Three-Dimensional Chaotic Advection in an Idealized Ocean Eddy

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MURI Charge

- Use DS theory/tools to learn something about 3D, time-varying ocean features.
- How much of the stirring is due to small scale motion and how much is simply due to stirring by the coherent features?

3D: $\frac{\partial w}{\partial z}$ is significant in $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \boxed{\frac{\partial w}{\partial z}} = 0$

Plan

- Larry: Background on Rotating Can
- Irina: Resonant Width
- Peng/Tamay: Time Dependence

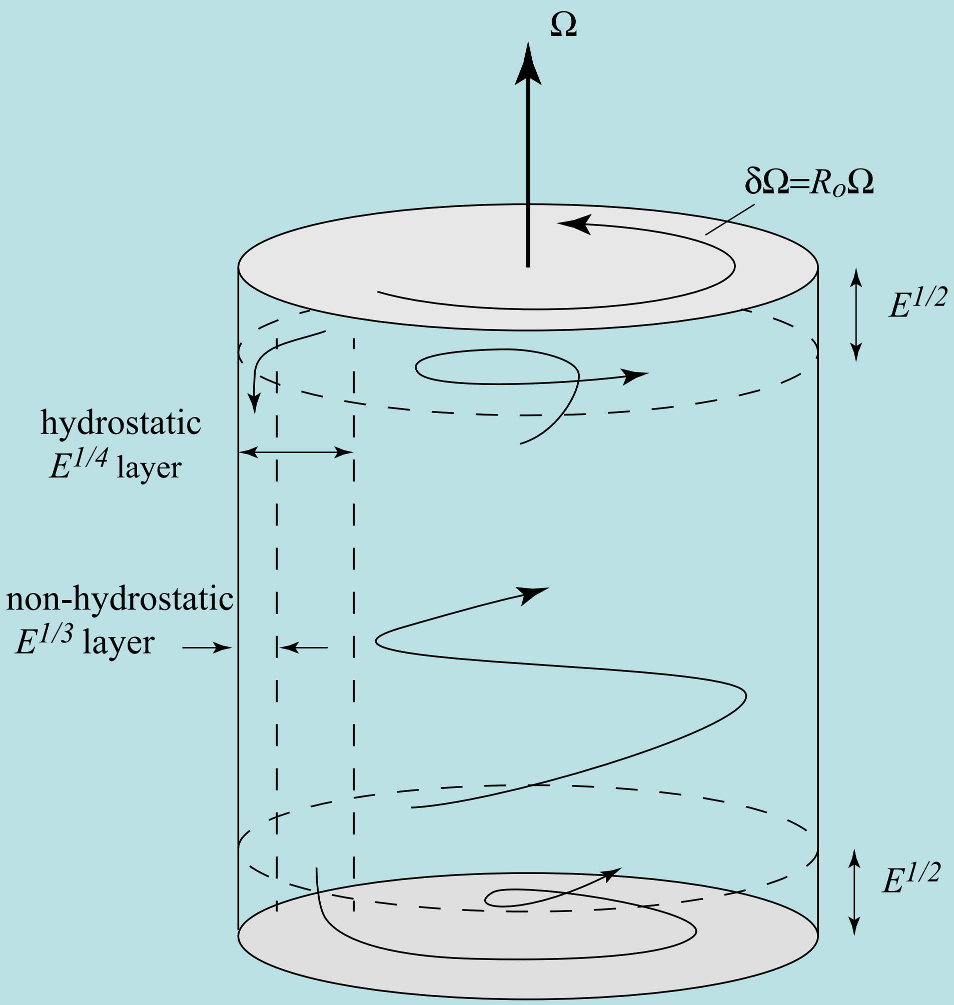


Figure 1. Rotating can with no-slip boundaries and small Ekman number. The flow is driven by a differentially rotating lid.

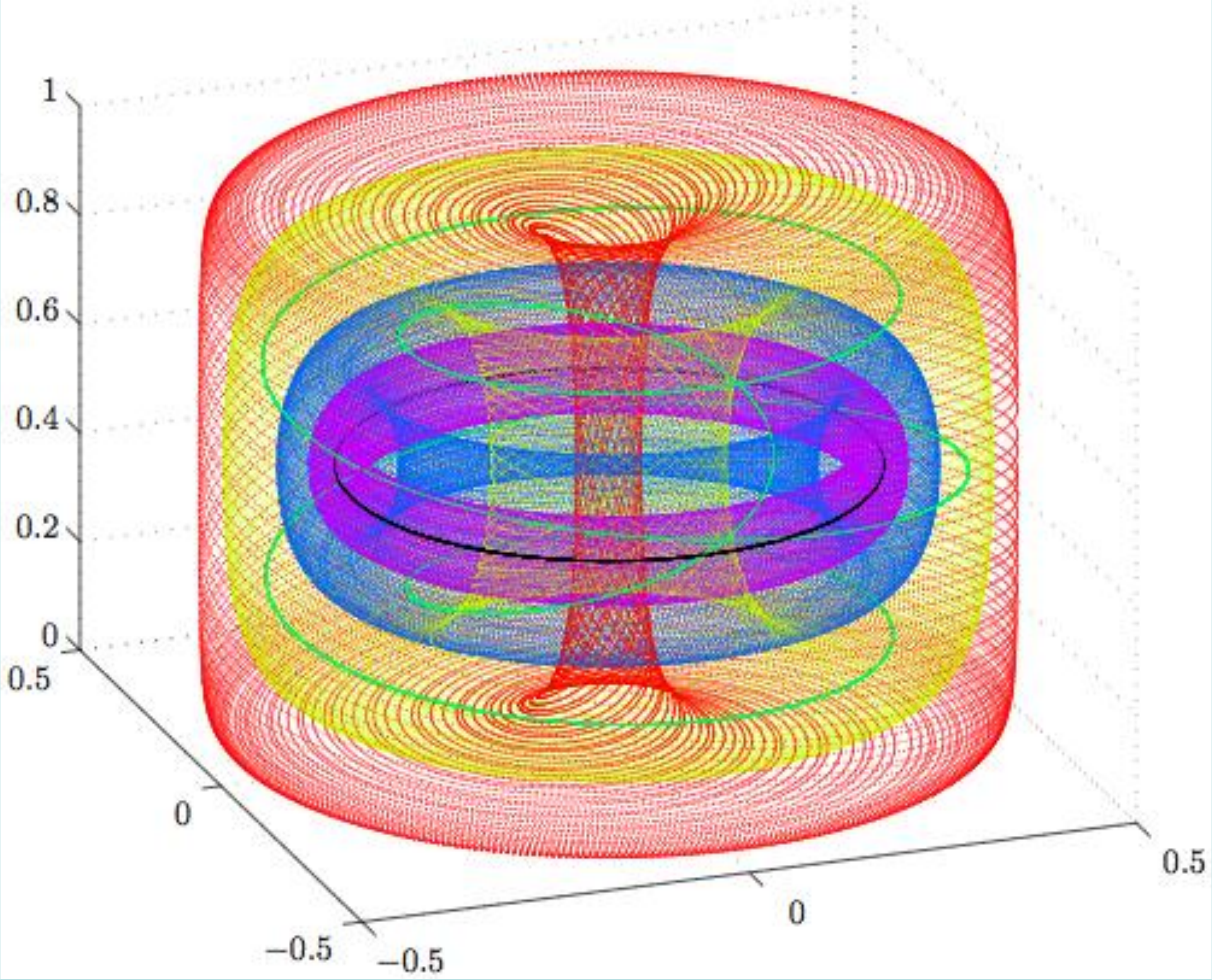
Velocity Fields

1) Kinematic (3d velocity non-divergent but no dynamics)

2) Linear asymptotic solution with $E = \frac{\nu}{\Omega H^2} \ll 1$

vertical velocity: $w \sim E^{1/2}$

3) Nonlinear numerical model.



1
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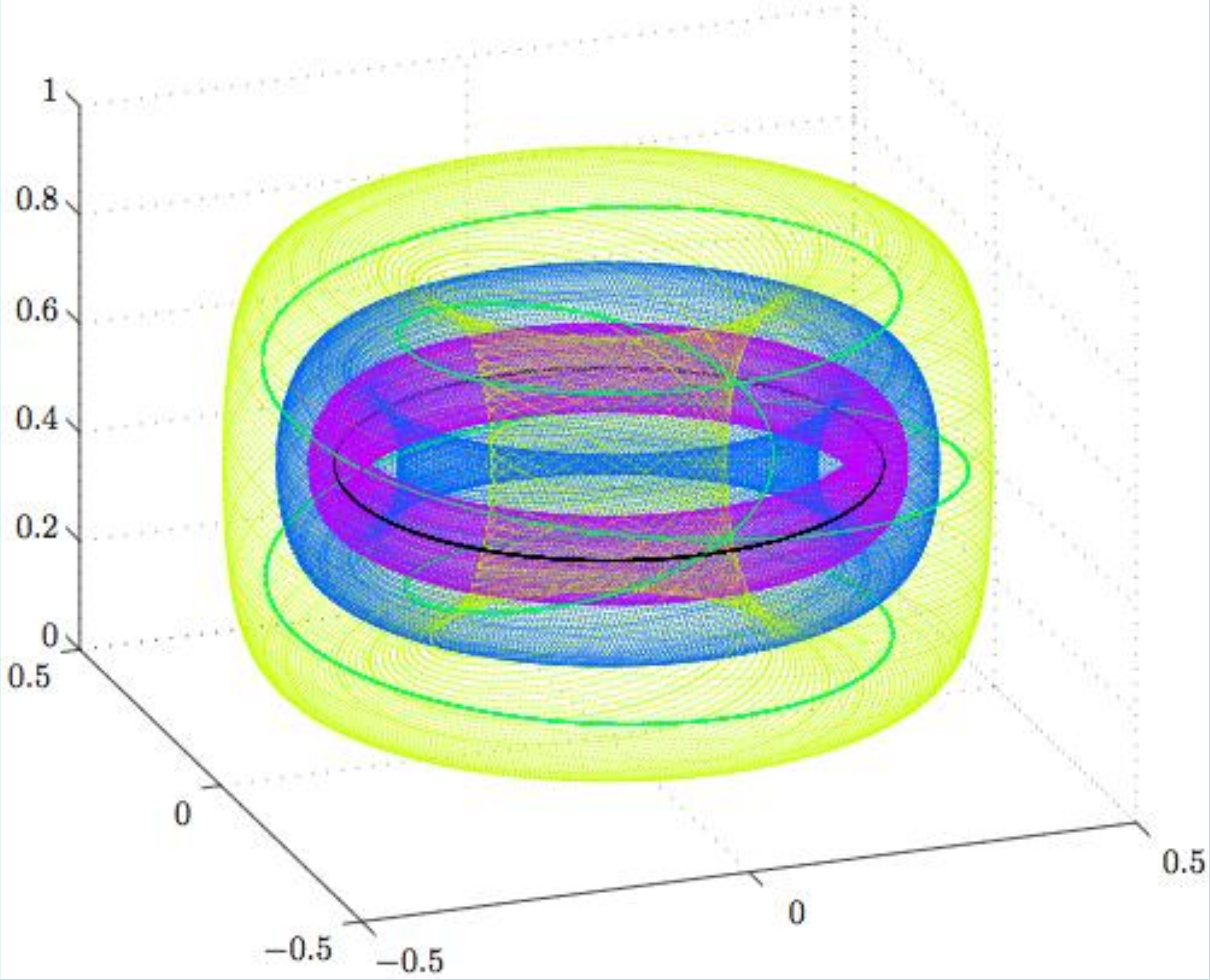

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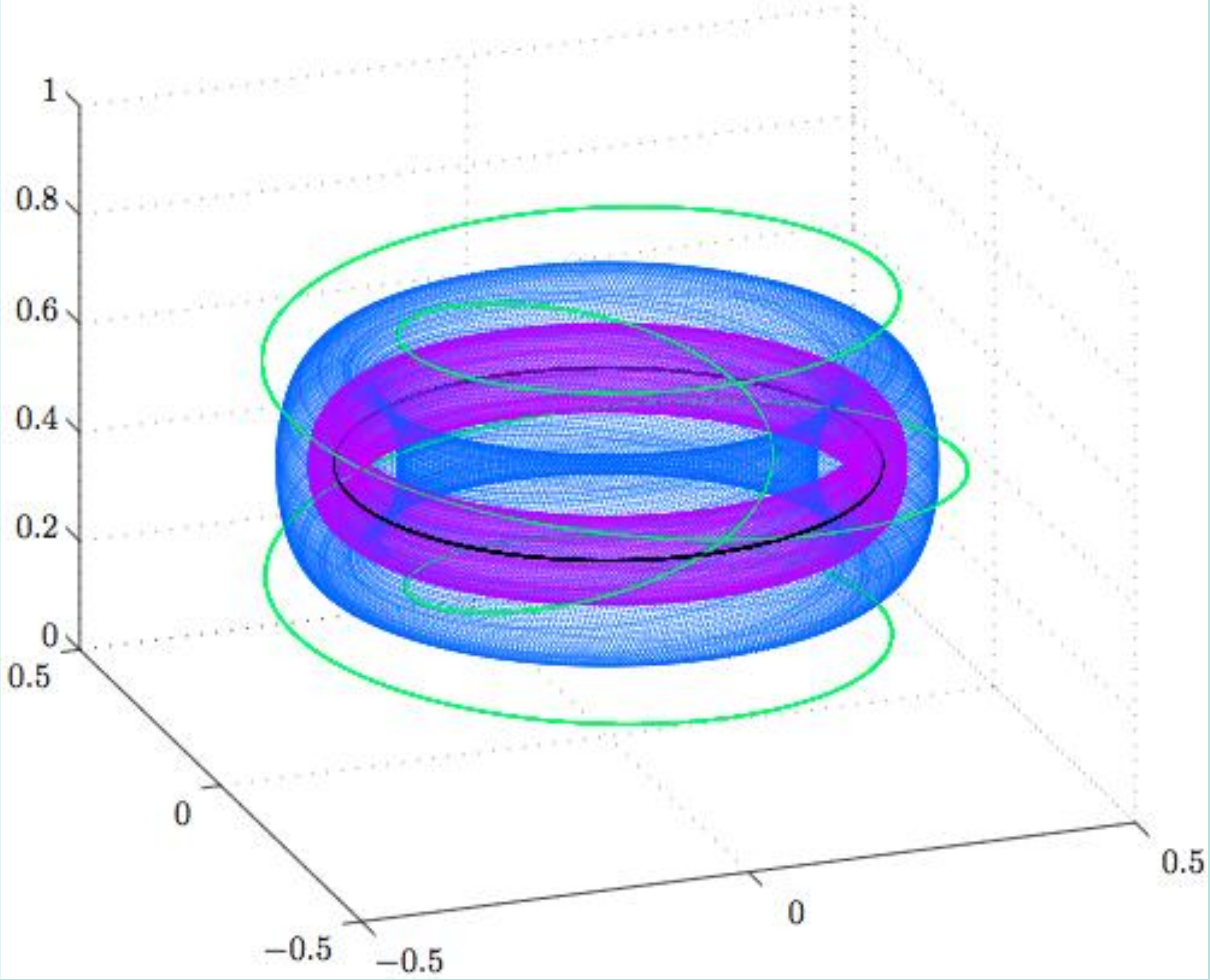

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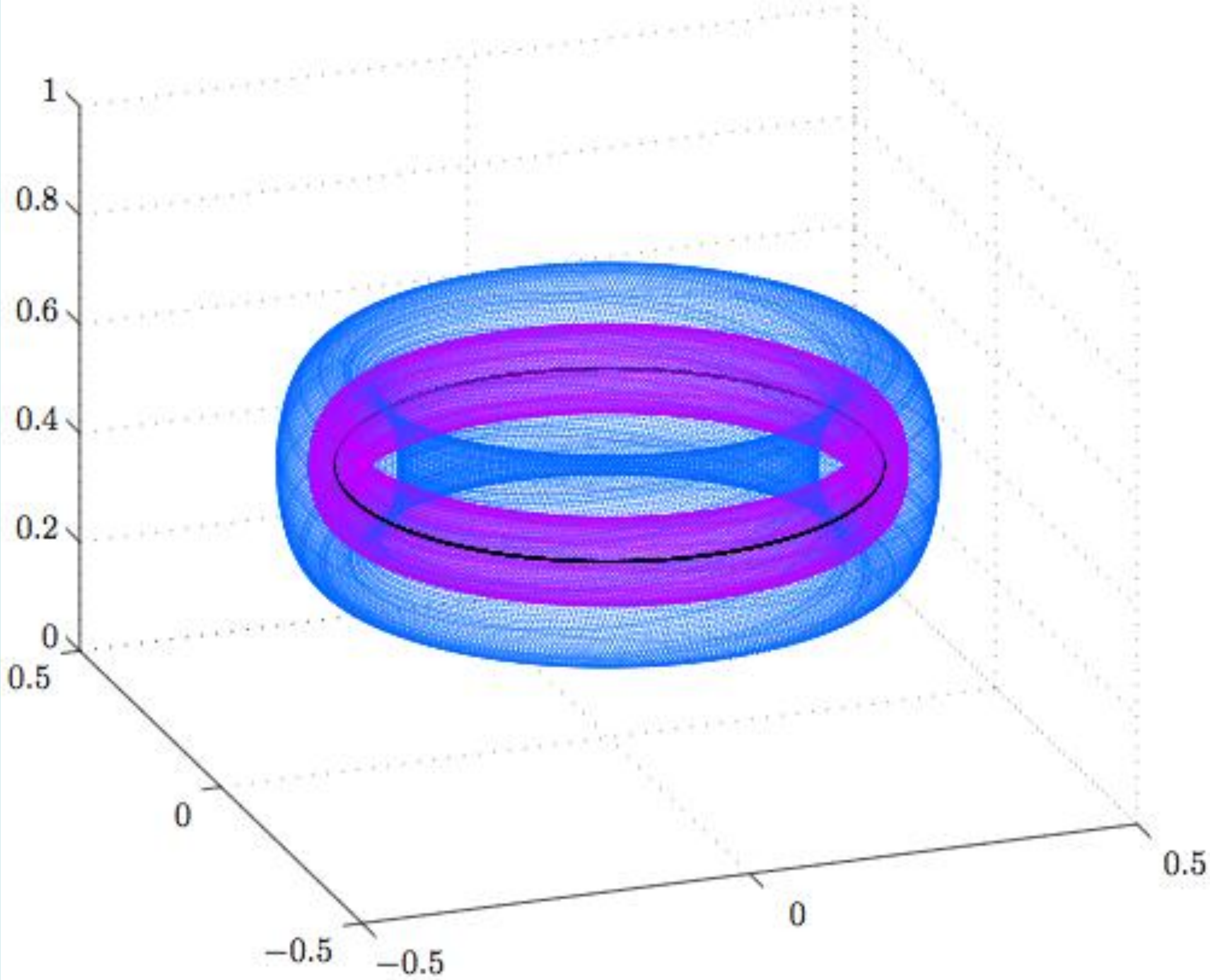
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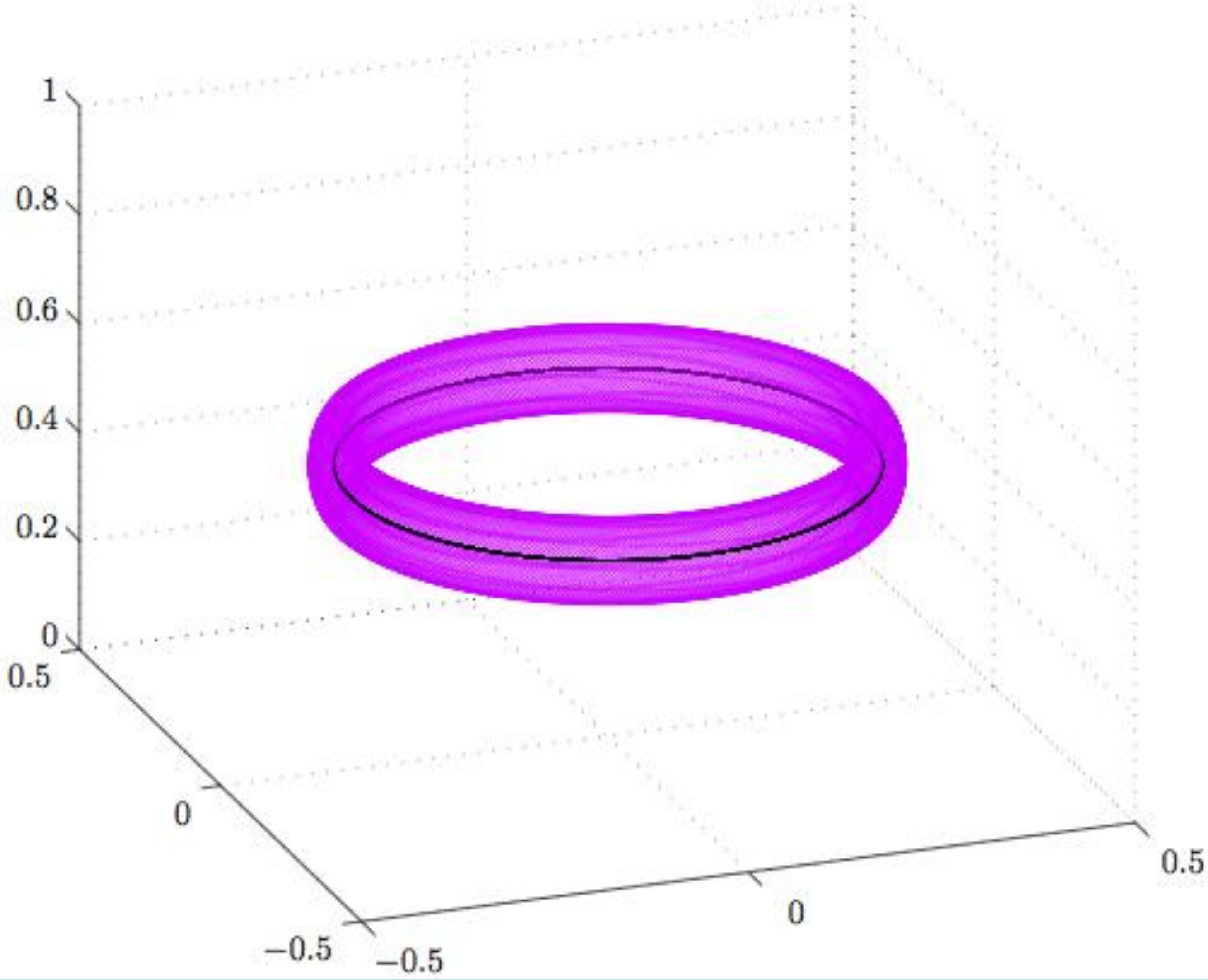

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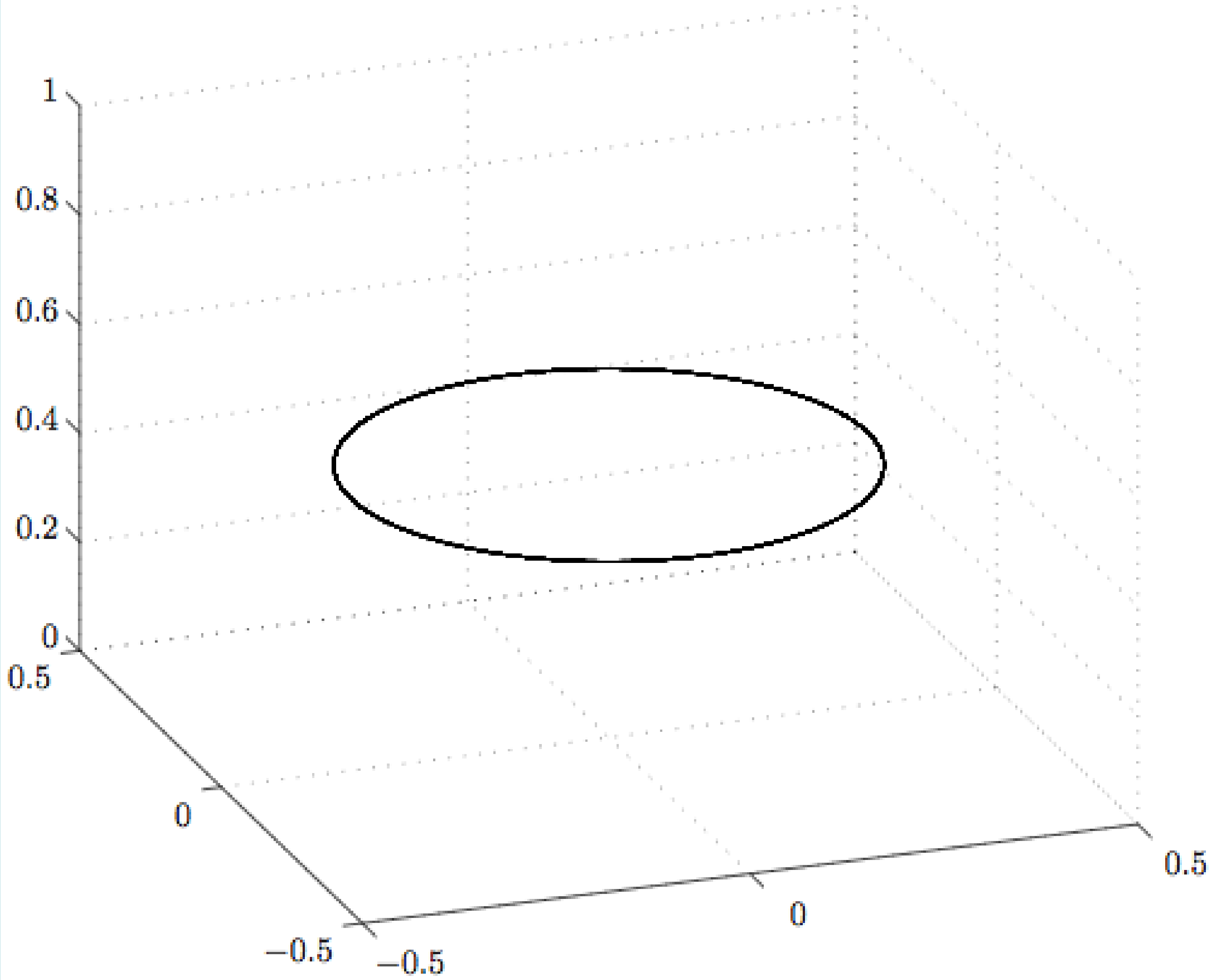

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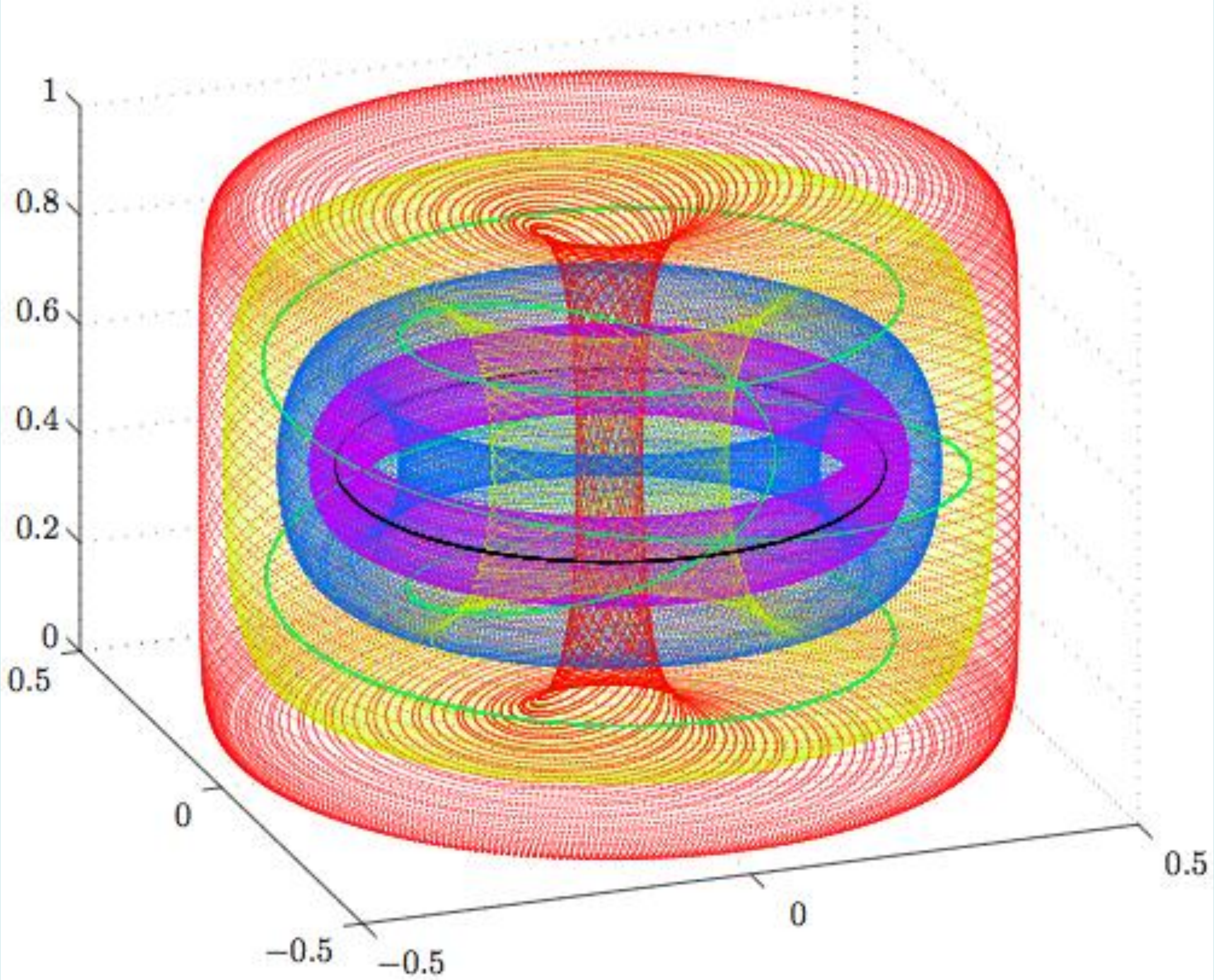

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0.6
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0.5
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−0.5
−0.5
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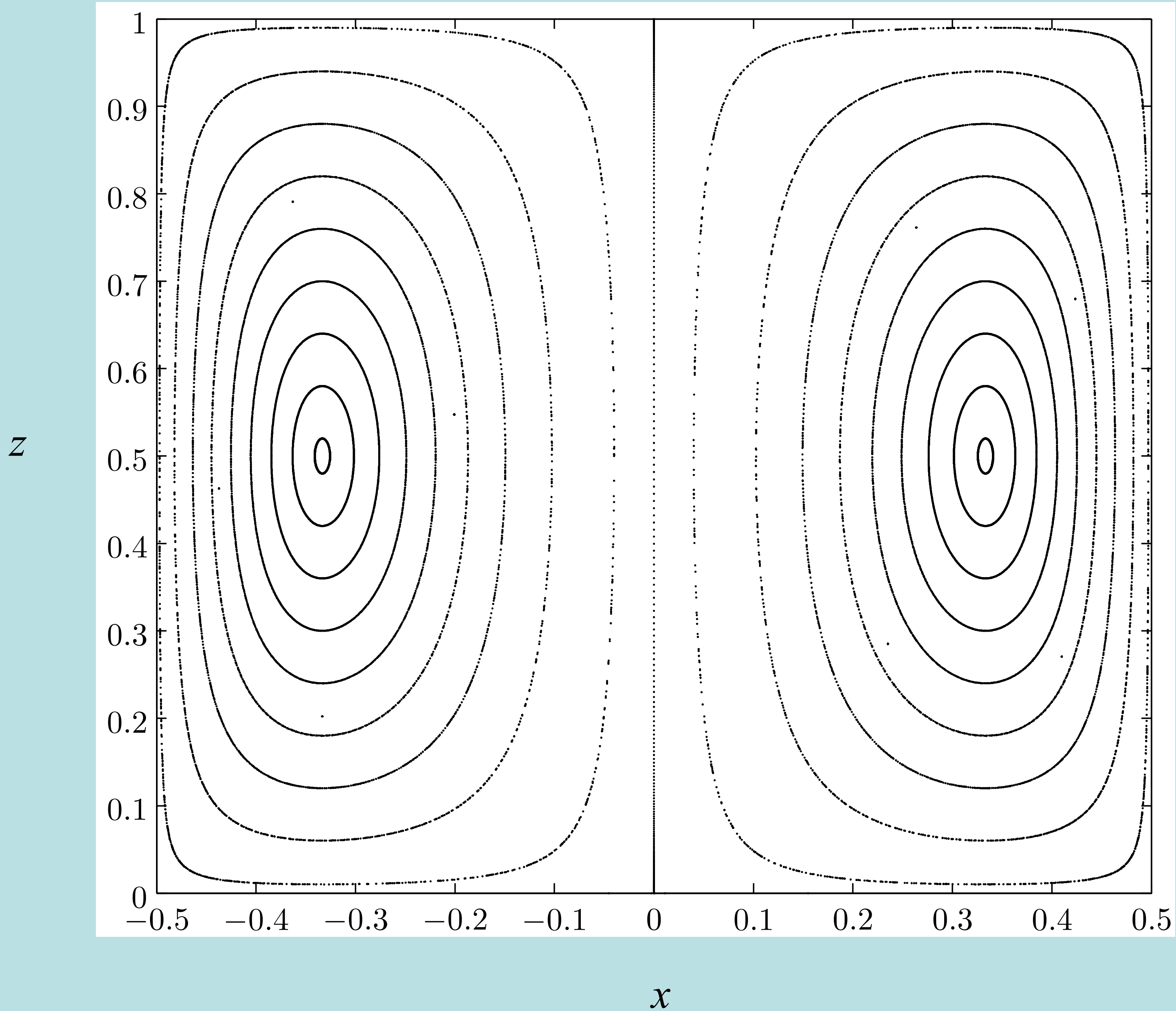


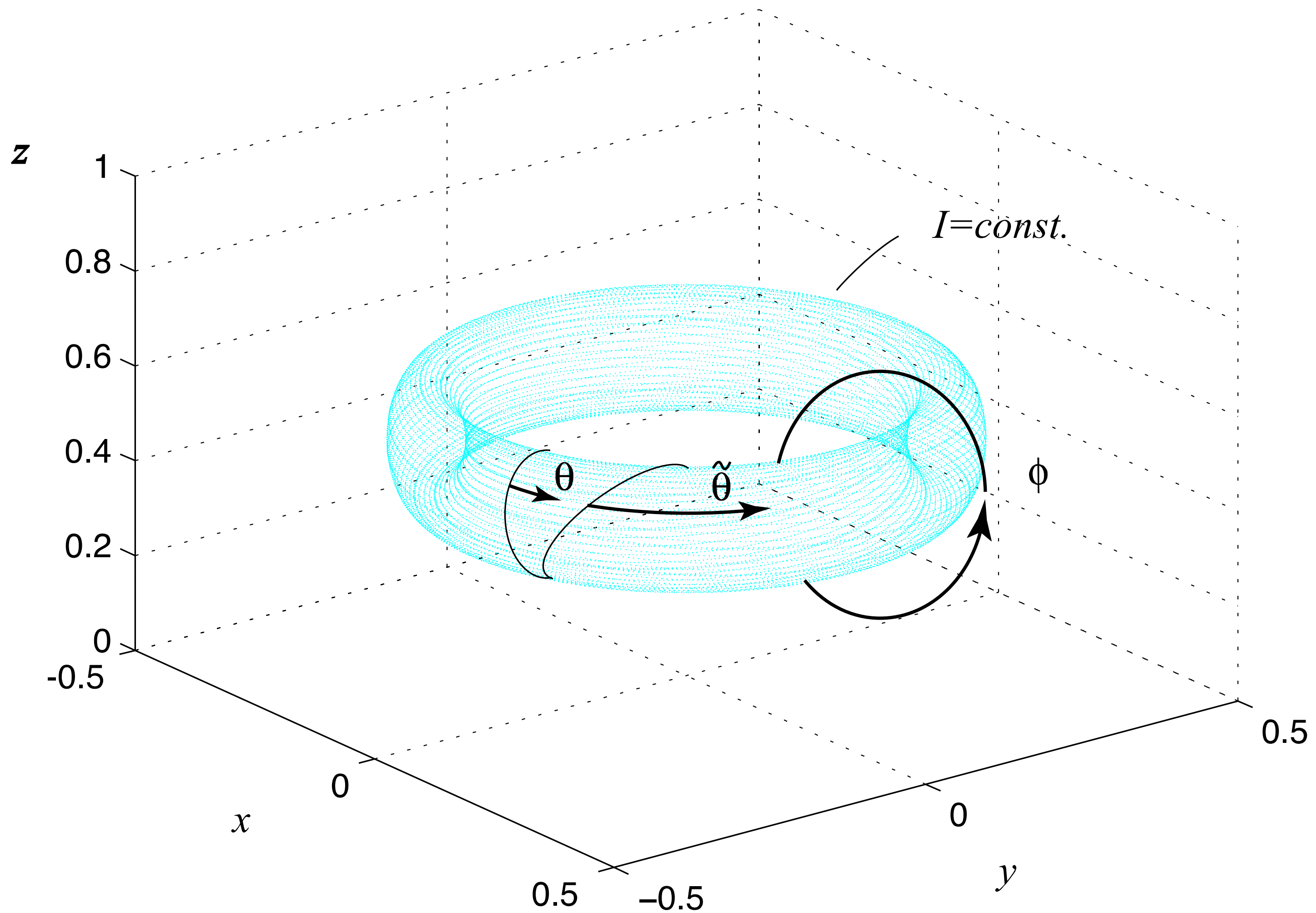
Fig 3

Action-Angle-Angle System (Mezic and Wiggins 1994)

$$\frac{d\tilde{q}}{dt} = \mathrm{W}_q(I)$$

$$\frac{df}{dt} = \mathrm{W}_f(I)$$

$$\frac{dI}{dt} = 0$$



$$T_f = \frac{2p}{\mathrm{W}_f}$$

$$T_q = \frac{2p}{\mathrm{W}_q}$$

Theory for Steady Perturbation

Mezic and Wiggins (1994); Cheng and Sun (1990)

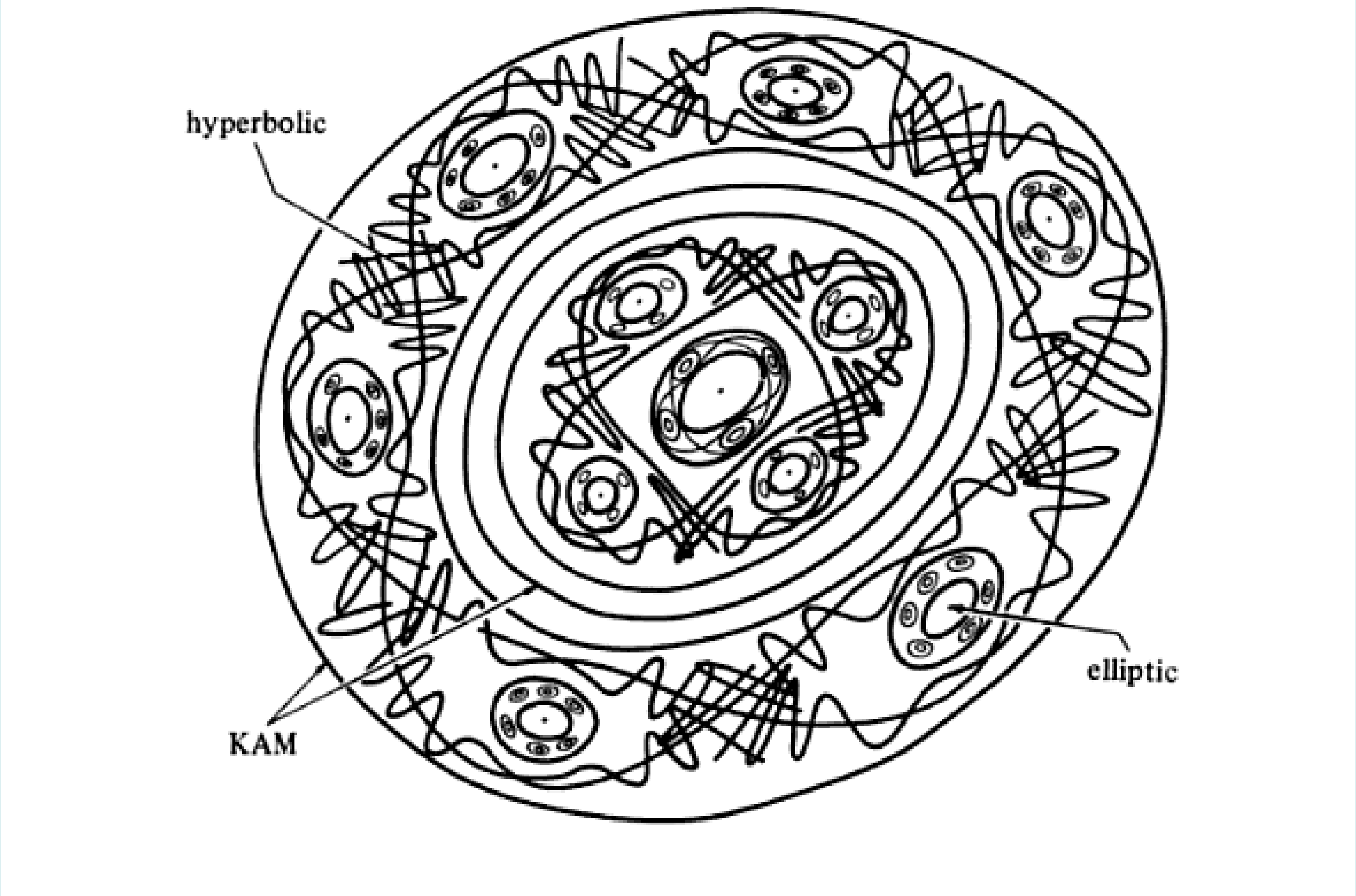
1) If $\dfrac{T_q(I)}{T_f(I)} = \dfrac{m}{n}$ (rational), trajectories on the torus I are periodic.

Trajectory Period= $\boxed{nT_q(I)}$ (also = $mT_f(I)$)

2) A torus with periodic trajectories is expected to break up, along with some of its neighboors.

3) KAM result (for small perturbation): Some tori for which $\dfrac{T_q}{T_f}$ is sufficiently irrational will survive.

The breakup of resonant contours.



$T_{forcing}/T_{winding} = m/n$ n=# of islands

(Ottino 1989)

T_q / T_f in our case

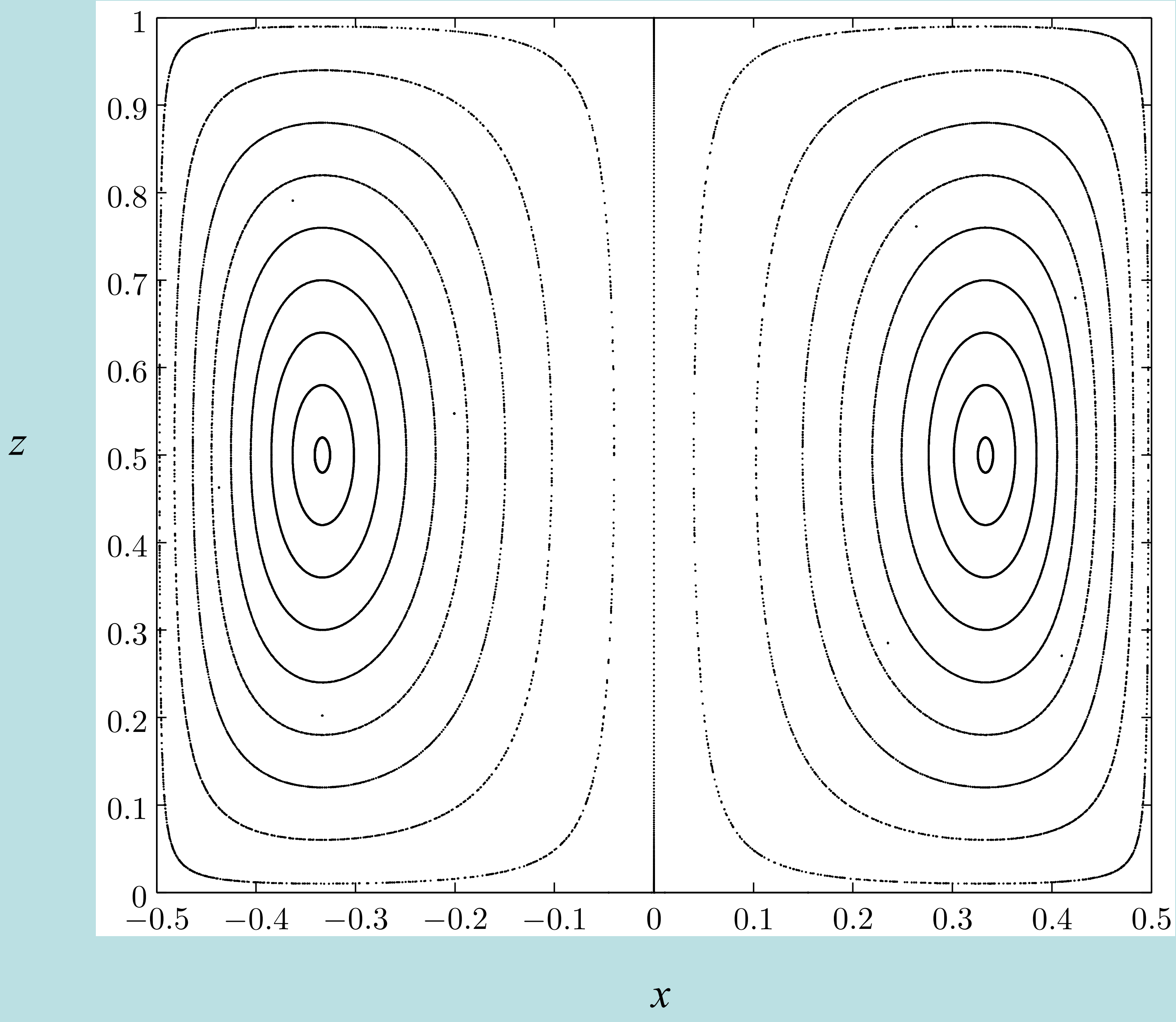


Fig 3

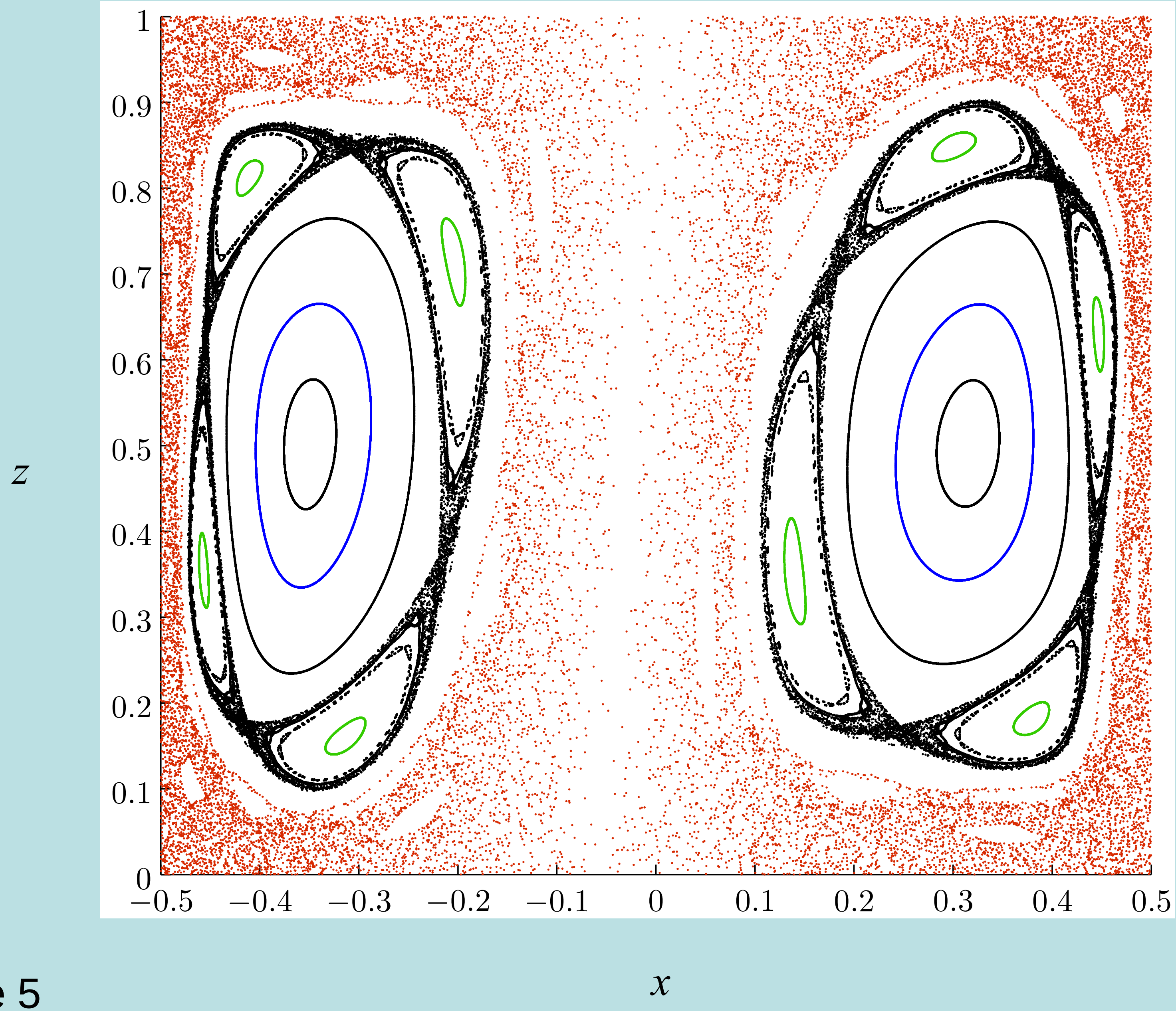


Figure 5

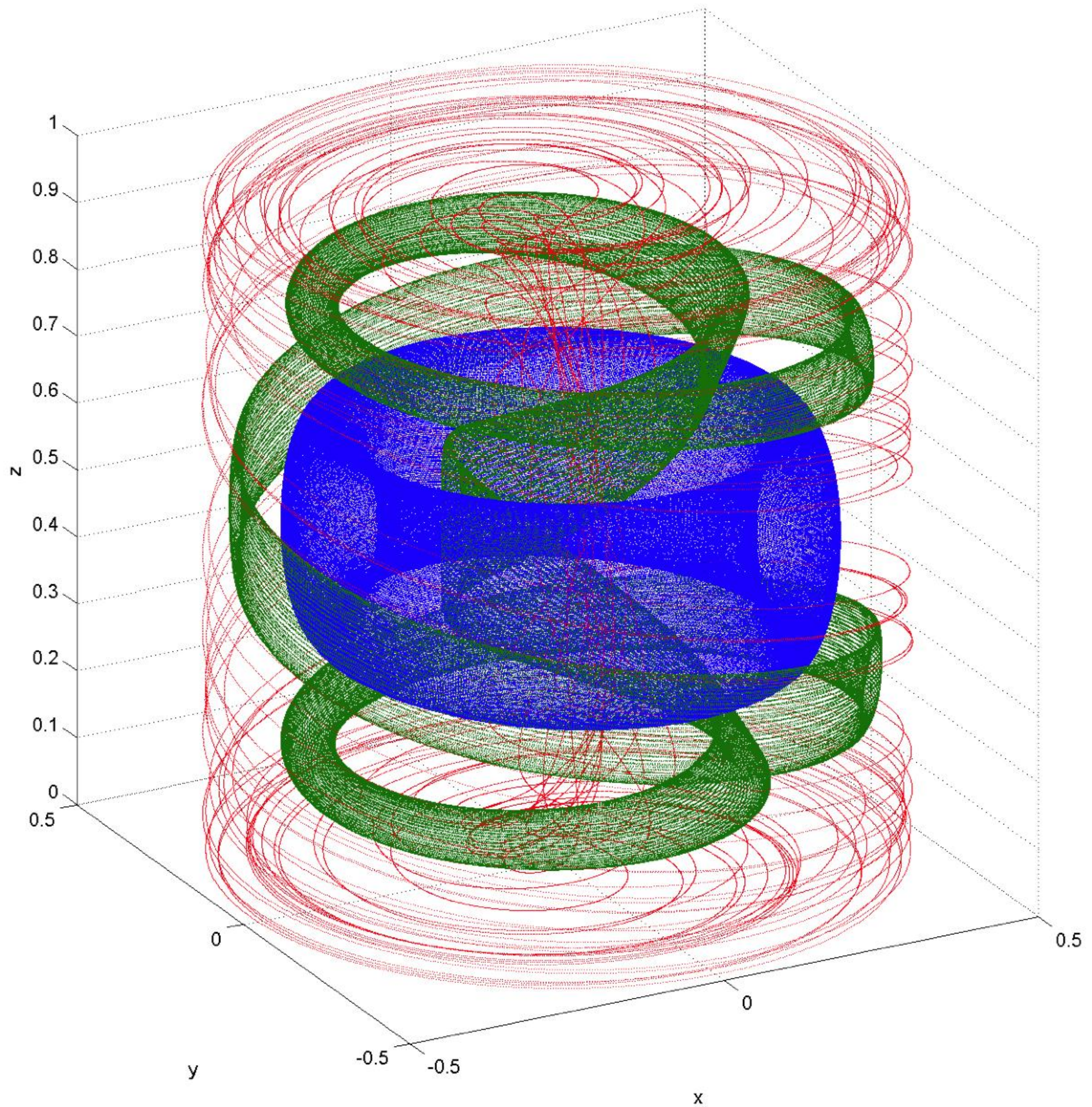
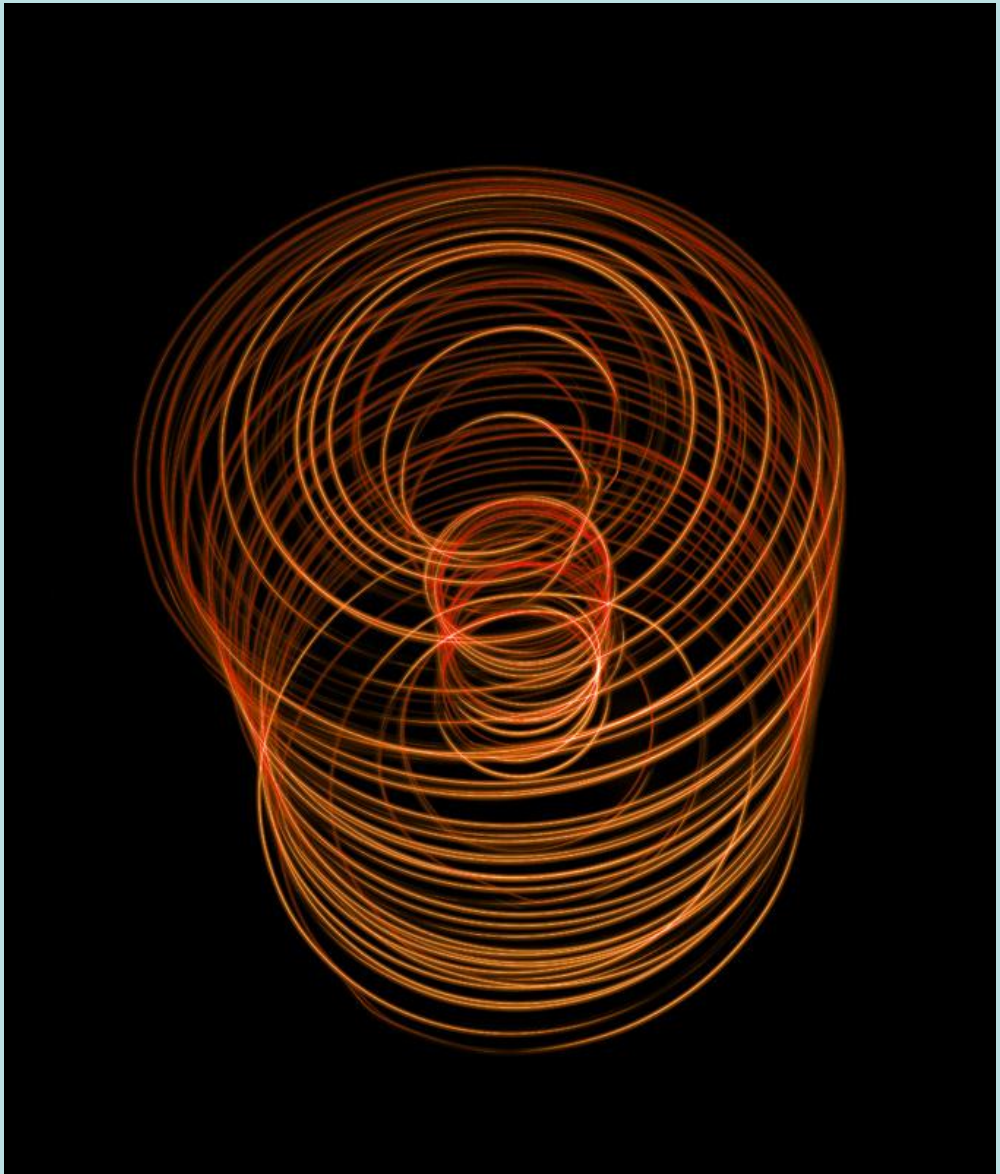
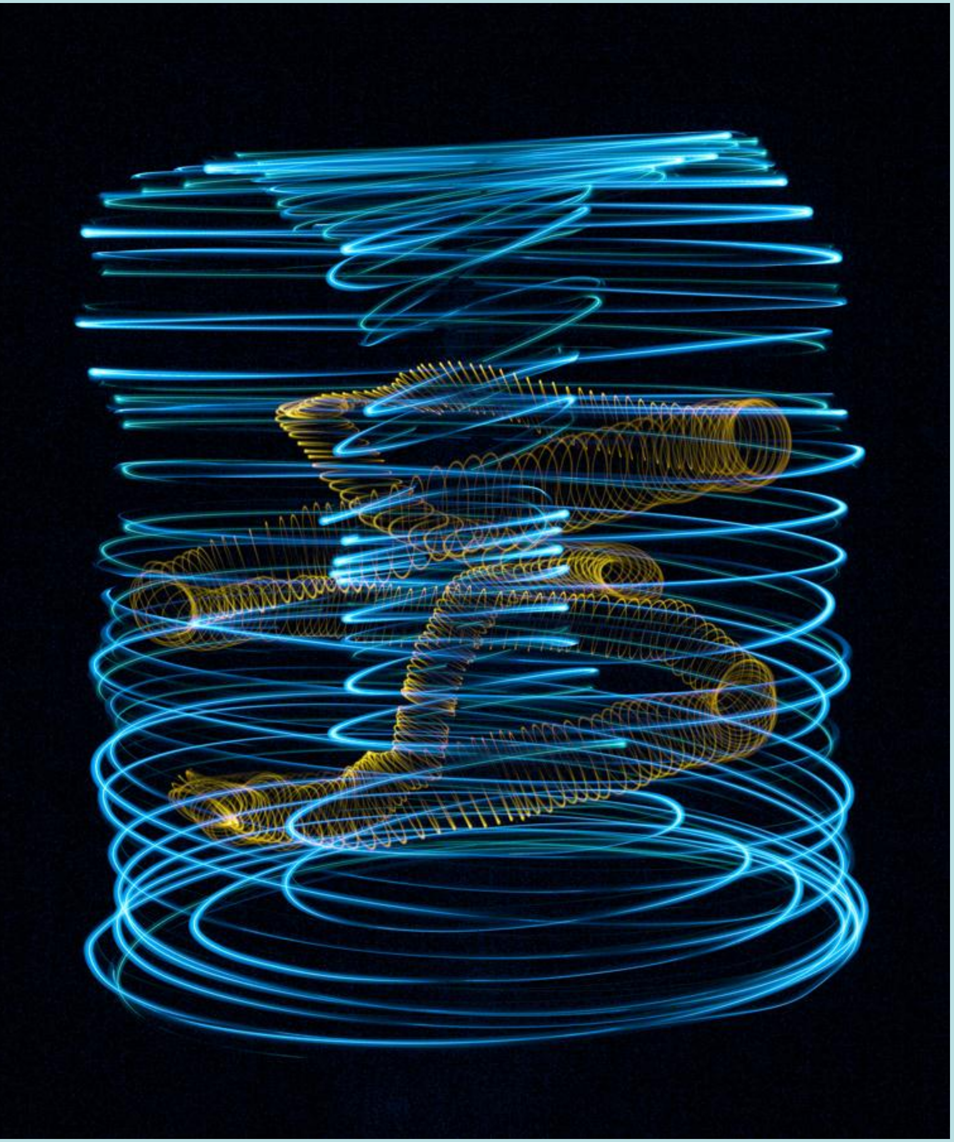


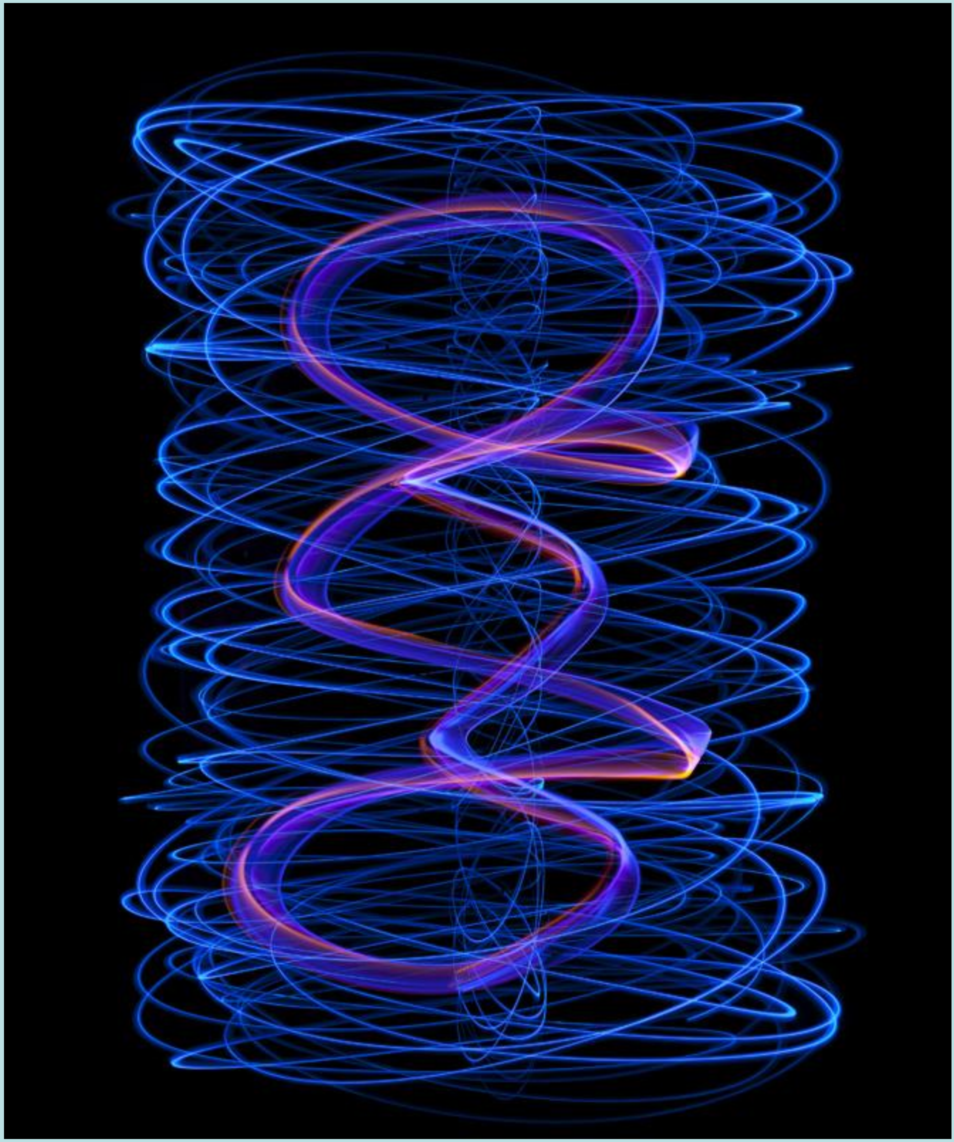
Fig. 6

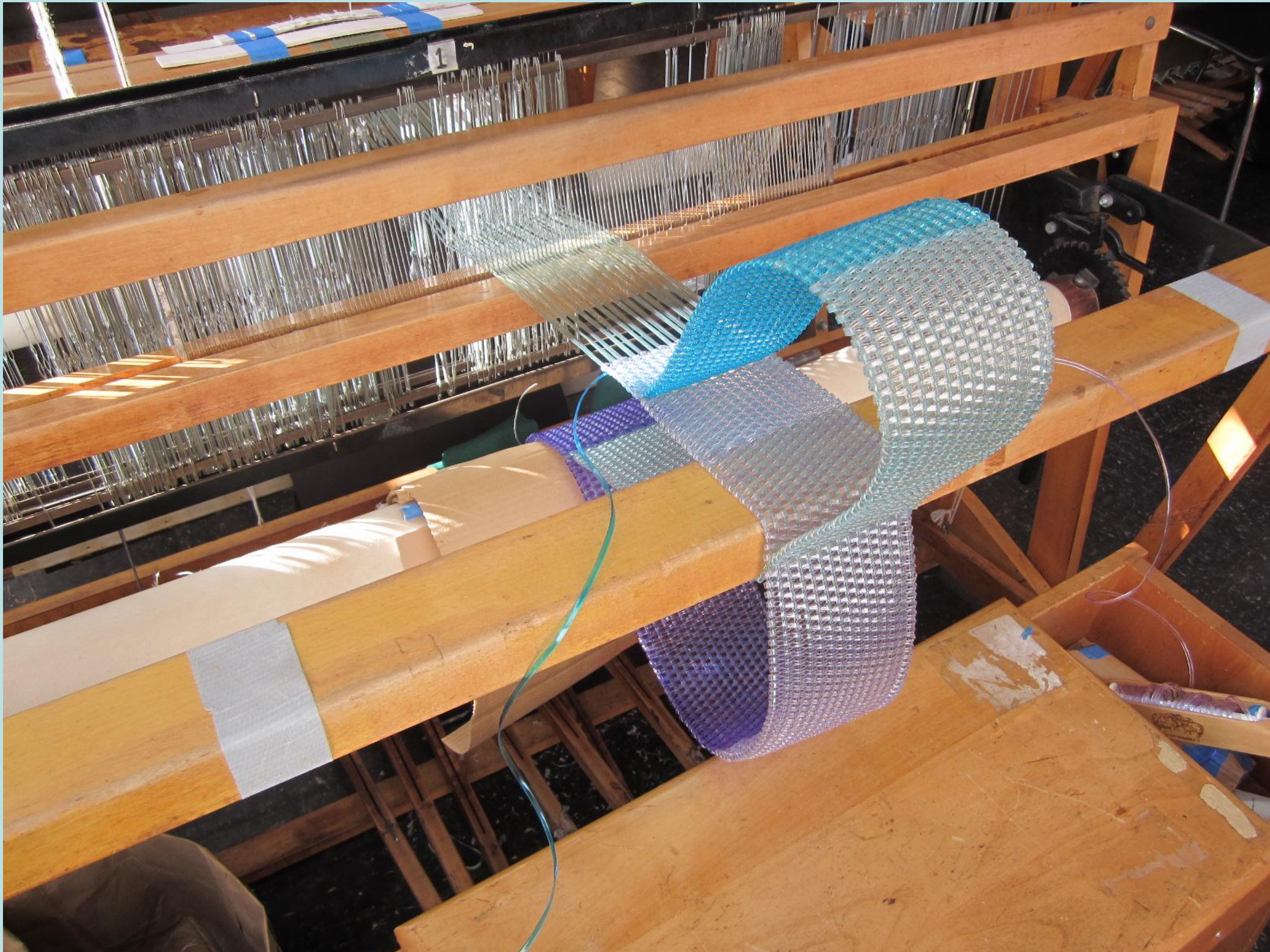
Museum of Science Boston Exhibit

(with Anastasia Azure: opens Feb. 17)

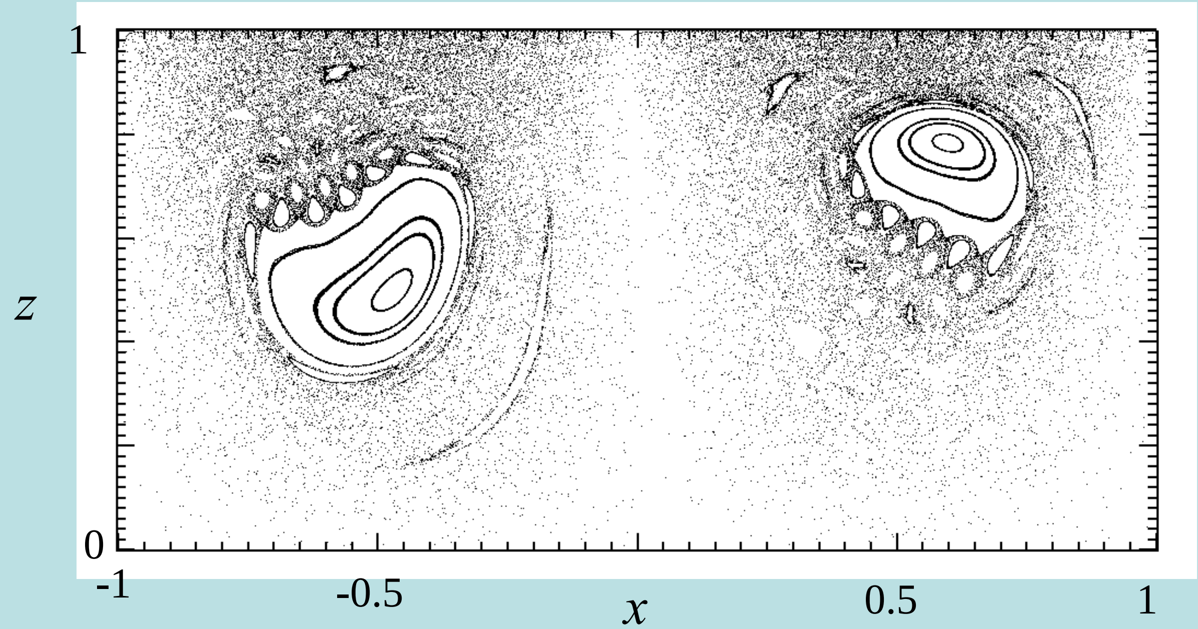







1

Re=20, Ro=1, x0=-0.02



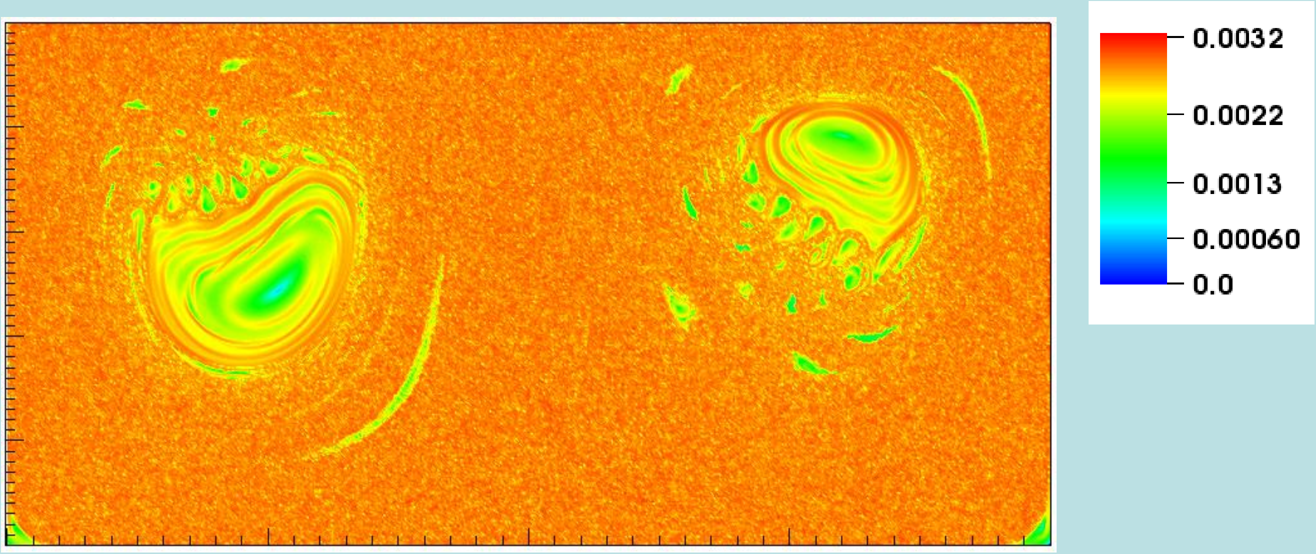
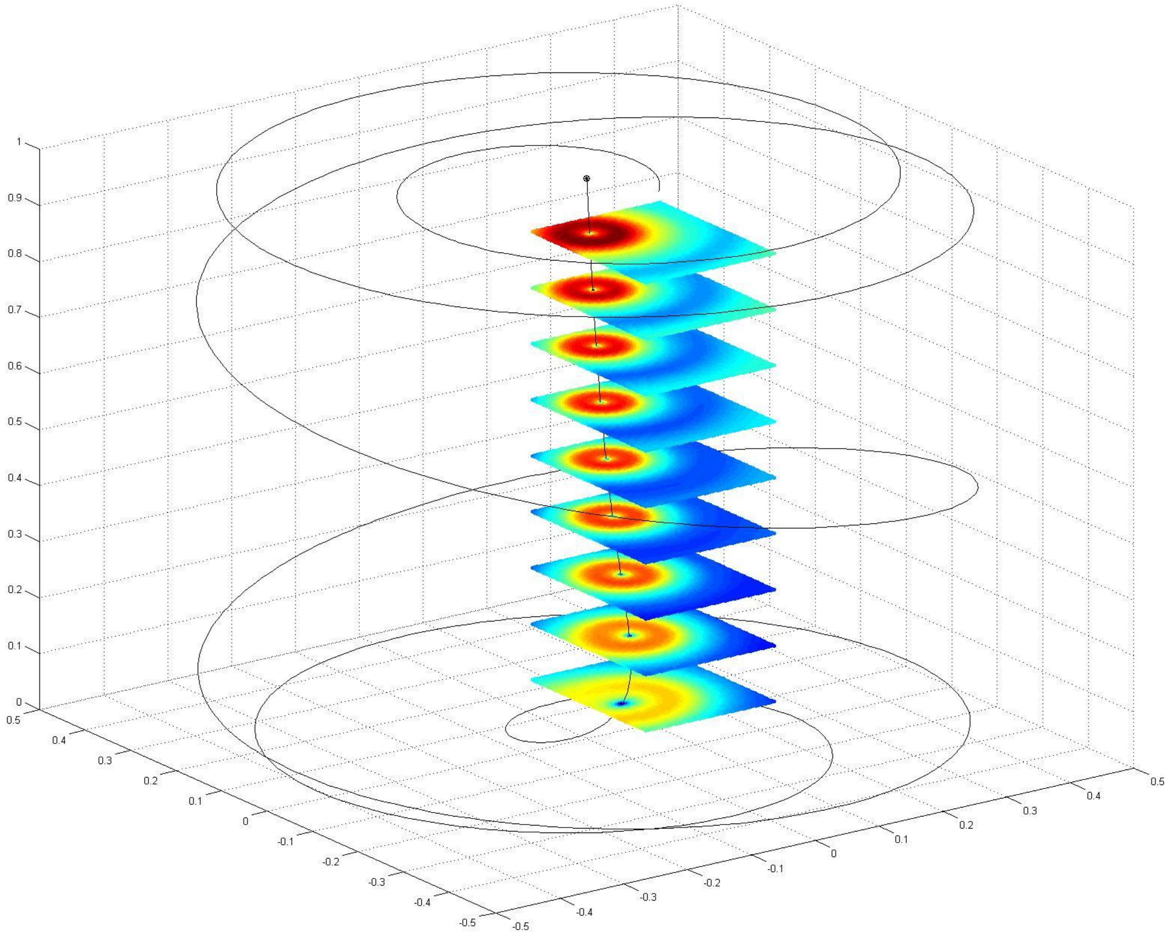
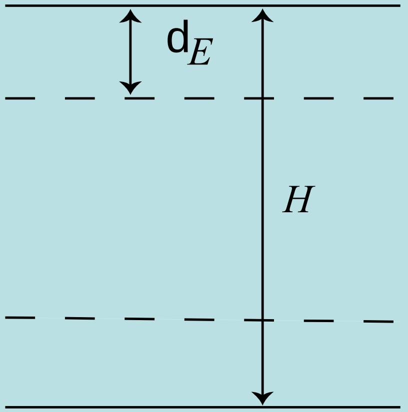


Figure 8

Stable Manifold



Parameters (Numerical Model)



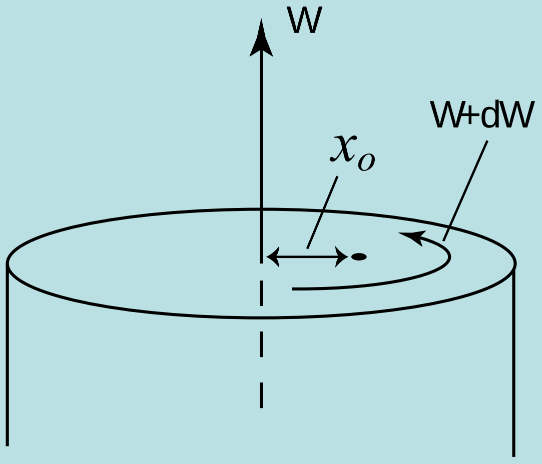
Ekman Number $E = \frac{\nu}{\Omega H^2} = \left(\frac{\delta_E}{H}\right)^2$

$$1/2000 < E < 1$$

Rossby Number $R_o = \delta\Omega / \Omega$

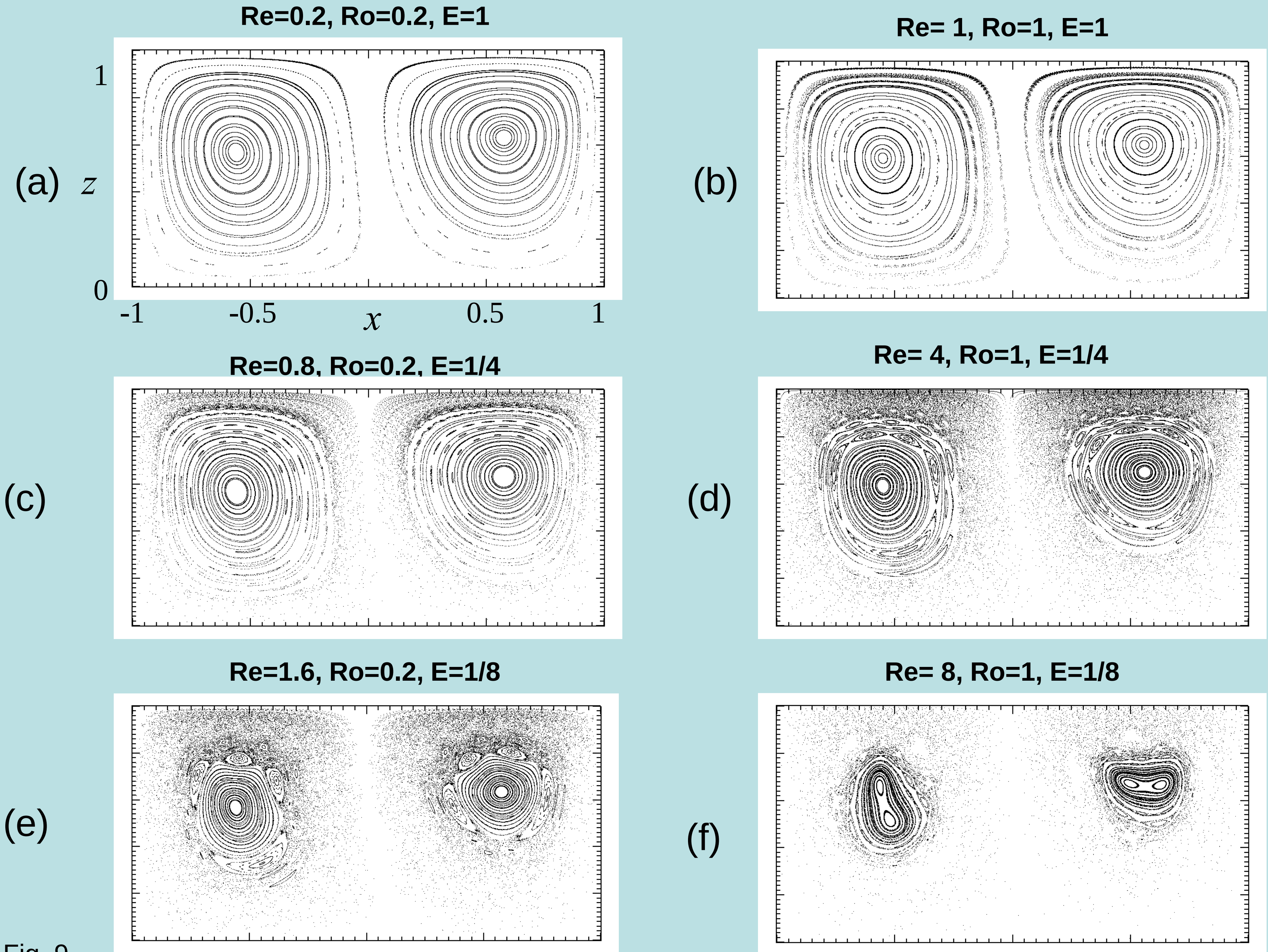
$$0.2 \le R_o \le 1$$

Perturbation Amplitude ξ_o



Aspect Ratio H/R=1

Note: $R_e = R_o / (EH^2 / R^2)$



Re=0.2, Ro=0.2, E=1
(a)
z
1
0
-1
-0.5
x
0.5
1
Re= 1, Ro=1, E=1
(b)
Re=0.8, Ro=0.2, E=1/4
(c)
Re= 4, Ro=1, E=1/4
(d)
Re=1.6, Ro=0.2, E=1/8
(e)
Re= 8, Ro=1, E=1/8
(f)

Fig. 9

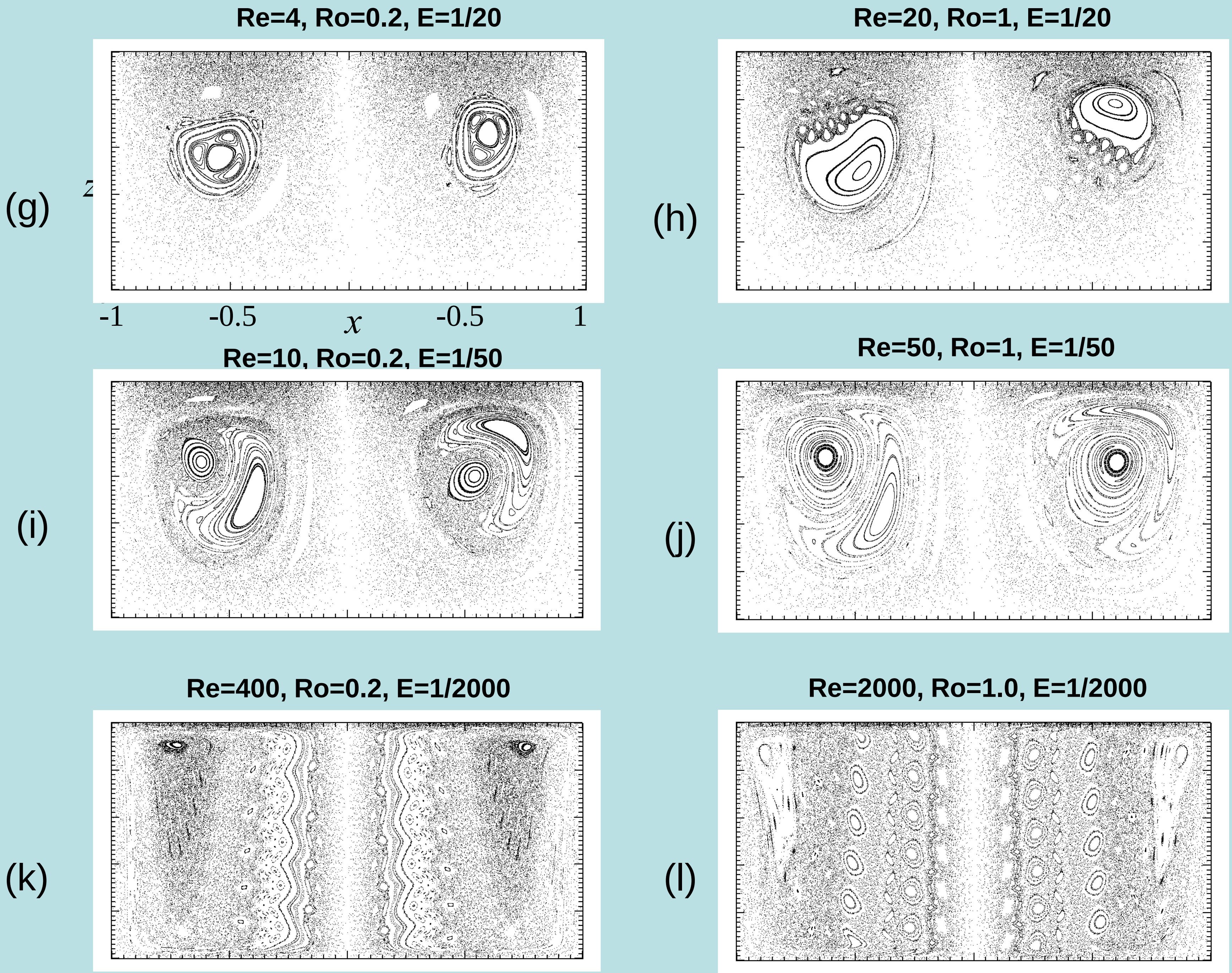
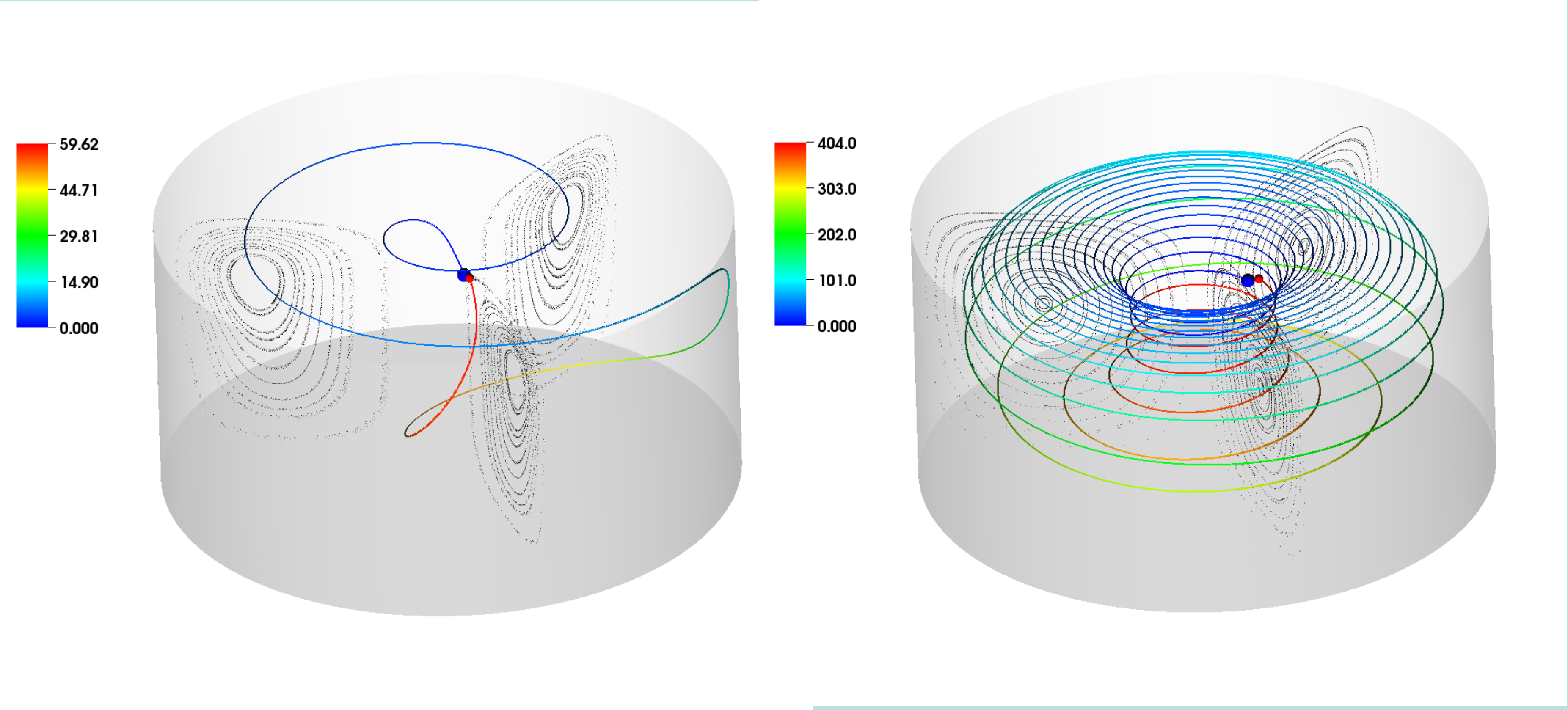


Figure 9 (continued)

Ro=1, *E*=1/100

Ro=1, *E*=1

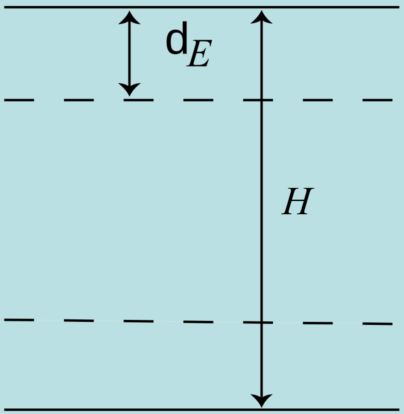


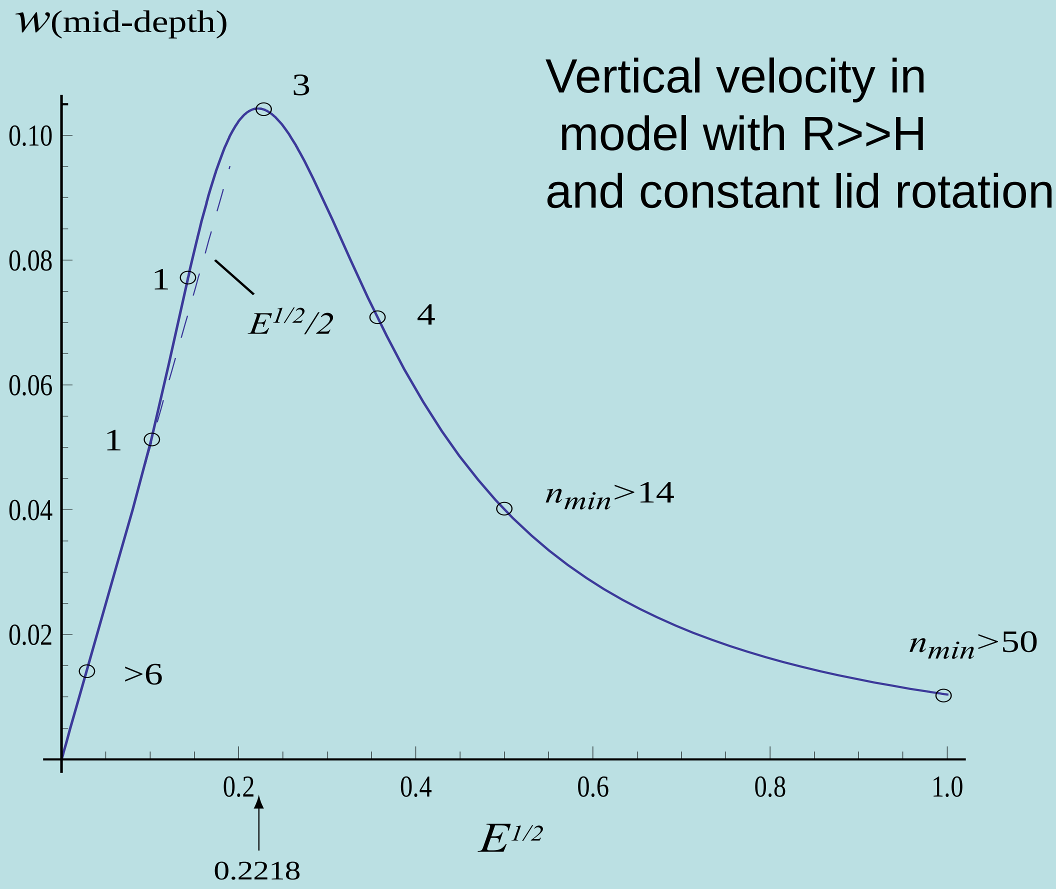
(a)

(b)

Figure 12

$$\frac{T_q}{T_f} = O\left(\frac{w/H}{v/R}\right) = O(w) \qquad \text{For } E \ll 1,\ \ w = O(E^{1/2})$$





Vertical velocity in
model with R>>H
and constant lid rotation

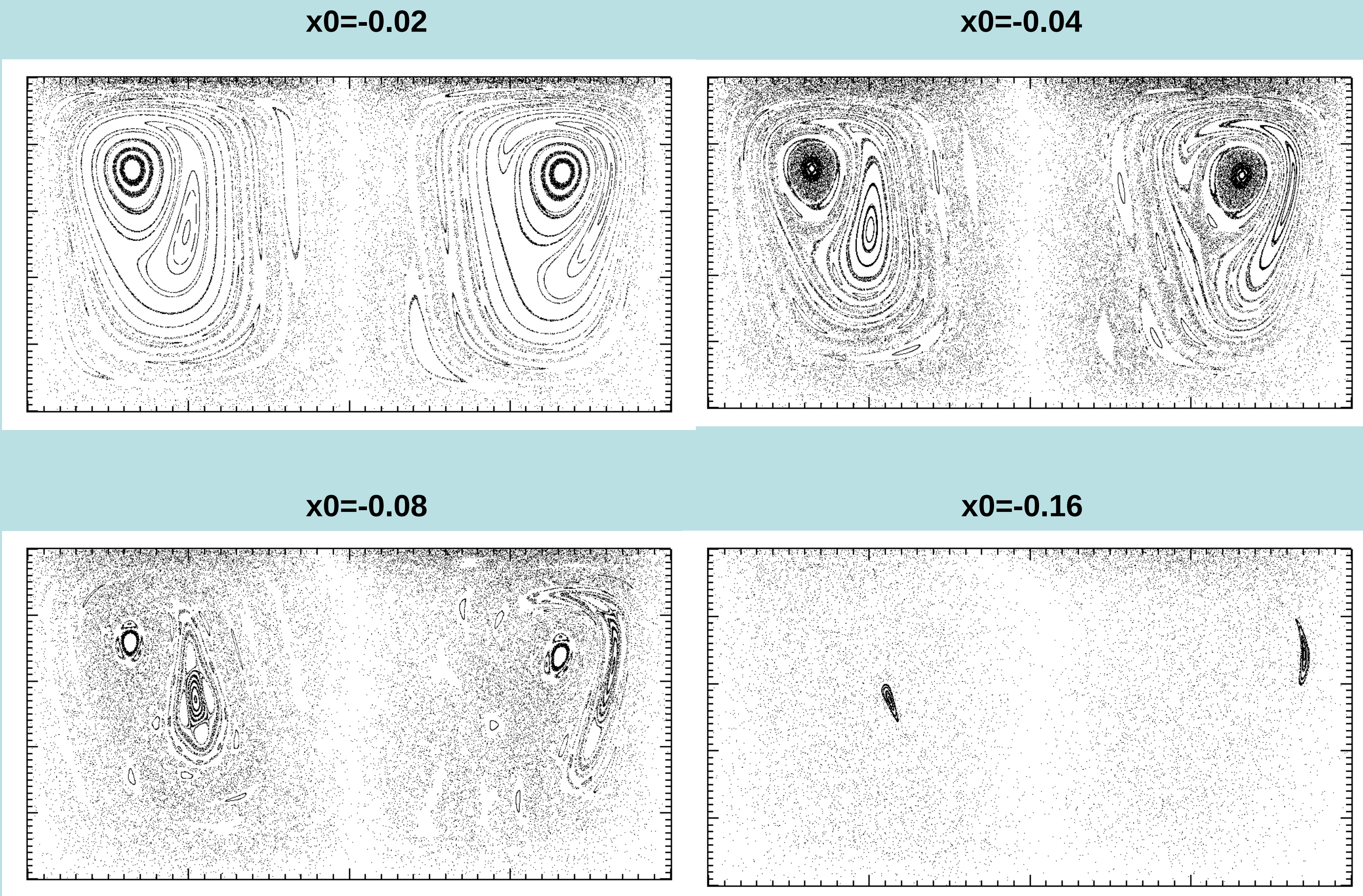


Figure 10

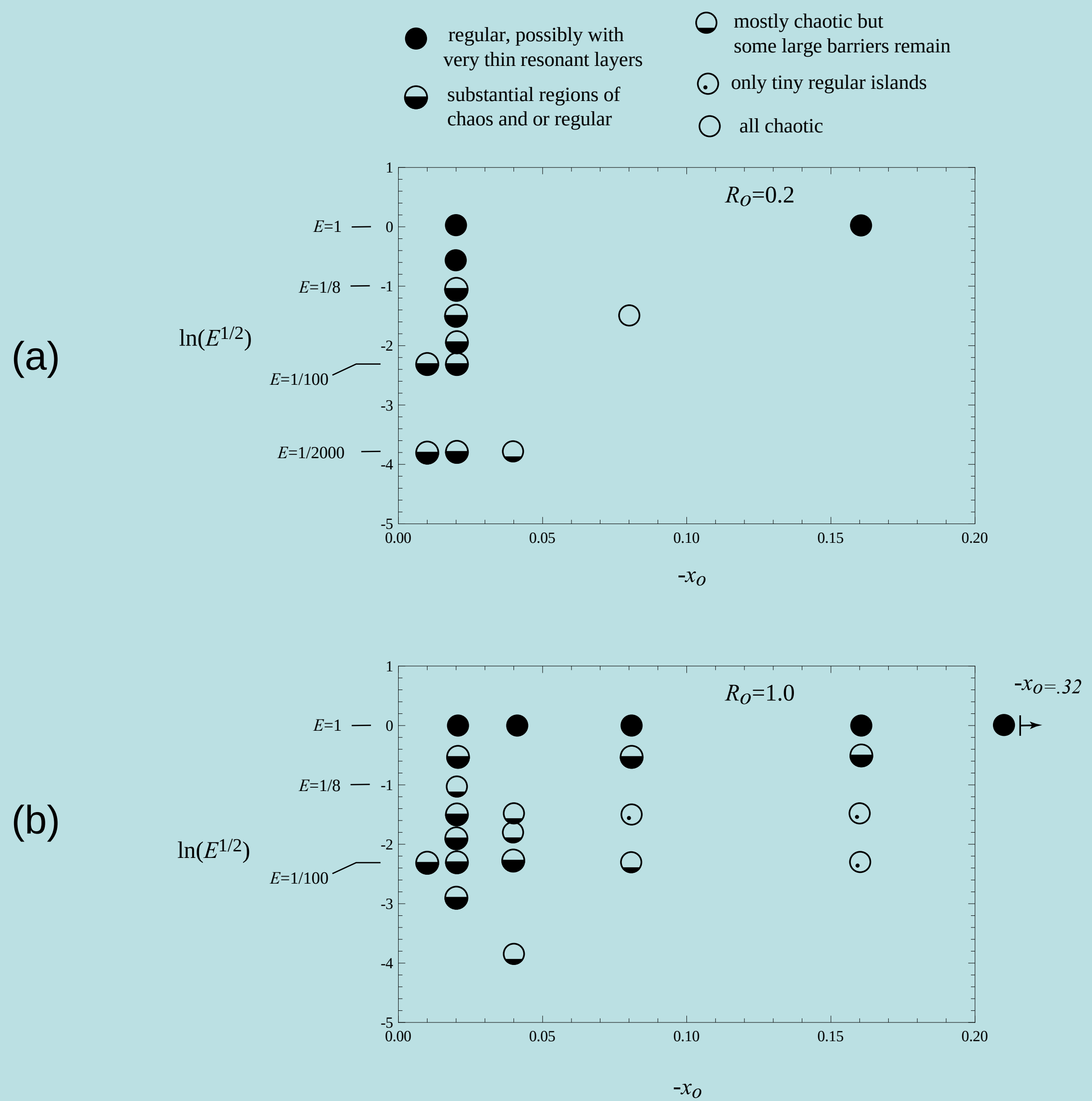
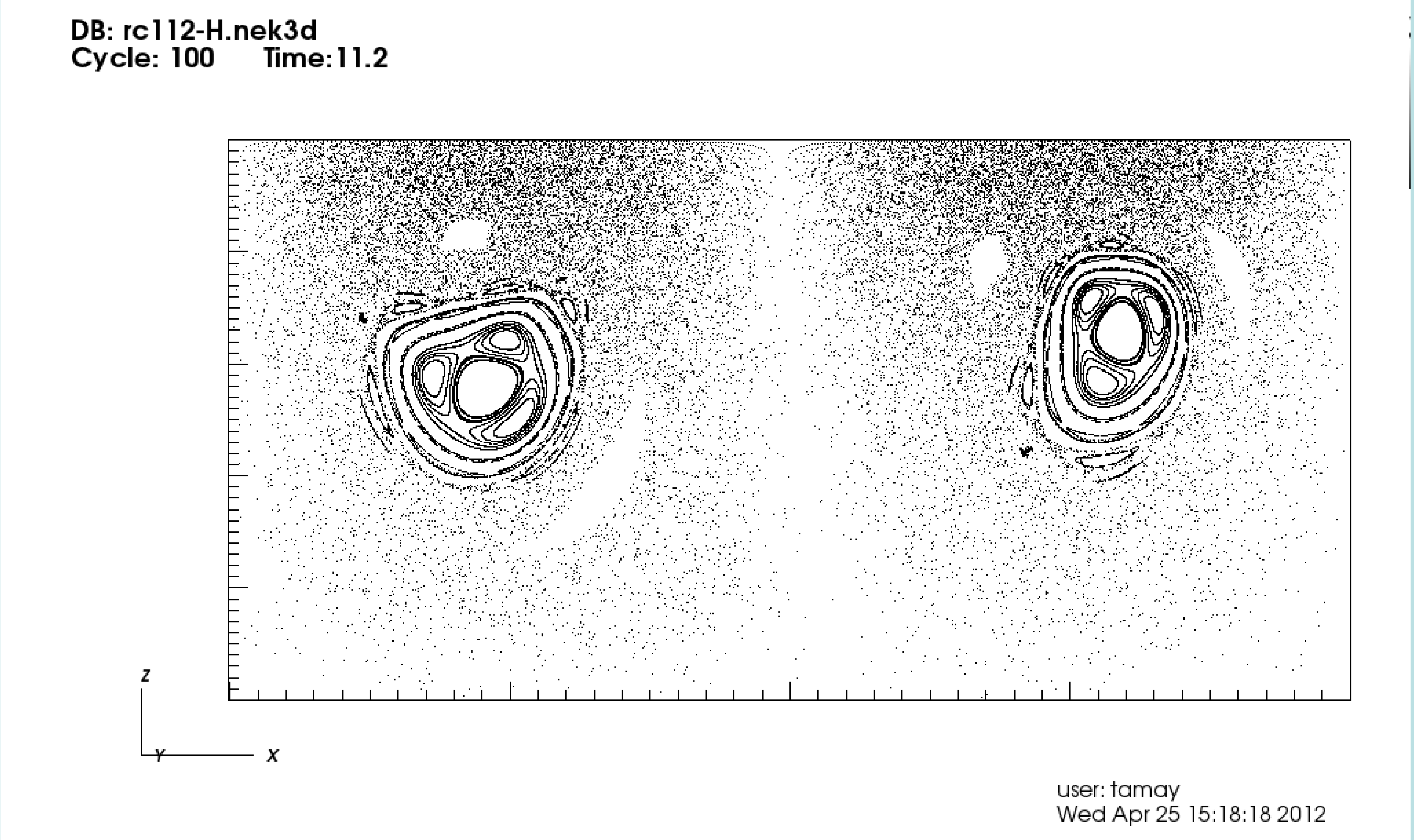

regular, possibly with very thin resonant layers
substantial regions of chaos and or regular
mostly chaotic but some large barriers remain
only tiny regular islands
all chaotic
(a)
R_O=0.2
$\ln(E^{1/2})$
E=1
E=1/8
E=1/100
E=1/2000
1
0
-1
-2
-3
-4
-5
0.00
0.05
0.10
0.15
0.20
$-x_O$
(b)
R_O=1.0
$-x_O$=.32
$\ln(E^{1/2})$
E=1
E=1/8
E=1/100
1
0
-1
-2
-3
-4
-5
0.00
0.05
0.10
0.15
0.20
$-x_O$

Fig. 11

Resonance Width

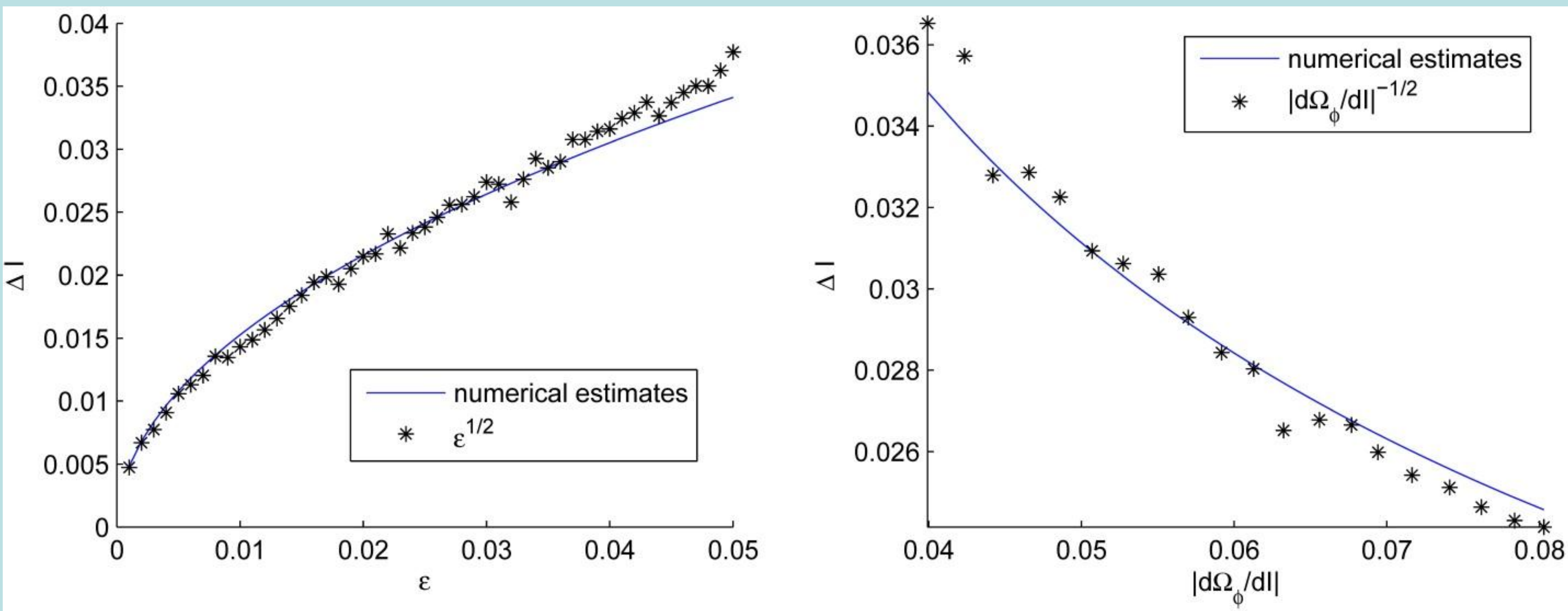


Resonance Width

$$\mathrm{D}I = \sqrt{\frac{eF^{o}_{nm}}{\left| n\mathrm{W}_f \left[\frac{d}{dI} \ln\left(\frac{T_q}{T_f} \right) \right] \right|_{I=I_o}}}$$

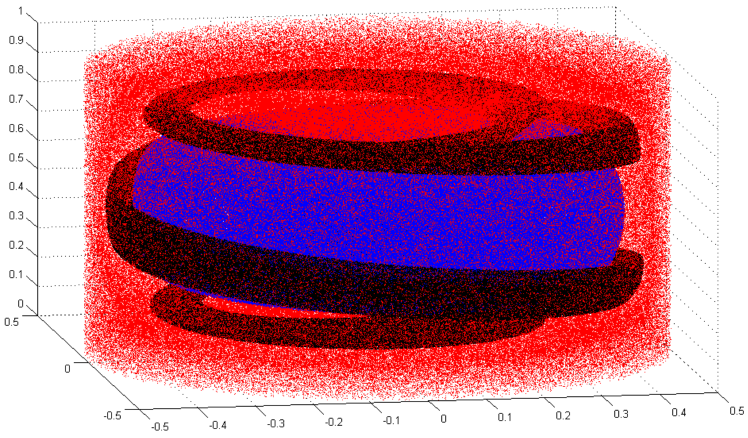
e= amplitude of perturbation

F^{0}_{nm} = resonant Fourier component of perturbation

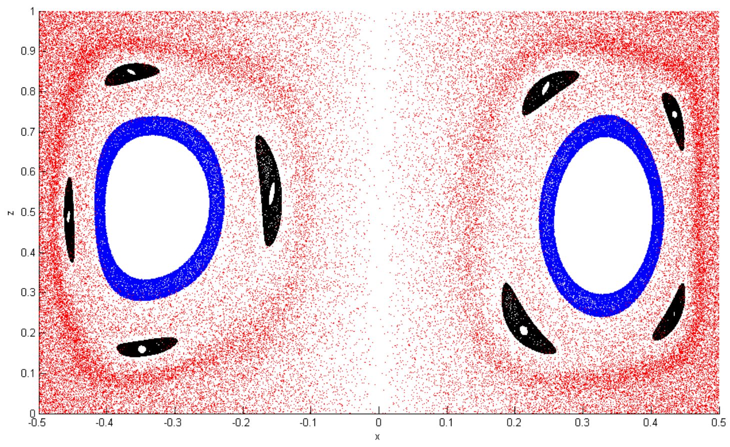


(a) (b)

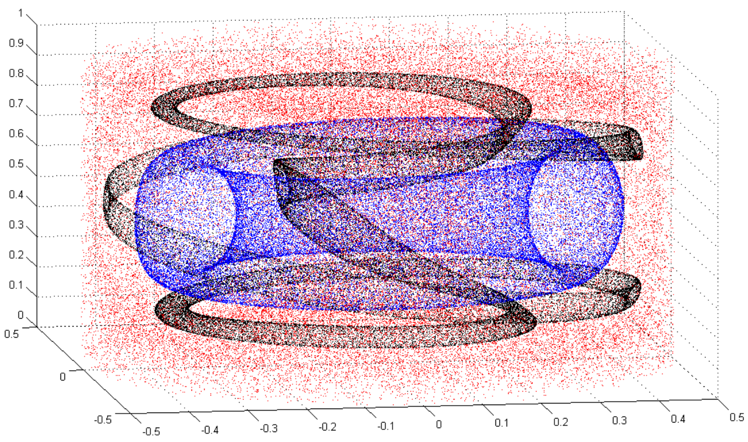
Perturbation with periodic time-dependence.



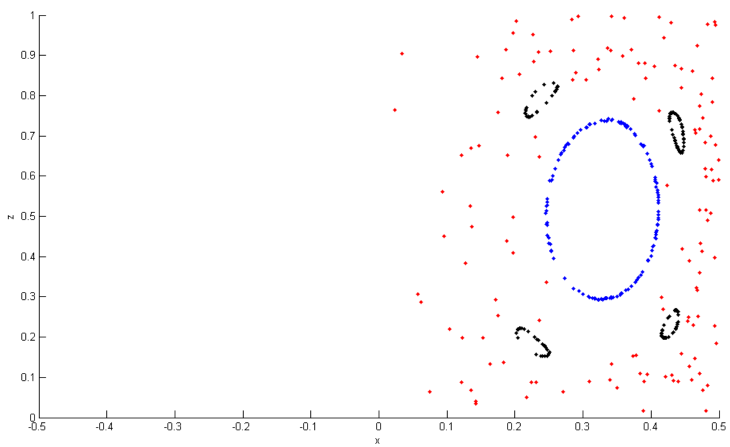
trajectories



Poincare' map in θ only



Snap shot of time-dependent tori



Double Poincare' map

Discussion: Where could this apply?

Mesoscale eddies?

$5 < w < 40$ cm/day H=50-1000s of meters: $T_{overturn}$=1-2 years

Hurricanes? $T_{overturn}$ =order of life of storm

Submesoscale? w=.02m/s H=30m: $T_{overturn}$=hrs to days

Langmuir circulations?

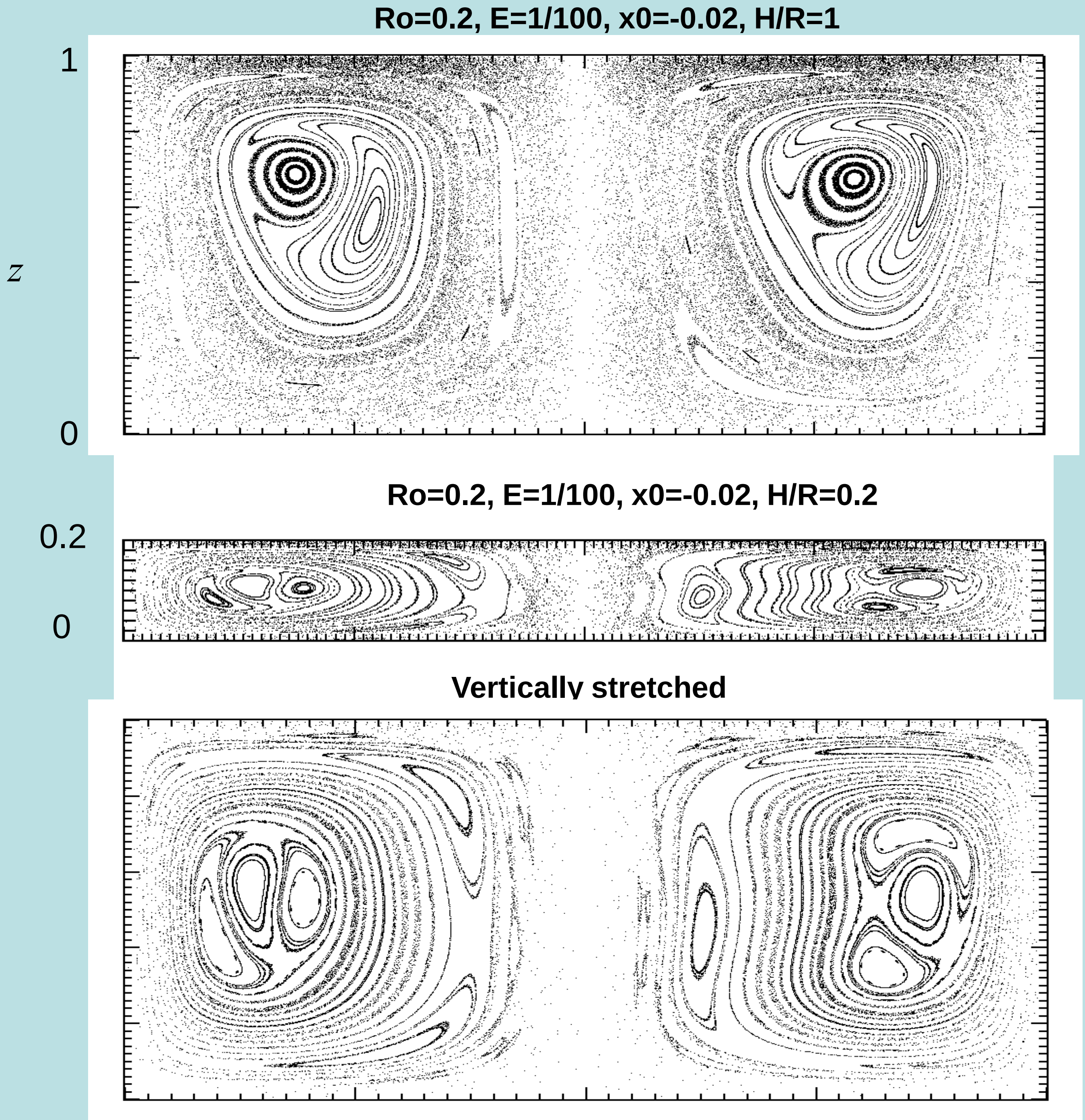

Ro=0.2, E=1/100, x0=-0.02, H/R=1
1
z
0
Ro=0.2, E=1/100, x0=-0.02, H/R=0.2
0.2
0
Vertically stretched

Fig. 15