Controlled vehicles, en route assimilation, conclusions and connections

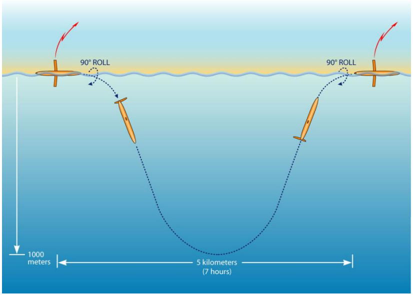
Elaine Spiller, Adam Mallen

Marquette University

Gliders

Instrument

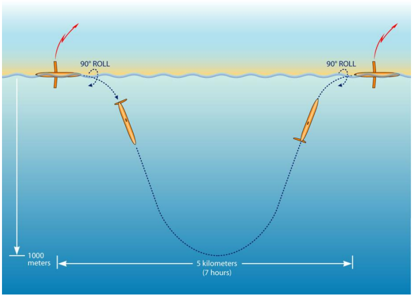
Profile



- collect standard (salinity, temperature, pressure) data
- can be fit with chemical/optical/biological sensors
- guided/steered by manipulation of buoyancy and battery location on pre-programmed flight plan, $\mathbf{u_c}$

Instrument

Profile

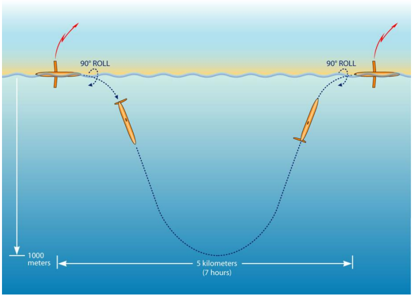


$$\begin{pmatrix} dx/dt \\ dy/dt \\ dz/dt \end{pmatrix} = \mathbf{u}(x, y, z) + \mathbf{u_c}$$

Gliders

Instrument

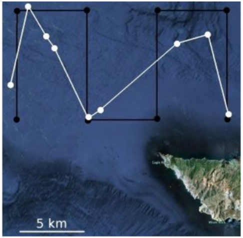
Profile



- good kinematic model of motion (Leonard & others) based on balancing forces during equilibrium glide
- velocity field uncertainty $>>$ glider model uncertainty
- path estimates available via dead reckoning – small velocity errors get integrated into large position errors

Challenges and objectives

Targeted vs actual surfacings



(Smith *et al* 2011)

Challenges

- glider path semi-Lagrangian
- ocean velocity, $\mathbf{u}(x, y, z)$, needed to plan flight
- data collected along unknown/poorly estimated paths

Data assimilation objectives:

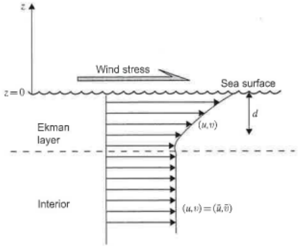
- probabilistic description of glider paths
  - better estimates of glider paths and uncertainty
- update estimates of $\mathbf{u}(x, y, z)$ for next flight planning
  - parameterization of model
  - model discovery

Test problem – Ekman Layer

- d – Ekman layer depth (unknown, a priori)
- $\bar{\mathbf{u}}$, – background ocean velocity
- τ/ρ – surface wind stress
- f – Coriolis parameter

$$u(x,y,z) = \bar{u}(x,y) + \frac{\sqrt{2}}{fd} e^{z/d} \left[\frac{\tau^x(x,y)}{\rho} \cos\left(\frac{z}{d} - \frac{\pi}{4}\right) - \frac{\tau^y(x,y)}{\rho} \sin\left(\frac{z}{d} - \frac{\pi}{4}\right) \right]$$

$$v(x,y,z) = \bar{v}(x,y) + \frac{\sqrt{2}}{fd} e^{z/d} \left[\frac{\tau^x(x,y)}{\rho} \sin\left(\frac{z}{d} - \frac{\pi}{4}\right) + \frac{\tau^y(x,y)}{\rho} \cos\left(\frac{z}{d} - \frac{\pi}{4}\right) \right]$$



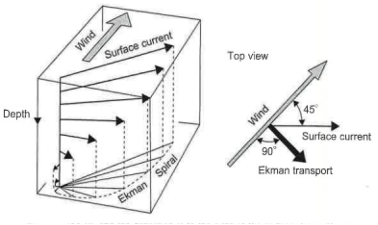


Figure: (Taken from Cushman-Roison '*Introduction to Geophysical Fluid Dynamics*')

Ekman test prob (ongoing/short term): experiment & goals

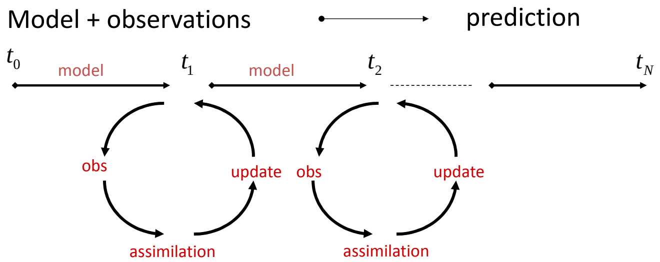
Experiment:

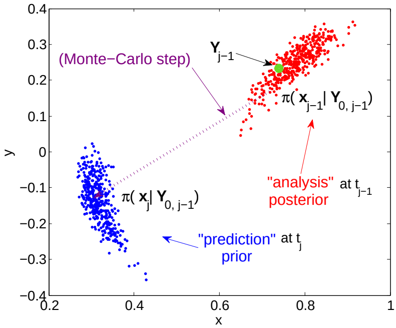
- twin-experiments, observe “true” surfacing location
- assimilate using particle filter
- use assimilated parameterization of velocity field to guide next flight

Goals:

- posterior distributions of parameters in velocity model
- probabilistic description of 3D glider paths
- improve glide path and velocity estimates
- reduce and accurately describe uncertainty in those estimates

Particle filters: from t_{j-1} to t_j





discrete approx:

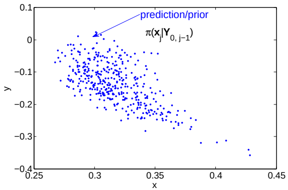
Particles are the support of the discrete approximations to these distributions

Each particle is associated with a weight, $w_j(x_j)$

Particle filters: update/analysis at $t = t_j$

Know (discrete approximation):

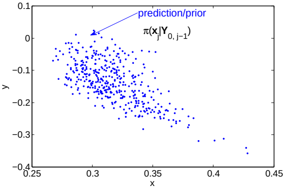
$\pi(x_j | Y_{0,j-1})$ (from last page)



Particle filters: update/analysis at $t = t_j$

Know (discrete approximation):

$$\pi(x_j | Y_{0,j-1}) \text{ (from last page)}$$



Bayes:

$$\pi(x_j | Y_{0,j}) \propto g(Y_j | x_j) \pi(x_j | Y_{0,j-1})$$

Particle filters: update/analysis at $t = t_j$

Know (discrete approximation):

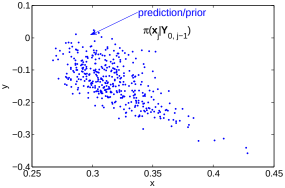
$$\pi(x_j|Y_{0,j-1}) \text{ (from last page)}$$

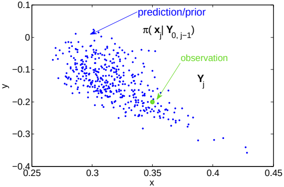
Bayes:

$$\pi(x_j|Y_{0,j}) \propto g(Y_j|x_j)\pi(x_j|Y_{0,j-1})$$

Likelihood:

$$g(Y|x) = \exp[\frac{H(x)\cdot Y}{\theta^2} - \frac{|H(x)|^2}{2\theta^2}]$$

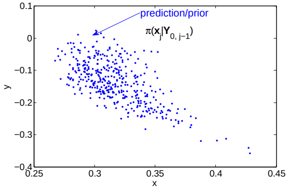




Particle filters: update/analysis at $t = t_j$

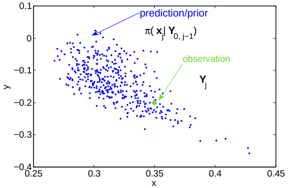
Know (discrete approximation):

$$\pi(x_j | Y_{0,j-1}) \text{ (from last page)}$$



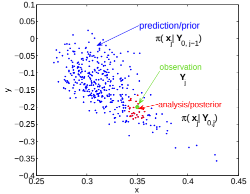
Bayes:

$$\pi(x_j | Y_{0,j}) \propto g(Y_j | x_j) \pi(x_j | Y_{0,j-1})$$



Likelihood:

$$g(Y|x) = \exp\left[\frac{H(x) \cdot Y}{\theta^2} - \frac{|H(x)|^2}{2\theta^2}\right]$$

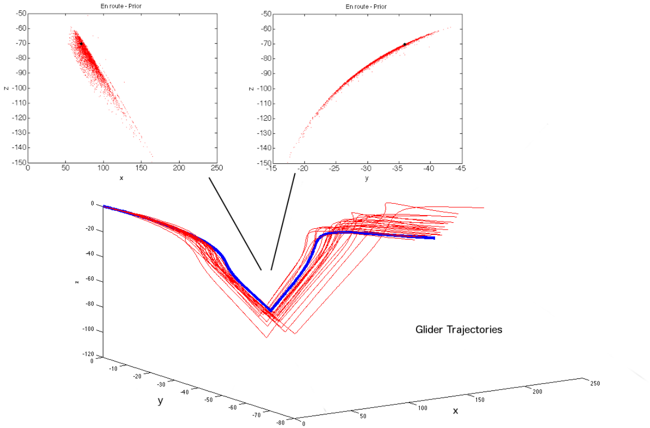


Update (discrete Bayes):

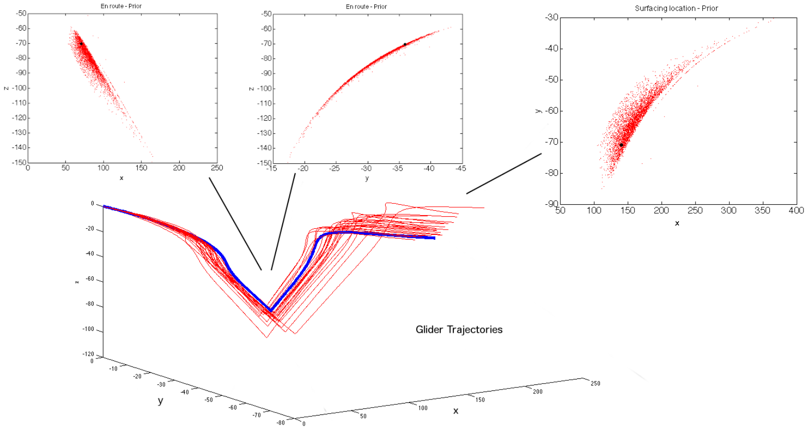
$$w_j(x_j) \propto g(Y_j | x_j) w_j^p(x_j)$$

$$\pi(x_j | Y_{0,j}) = \{x_j, w_j(x_j)\}$$

Glider DA

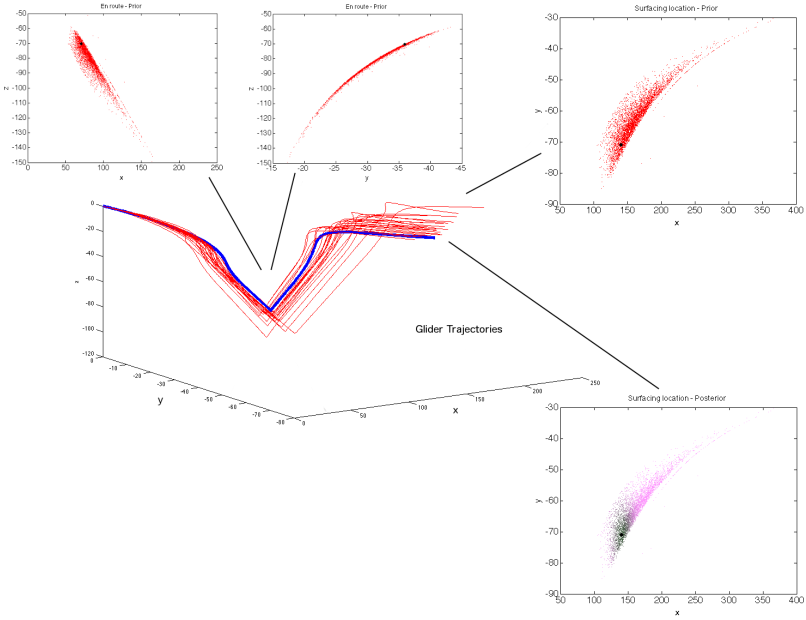


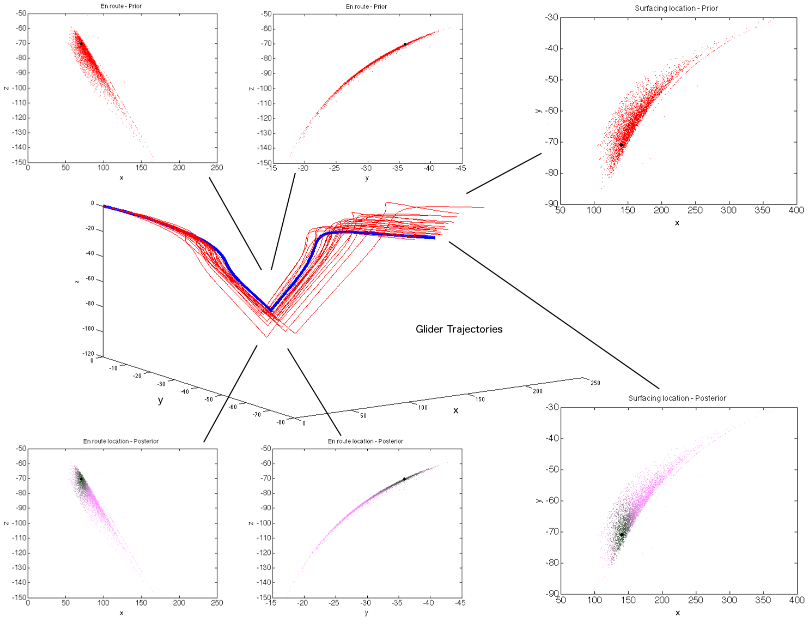
Glider DA



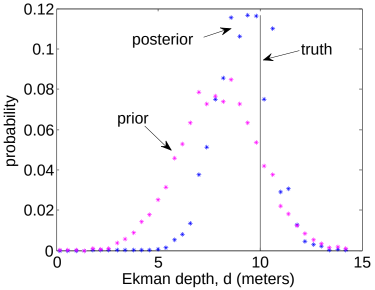
En route - Prior
En route - Prior
Surfacing location - Prior
Glider Trajectories
x
y
z

Glider DA


En route - Prior
x
z
En route - Prior
y
z
Surfacing location - Prior
x
y
Glider Trajectories
x
y
z
Surfacing location - Posterior
x
y


En route - Prior
x
z
En route - Prior
y
z
Surfacing location - Prior
x
y
Glider Trajectories
x
y
z
En route location - Posterior
x
z
En route location - Posterior
y
z
Surfacing location - Posterior
x
y

Assimilated Ekman depth



- quantify uncertainty in parameterization of $\mathbf{u}(x, y, z)$
- iterate, posterior of first dive becomes prior of second
- reduced uncertainty/improved accuracy of glider paths in subsequent dives
- use estimates of $\mathbf{u}$ to help choose $\mathbf{u_c}$

Glider DA: experiments & objectives

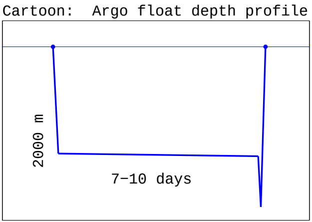
- twin-experiments
  - observe “true” surfacing location
  - observe vertical, $z(t)$, paths
- spatially or time varying background velocity and/or wind stress
- use LADA with model-discovery to determine distribution of most-likely velocity fields
- probabilistic description of 3D glider paths

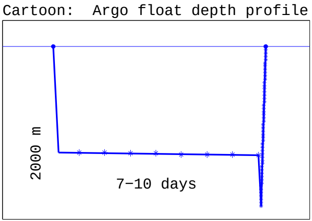
En-route data assimilation

Argo float

- Lagrangian instruments collect data en route (salinity, temperature, pressure)
- Observations depend on unknown drifter paths
- What to do with that data?

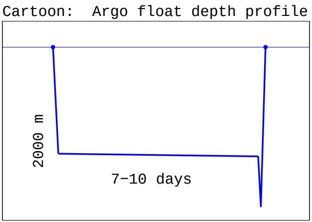
Float depth profile

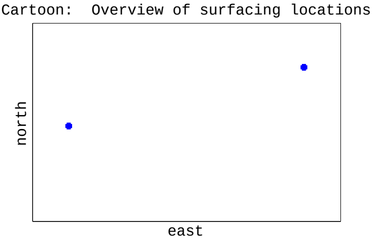




- 7-10 day float results in O(10)-O(100) km traveled
- high frequency data in dive/ascent just before surfacing in water column beneath "surfacing location"
- low frequency en-route measurements at depth, no latitude/longitude information
- en-route measurements averaged, not used in assimilation

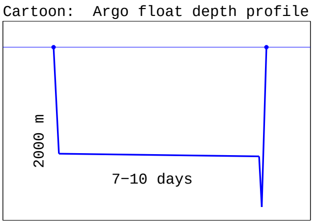
Float depth and overview

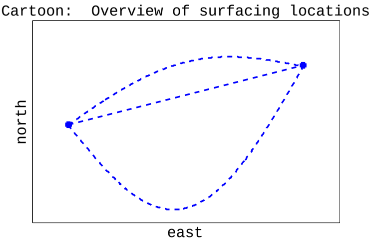




- Lagrangian DA can help ascertain velocities w/o averaging

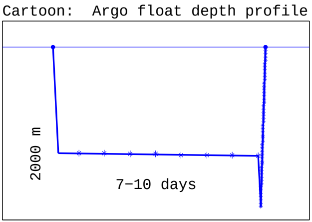
Float depth and overview

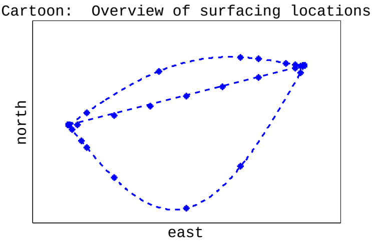




- Some possible Lagrangian paths

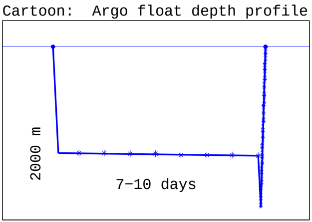
Float depth and overview

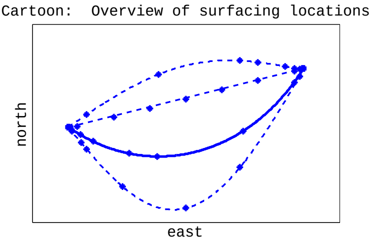




- need path & speed for subsurface observation locations

Float depth and overview





- Can en-route observations help Lagrangian DA?

Assimilated 3-D Lagrangian paths are (possibly) useful for

- aid in resolving Lagrangian structures
- assimilating data into high resolution models
- avoiding averaging via determining en-route data collection locales along paths which cross multiple grid cells

Inviscid linearized Shallow Water Equations, periodic BCs

Non-dimensional velocity fields

$$\begin{aligned}\frac{\partial u}{\partial t} &= \quad v - \frac{\partial h}{\partial x}\\ \frac{\partial v}{\partial t} &= -u - \frac{\partial h}{\partial y}\\ \frac{\partial h}{\partial t} &= -\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\end{aligned}$$

Lagrangian trajectories

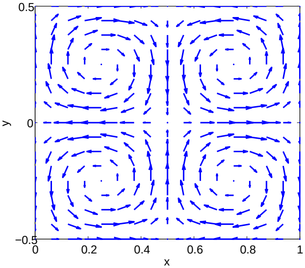
$$\begin{aligned}\dot{x}(t) &= u[x(t), y(t), t]\\ \dot{y}(t) &= v[x(t), y(t), t]\end{aligned}$$

Decomposition into Fourier Modes

$$\begin{aligned}u(x,y,t) &= -2\pi \sin(2\pi x)\cos(2\pi y)u_o + \cos(2\pi y)u_1(t)\\ v(x,y,t) &= 2\pi \cos(2\pi x)\sin(2\pi y)u_o + \cos(2\pi y)v_1(t)\\ h(x,y,t) &= \sin(2\pi x)\sin(2\pi y)u_o + \sin(2\pi y)h_1(t)\end{aligned}$$

Cellular flow field

If $u_1 = v_1 = h_1 = 0$,
flow field is constant & tracers stay w/in cells



Cellular flow field

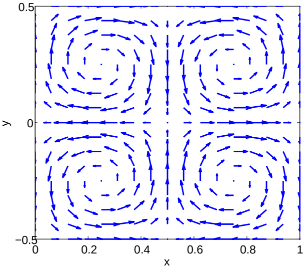
If $u_1 = v_1 = h_1 = 0$,
flow field is constant & tracers stay w/in cells

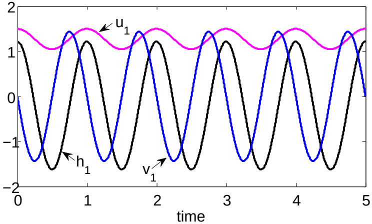
otherwise,

$$\dot{u}_o = 0$$
$$\dot{v}_1 = -u_1 - 2\pi h_1$$
$$\dot{u}_1 = v_1$$
$$\dot{h}_1 = 2\pi v_1$$

w/initial conditions

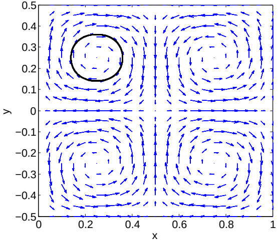
$$[u_o(0), u_1(0), v_1(0), h_1(0)]$$





A few trajectories

Steady



$u_1(0) = 0$

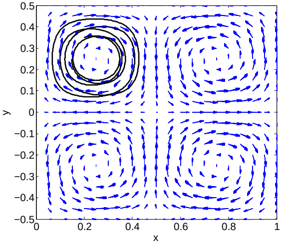
$v_1(0) = 0$

$h_1(0) = 0$

$x(0) = 0.2$

$y(0) = 0.3$

Center



$u_1(0) = 0.5$

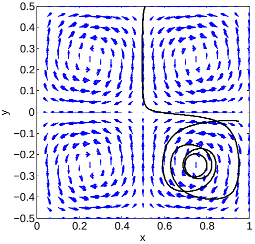
$v_1(0) = 0.5$

$h_1(0) = 0.5$

$x(0) = 0.2$

$y(0) = 0.3$

Saddle



$u_1(0) = 0.2$

$v_1(0) = 1.3$

$h_1(0) = 1.4$

$x(0) = 0.51$

$y(0) = 0.498$

Particle filter for standard LADA

Test problem:

- $u_1(0) = v_1(0) = h_1(0) = 0.5,\ x(0) = .2,\ y(0) = .3$
- broad priors on (u_1, v_1, h_1), tight on (x, y) at $t = 0$
- run to $t = T$ (1 period of coefficients)
- 5 noisy observations of drifter

Particle filter for standard LADA

Test problem:

- $u_1(0) = v_1(0) = h_1(0) = 0.5,\ x(0) = .2,\ y(0) = .3$
- broad priors on (u_1, v_1, h_1), tight on (x, y) at $t = 0$
- run to $t = T$ (1 period of coefficients)
- 5 noisy observations of drifter

Goal:

- learn about $u_1(0), v_1(0), h_1(0)$ from Lagrangian observations

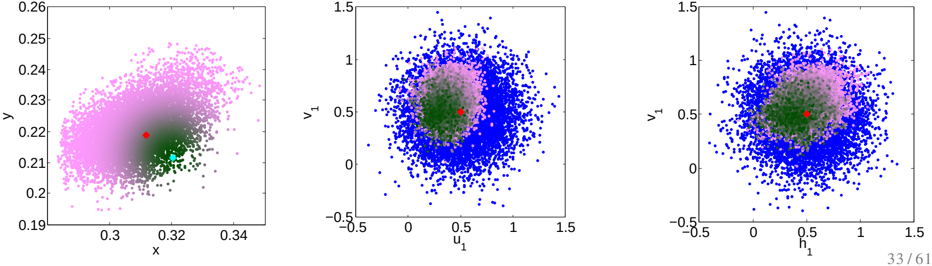
Particle filter for standard LADA

Test problem:

- $u_1(0) = v_1(0) = h_1(0) = 0.5$, $x(0) = .2$, $y(0) = .3$
- broad priors on (u_1, v_1, h_1), tight on (x, y) at $t = 0$
- run to $t = T$ (1 period of coefficients)
- 5 noisy observations of drifter

Goal:

- learn about $u_1(0), v_1(0), h_1(0)$ from Lagrangian observations



En route Lagrangian data – a test problem

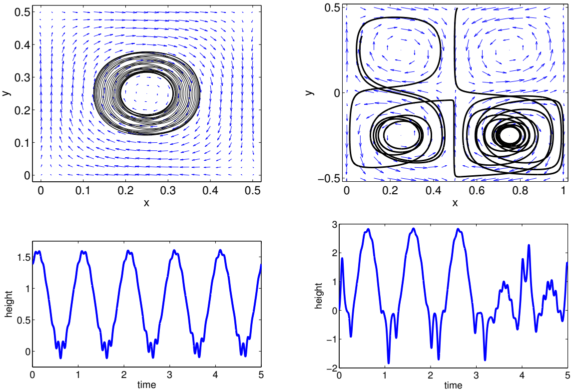
- Idea treat height, $h(x, y, u_1, v_1, h_1)$, as proxy for temperature – typical quantity measured en route

En route Lagrangian data – a test problem

- Idea treat height, $h(x, y, u_1, v_1, h_1)$, as proxy for temperature – typical quantity measured en route
- Sample height, $\hat{h}(t) = h(x^d(t), y^d(t), t) + \textit{noise}$ between "surfacings", e.g. traditional observation instants t_j

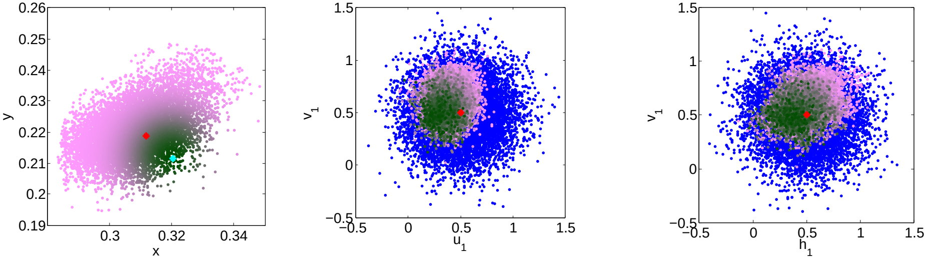
En route Lagrangian data – a test problem

- Idea treat height, $h(x, y, u_1, v_1, h_1)$, as proxy for temperature – typical quantity measured en route
- Sample height, $\hat{h}(t) = h(x^d(t), y^d(t), t) + \textit{noise}$ between "surfacings", e.g. traditional observation instants t_j



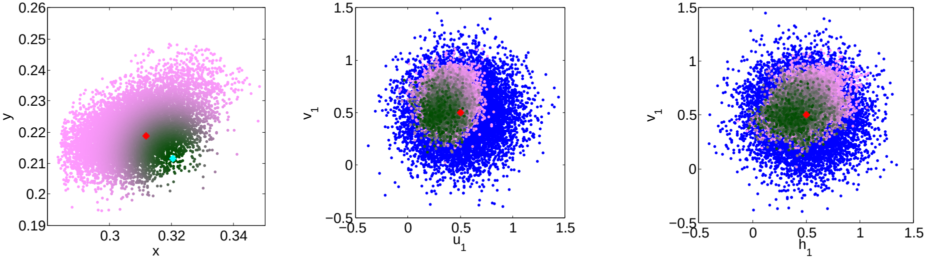
Particle filter w/en route observations

"traditional" LADA:

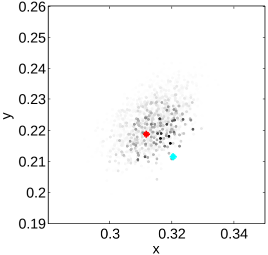


Particle filter w/en route observations

<u>“traditional” LADA</u>:

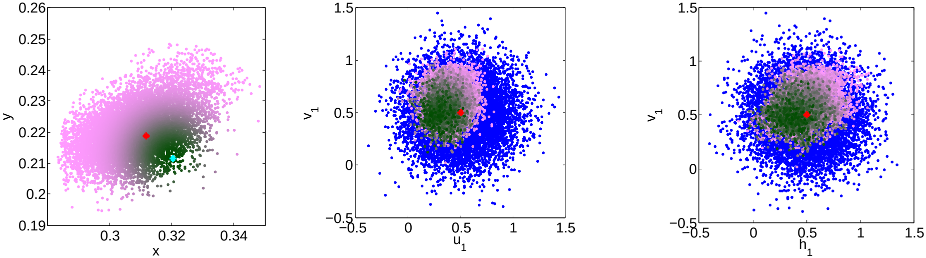


<u>en route LADA</u>:

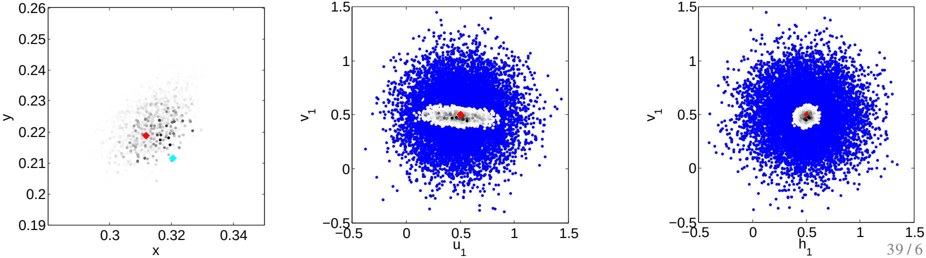


Particle filter w/en route observations

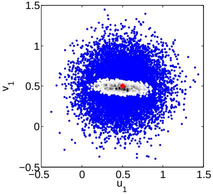
<u>“traditional” LADA</u>:

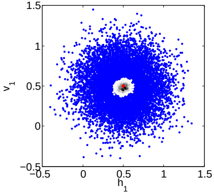


<u>en route LADA</u>:



Characterizing improvement





Characterizing improvement:
compare covariance matrices of prior and posterior distribution

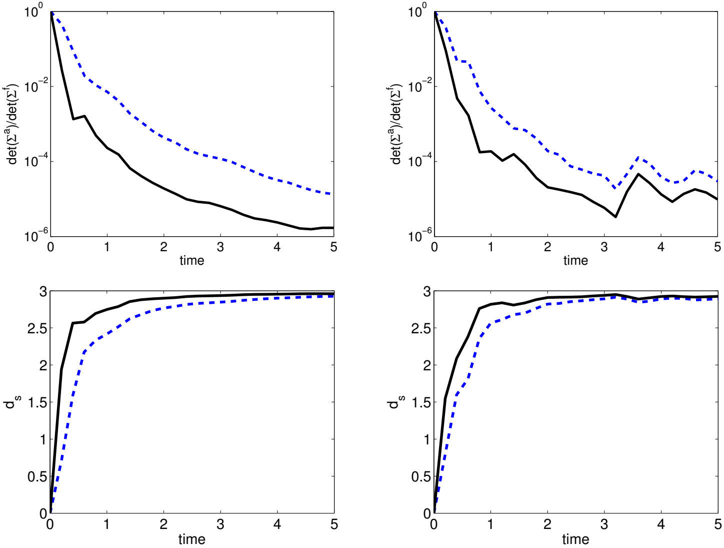
$$d_s(t) = tr[\mathbf{I} - \Sigma_F^a(t)(\Sigma_F^f)^{-1}]$$ (Zupanski, 2007)

$$r(t) = det(\Sigma_F^a(t))/det(\Sigma_F^f)$$

Improvements w/assimilating en-route data

left column: center

right column: saddle



(S, Apte, Jones, submitted 2012)

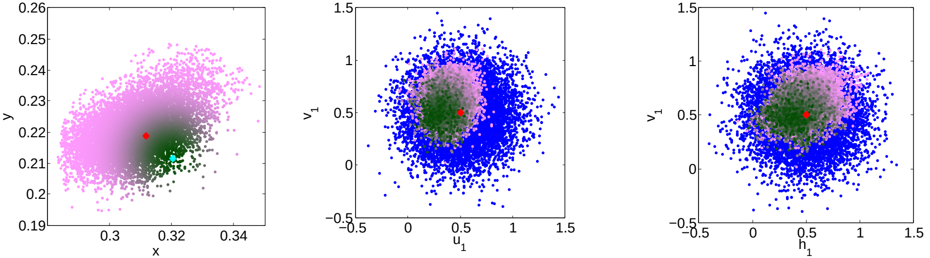
Looking forward

- Consider two-layer or 3D model w/observable-at-depth, spatially dependent variable
  - collect en-route observations on bottom layer
  - traditional Lagrangian observations, less frequent
- Improvements from en-route assimilation?
- Can we estimate Lagrangian paths at depth?
- How dependent is this on coupling strength?

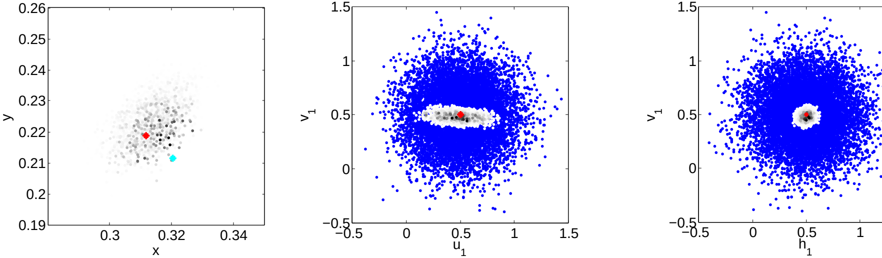
Conclusions and Connections

Observing unknown locations: en-route assimilation

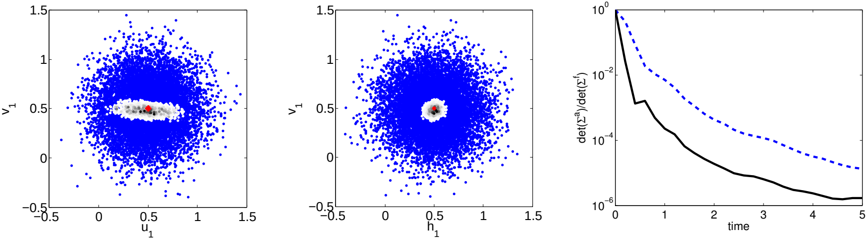
“traditional” LADA:



en route LADA:

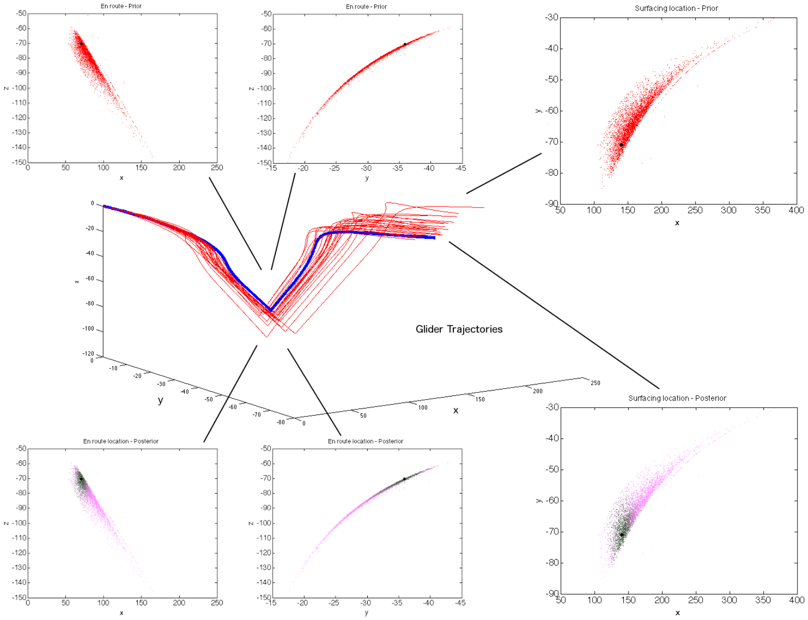


Observing unknown locations: en-route assimilation

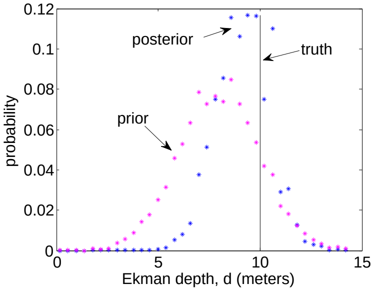


- developed strategies which fully exploit data collected at unknown locations
- marked improvement over assimilating only instrument location data

Subsurface data via controlled vehicles: gliders

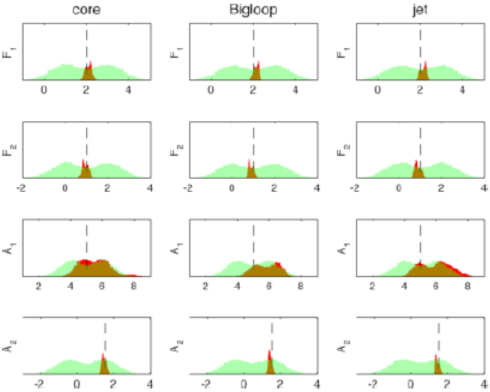


Subsurface data via controlled vehicles: gliders



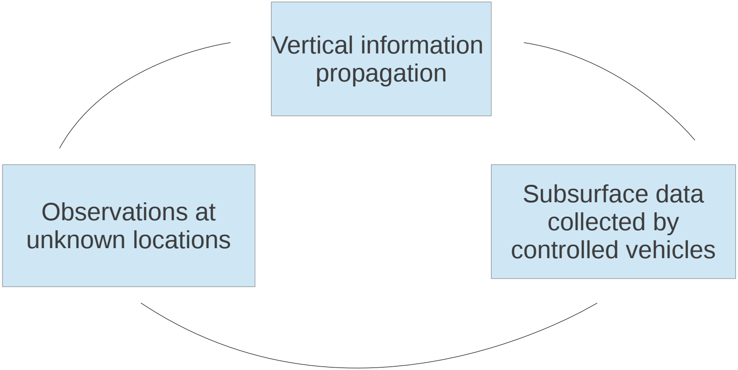
- probabilistic description of 3D glider paths
- improve glide path and z-dependent velocity estimates
- reduce and accurately describe uncertainty in those estimates

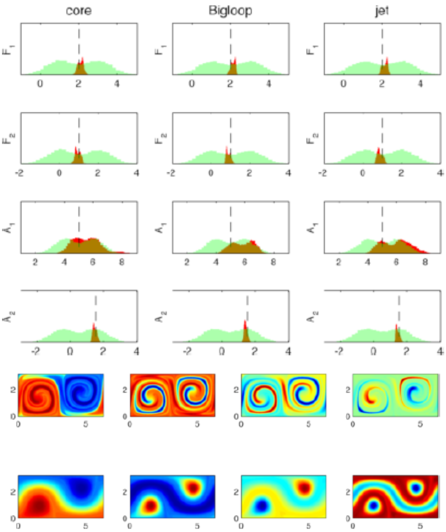
Vertical information propagation



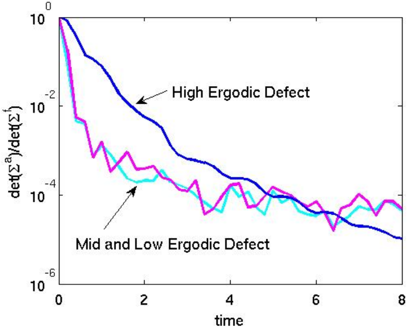
- Ambiguity resolved by more observations and, in some cases, uncertainty, but generally not inaccuracy
- Baroclinic perturbations to barotropic flow are not easily resolved by observations on more layers

Connections: between DA projects





- Identified relationship between accuracy/uncertainty and complexity of coherent sets as encoded in Koopman eigenfunctions
- Need to further explore and understand this this relationship



three different trajectories, each w/different ergodicity defect

- lower values mean greater information content through DA
- how does complexity of trajectories relate to information gain through assimilation?

Connections: Rotating can as testbed for model error DA

Two models:

- kinematic
  3D velocity non-divergent but no dynamics
- CFD
  nonlinear numerical model, Navier-Stokes equations

Connections: Rotating can as testbed for model error DA

Two models:

- kinematic
  3D velocity non-divergent but no dynamics
- CFD
  nonlinear numerical model, Navier-Stokes equations

Strategy:

- treat CFD as "reality" and take observations
- treat kinematic model as "model"

Connections: Rotating can as testbed for model error DA

Two models:

- kinematic
  3D velocity non-divergent but no dynamics
- CFD
  nonlinear numerical model, Navier-Stokes equations

Strategy:

- treat CFD as “reality” and take observations
- treat kinematic model as “model”

Problem:

- kinematic model can’t reproduce CFD trajectories even with “best” choice of parameters

Connections: Rotating can as testbed for model error DA

Two models:

- kinematic
  3D velocity non-divergent but no dynamics
- CFD
  nonlinear numerical model, Navier-Stokes equations

Strategy:

- treat CFD as “reality” and take observations
- treat kinematic model as “model”

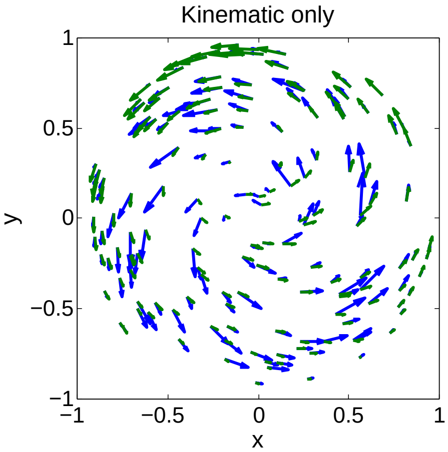
Problem:

- kinematic model can’t reproduce CFD trajectories even with “best” choice of parameters

Assimilate:

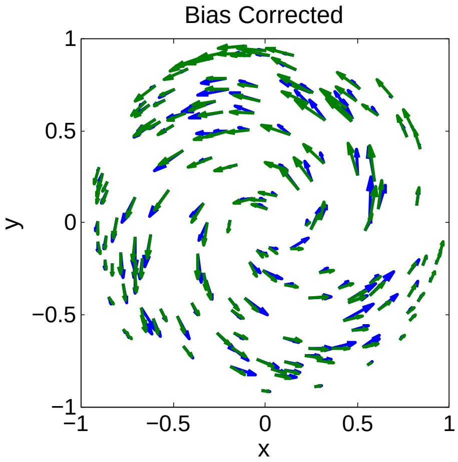
- data from CFD to “discover” model bias

$$\mathbf{u}_{CFD}(x, y, z) = \mathbf{u}_{KM}(x, y, z) + \mathbf{u}_B(x, y, z)$$



- CFD field and"best fit" Kinematic field
- (u, v) projection of top of can, $z > 0.8$

Rotating can: comparison of velocity fields



- CFD field and bias corrected kinematic field
- (u, v) projection of top of can, $z > 0.8$

Rotating can: comparisons of trajectories

- Kinematic model

$$\frac{dx}{dt} = u_{KM}, \quad \frac{dy}{dt} = v_{KM}, \quad \frac{dz}{dt} = w_{KM}$$

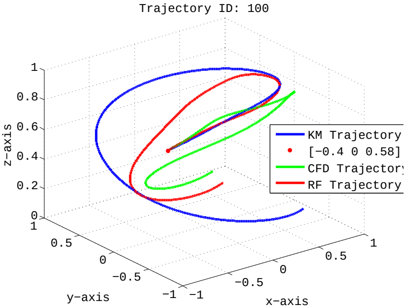
- CFD model

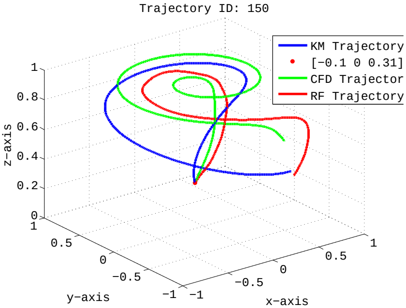
$$\frac{dx}{dt} = u_{CFD}, \quad \frac{dy}{dt} = v_{CFD}, \quad \frac{dz}{dt} = w_{CFD}$$

- Bias corrected kinematic model (labeled RF)

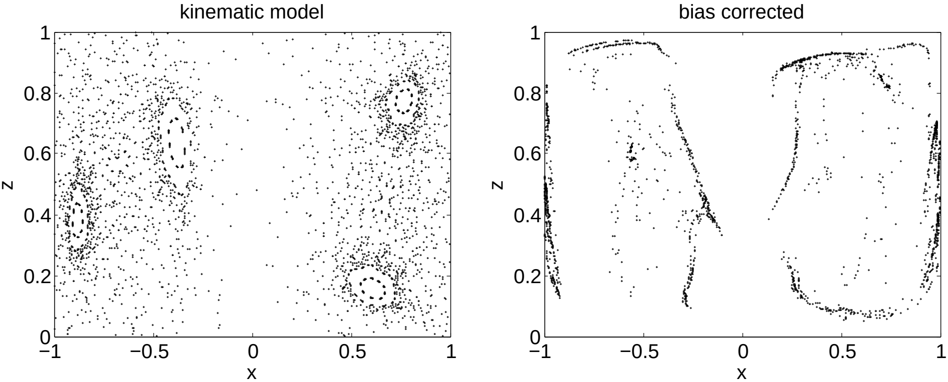
$$\frac{dx}{dt} = u_{KM} + \hat{u}_B, \quad \frac{dy}{dt} = v_{KM} + \hat{v}_B, \quad \frac{dz}{dt} = w_{KM} + \hat{w}_B$$

Rotating can: comparison of trajectories

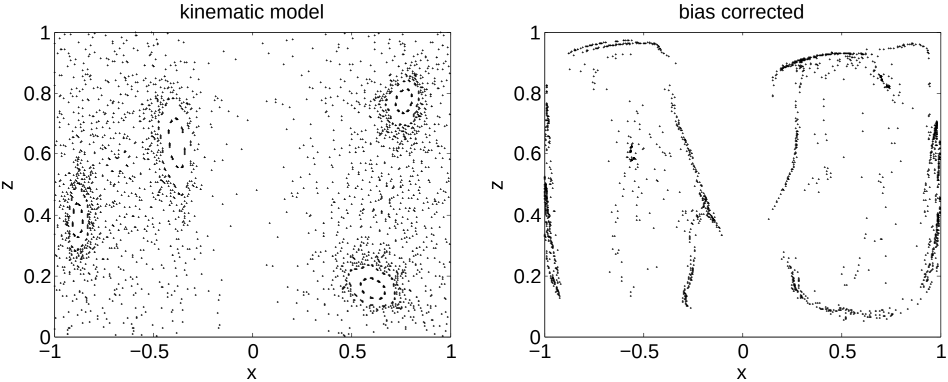




Proof of concept: Poincaré Sections, bias corrected model



Proof of concept: Poincaré Sections, bias corrected model



- what are we seeing on the right?
- lots of computational experiments to be done
- what is we move to non-steady case?