Dynamical Systems Theory and Lagrangian Data Assimilation in 4D Geophysical Fluid Dynamics

Larry Pratt (PI), Woods Hole Oceanographic Inst.



http://www.whoi.edu/ocean3dplus1/

Scott Harper (ONR Program Manager)

Participants

Woods Hole Oceanographic Institution: Larry Pratt, Irina Rypina

University of Delaware: Denny Kirwan, Bruce Lipphardt, Helga Huntley

University of North Carolina: Chris Jones

City University of New York, Staten Island: Drew Poje

UC Santa Barbara: Igor Mezic

SIO: Stefan Lewellyn Smith

Marquette: Sherry Scott and Elaine Spiller

University of Miami: Tamay Ozgokmen, Angelique Heza and Annalisa Griffa

Plus 5 postdocs, and 7 graduate.



Workshops

Oct. 2010 Chicago Feb. 2012 Wilmington Feb. 2013 Chapel Hill

Publications

Amey, C., B. Sundaram, and A. C. Poje. Mixing in mixed phase spaces. To be published in *Physics of Fluids*, May 2013.

Apte, A., and C.K.R.T. Jones. The impact of nonlinearity on Lagrangian data assimilation. *Nonlinear Processes in Geophysics*, accepted.

Apte, A., E. Spiller, and C.K.R.T. Jones. Assimilating en-route Lagrangian observations. Submitted to *Tellus A*.

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Budisic, Marko, and Igor Mezic, 2012. Geometry of the ergodic quotient reveals coherent structures in flows. *Physica D*, **241**(15), 1255-1269, 10.1016/j.physd.2012.04.006.

Chabreyrie, R., and S. G. Lewellyn Smith. Short- and Long-Time Structures in a Three-Dimensional Time Dependent Flow. Submitted to *Physics of Fluids*.

Chini, G., K. Julien, E. Knobloch, and B. Maultsby. 2:1 Spatial Resonance in Langmuir Circulation, (GFD Summer Program Report), in final preparation for submission as a journal article.

Cox, G. Large-time uniqueness in a data assimilation problem for Burgers' equation. Submitted to *Inverse Problems*.

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Loire, S., P. Kauffmann, I. MeziÃ, ‡ and C. D. Meinhart, 2012. A theoretical and experimental study of AC electrothermal flows. *J. Phys. D: Appl. Phys.*, **45**, 185301.

Mallen, A., S. Lakshmivarahan, and E. T. Spiller. A forward sensitivity approach to Lagrangian Data Assimilation. (in preparation)

McDougall, D., and C.K.R.T. Jones. Decreasing flow uncertainty in Lagrangian data assimilation through drifter control. In final preparation.

Mensa, J., A. Griffa, Z. Garraffo, T.M. Özgökmen and M. Veneziani: Seasonality of the submesoscale dynamics in the Gulf Stream region. Ocean Dynamics, submitted: March 2013.

Özgökmen, T.M., A.C. Poje, P.F. Fischer, H. Childs, H. Krishnan, C. Garth, A. Haza, and E. Ryan, 2012. On multi-scale dispersion under the influence of surface mixed layer instabilities and deep flows. *Ocean Modelling*, **56**, 16-30 DOI: 10.1016/j.ocemod.2012.07.004.

Özgökmen, T. M., A. C. Poje, P. F. Fischer, and A. C. Haza. Large eddy simulations of mixed layer instabilities and sampling strategies, 2011. *Ocean Modelling*, **39**(3-4), 311-331, DOI: 10.1016/j.ocemod.2011.05.006.

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Santitissadeekorn, N., E.T. Spiller, C.K.R.T. Jones, R. Rutarindwa, L. Liu, and K. Ide. Observing System Simulation Experiments of Cross-Layer Lagrangian Data Assimilation. Submitted to *Dynamics of Atmospheres and Oceans*, January, 2013.

Schroeder, K., J. Chiggiato, A. C. Haza, A. Griffa, T. M. Özgökmen, P. Zanasca, A. Molcard, M. Borghini, P.M. Poulain, R. Gerin, Z. Zambianchi, P. Falco, and C. Trees, 2012. Targeted Lagrangian sampling of submesoscale dispersion at a coastal frontal zone. *Geophys. Res. Lett.*, **39**, L11608, doi.10.1029/2012GL051879.

Spiller, E. T., A. Apte, and C.K.R.T. Jones. Assimilating en Route Lagrangian Observations. Submitted to *Tellus*, December, 2012.

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Sulman, M. H. M., H. S. Huntley, B. L. Lipphardt, Jr., P. Hogan, G. Jacobs, and A. D. Kirwan, Jr. Three Dimensional Aspects of Loop Current Ring Formation. To be submitted to *Nonlinear Processes in Geophysics*.

Valentine, D. A., I. Mezic, Senka Maćešić, Nelida ÄÂŒrnjarić-ÂÃ,ÂŽic, Stefan Ivić, P. J. Hogan, V. A. doi: 10.1073/pnas.1108820109 water hydrocarbon irruption. *Proceedings of the National Academy of Sciences,* doi: 10.1073/pnas.1108820109.

Objective

Develop and exploit dynamical systems methodology in order to improve our understanding of, and predictive capability for, 3D, time-dependent features of ocean circulation.



Lagrangian perspective: follow fluid elements or 'particles'.

Dynamical Systems (DS)
$$\frac{dx}{dt} = u(x, y, z, t); \frac{dy}{dt} = v(x, y, z, t); \frac{dz}{dt} = w(x, y, z, t);$$

Lagrangian Data Assimilation (LDA): assimilation of trajectory positions into models.

Lagrangian Chaos: nearby trajectories separate from each other exponentially fast in time.

Hyperbolic point/trajectory

Lagrangian Coherent Structures (LCS): refers to a collection of distinguished objects (hyperbolic trajectories, manifolds, material curves, material surfaces) that form a template or skeleton for chaotic transport.

Chlorophyll in the North Sea



History

- 1980s: Lagrangian chaos found in very simple 2D point vortex flows; chaotic transport theory developed in applied math community.
- 1990s: With encouragement from ONR (Reza Malek-Madani), mathematicians and physical oceanographers meet. Leads to Little Compton meeting, circa 1994.
- 1995-2005: series of ONR initiatives (Manny Fiadeiro, Reza...);
 2D+time, Lagrangian data assimilation developed in parallel.
- Mid 2000s: methods popular; many applications to ocean surface flows.



Applications to 2D or quasi-2D flows.



Monterey Bay: Marsden & colleagues



Loop Current Eddies (Branicki and Kirwan)



Dipoles (Rypina, et al.)



Gulf Mexico (Lipphardt et al.)

Why 3D+1? Why now?

- Modern observations better able to resolve 3D structures.
- Interest in 'sub-mesoscale': eddies, fronts, jets scales 100s m to 10s km. Vertical velocity more important at these scales.
- Oceanographic sampling platforms (unmanned underwater vehicles) can operate in 3D.





3D flow have a richer variety of hypbolicity



Main Thrusts

 Identify the Lagrangian structures that guide transport and mixing processes in idealized 3D+1 flow fields.



2) Establish methodology that will help us map out organizing Lagrangian structures in more realistic models and data.



Methods

- Measures of rate of trajectory separation (Lyapunov exponents: FTLE and FSLE)
- Measures of hyperbolicity along trajectories (mesohyperbolicity)
- Measures of trajectory complexity (correlation dimension, ergodicity defect)
- Measures based on averages taken along trajectories (ergodic quotient, Koopman modes)
- (All are valid in 3D+1: when does 2D+1 suffice.

3) Lagrangian data assimilation in 3D+1: how do we optimize the use of Lagrangian data?



(from Glad program)

Schedule

- 1. Theory and simple models.
- 2. Ocean models and 3D+1 transport.
- 3. Lagrangian data assimilation.
- 4. Future plans.

Workshops

Oct. 2010 Chicago Feb. 2012 Wilmington Feb. 2013 Chapel Hill

Budget

Period	months	original start date	received funds on	amount
1	2	08/2010	10/2010	\$352,549.
2	12	10/2010	5/2011	\$1,354,449.
3	12	10/2011	1/2012	\$1,392,056.

We are midway through this 3rd period, at the 19th month

4 10 10/1212 \$1,278,214.

5,6,7 \$3,123,393. (year 4 and 5 extension) \$3,123,393.