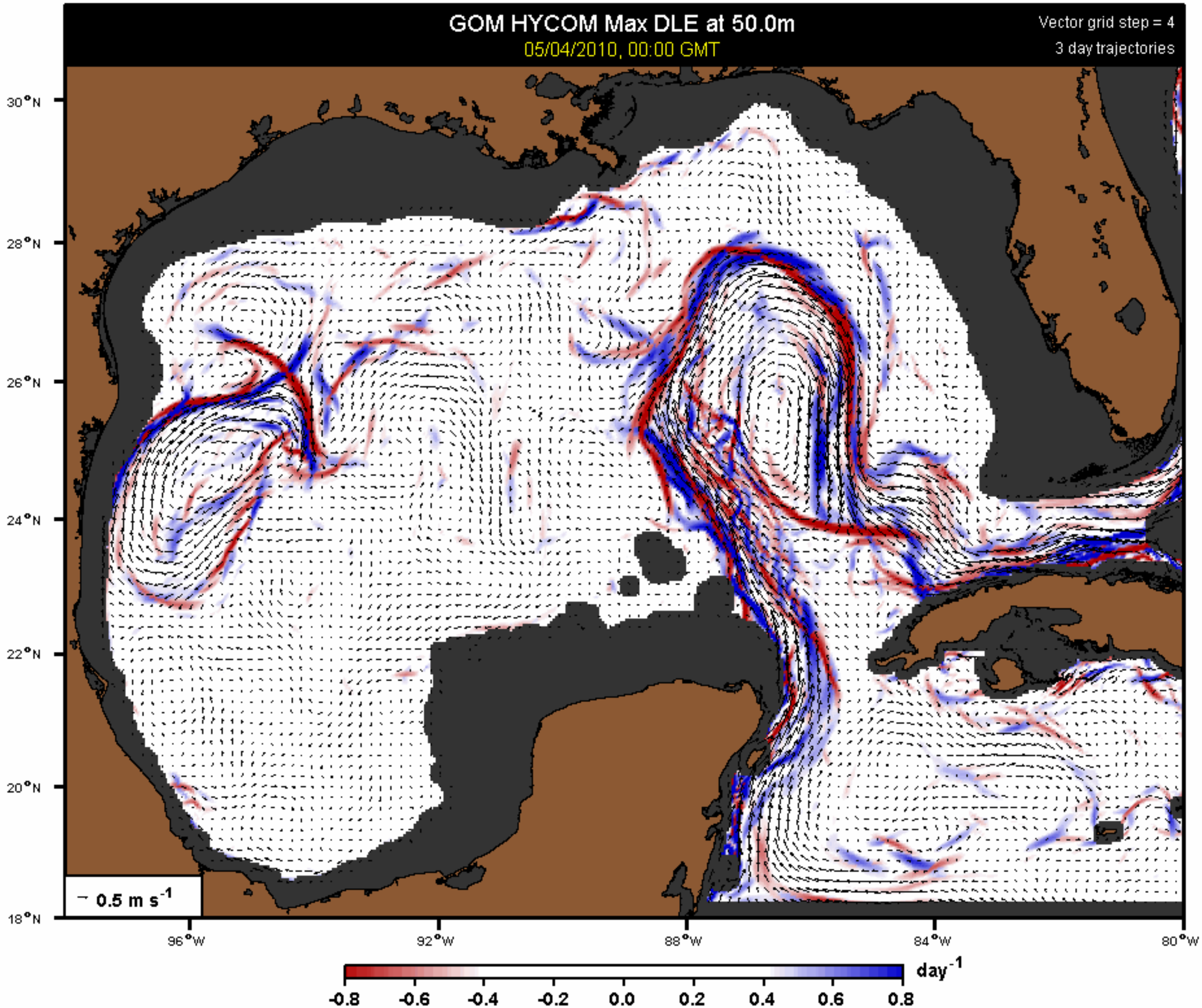
- Assumption: Separation characterized by robust DHT. Transport barriers between LCR and Loop Current
- ***Applicable at any depth***
- Science Questions
  - Separation baroclinic or barotropic?
  - Quantify Separation/Reattachment?

The Model: GoM-HYCOM

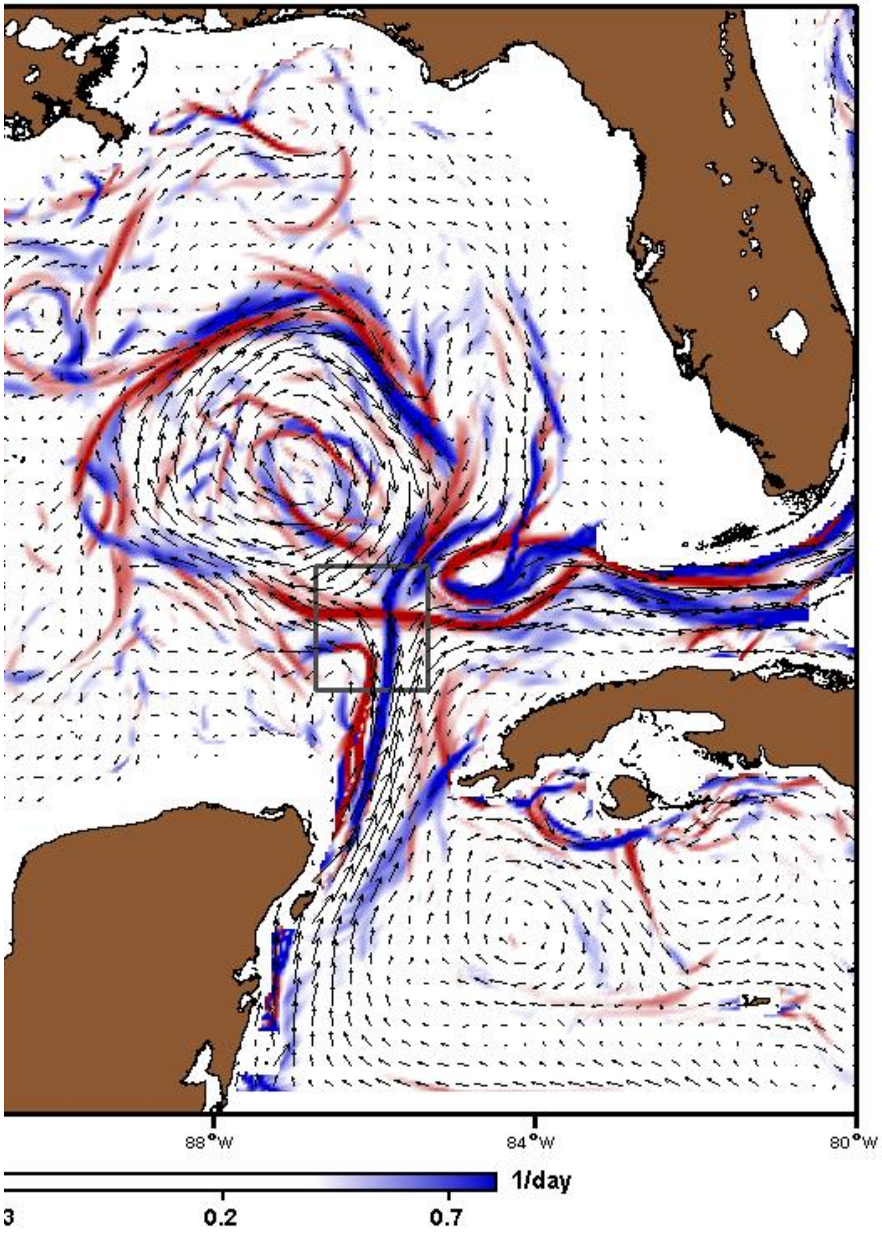


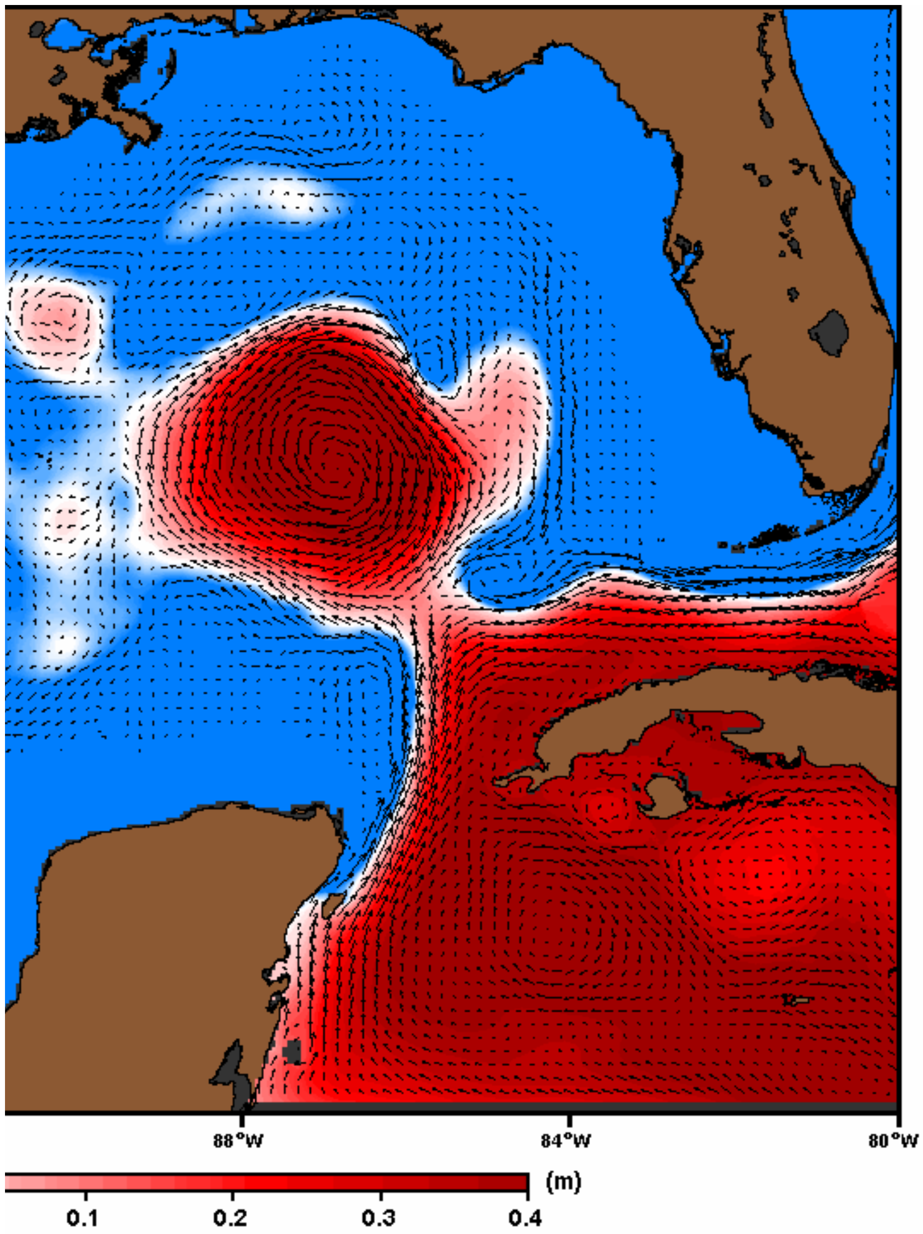
Gulf of Mexico – Hybrid Coordinate Ocean Model

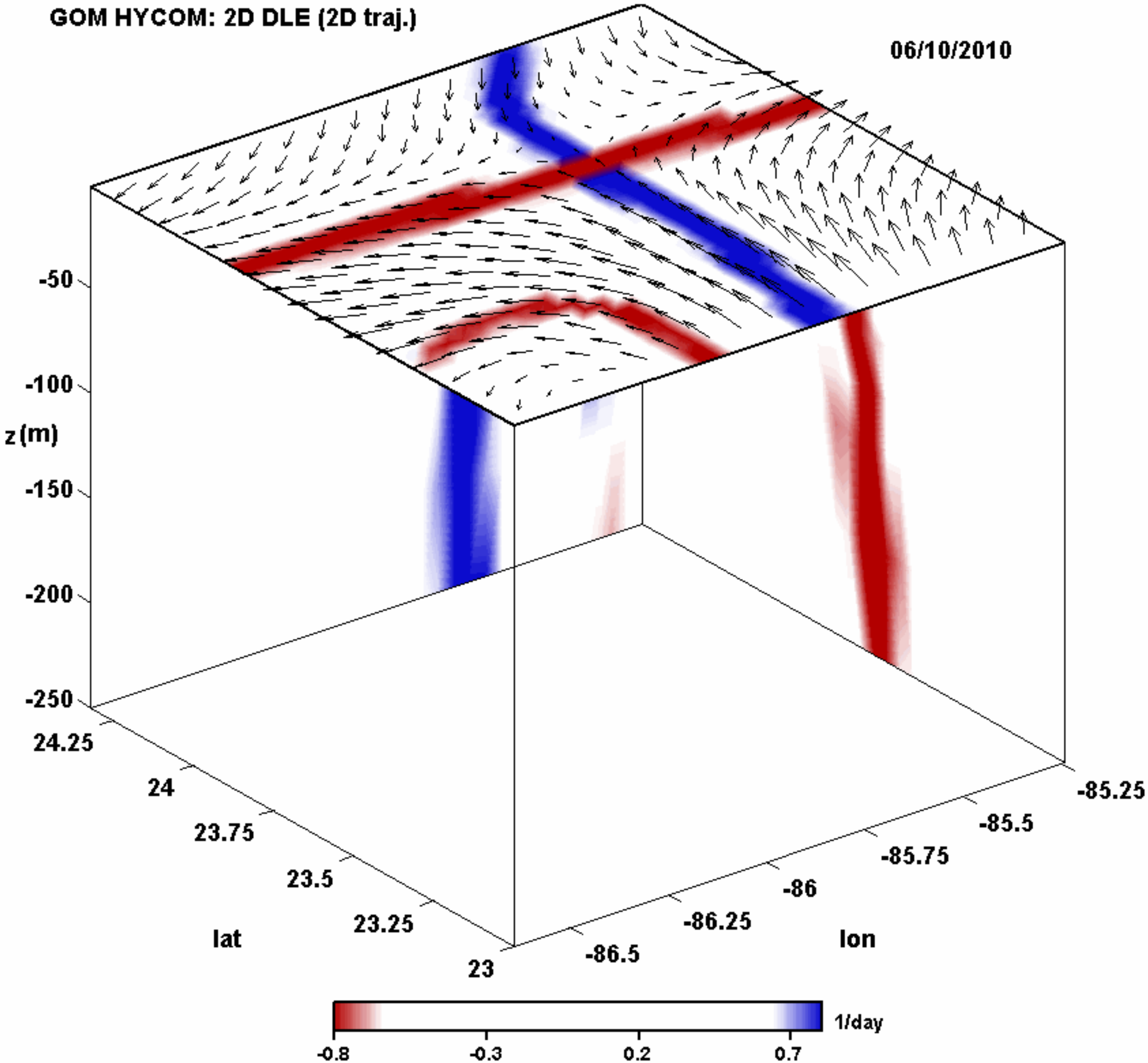
- Quasi-operationally run at NRL-Stennis
- 1/25 degree resolution (~ 4 km)
- Hourly archiving
- Boundary conditions from global HYCOM
- Daily data assimilation
- 5-day forecasts

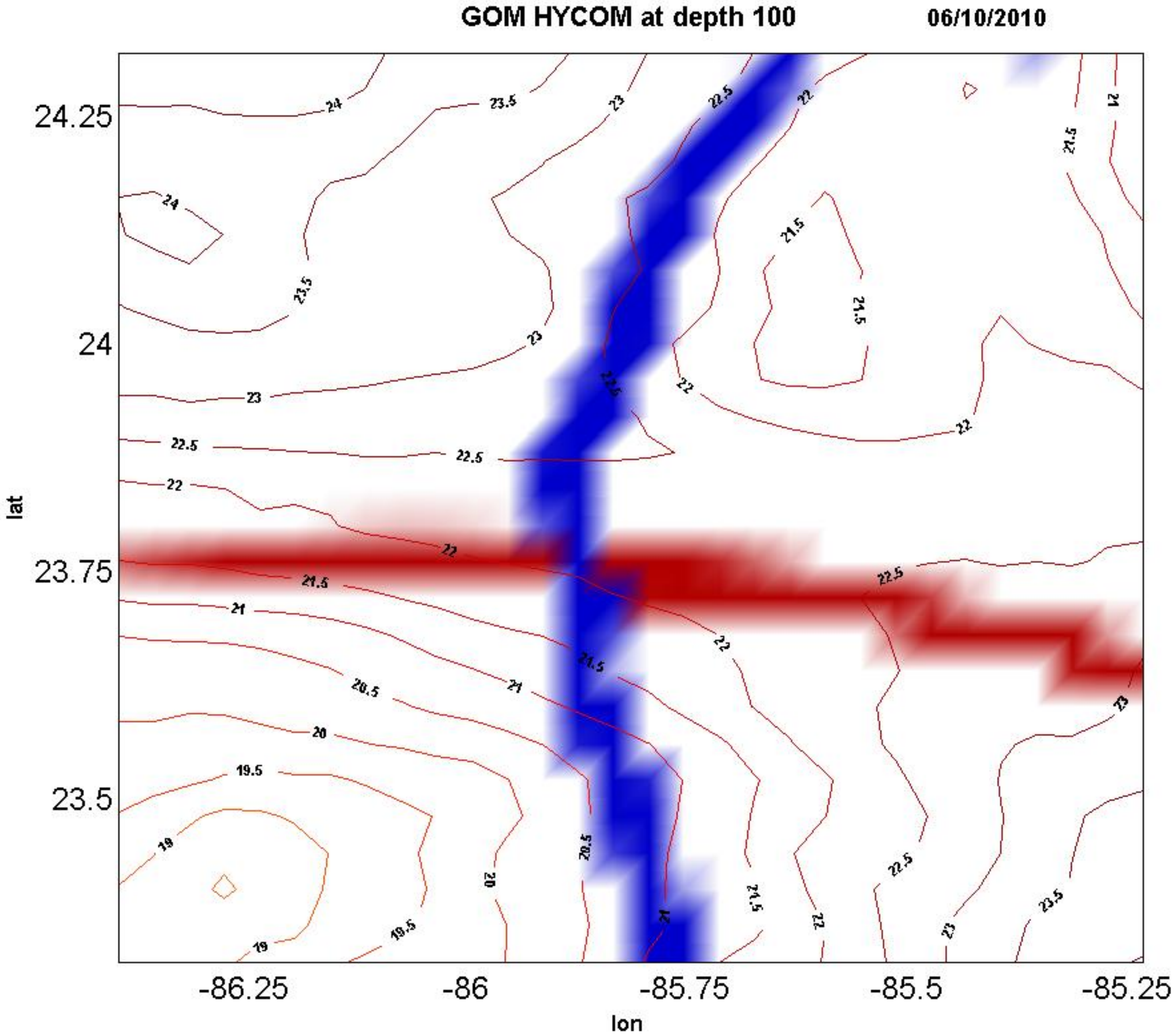

GOM HYCOM Max DLE at 50.0m
05/04/2010, 00:00 GMT
Vector grid step = 4
3 day trajectories
30°N
28°N
26°N
24°N
22°N
20°N
18°N
96°W
92°W
88°W
84°W
80°W
0.5 m s-1
day-1
-0.8
-0.6
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8

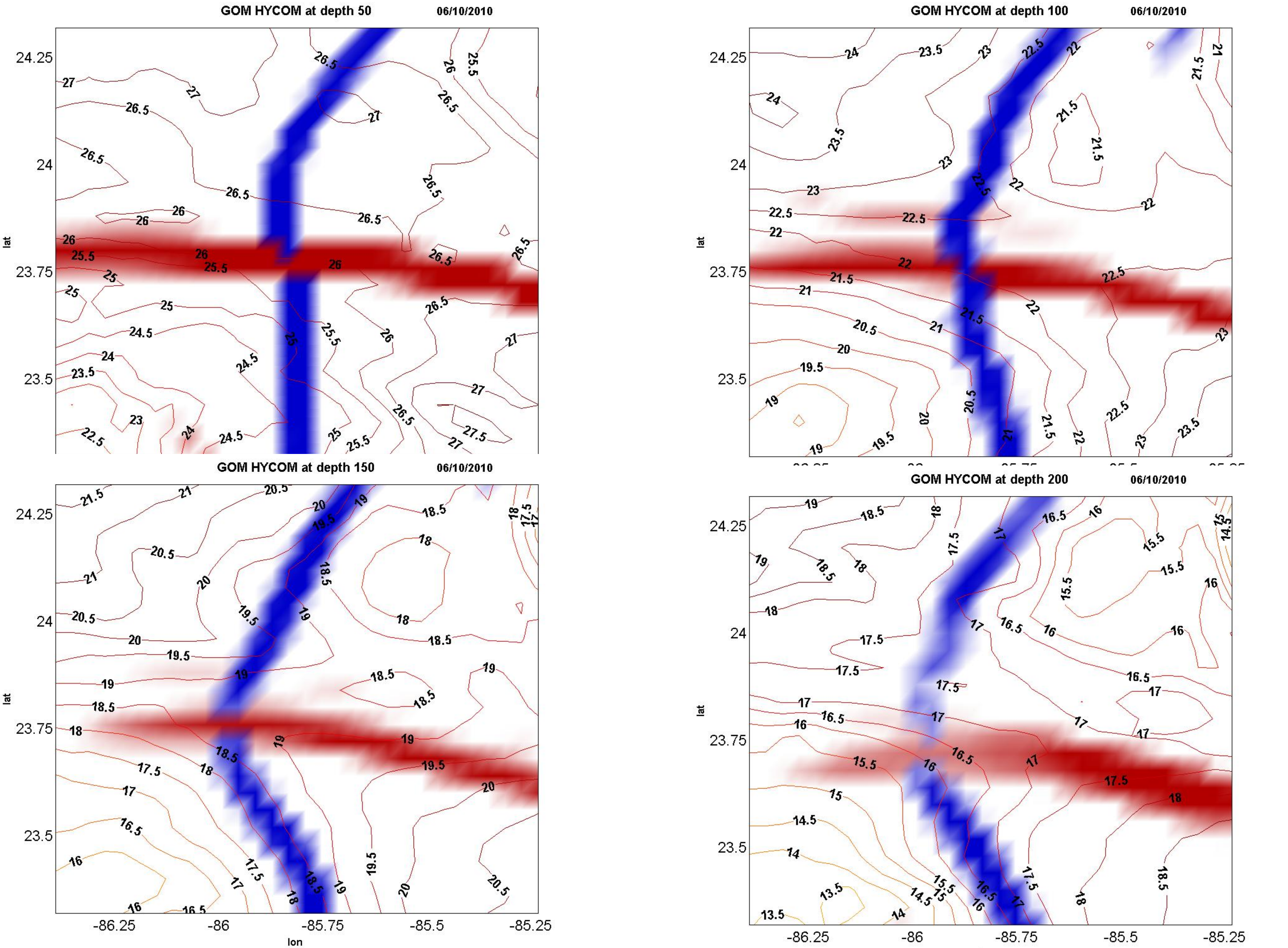
Eddy Franklin 10 June DLE & SSH





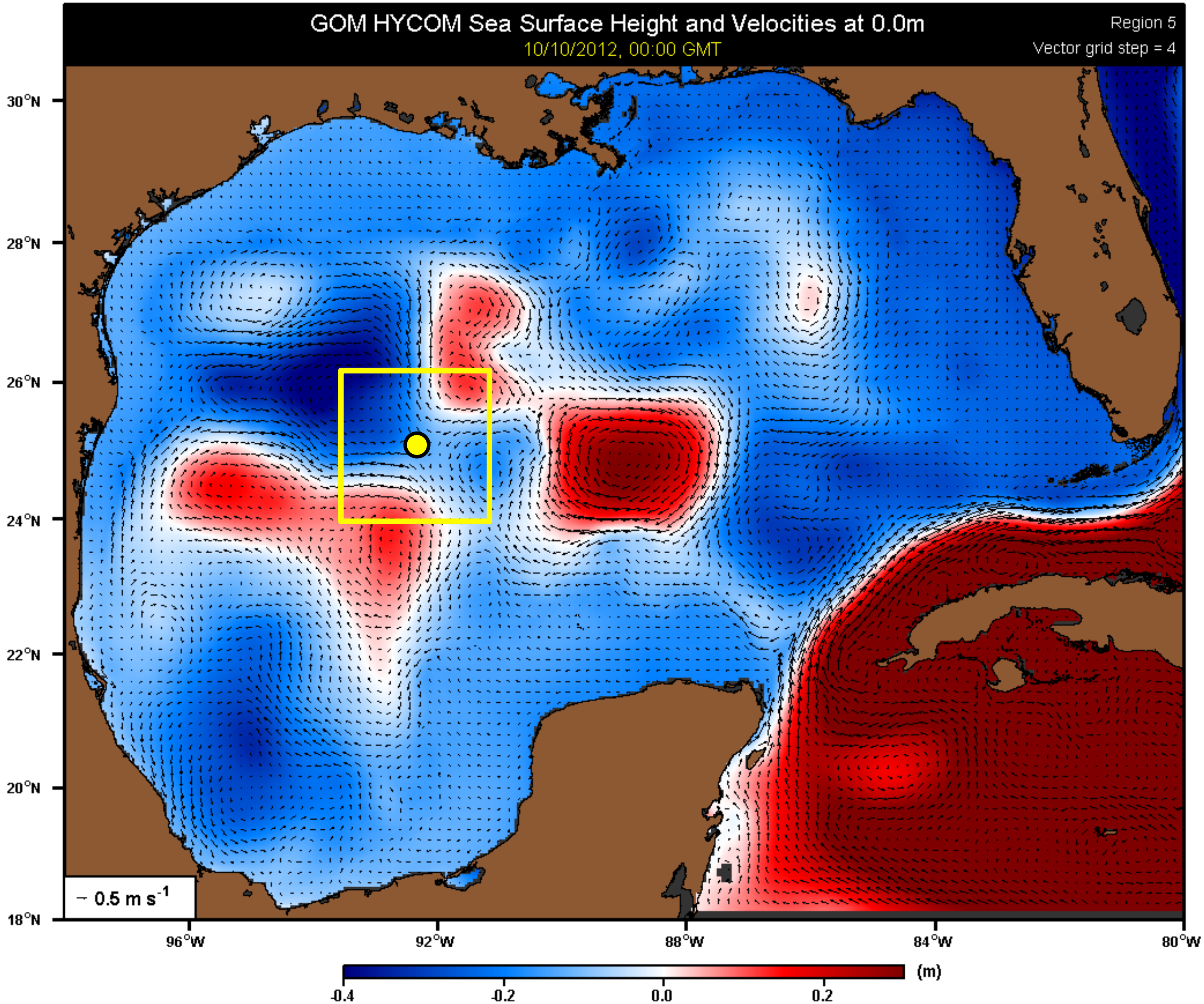

GOM HYCOM: 2D DLE (2D traj.)
06/10/2010
z (m)
-50
-100
-150
-200
-250
24.25
24
23.75
23.5
23.25
23
lat
-86.5
-86.25
-86
-85.75
-85.5
-85.25
lon
-0.8
-0.3
0.2
0.7
1/day


GOM HYCOM at depth 100
06/10/2010
lat
24.25
24
23.75
23.5
lon
-86.25
-86
-85.75
-85.5
-85.25

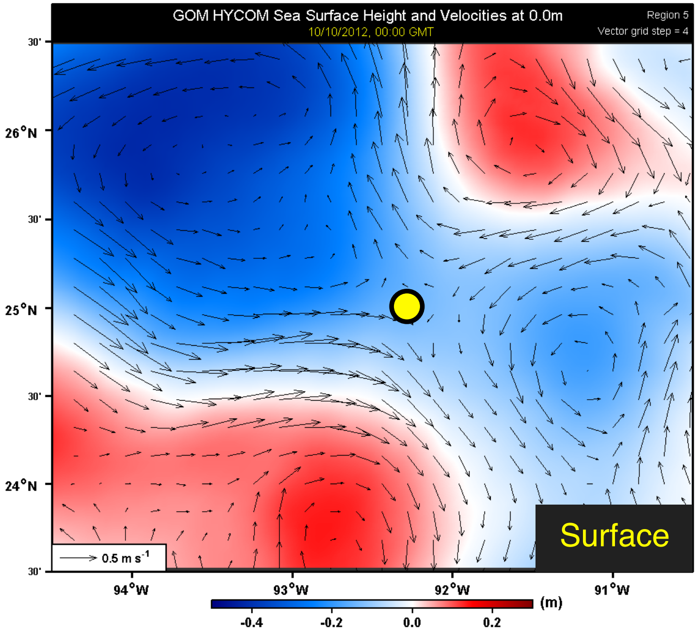

GOM HYCOM at depth 50
06/10/2010
GOM HYCOM at depth 100
06/10/2010
GOM HYCOM at depth 150
06/10/2010
GOM HYCOM at depth 200
06/10/2010
lat
lon
24.25
24
23.75
23.5
-86.25
-86
-85.75
-85.5
-85.25

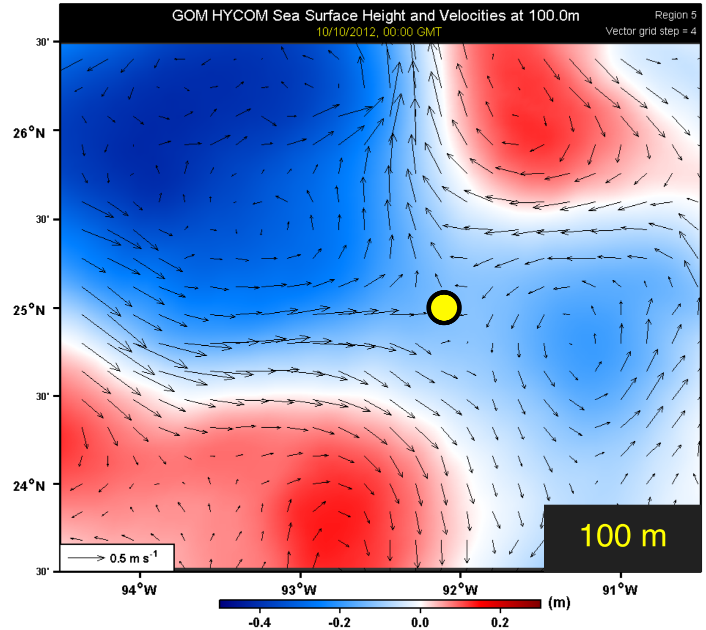
Western GoM Quadrupole

October 2012

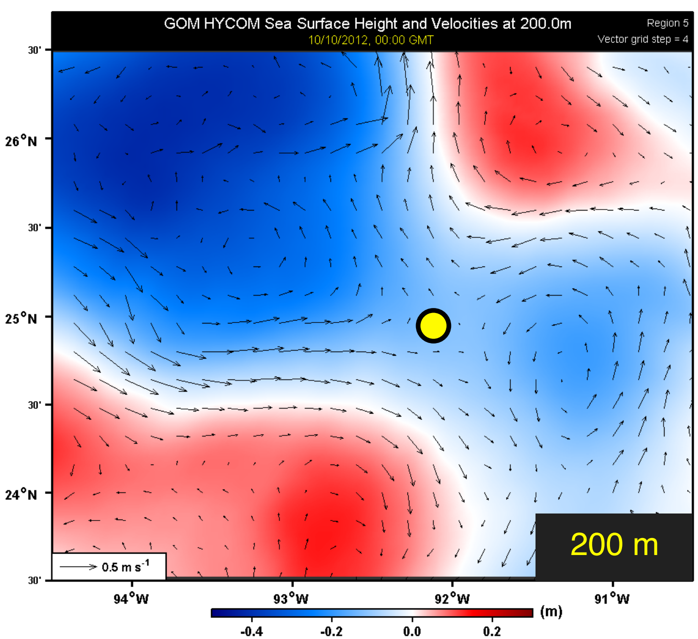


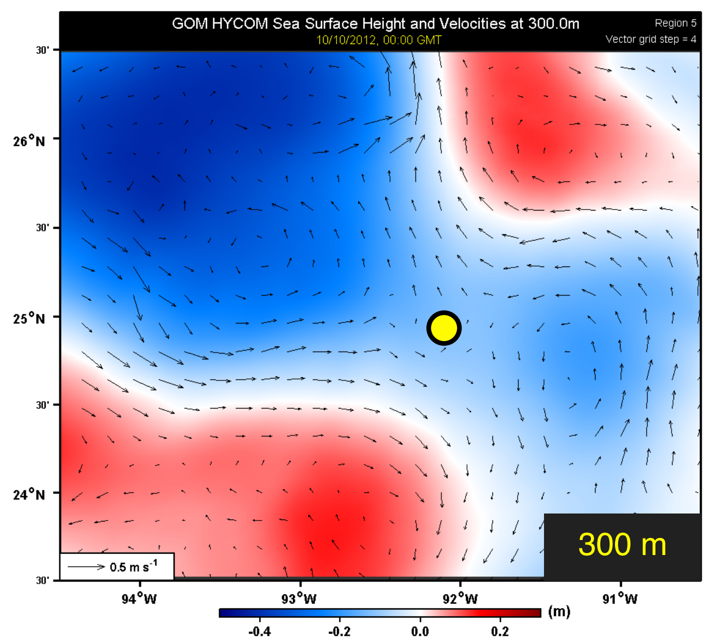
HYCOM 10 October 2012





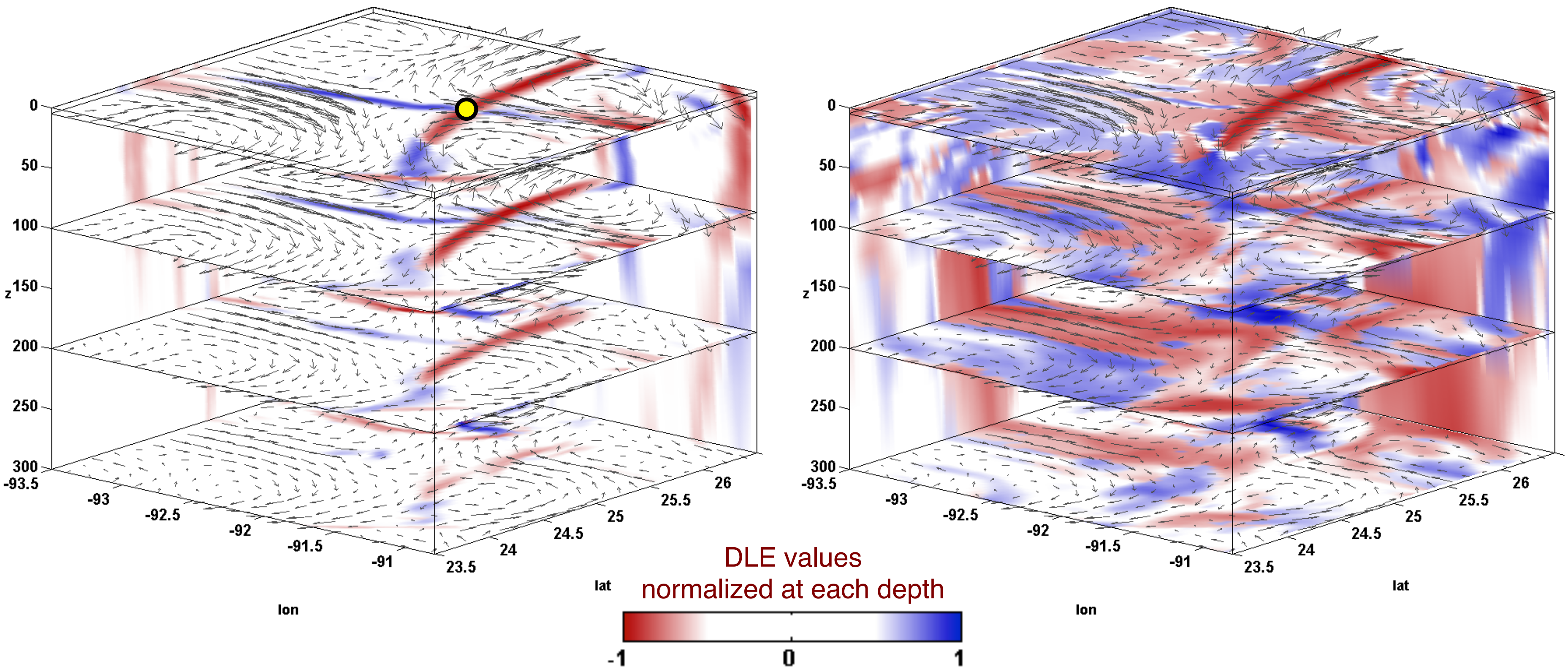
HYCOM 10 October 2012





2D DLE

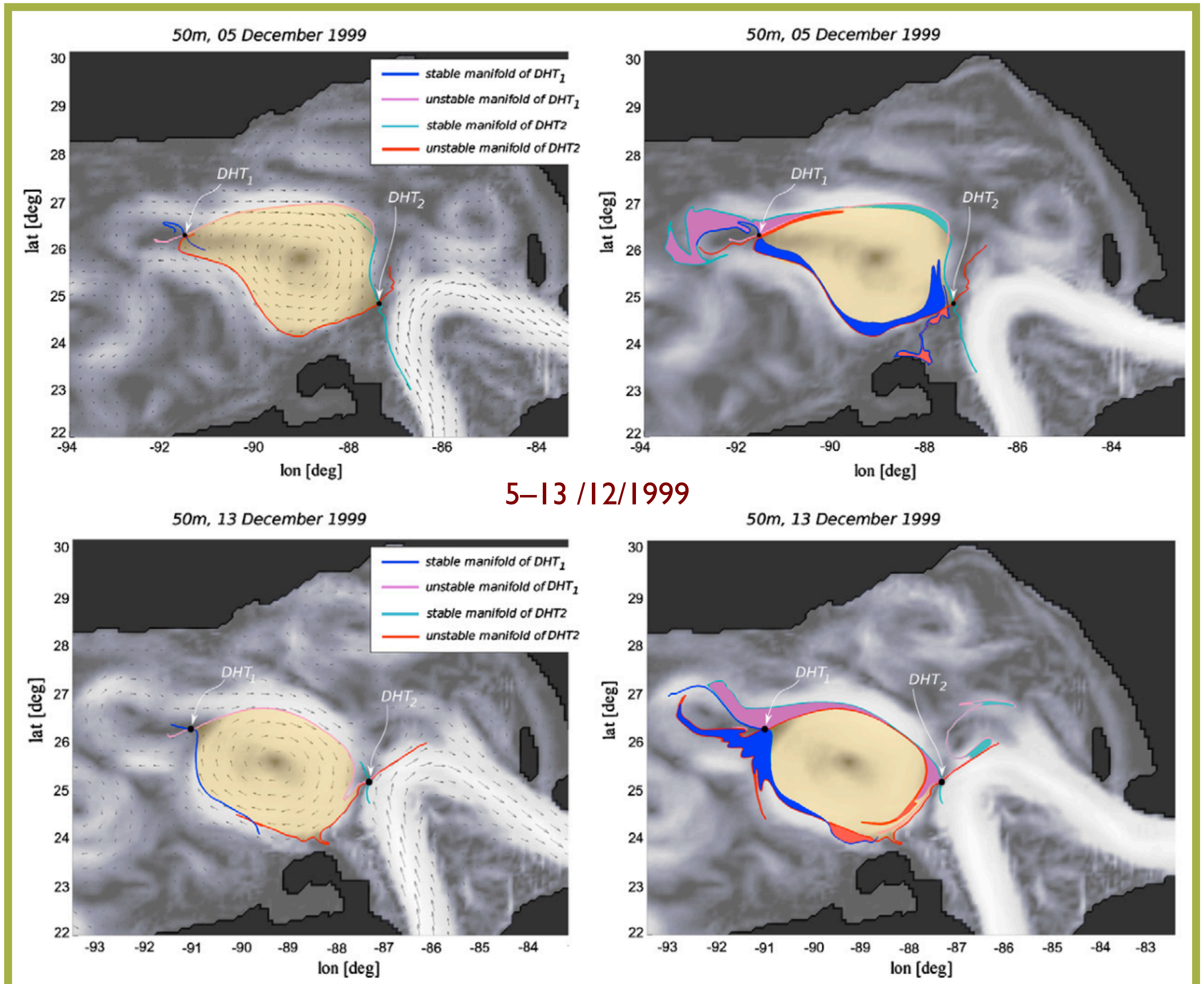
3D DLE



Is the 3D DLE revealing important new complexity, or is it simply obscuring the most important mixing structures?

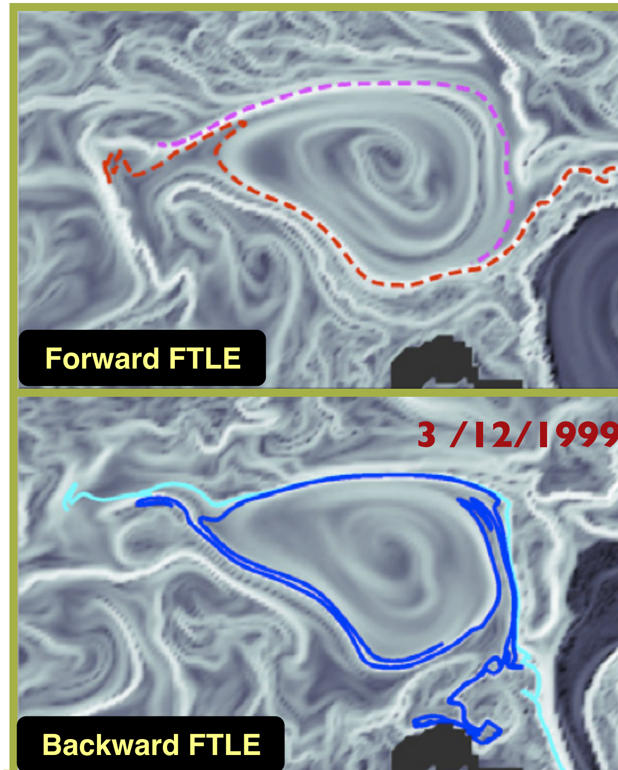
HYCOM 10 October 2012

Loop Current Ring Exchange

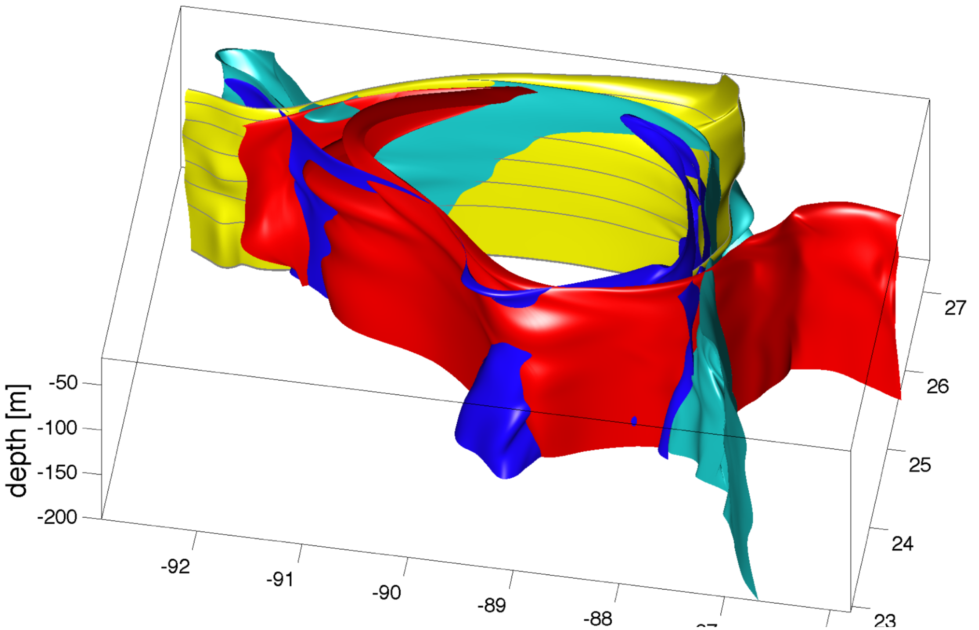


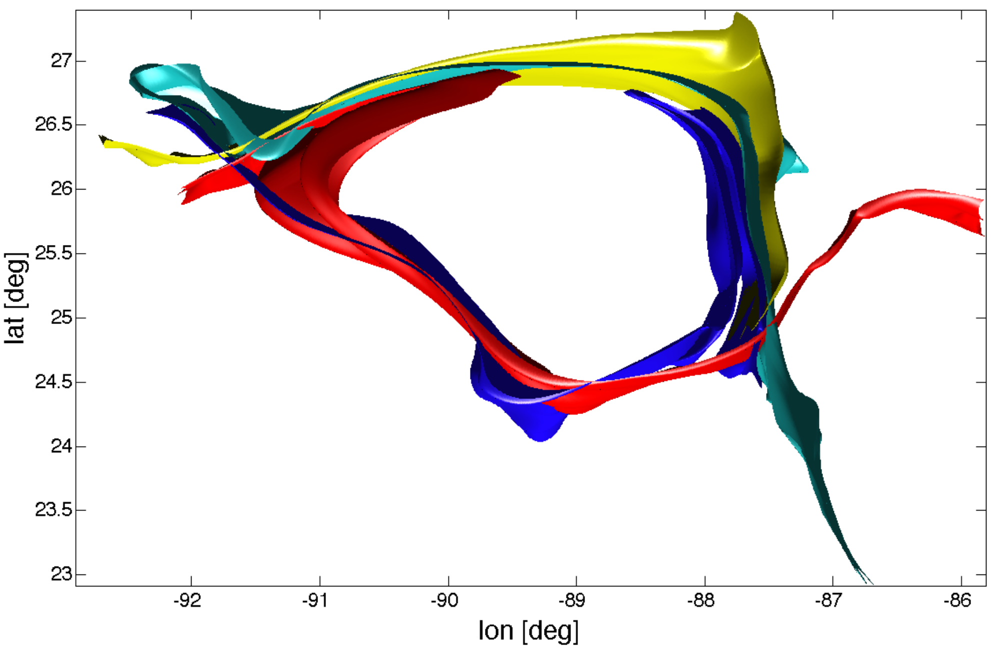
DHTs, manifolds, and lobes near the eddy

Eddy Juggernaut



How do manifolds compare with FTLEs ?





J.H. Bettencourt et al. / Ocean Mo

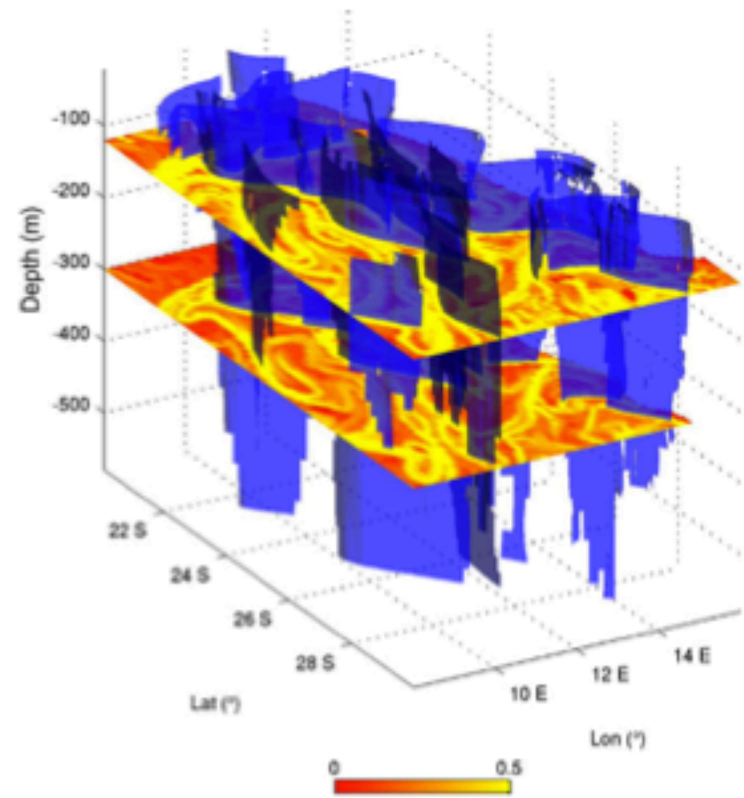


Fig. 3. Attracting LCSs (blue) for day 1 of the calculation period, together with horizontal slices of the backward FSLE field at 120 m and 300 m depths. The color bar refers to the color map of horizontal slices. The units of the color bar are day^{-1}. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Ocean 3D + 1 Hypotheses

- Ring formation is a barotropic process
- B&K, Bettencourt et al, Ring formation, QP in GoM all imply transport barriers are nearly vertical curtains
- Saddle points in temperature/density fields accompany DHTs?

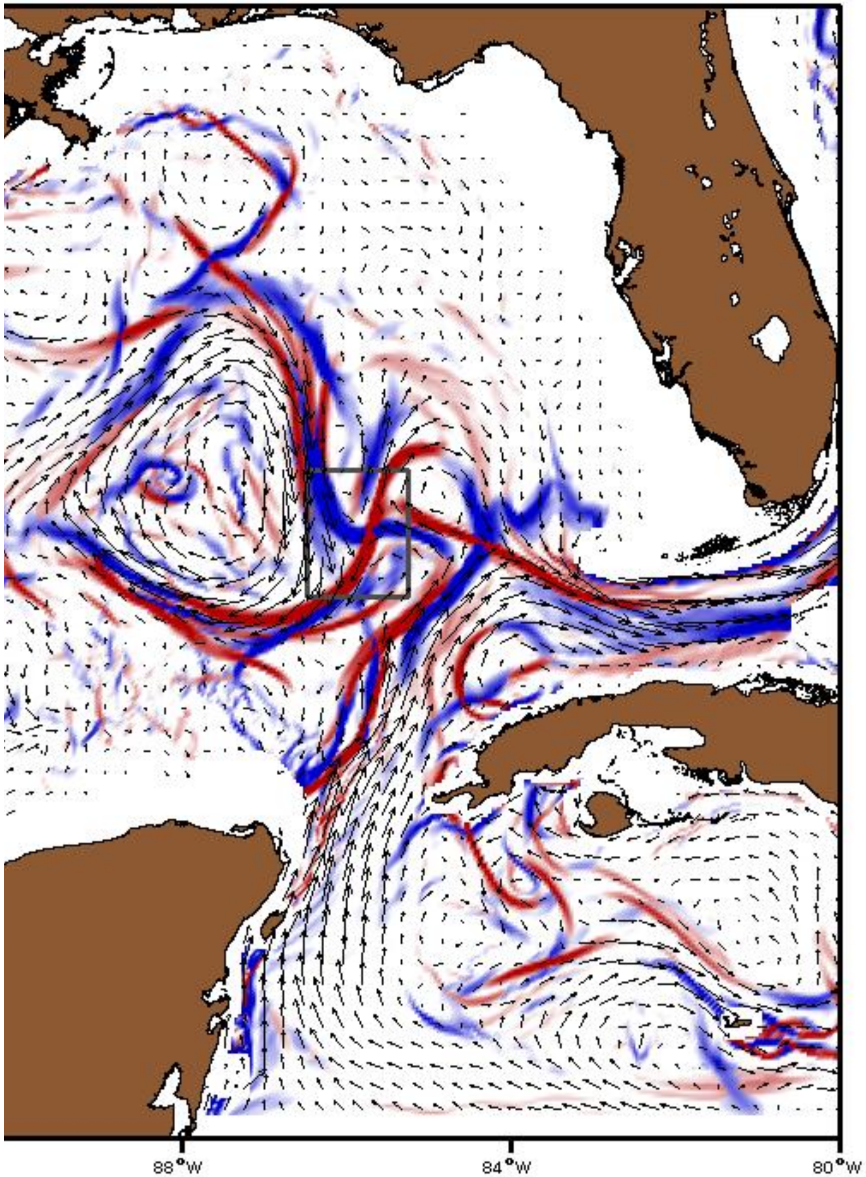
Now What?

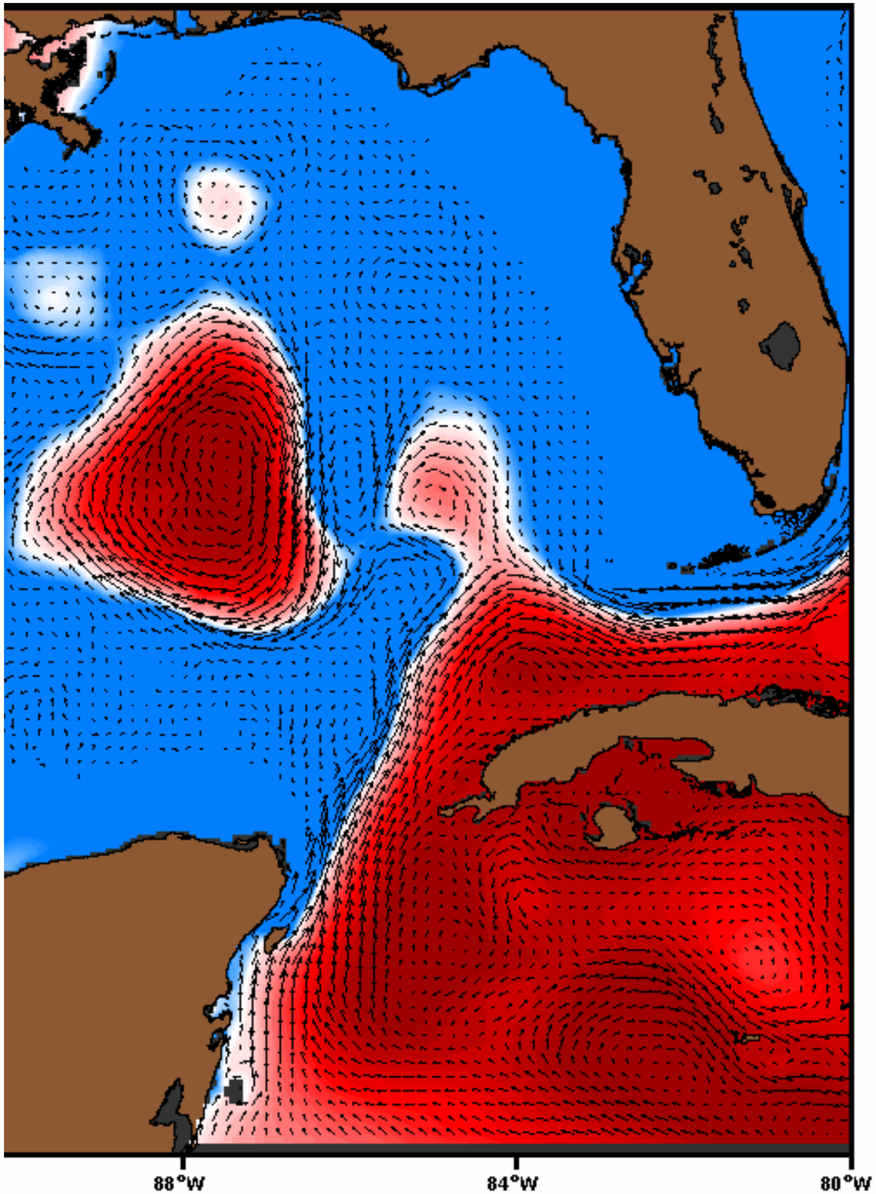
- Ring formation paper (Test hypotheses)
- Look at other oceanic scenarios with other LCS diagnostics (Test hypotheses)
- Sensitivity of transport barriers
- Dynamics of vertical transport barriers

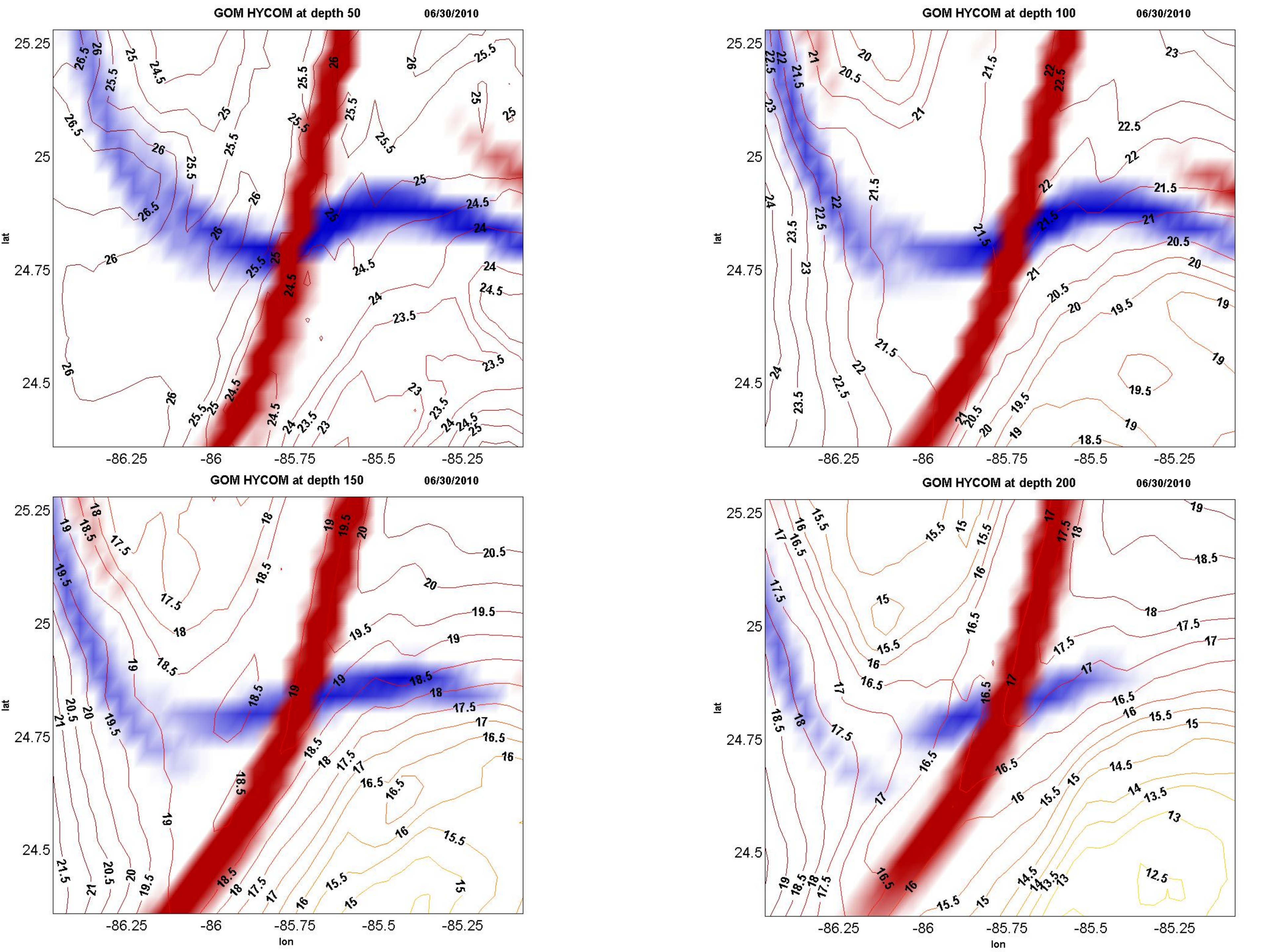
Work-in-Progress Sidebar

- Tedious calculation and data management issues
- Lot of physics going on near ring pinch-off
- Visualization, Visualization, Visualization

30 June DLE & SSH







GOM HYCOM at depth 50
06/30/2010
GOM HYCOM at depth 100
06/30/2010
GOM HYCOM at depth 150
06/30/2010
GOM HYCOM at depth 200
06/30/2010
lat
lon
25.25
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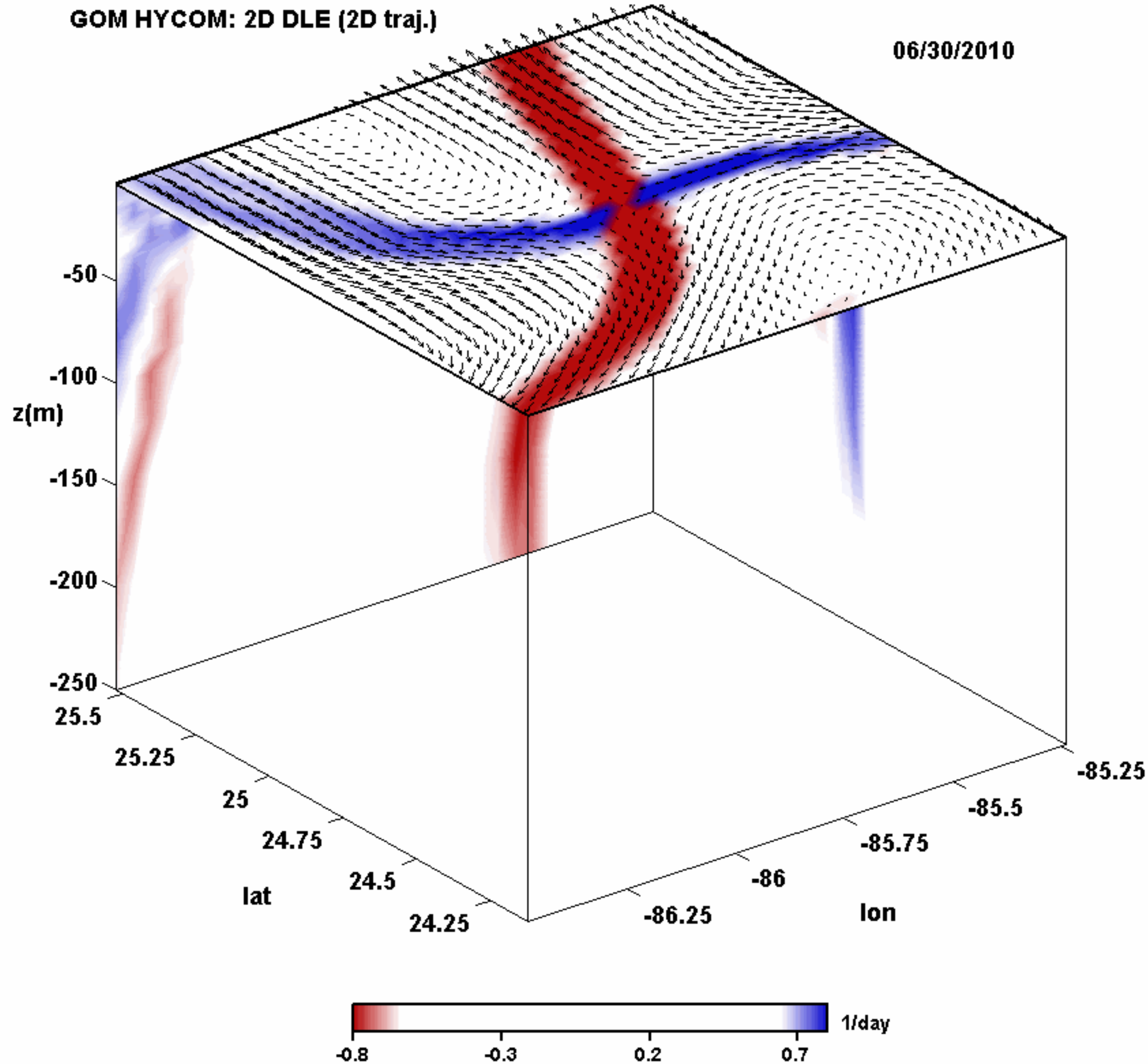
24.75
24.5
-86.25
-86
-85.75
-85.5
-85.25



GOM HYCOM: 2D DLE (2D traj.)
06/30/2010
z(m)
-50
-100
-150
-200
-250
25.5
25.25
25
24.75
24.5
24.25
lat
-86.25
-86
-85.75
-85.5
-85.25
lon
1/day
-0.8
-0.3
0.2
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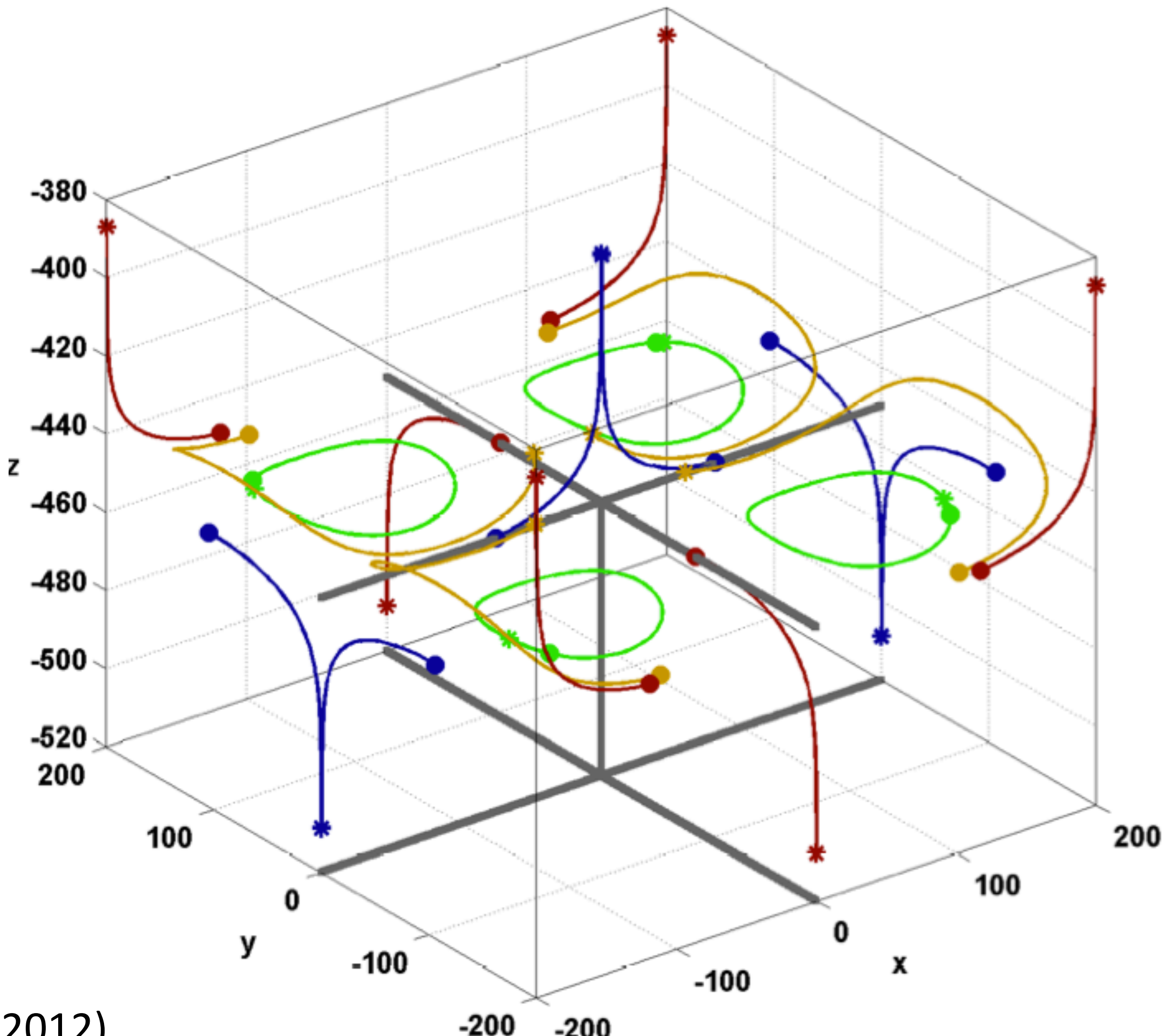
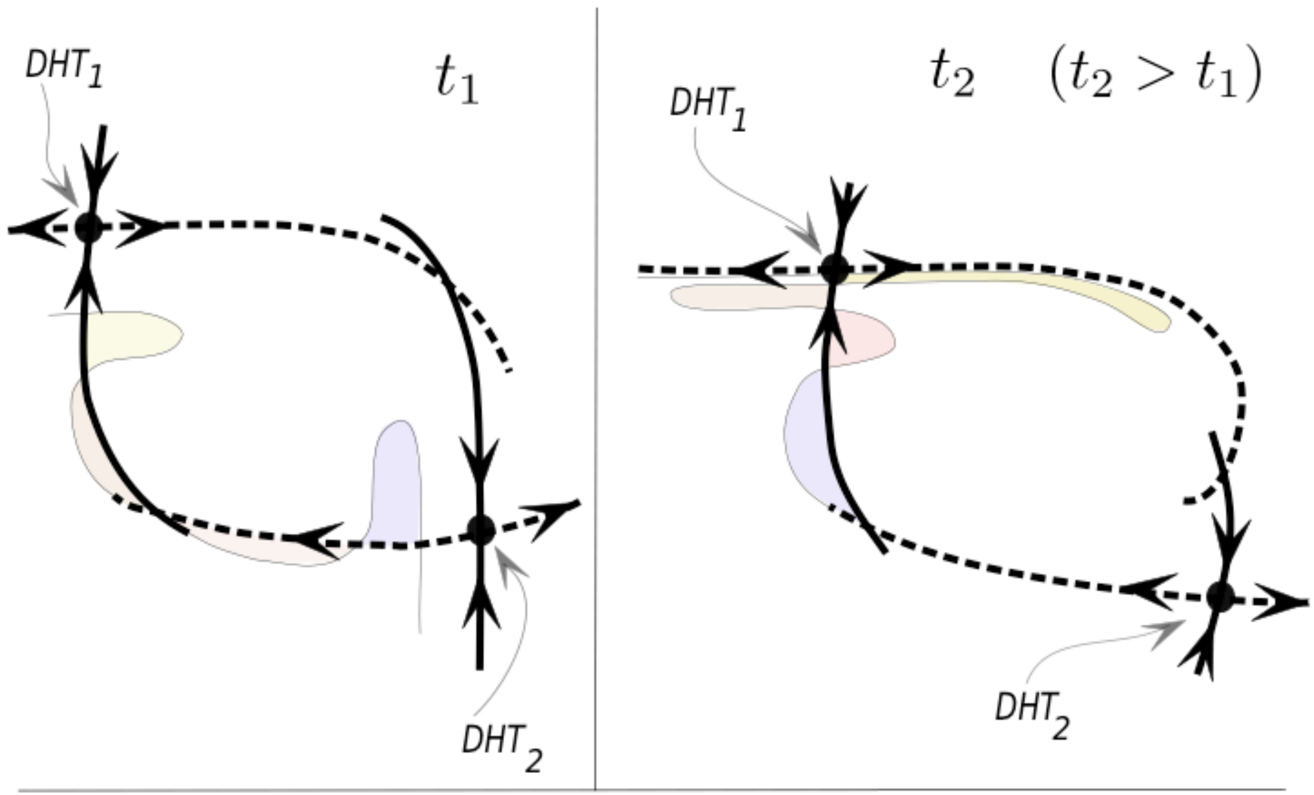
I have some theoretical stuff?

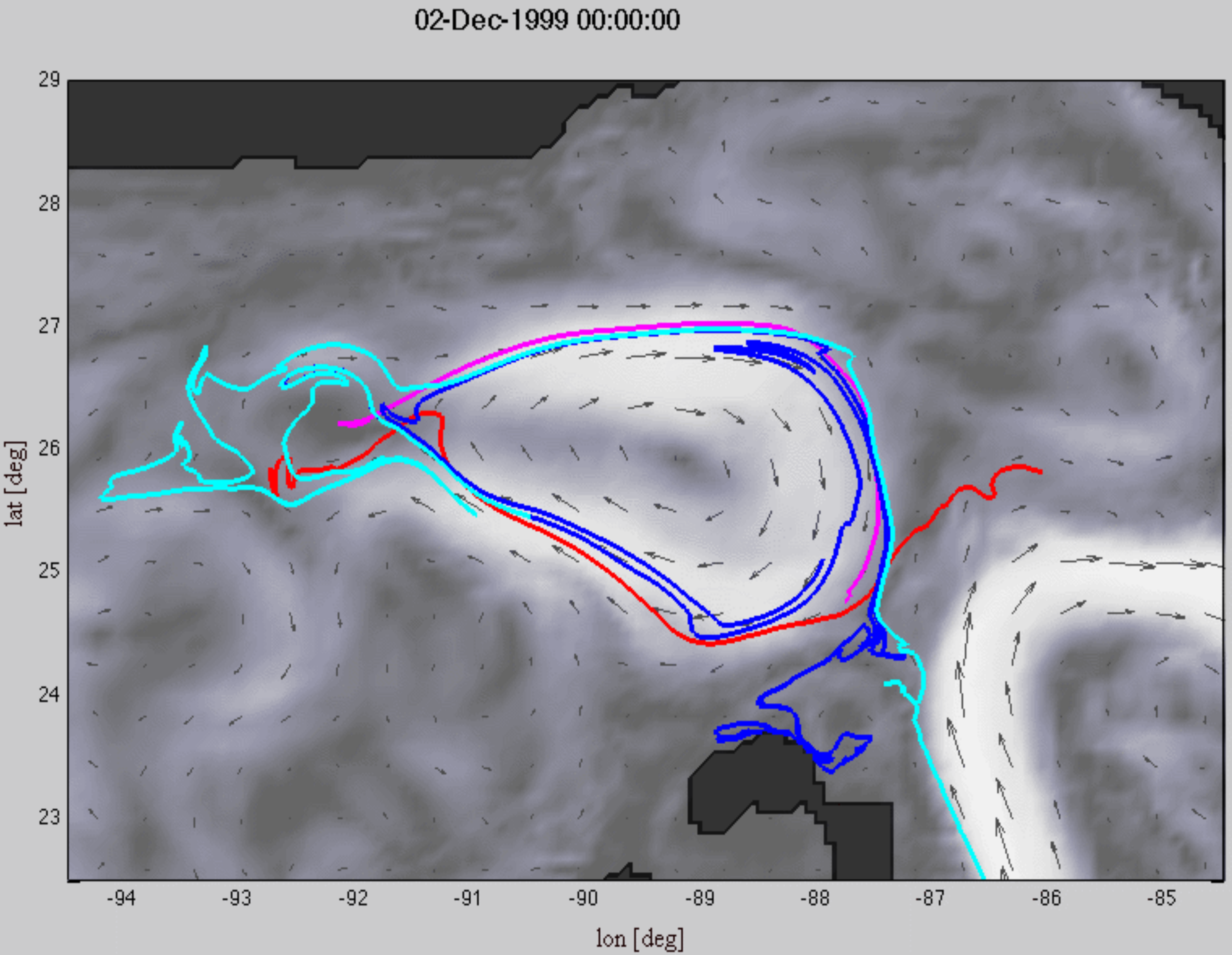
Trajectories in QP*



*Sulman et al (2012)

Dynamical Systems Approach





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24
23
lat [deg]
-94
-93
-92
-91
-90
-89
-88
-87
-86
-85
lon [deg]

Branicki & Kirwan (IJES 2010)

- FTLE located DHTs
- 1D manifolds using 2D velocities (Ide et al, 2002) from 0 to 250m for eddy Juggernaut
- Stitched 1D manifolds into 2D material surfaces

B&K Conclude

- Material surfaces stitched from 2D analysis revealed coherent lobes with depth
- Material surfaces drop nearly vertically. No evidence of eddy lens structure
- Net inflow at bottom, outflow at top

Realistic, or artifacts of stitching and/or data assimilation?