3D+1 Transport in a Quadrupole: Dynamics

- Henry Chang
- Helga S. Huntley
- A. D. Kirwan, Jr

Special thanks to

- Stefan for hosting this meeting
- CARTHE CONSORTIUM for ADVANCED RESEARCH on TRANSPORT of HYDROCARBON in the ENVIRONMENT
- Bruce Lipphardt & Mohamed Sulman

Outline

- Review of recent observations - motivation
- Model physics
- Time dependent “perturbation” solution
- Discussion

Henry will give more detail on vertical structure of transport boundaries & mixing from composite flow

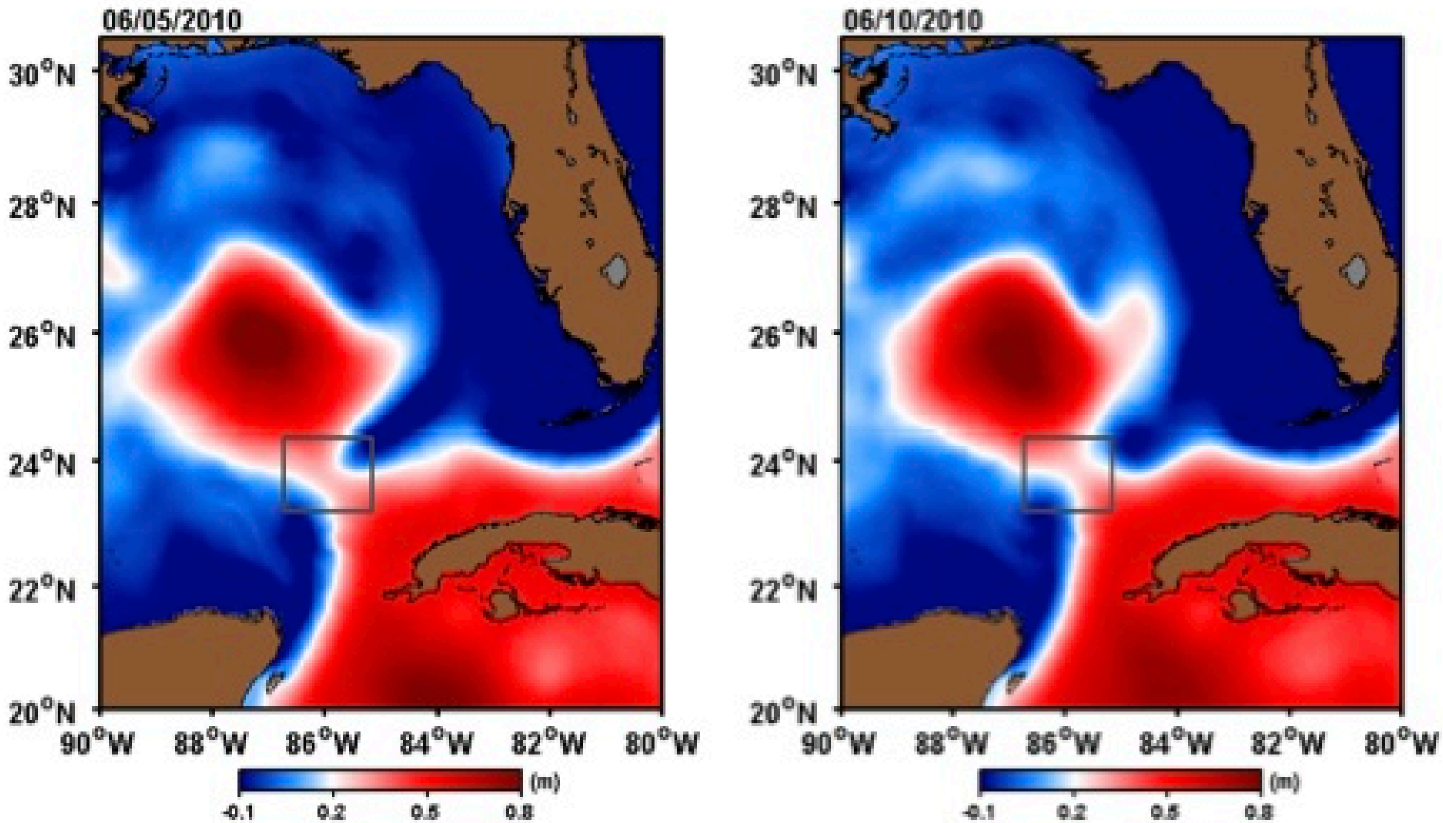
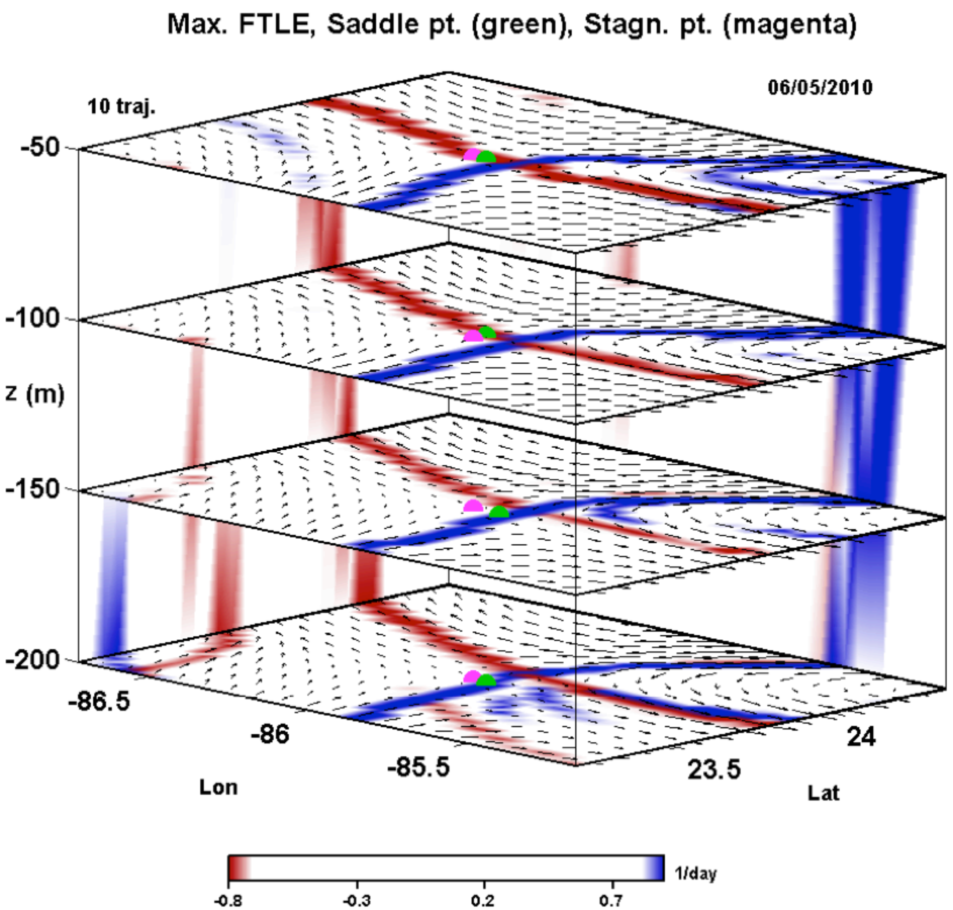


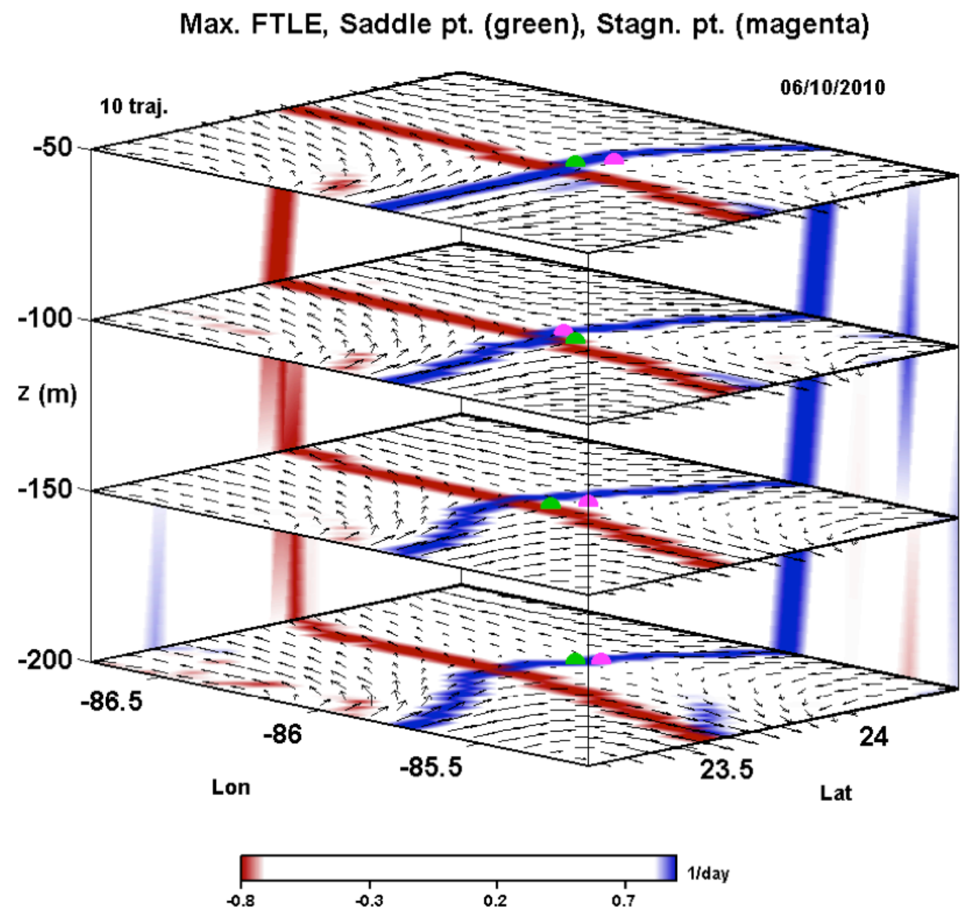
Fig. 1. Sea surface height anomaly (SSHa) of the Loop Current in the Gulf of Mexico during a ring formation event in June 2010. The left panel shows SSHa on 5 June and the right panel shows SSHa five days later on 10 June. The saddle region in the small box is the focus of this study.

Sulman et al., NPG 20, 883-892, 2013

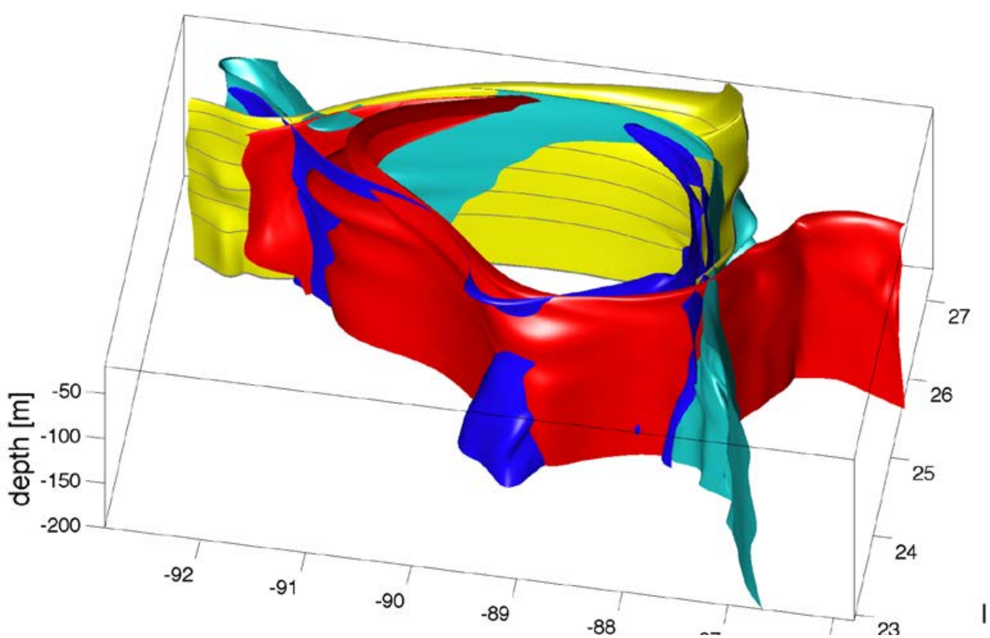
Ocean 3D +1 Mixing Boundaries

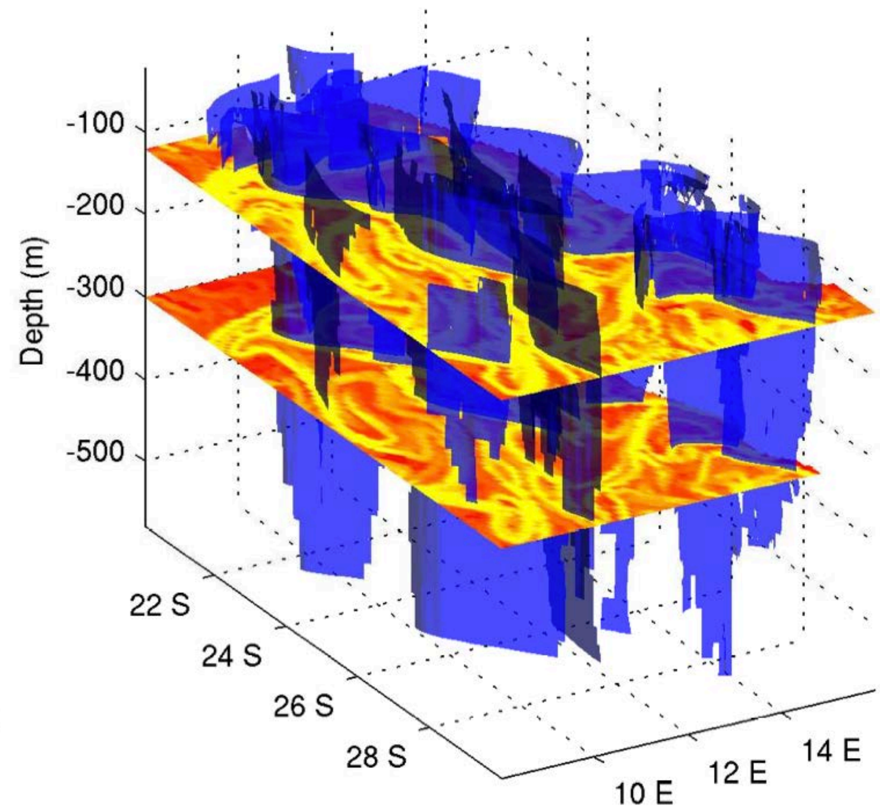
5 June 2010 ← 25 Kilometers → 10 June 2010





Ocean 3D Mixing Boundaries: Curtains?





CUPOM: Loop Current Ring Juggernaut
Branicki and Kirwan (2010)

ROMS: Benguela Current
Bettencourt et al. (2012)

OTOH

Koopmanism applied to the Gulf Stream

by Fabregat, Poje, and Mezić

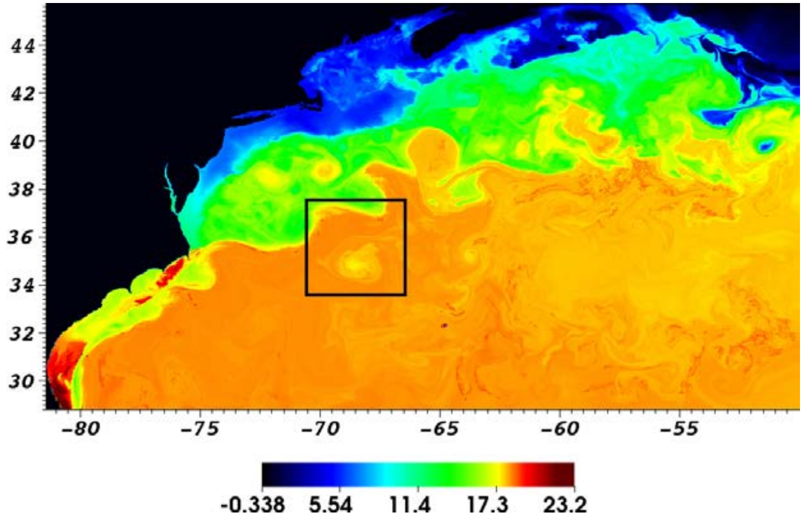


Figure: North Atlantic HYCOM numerical simulation snapshot showing temperature at isopycnal 12 for day 80. The black box shows the region of interest defined by initial extent of the Lagrangian particle patch.

alinity fields for days 70, 75, 80, 85, 90 and 95 at isopycnal 12.

(d)

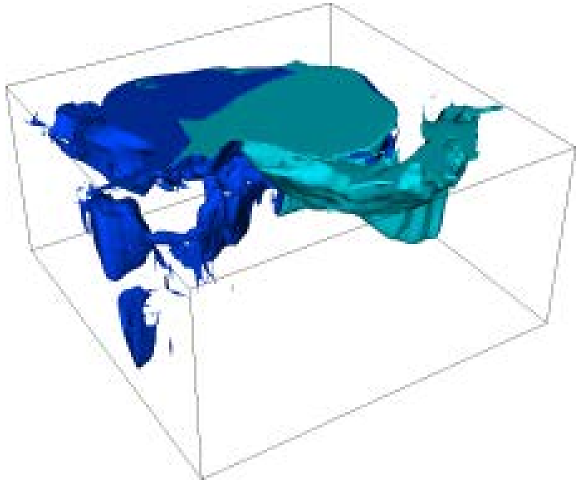


Figure: Days 73, 78, 84 and 90.

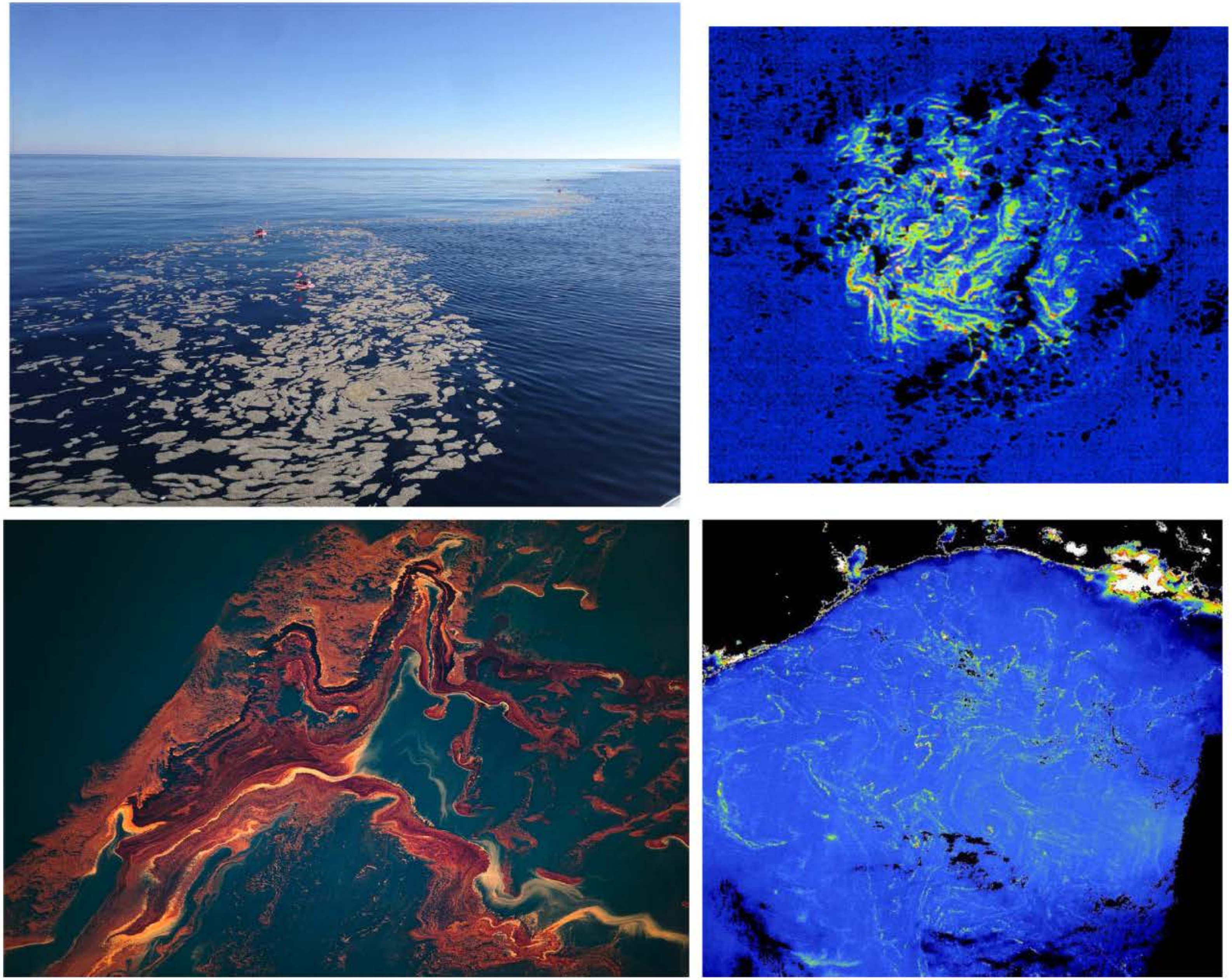


Figure 1. (top left) A cluster of surface drifters and Sargassum along a front in the northeastern Gulf of Mexico. (Image courtesy Tamay Özgökmen.) (bottom left) Accumulating oil from the *Deepwater Horizon* oil spill in the Gulf of Mexico. (Photo by Daniel Beltrá.) (top right) Patch of sargassum off the east coast of U.S., as observed by Envisat. (Image courtesy ESA.) (bottom right) Streaks of sargassum in the western Gulf of Mexico, as observed by MERIS. (Image courtesy ESA.)

Clusters, deformation, and dilation: Diagnostics for material accumulation regions- H. S. Huntley et al, in press, JGR 2015

$$\text{SVs of flow map} = \mu_1,\ \mu_2$$

$$\text{dilation rate } \Delta \equiv \frac{\log(\mu_1\mu_2)}{T} = \left(\frac{\log\mu_1}{T} + \frac{\log\mu_2}{T}\right)$$

$$\text{stretch rate } \Sigma \equiv \frac{(\mu_1/\mu_2)}{T} = \left(\frac{\log\mu_1}{T} - \frac{\log\mu_2}{T}\right)$$

$$\text{FTLE} \equiv \frac{\log\mu_1}{T} = \frac{\Delta+\Sigma}{2}$$

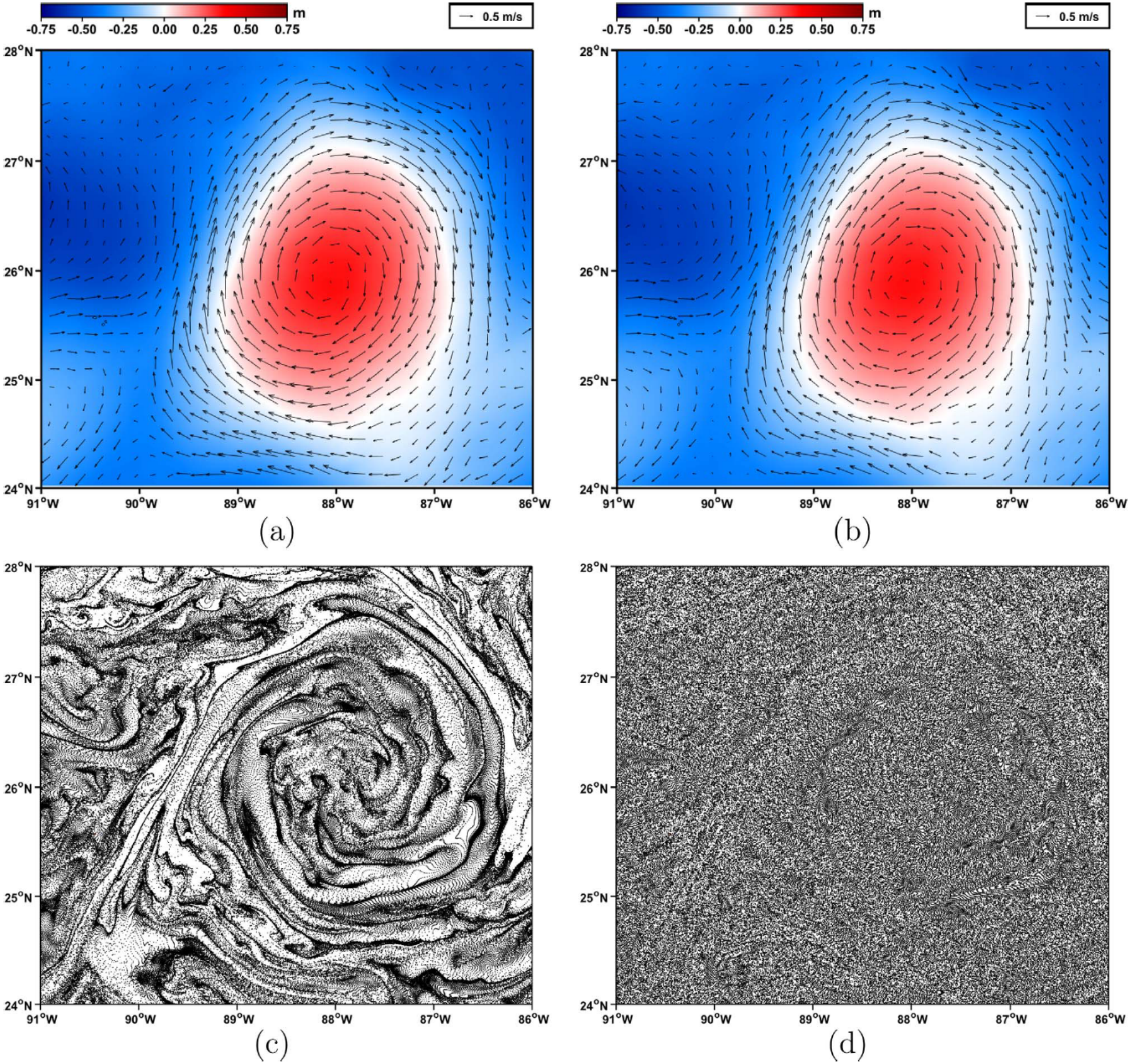


Figure 4. (top) Sea surface height anomaly on 20 July 2012 00:00 with (a) the full velocity field and (b) the geostrophic velocity field. (bottom) Positions of particles launched on a 1 grid on 15 July 2012 00:00, after 10 days of integration of (c) the full velocity field and (d) the geostrophic velocity field.

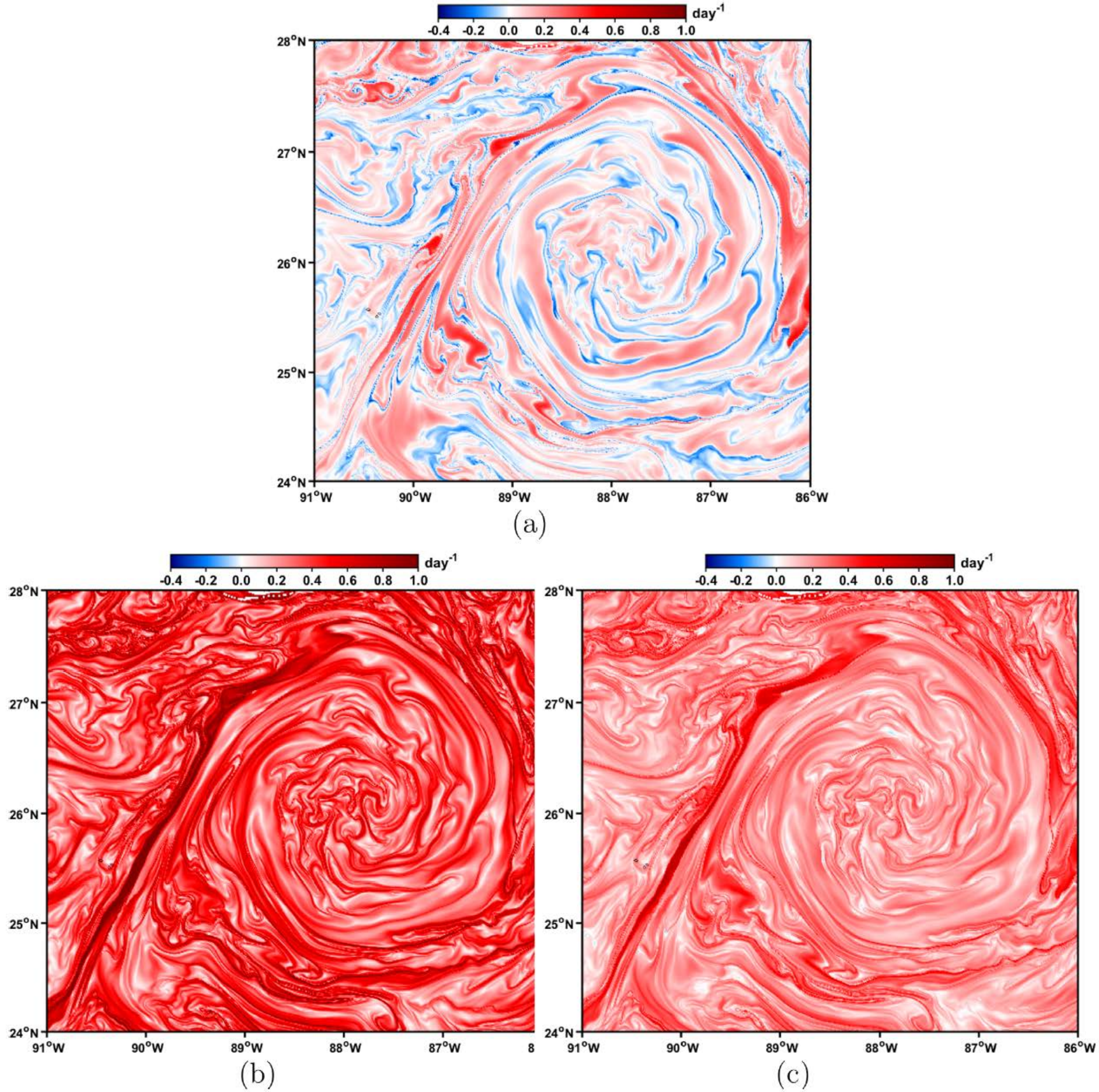


Figure 7. (a) Dilation rate, (b) stretch rate, and (c) FTLE for the time period 15 July 2012 00:00 to 25 July 2012 00:00, computed on a 1 km grid.

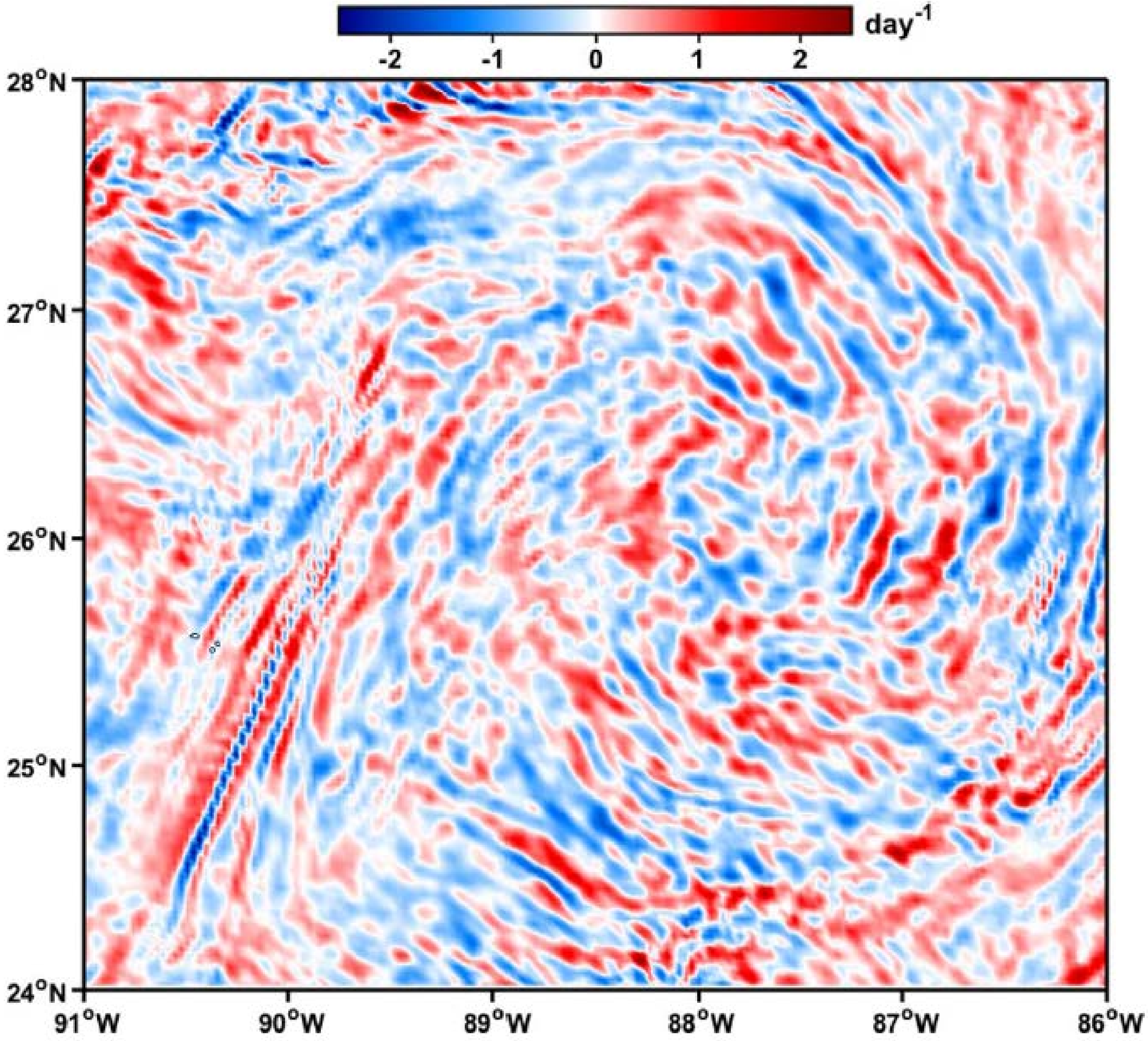


Figure 10. Instantaneous divergence on 25 July 2012 00:00.

Motivation

- Pretty pictures – but where's the physics?
- Transport feature predictions - dynamical balances in evolution of pathways?
- 2D+1 velocities from assimilating models - role of horizontal divergence/vertical velocities & superinertial oscillations?
- Visualization of 2D+1 transport boundaries in a 3D+1 ocean

Physics & Dynamical Systems

- ABC flow – Haller – Physica D 2001
- ABC & Kinematic QP – Sulman et al – Physica D 2013
- Steady 3D Ekman flow – Pratt et al – JFM 2014

Here: QP Model

- Time dependent, 3D incompressible, stratified linear Euler equations on f plane
- Pressure determined by eigenvalue problem
- Periodic in x, y space & t
- Near-inertial oscillations not seen in 2D+1 models

Euler Equations

$$\frac{\partial u}{\partial t} - fv = -\rho_0^{-1}\frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t} + fu = -\rho_0^{-1}\frac{\partial p}{\partial y}$$

$$\frac{\partial w}{\partial t} = \rho_0^{-1}\left(\rho g - \frac{\partial p}{\partial z}\right)$$

$$\frac{\partial \rho}{\partial t} = \left(\frac{\rho_0 N^2}{g}\right) w$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Solution

$$u = -lA_g(z) \sin kx \cos ly - \sum_j \left(A_j(z) \frac{\partial \Psi_j(x,y,t)}{\partial y} - \frac{dB_j(z)}{dz} \frac{\partial \Phi_j(x,y,t)}{\partial x} \right)$$

$$v = kA_g(z) \cos kx \sin ly + \sum_j \left(A_j(z) \frac{\partial \Psi_j(x,y,t)}{\partial x} + \frac{dB_j(z)}{dz} \frac{\partial \Phi_j(x,y,t)}{\partial y} \right)$$

$$w = -\sum_j B_j(z) \nabla_h^2 \Phi_j(x,y,t) = \sum_j \left(k_j^2 + l_j^2\right) B_j(z) \Phi_j(x,y,t)$$

$$p = p_0(z) + p_g(z) \sin kx \sin ly + \sum_j p_j(z) \Psi_j(x,y,t)$$

$$\rho = \rho_0(z) + \rho_g(z) \sin kx \sin ly + \sum_j \rho_j(z) \Psi_j(x,y,t)$$

Dynamic Constraints

$$(f^2 - \omega_j^2)A_j = f\rho_0^{-1}p_j, \quad \frac{dB_j}{dz} = \left(\frac{\omega_j}{f}\right)A_j$$

$$(k_j^2 + l_j^2)B_j(N^2 - \omega_j^2) = \omega_j\rho_0^{-1}\frac{dp_j}{dz}$$

$$\omega_j\rho_j = \left(\frac{\rho_0 N^2}{g}\right)(k_j^2 + l_j^2)B_j$$

$$\omega_j^2 = f^2 + \frac{N^2H^2}{L_j^2}$$

$$\frac{\partial\Psi_j}{\partial t} = -\omega_j\Phi_j, \quad \frac{\partial\Phi_j}{\partial t} = \omega_j\Psi_j$$

Eigen relations

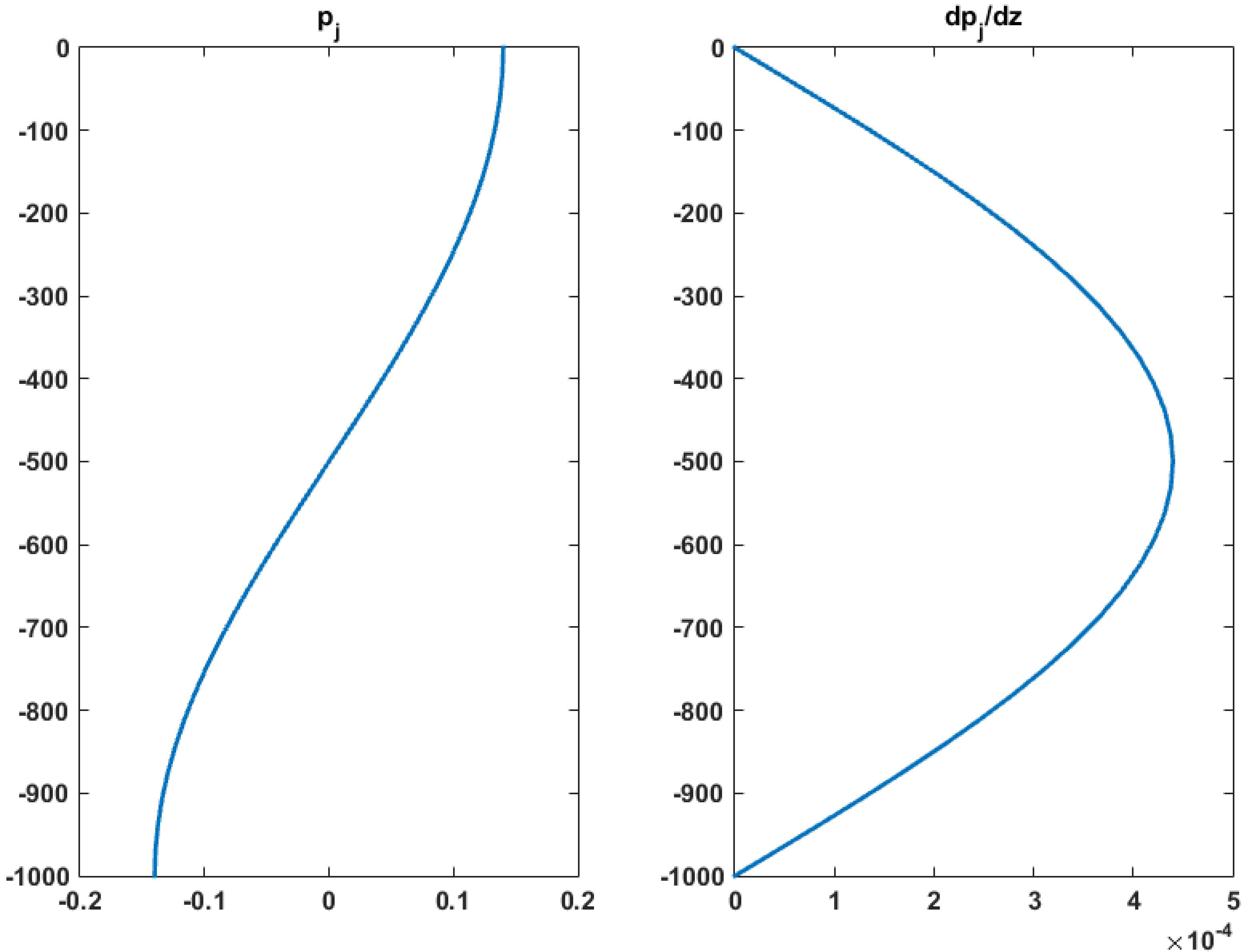
$$p(x, y, z, t) = p_0 + p_g \sin kx \sin ly + p_j \Psi_j$$

$$\left[\frac{d^2}{dz^2} + \left(\frac{N^2}{g} \right) \frac{d}{dz} + \left(\frac{N^2}{g h_j} \right) \right] p_j = 0$$

$$\left\{ \left[\nabla_h^2 - \left(\frac{\partial^2}{\partial t^2} + f^2 \right) (g h_j)^{-1} \right] + \beta \mathcal{L} \right\} \Psi_j = 0$$

β = 0

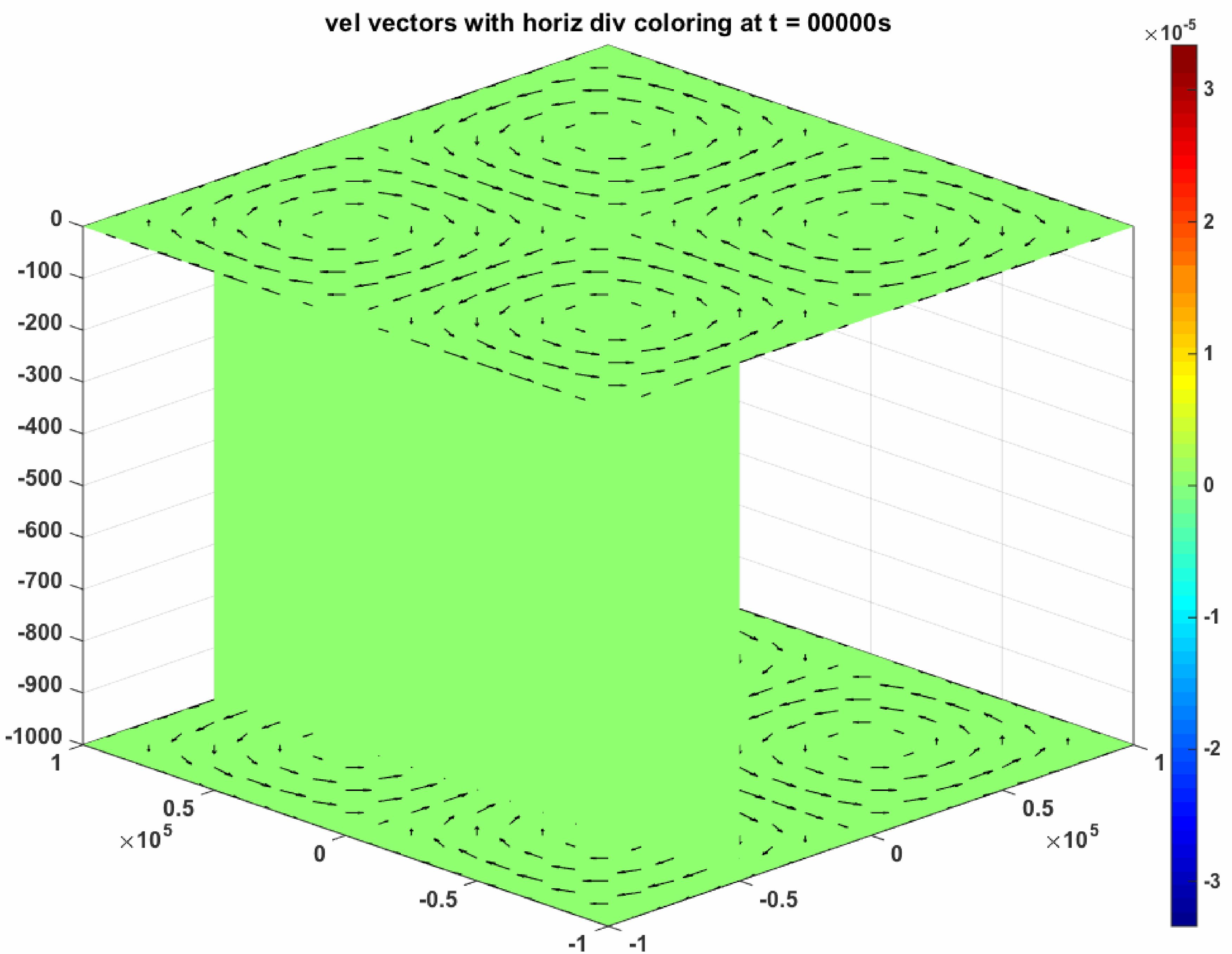
Perturbation pressure and gradient



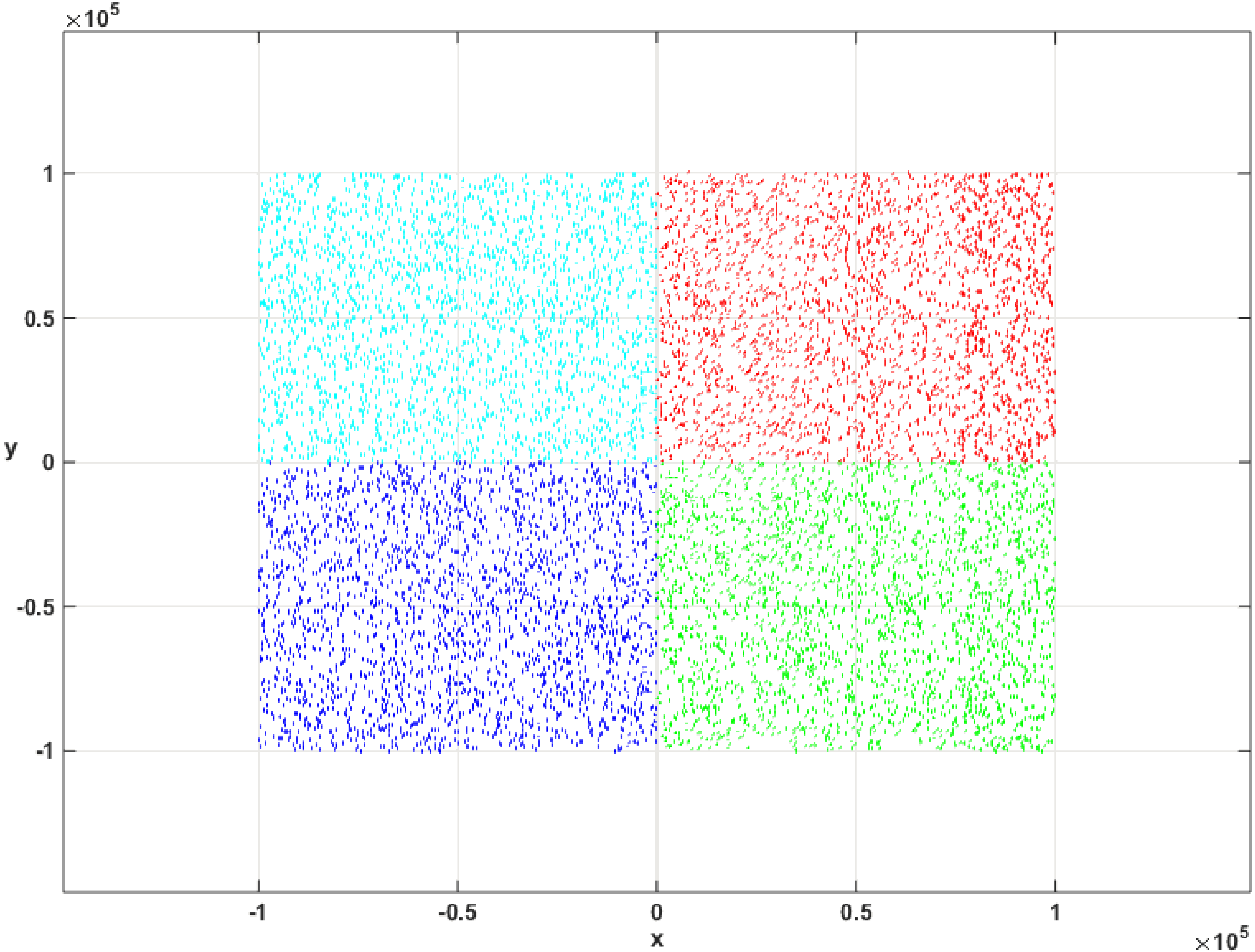
Parameters

- $L = L_j = 100$ km
- Ψ "standing wave" quadrupole
- $(f/N)^2 = 10^{-3}$
- $(H/L_j) = 10^{-2}$
- Geostrophic velocity scale = 1 m/s
- Perturbed horizontal velocity scale = 0.5 m/s

  Perturbed vertical velocity scale = 5×10^{-3} m/s
- $\omega_j = 1.08\, f$, $T \approx 58{,}000$ sec ≈ 16 hr

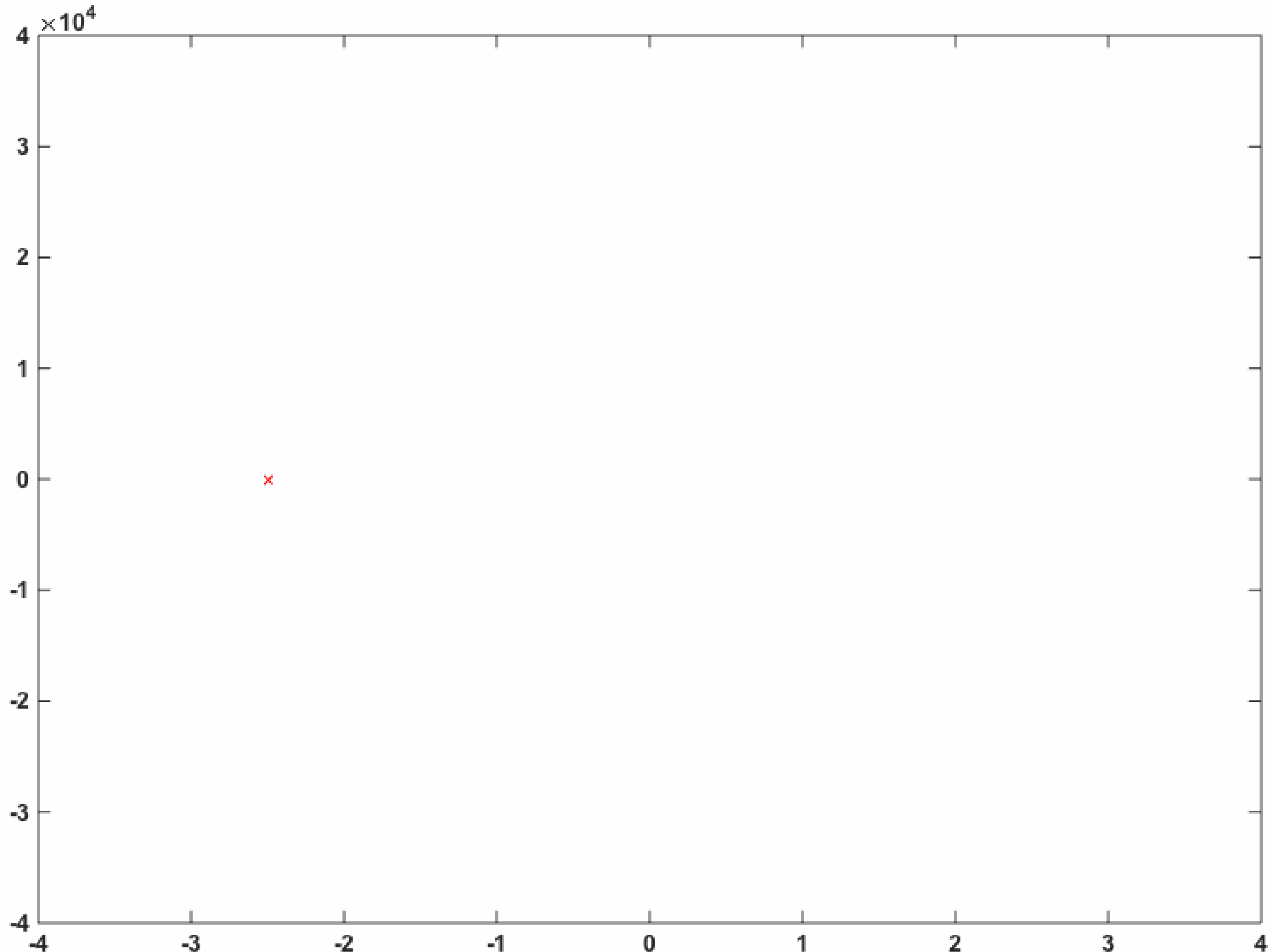


vel vectors with horiz div coloring at t = 00000s
×10^-5
3
2
1
0
-1
-2
-3
0
-100
-200
-300
-400
-500
-600
-700
-800
-900
-1000
1
0.5
×10^5
0
-0.5
-1
-1
-0.5
0
×10^5
0.5
1

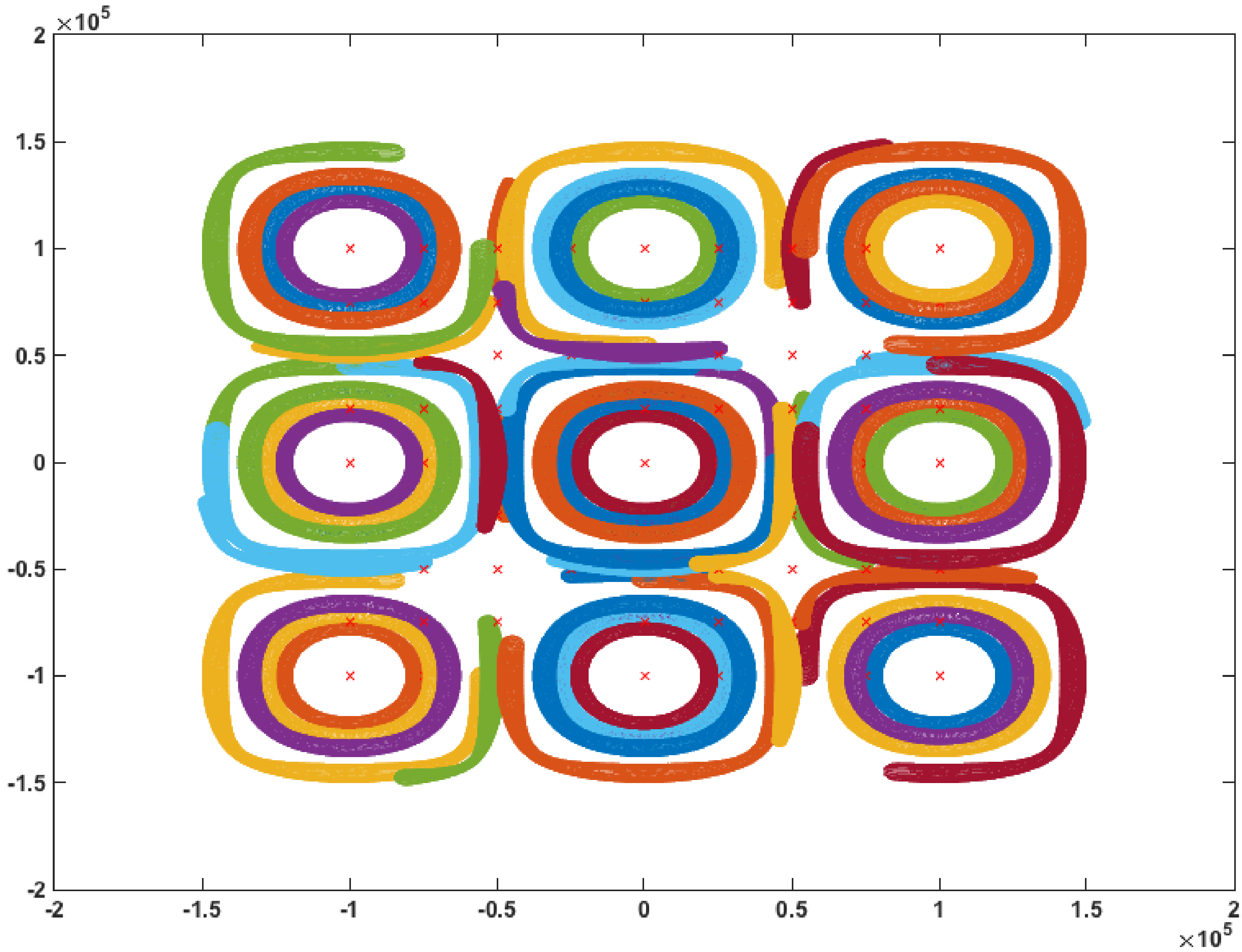


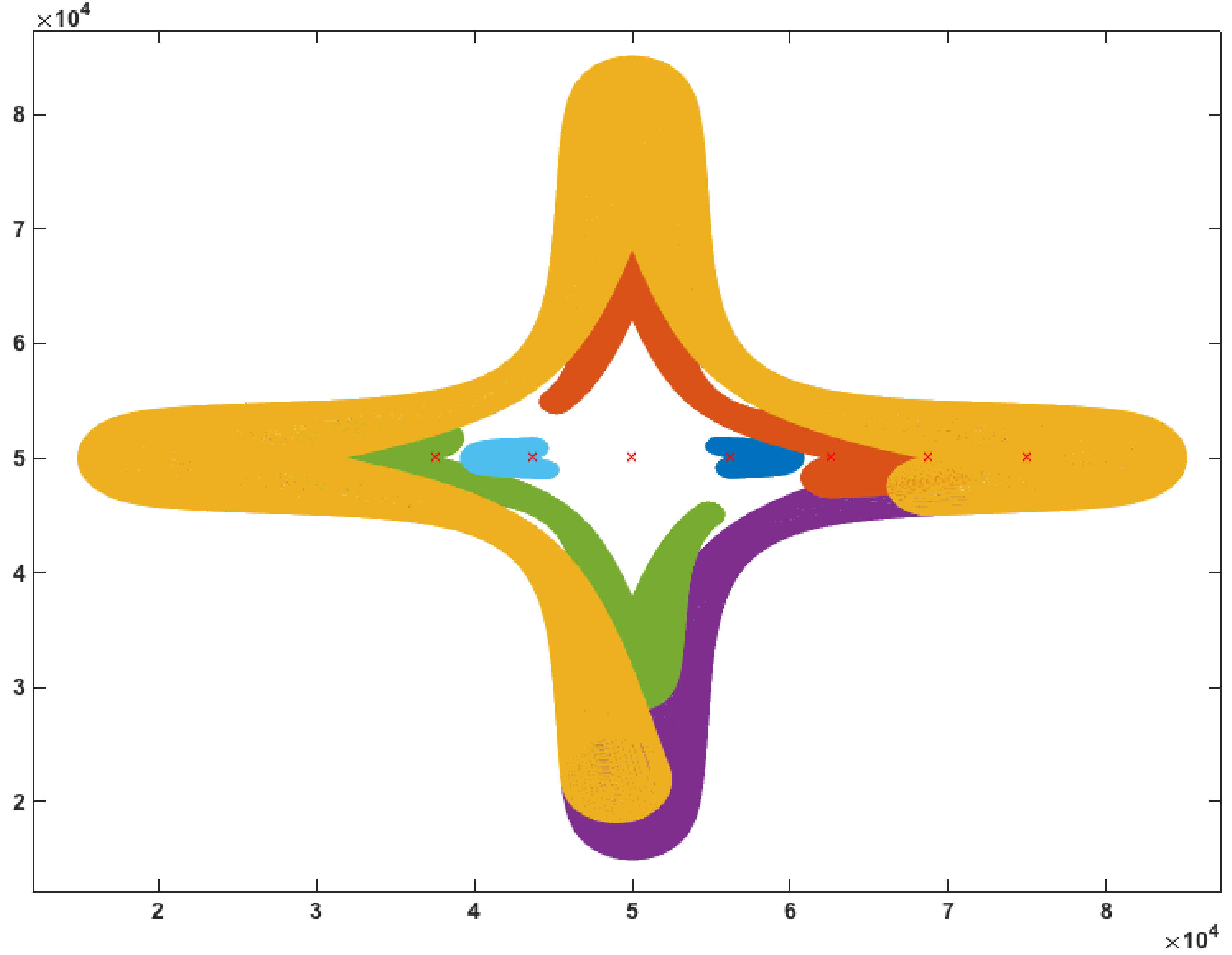
×10^5
1
0.5
0
-0.5
-1
y
-1
-0.5
0
0.5
1
x
×10^5

Typical surface trajectory

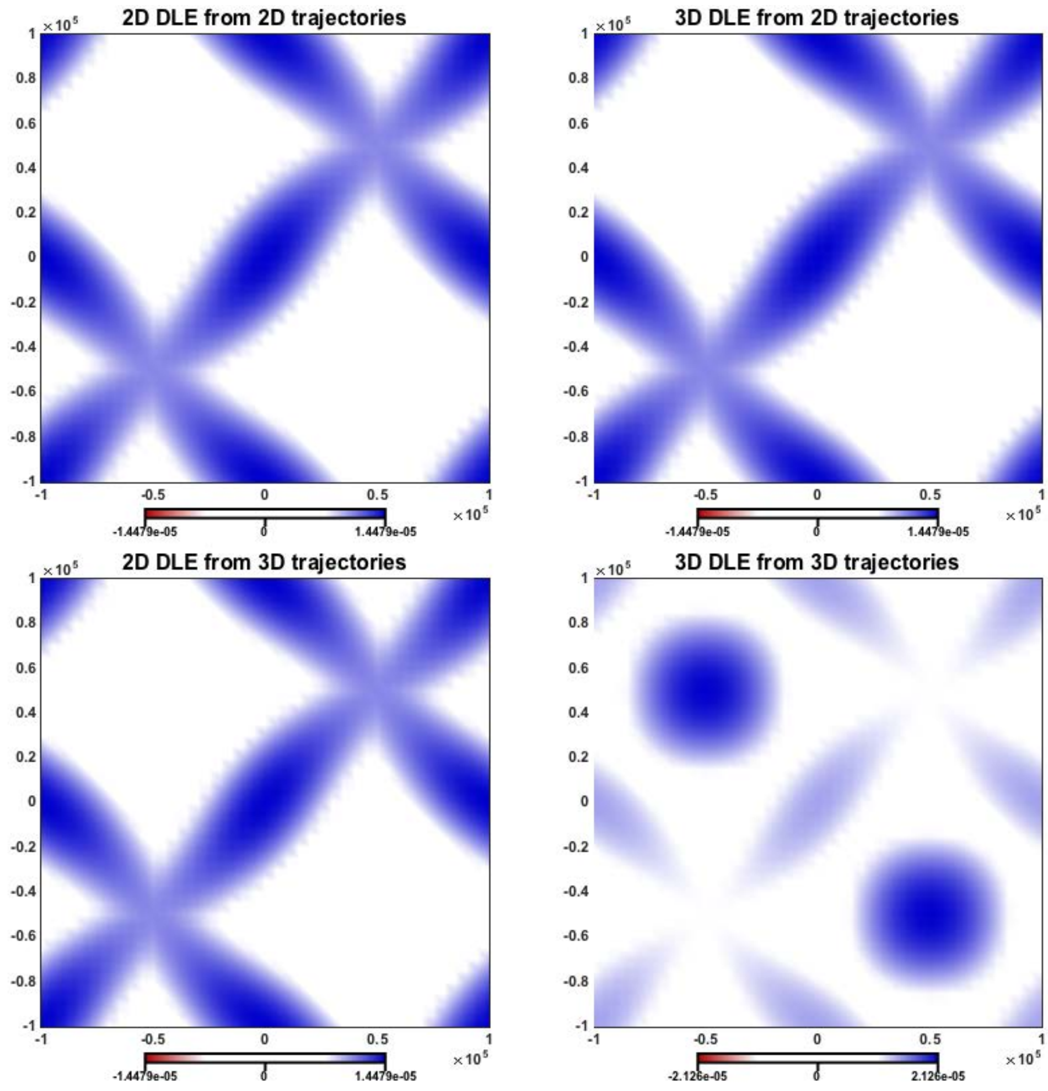


Surface Transport Boundaries

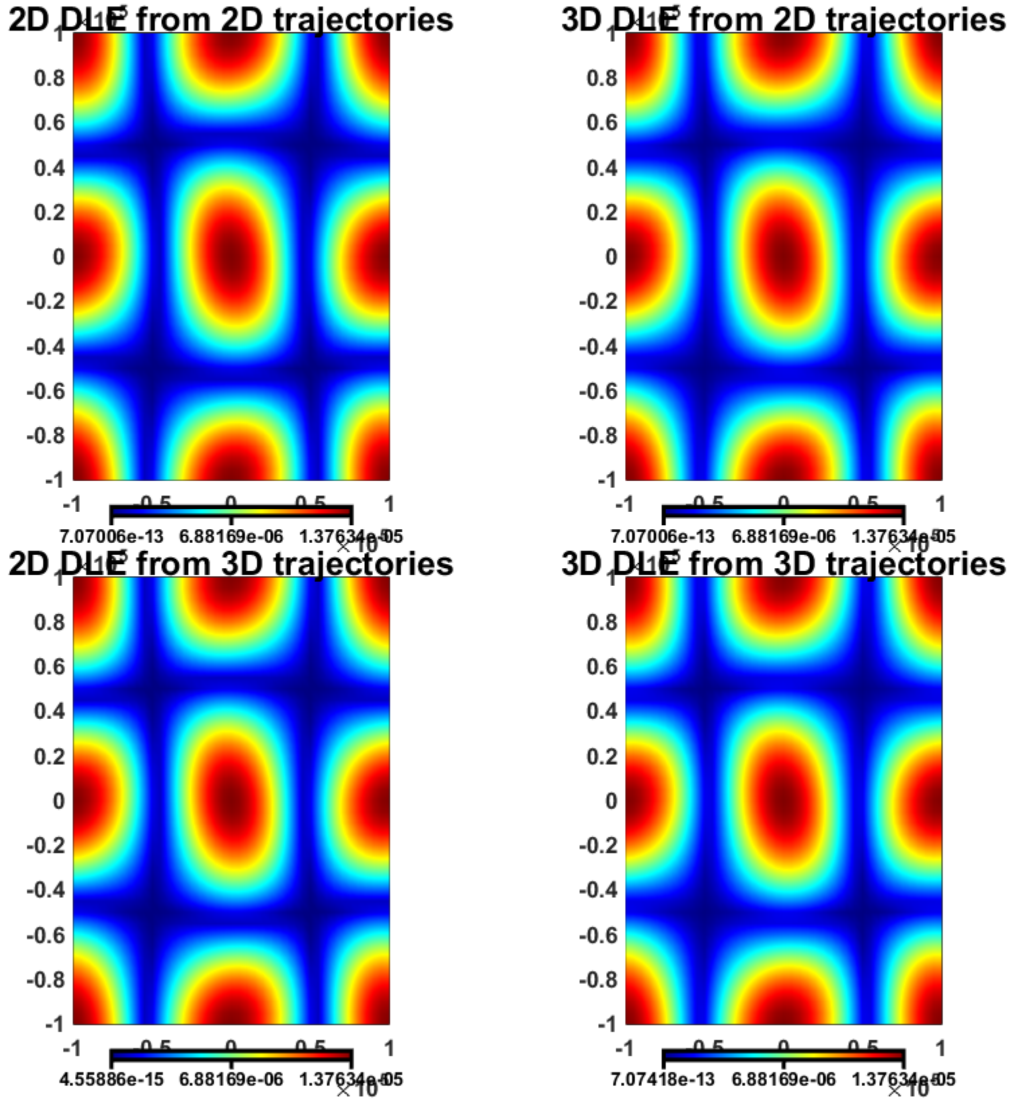



×10^4
8
7
6
5
4
3
2
2
3
4
5
6
7
8
×10^4

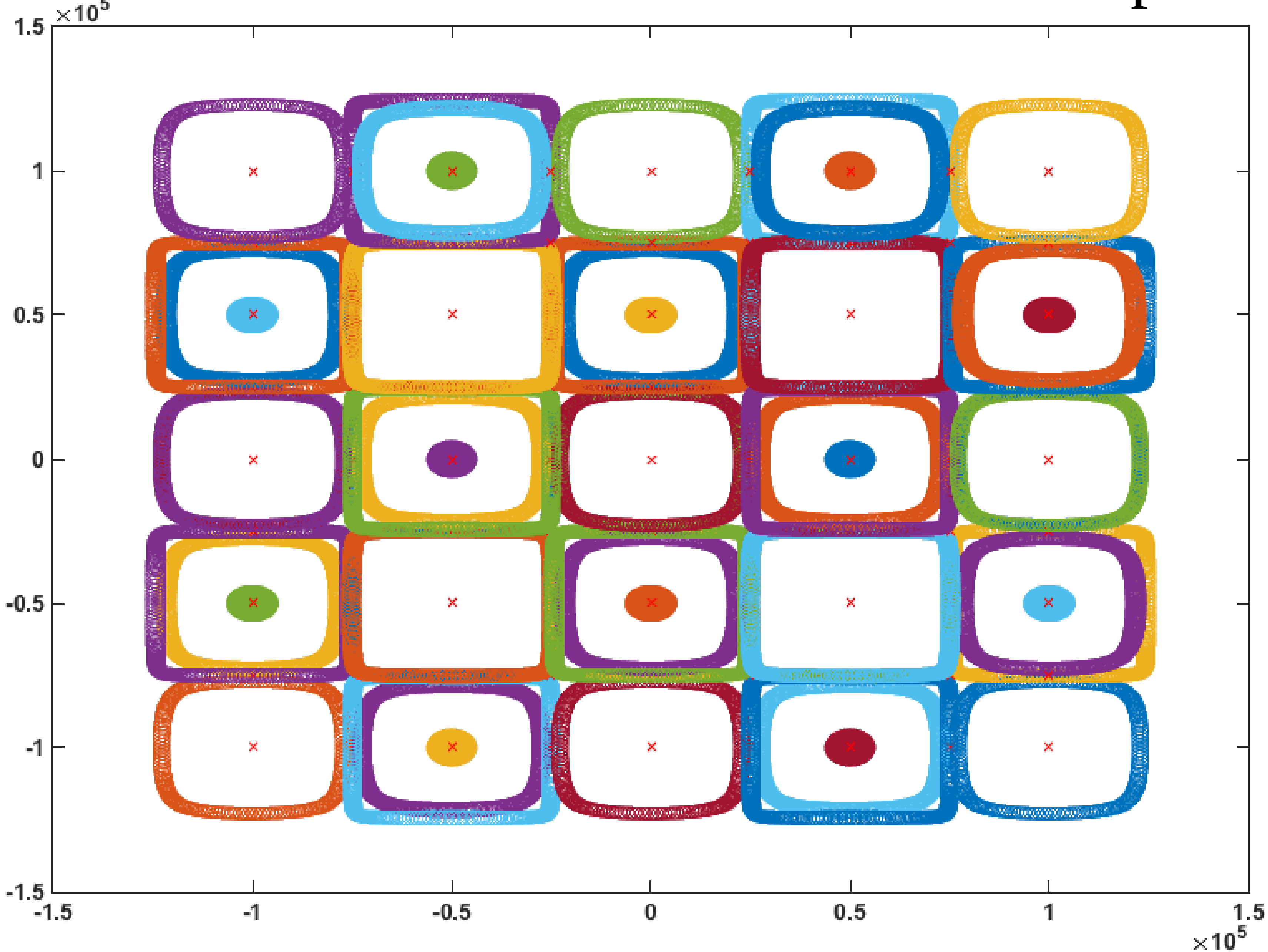
FTLE from 0 to T/4

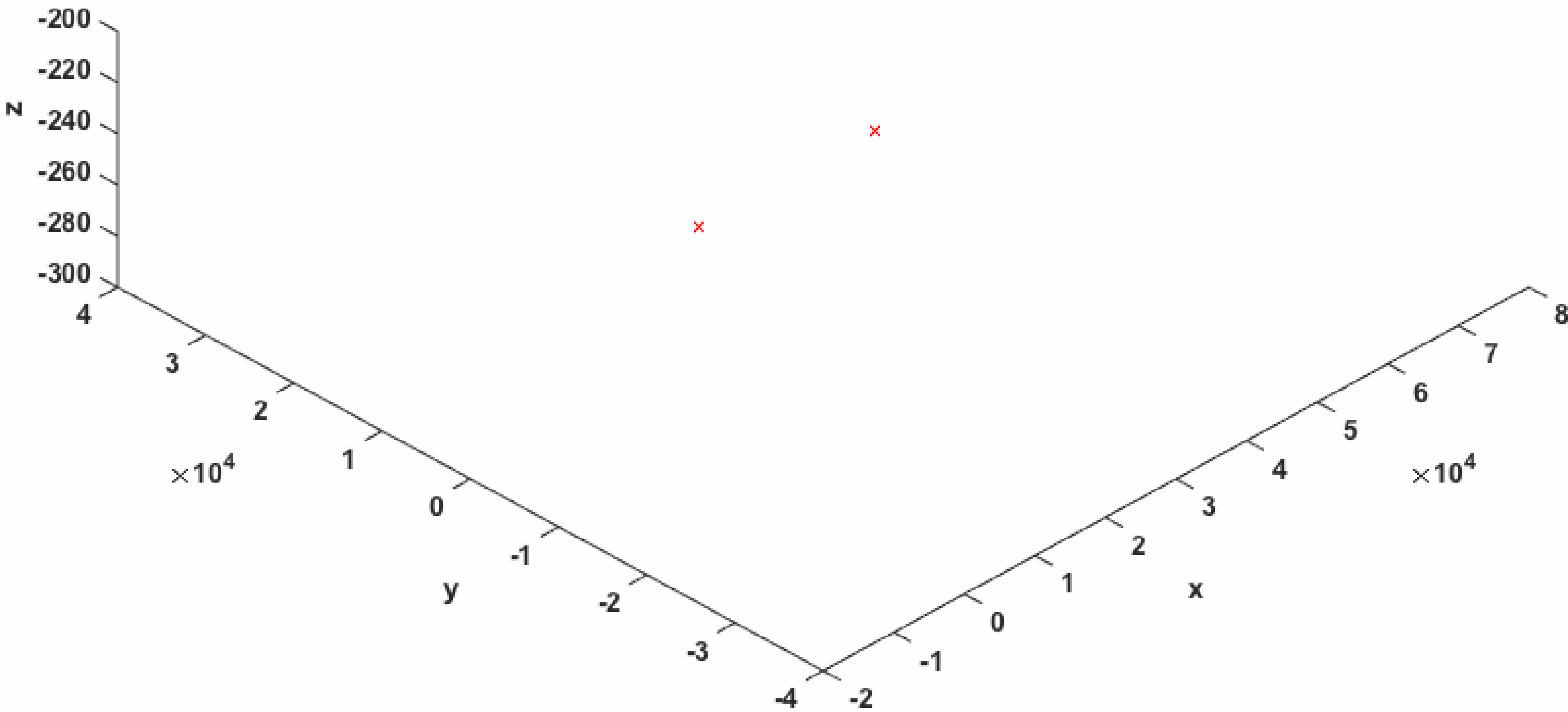


FTLE from – T/8 to T/8



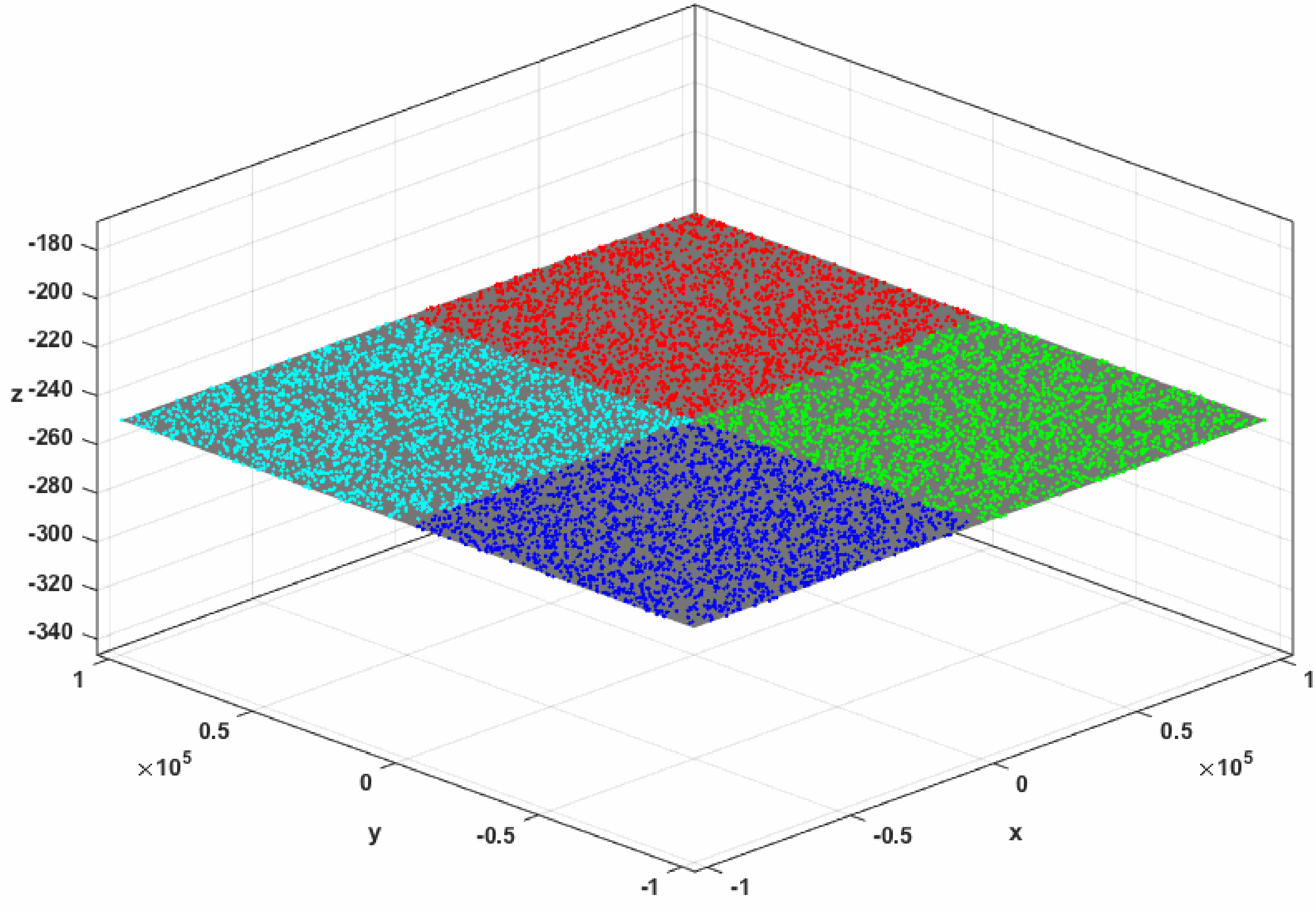
This makes for some confusion at depth



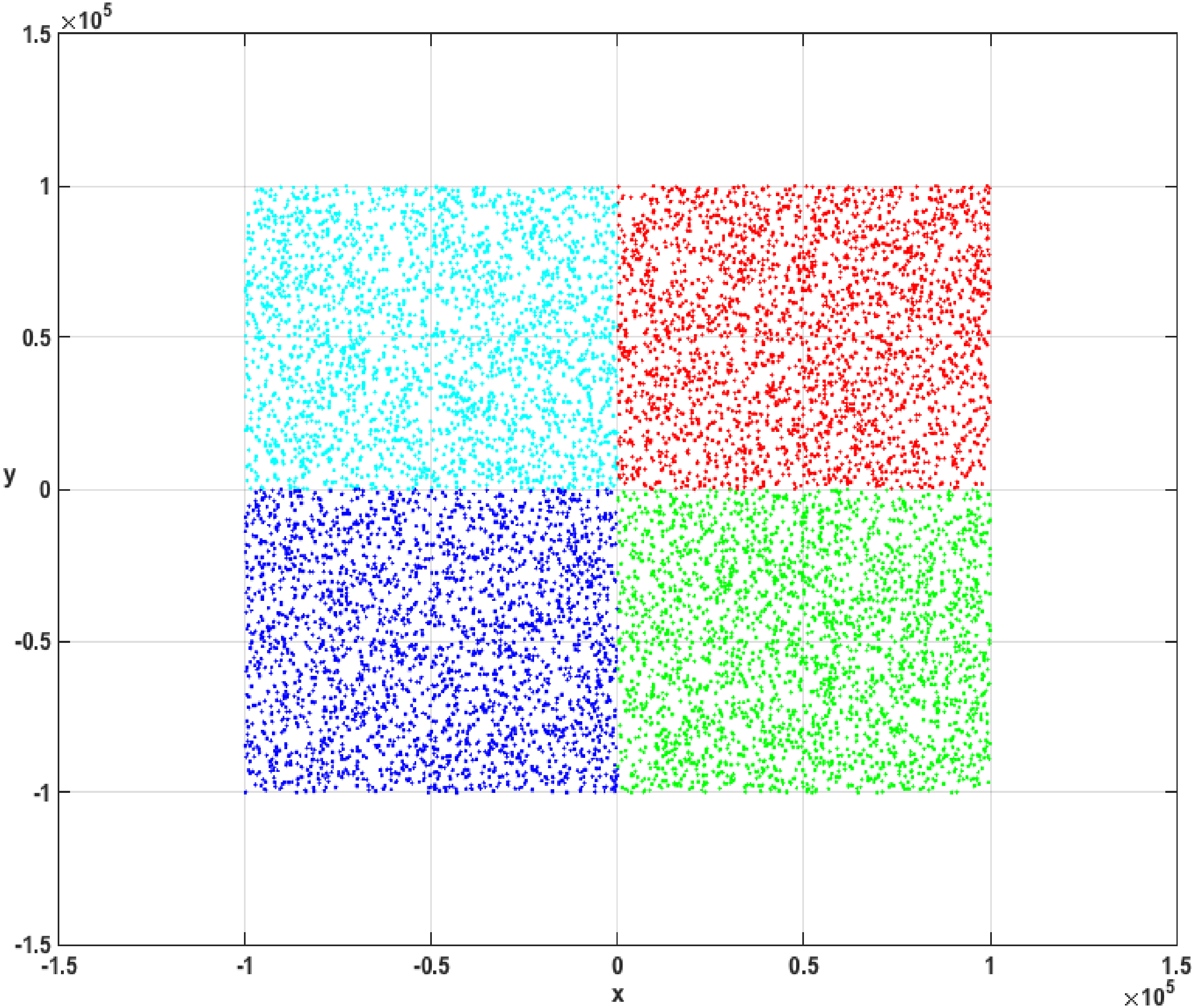


-200
-220
-240
-260
-280
-300
z
4
3
2
1
0
-1
-2
-3
-4
$\times 10^4$
y
-2
-1
0
1
2
3
4
5
6
7
8
$\times 10^4$
x

Large vertical excursions but particles want to follow density surfaces



Z = H/2



Conclusions

- Near inertial motion important. Vertical velocities unobservable but consequent horizontal divergence fundamental to transport boundary dynamics.
- Perturbation solution has weird transport boundaries. Eulerian bigots may interpret as mixing.
- FTLE not effective in detecting such boundaries
- Core regions centered about vortex centers
- Particles tend to stay on density surfaces.