Lagrangian Coherent Structures in an Idealized Model of a 3D, Time-Dependent Eddy

Larry Pratt and Irina Rypina (WHOI)

Tamay Ozgokmen and Peng Yang (RSMAS)

(I. Mezic, V. Garcia-Garrido, H. Childs and Y. Bebieva)

Pratt, Rypina, Ozgokmen, Wang,….(2013) J. Fluid Mech. 738, 143-183

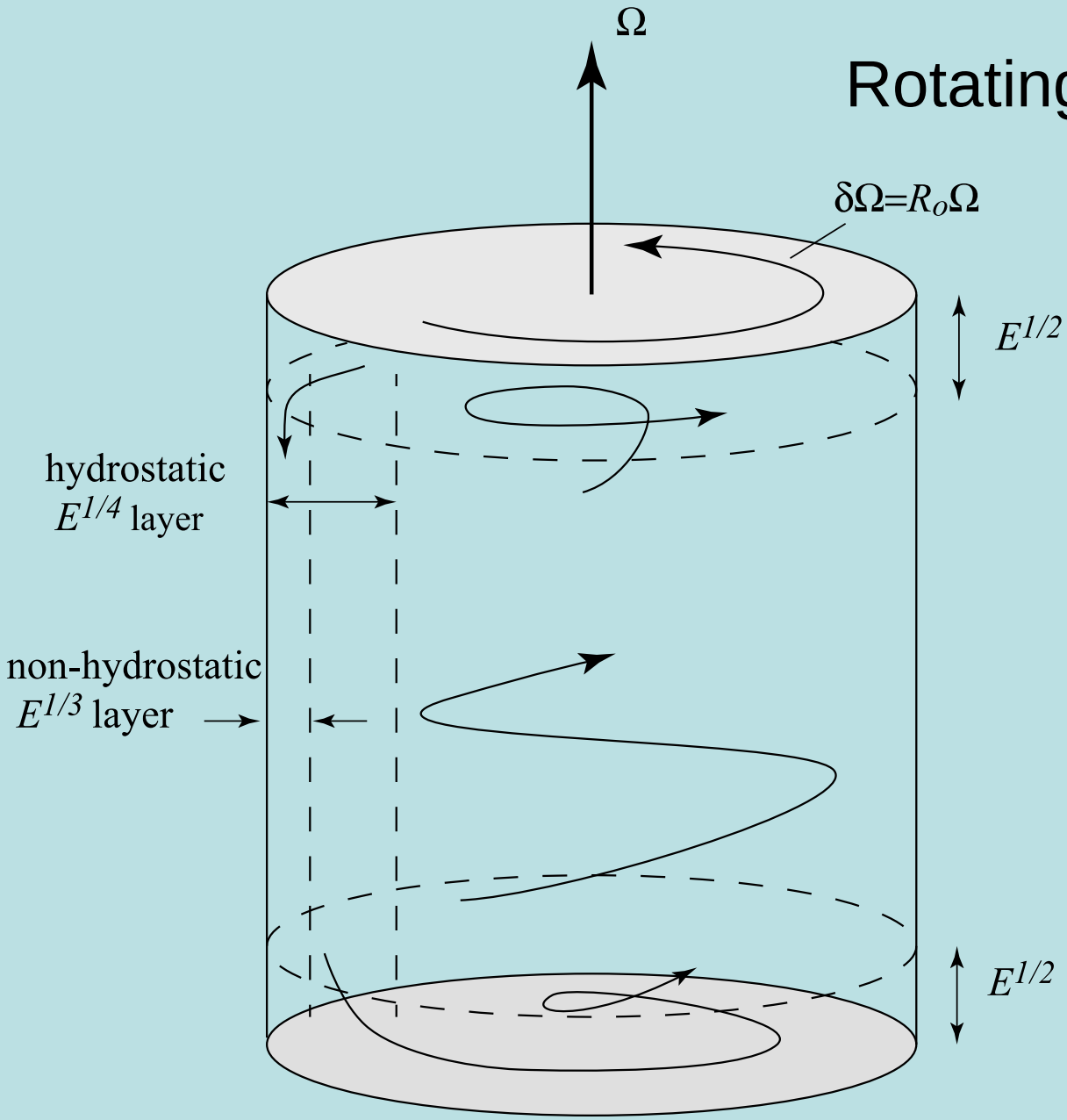
Rypina, Pratt, Wang, Ozgokmen (2015) J. *Chaos 25, 087401*

Chaotic Advection

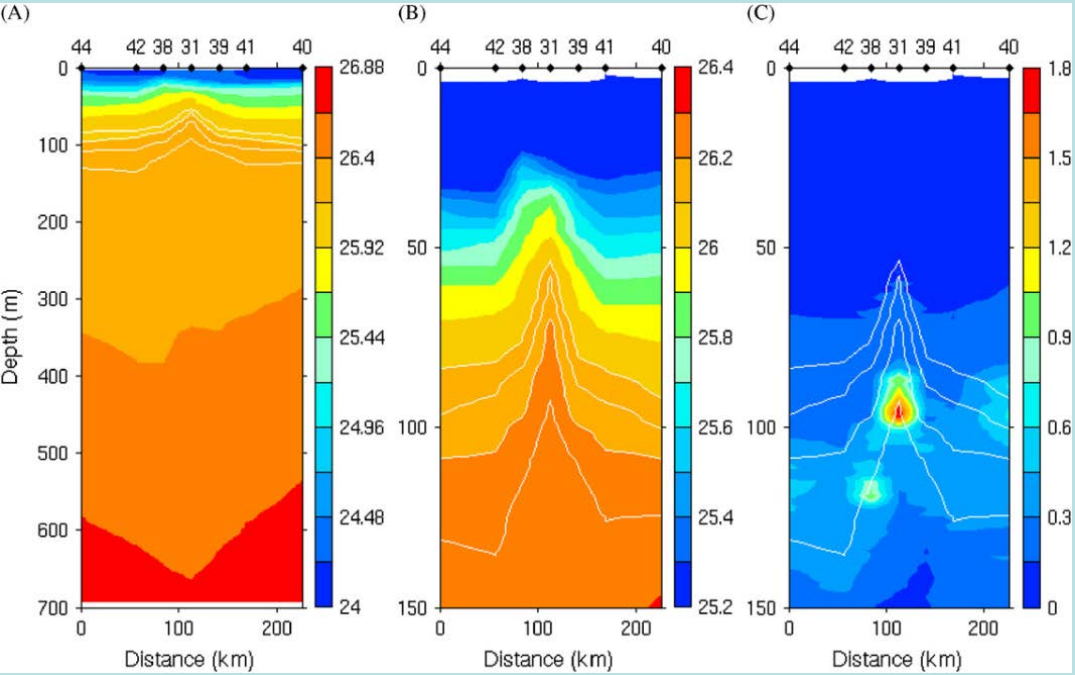
Aref and Pomphrey, 1982:
Chaotic trajectories in simple (and regular) 2D flow.

Hassan Aref
(1950-2011)

Rotating Can Flow

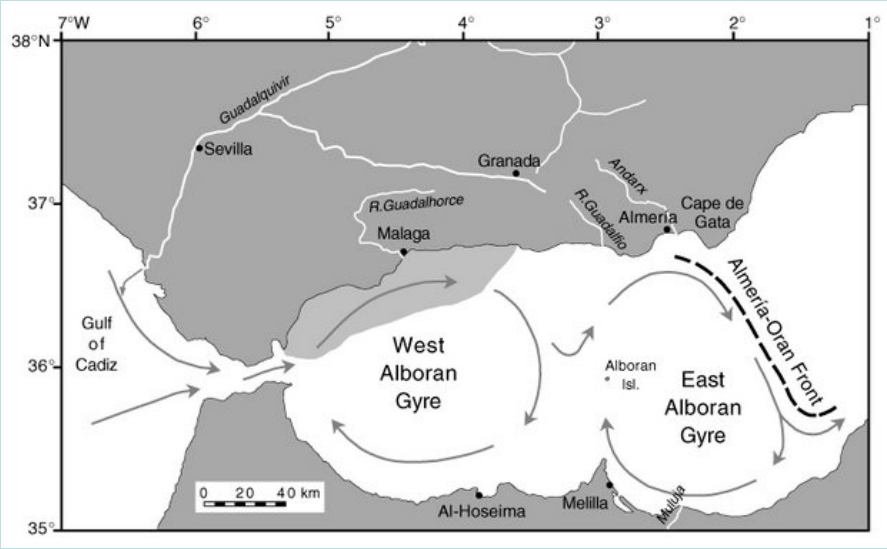


$$E = \frac{\nu}{\Omega H^2} = \left(\frac{\delta_E}{H}\right)^2$$

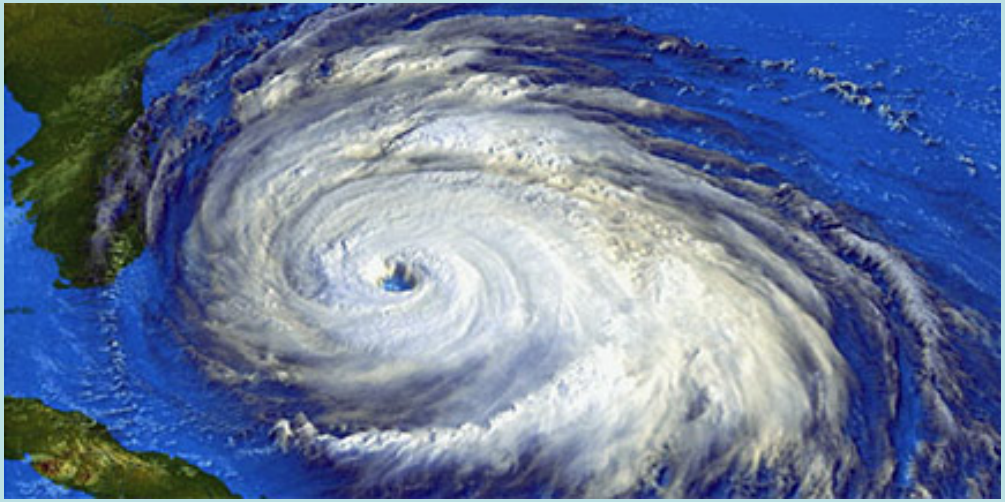


From Ledwell, McGillicuddy and Anderson DSR-II (2008)

Langmuir turbulence



Alboran Gyre (Jay Brett's thesis work)



hurricanes

Velocity Fields

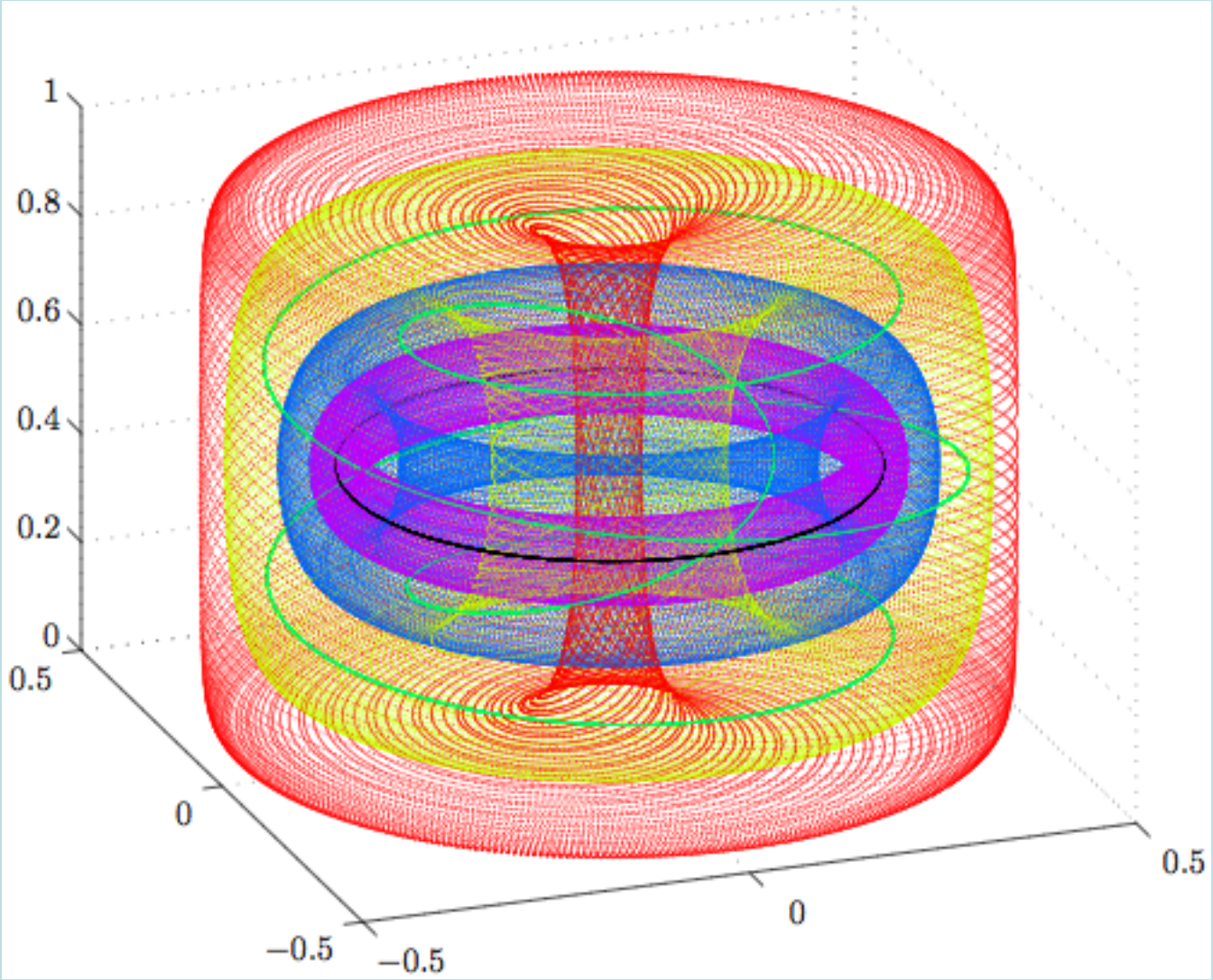
1) Navier-Stokes integration (Nek5000 model)

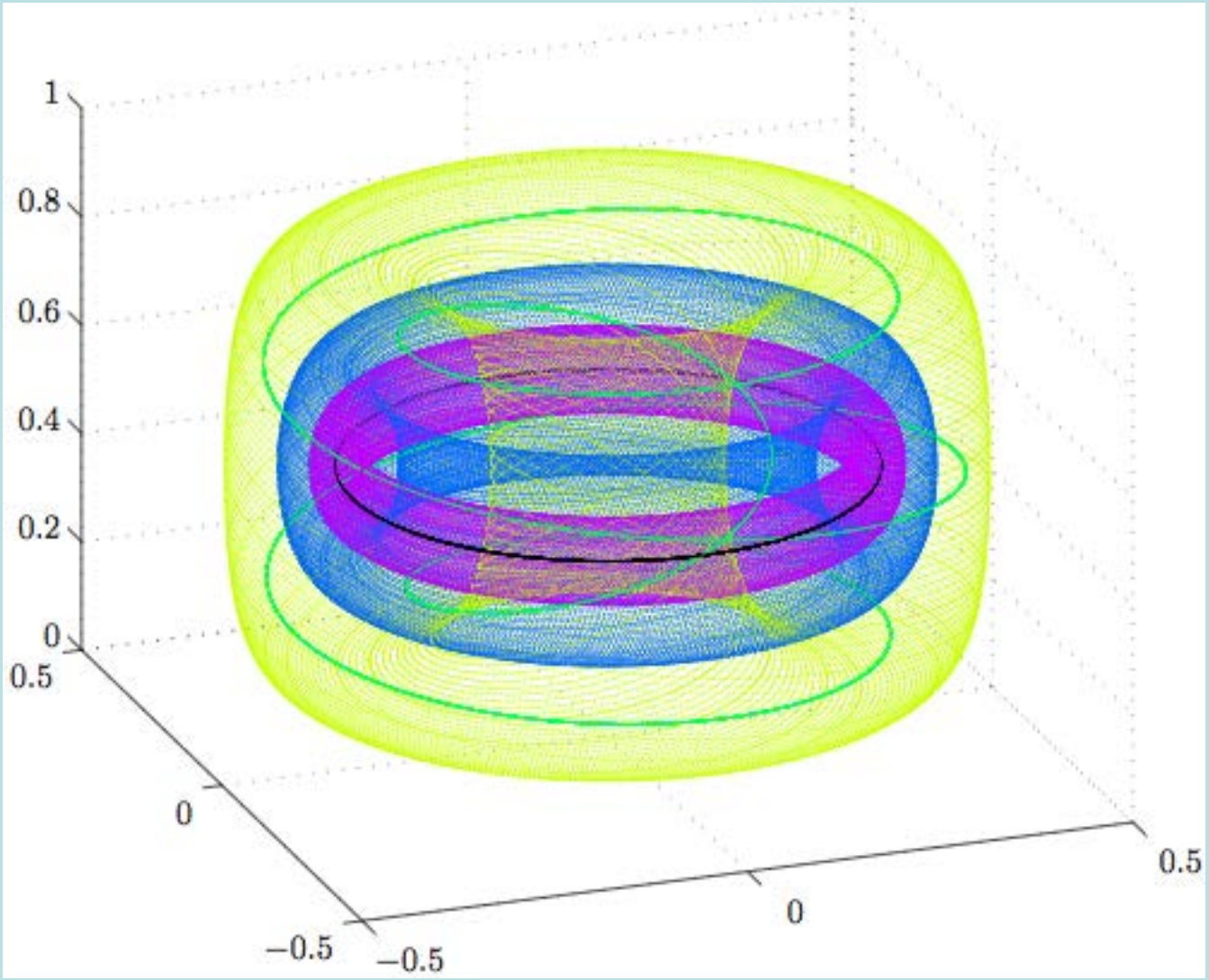
2) Kinematic (3D velocity: non-divergent but no dynamics)

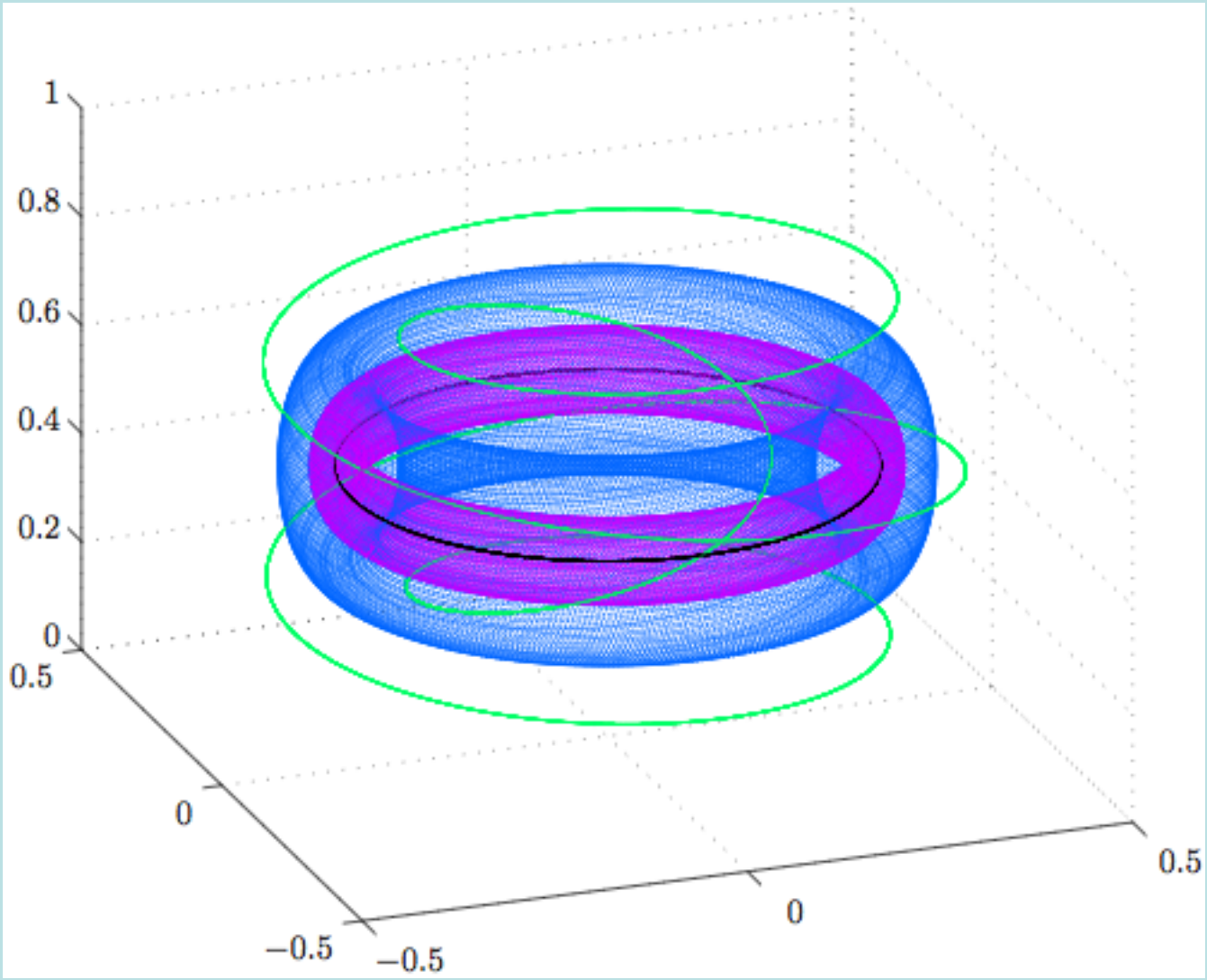
$$u = -bx(1-2z)\frac{R-r}{3} - ay(c+z^2) + \varepsilon\left[y(y - y_o + \gamma\cos(\sigma t) - \frac{R^2 - r^2}{2}\right](1-\beta z)$$

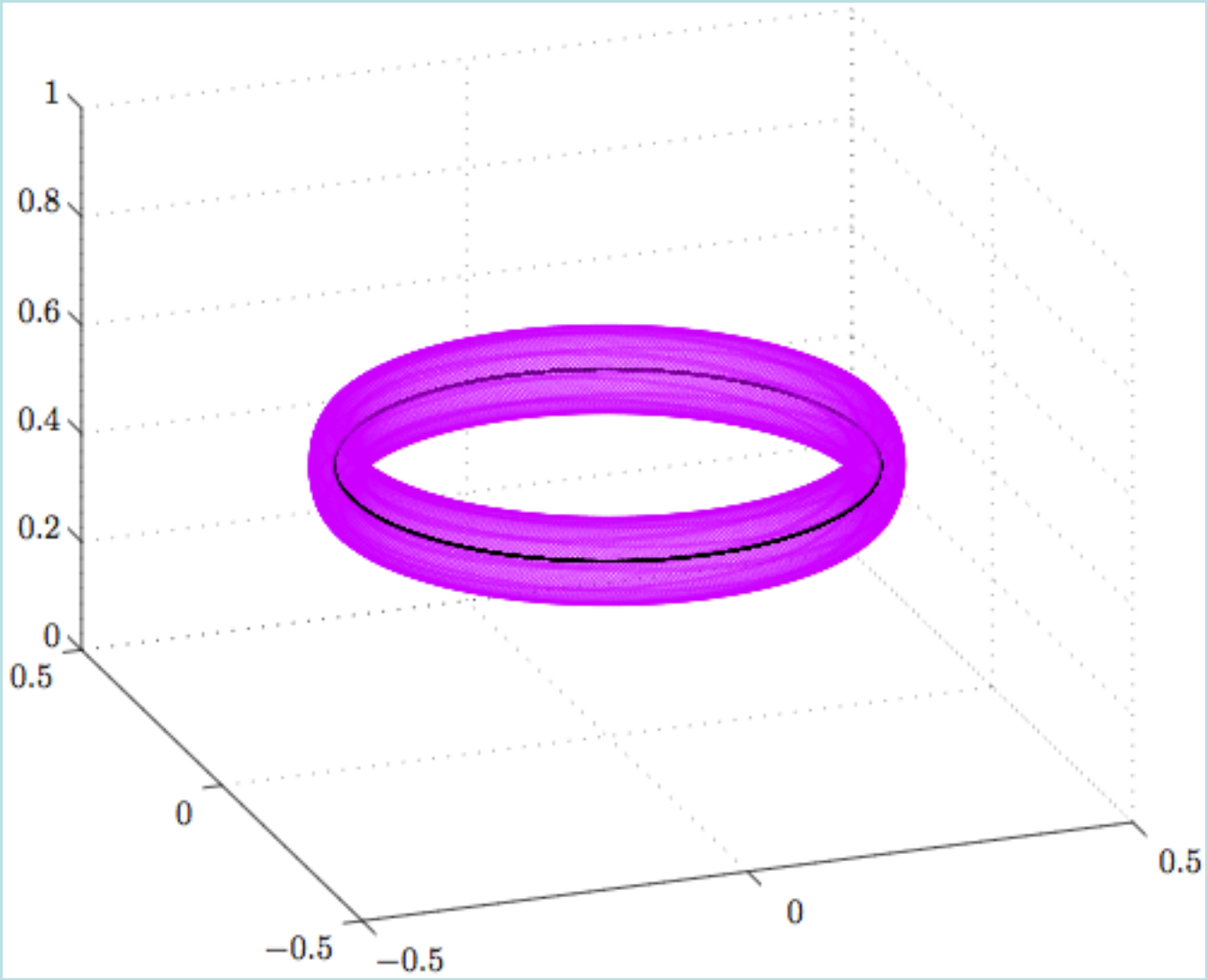
$$v = -by(1-2z)\frac{R-r}{3} + ax(c+z^2) - \varepsilon[y - y_o + \gamma\cos(\sigma t)](1-\beta z)$$

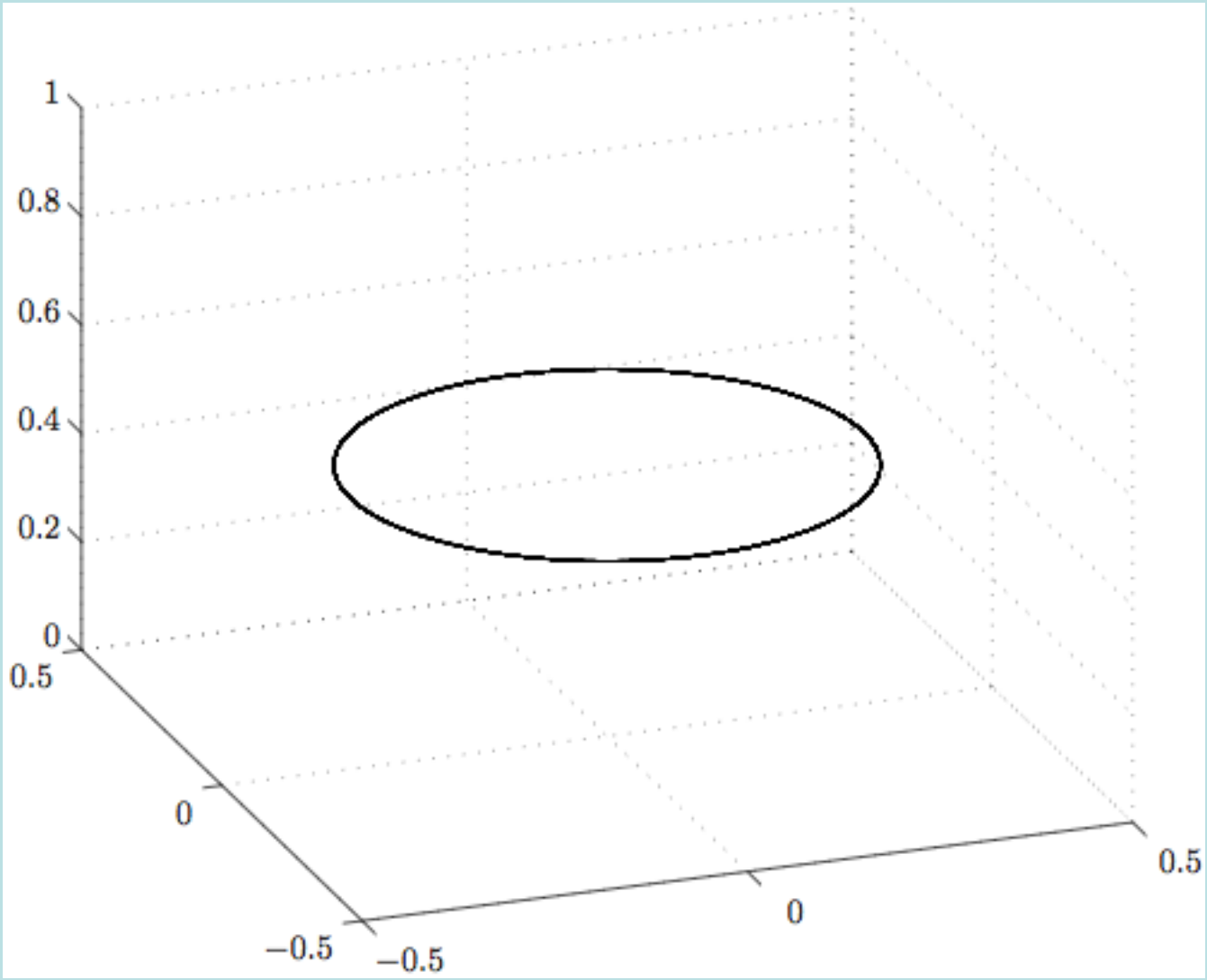
$$w = bz(1-z)\left[\frac{2R}{3} - r\right]$$

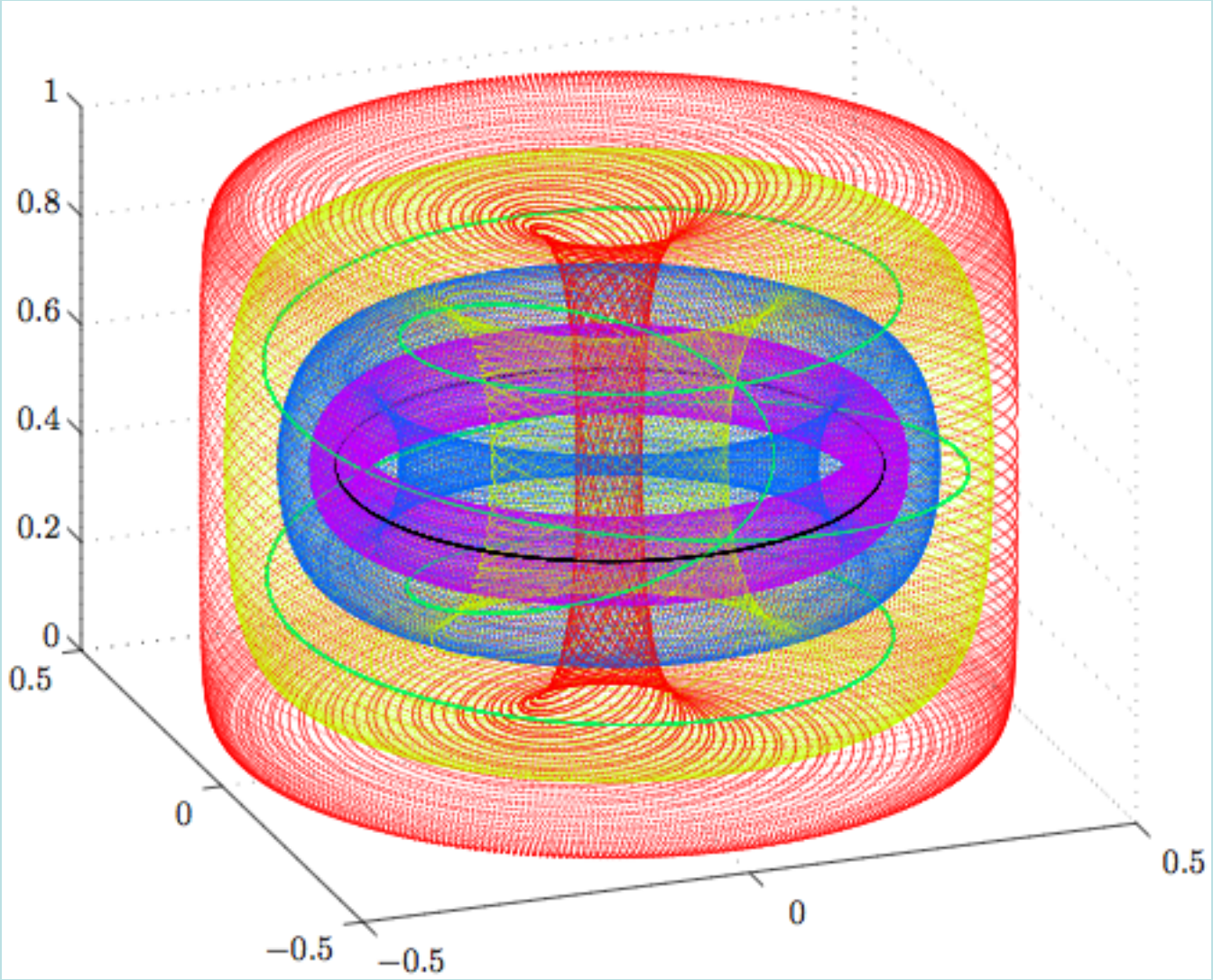

1
0.8
0.6
0.4
0.2
0
0.5
0
−0.5
−0.5
0
0.5

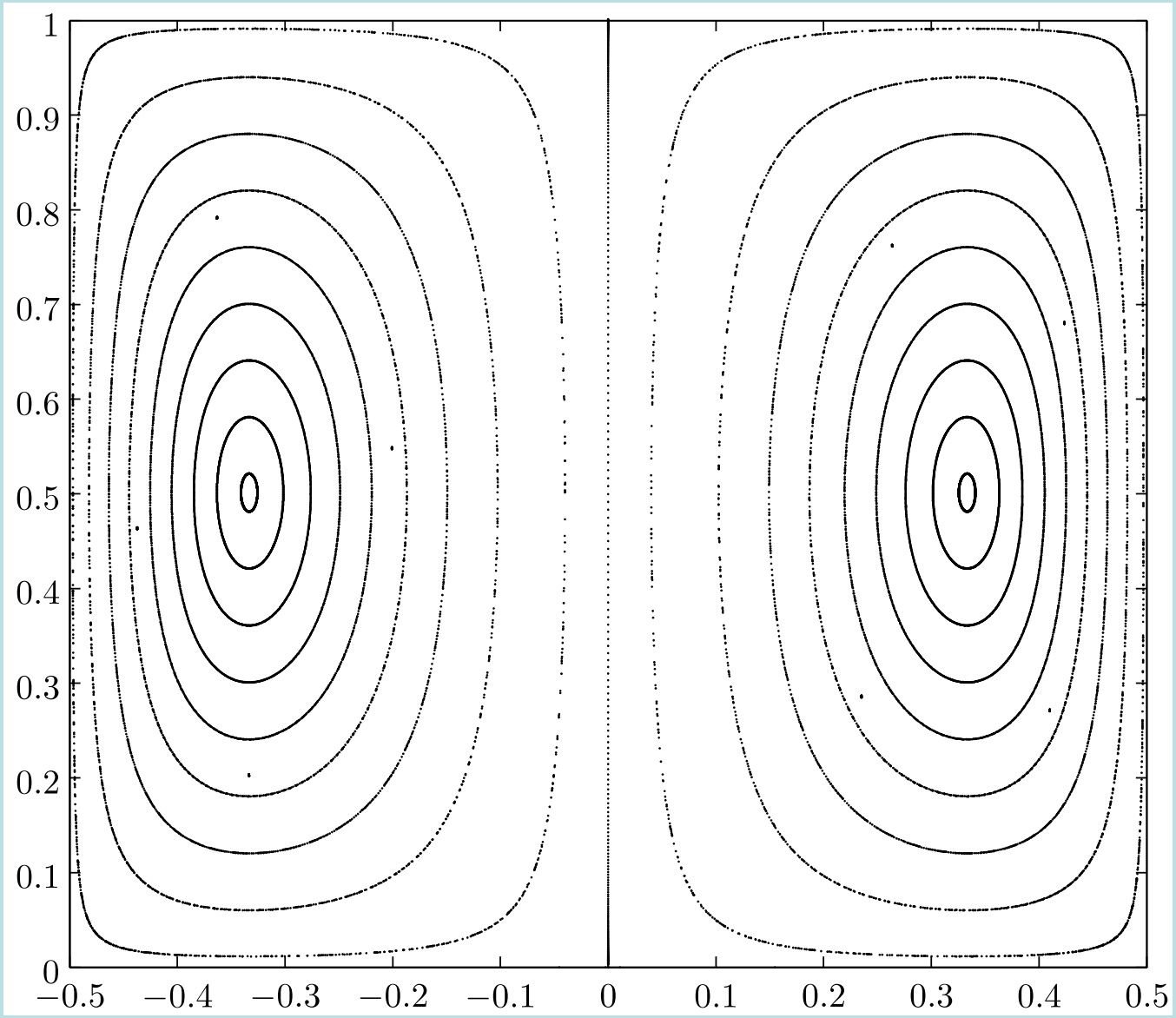

1
0.8
0.6
0.4
0.2
0
0.5
0
−0.5
−0.5
0
0.5

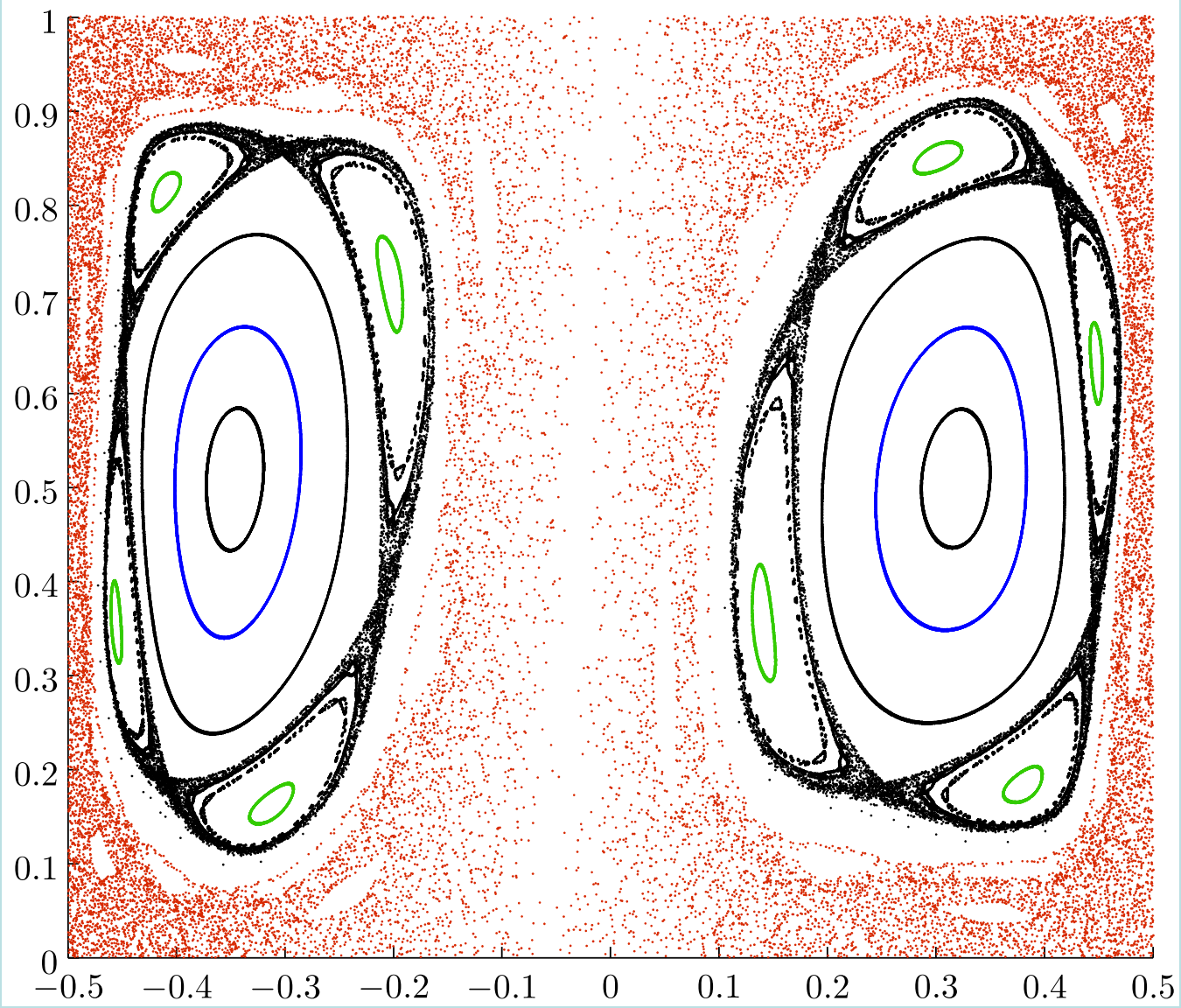

1
0.8
0.6
0.4
0.2
0
0.5
0
−0.5
−0.5
0
0.5

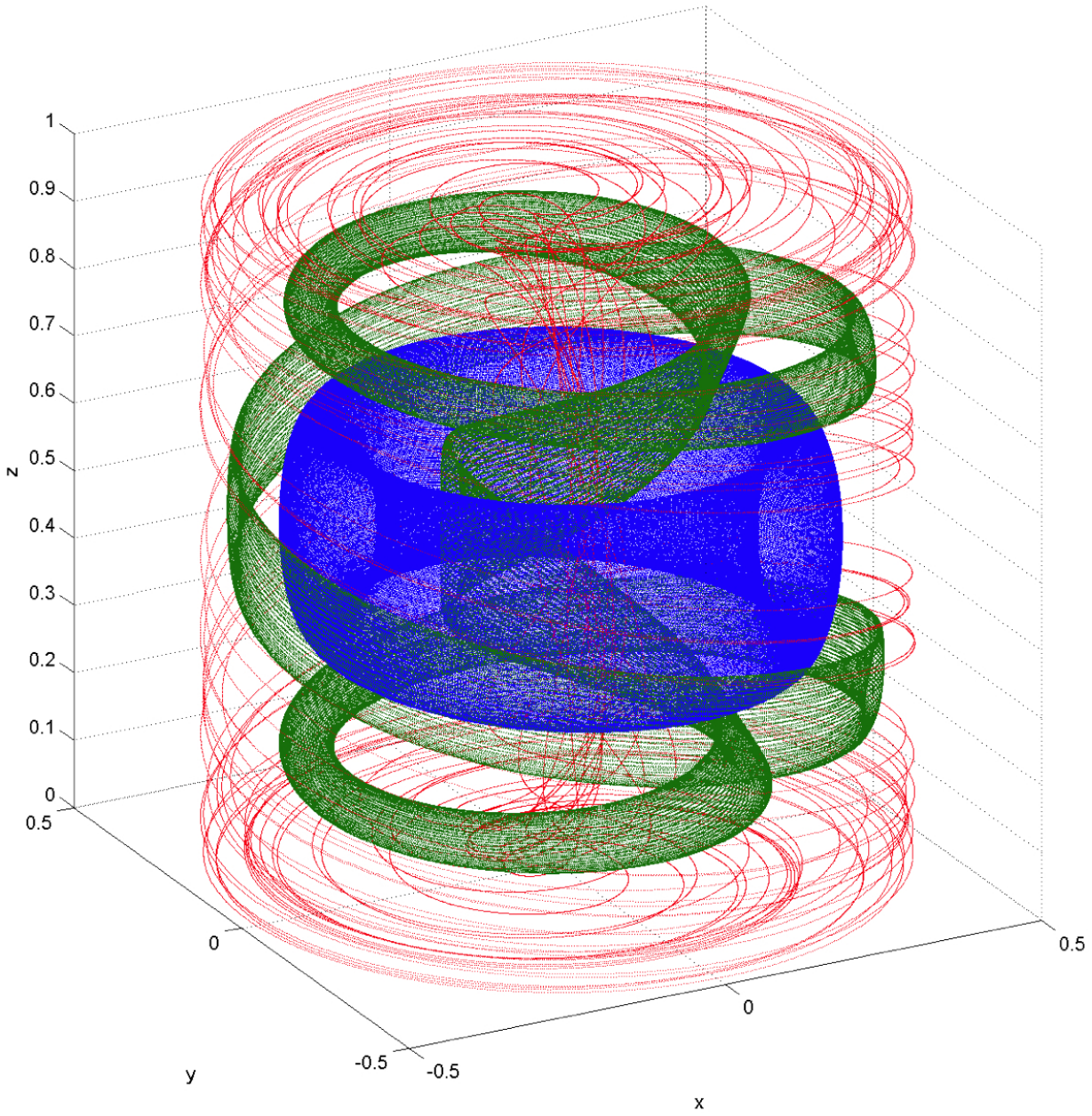

1
0.8
0.6
0.4
0.2
0
0.5
0
−0.5
−0.5
0
0.5


1
0.8
0.6
0.4
0.2
0
0.5
0
−0.5
−0.5
0
0.5

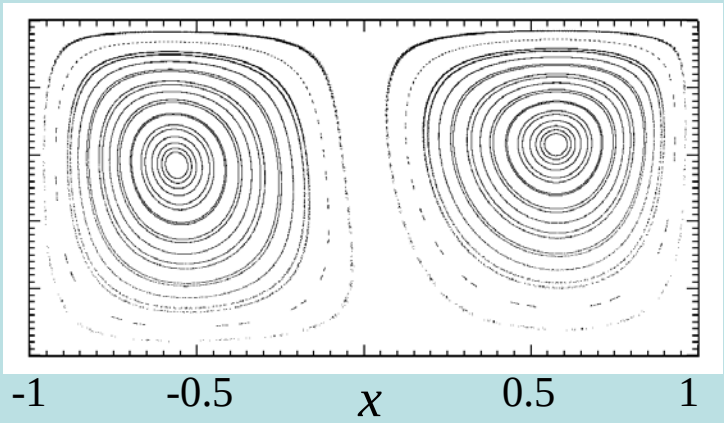

1
0.8
0.6
0.4
0.2
0
0.5
0
−0.5
−0.5
0
0.5


z
x
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.5
−0.4
−0.3
−0.2
−0.1
0
0.1
0.2
0.3
0.4
0.5

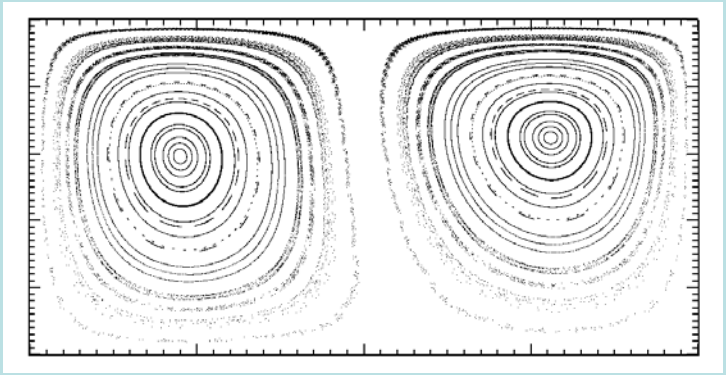

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
z
−0.5
−0.4
−0.3
−0.2
−0.1
0
0.1
0.2
0.3
0.4
0.5
x


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
z
0.5
0
-0.5
y
-0.5
0
0.5
x

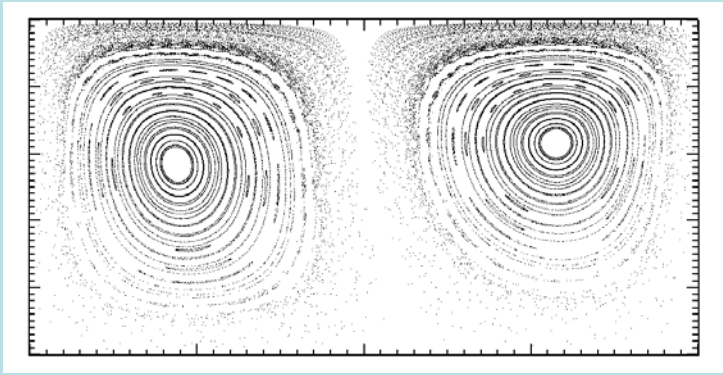
(a)



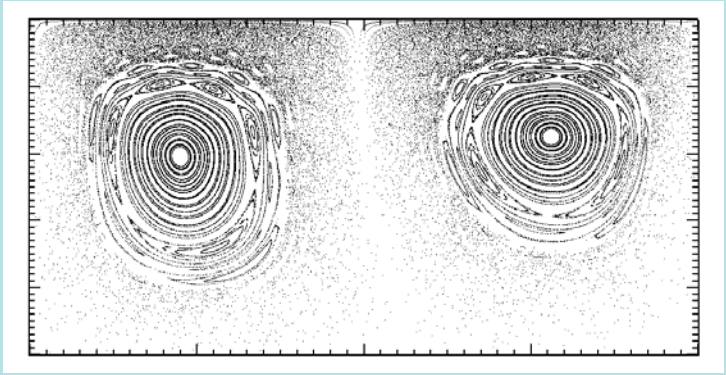
(b)



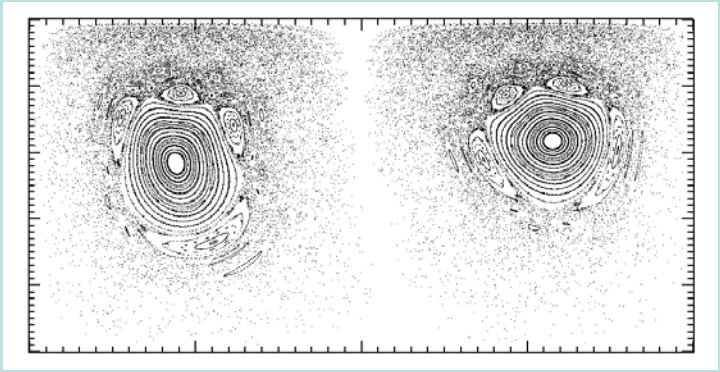
(c)



(d)



(e)



(f)

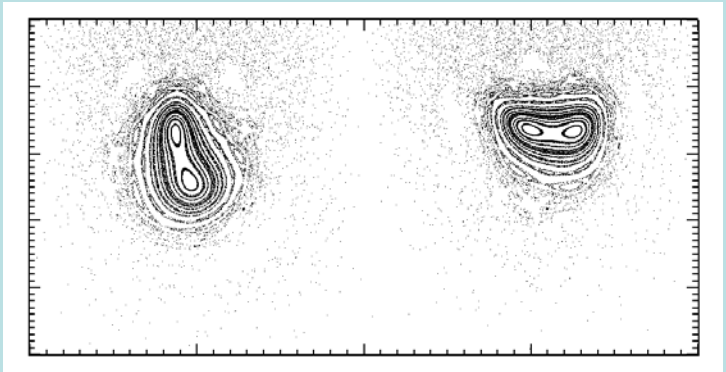
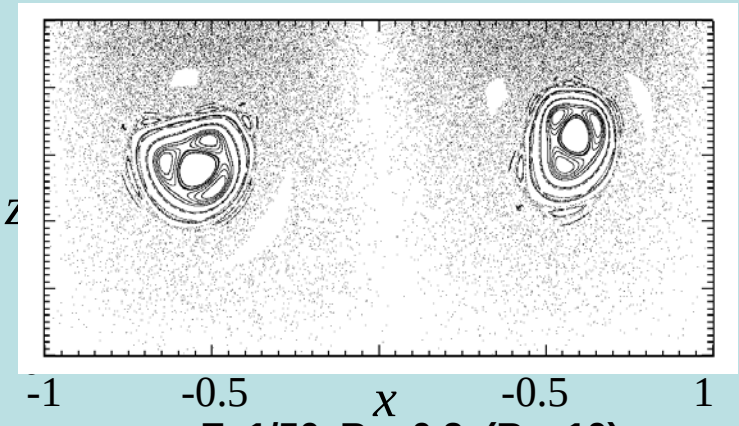


Fig. 10a-f

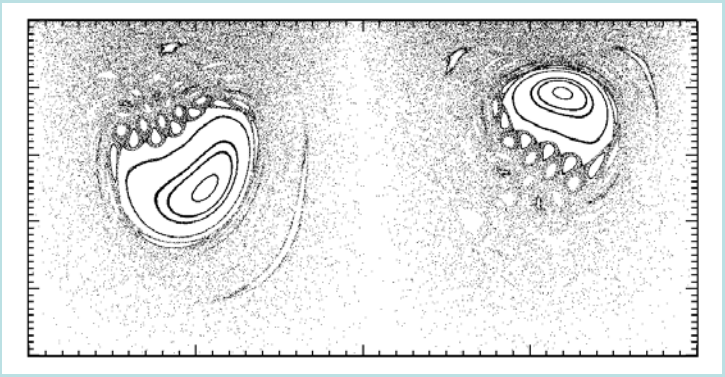
E=1/20, Ro=0.2 (Re=4)

(g)



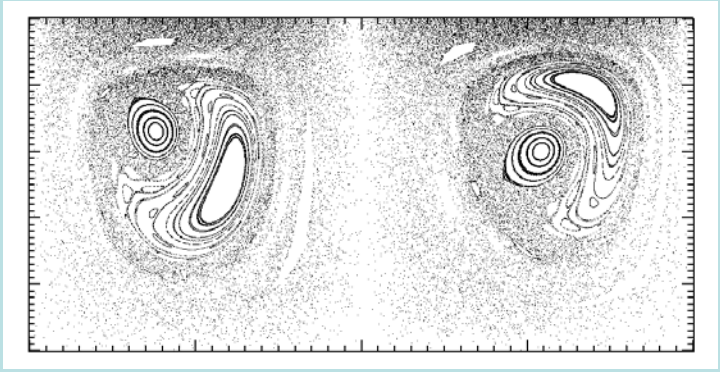
E=1/20, Ro=1 (Re=20)

(h)



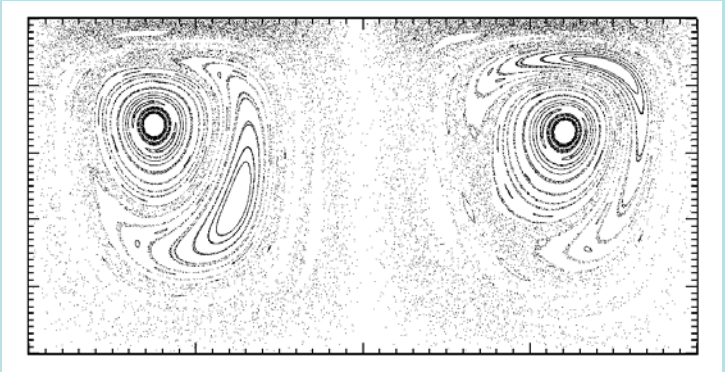
E=1/50, Ro=0.2 (Re=10)

(i)



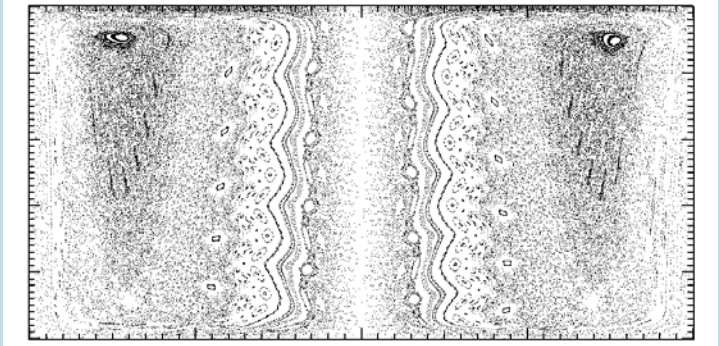
E=1/50, Ro=1 (Re=50)

(j)



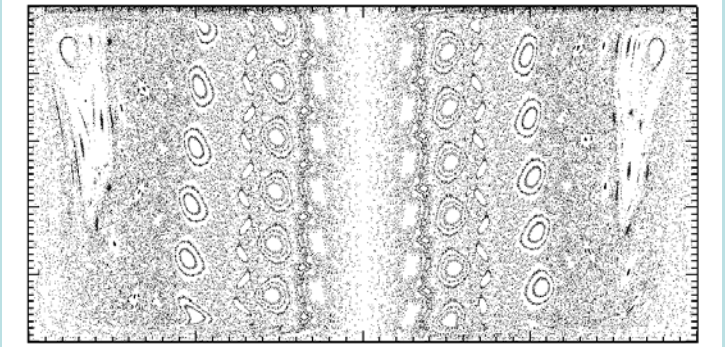
E=1/2000, Ro=0.2 (Re=400)

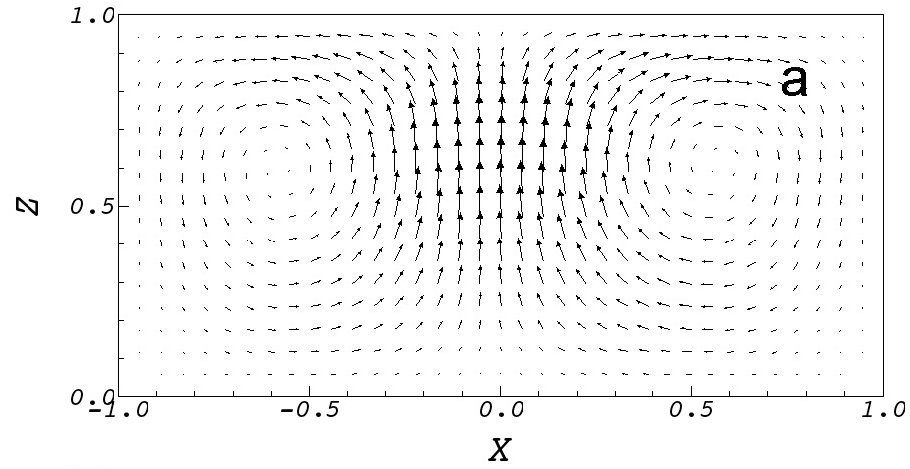
(k)

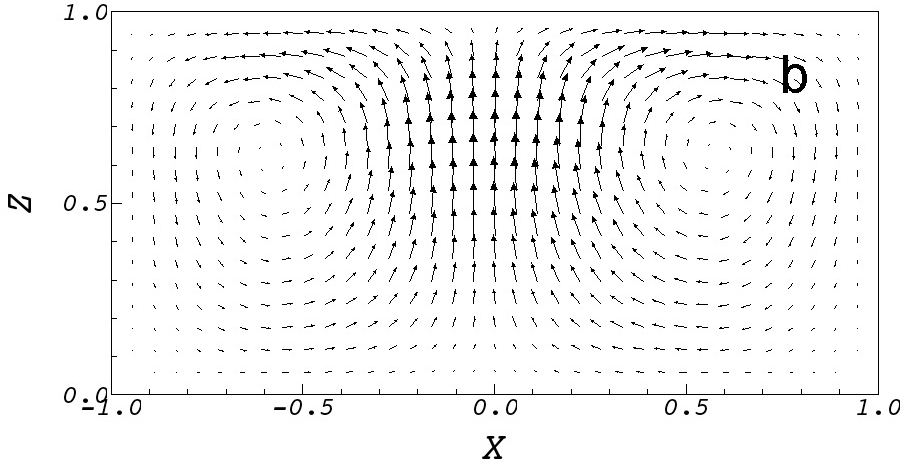


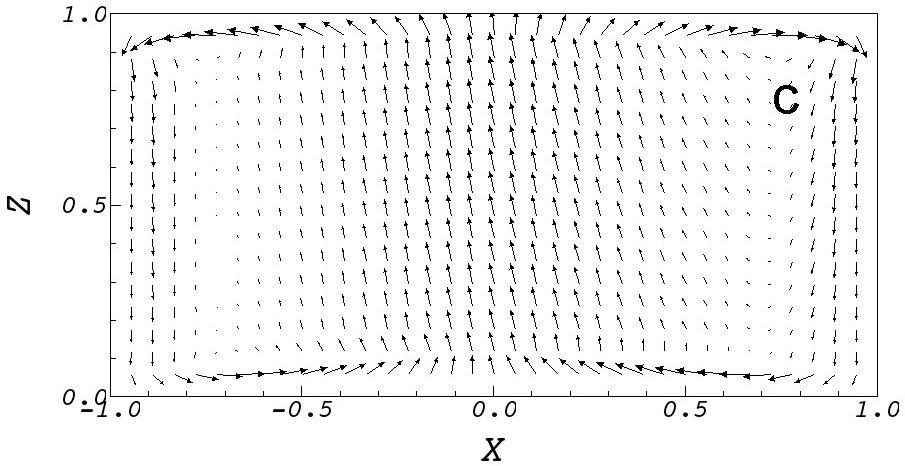
E=1/2000, Ro=1.0 (Re=2000)

(l)

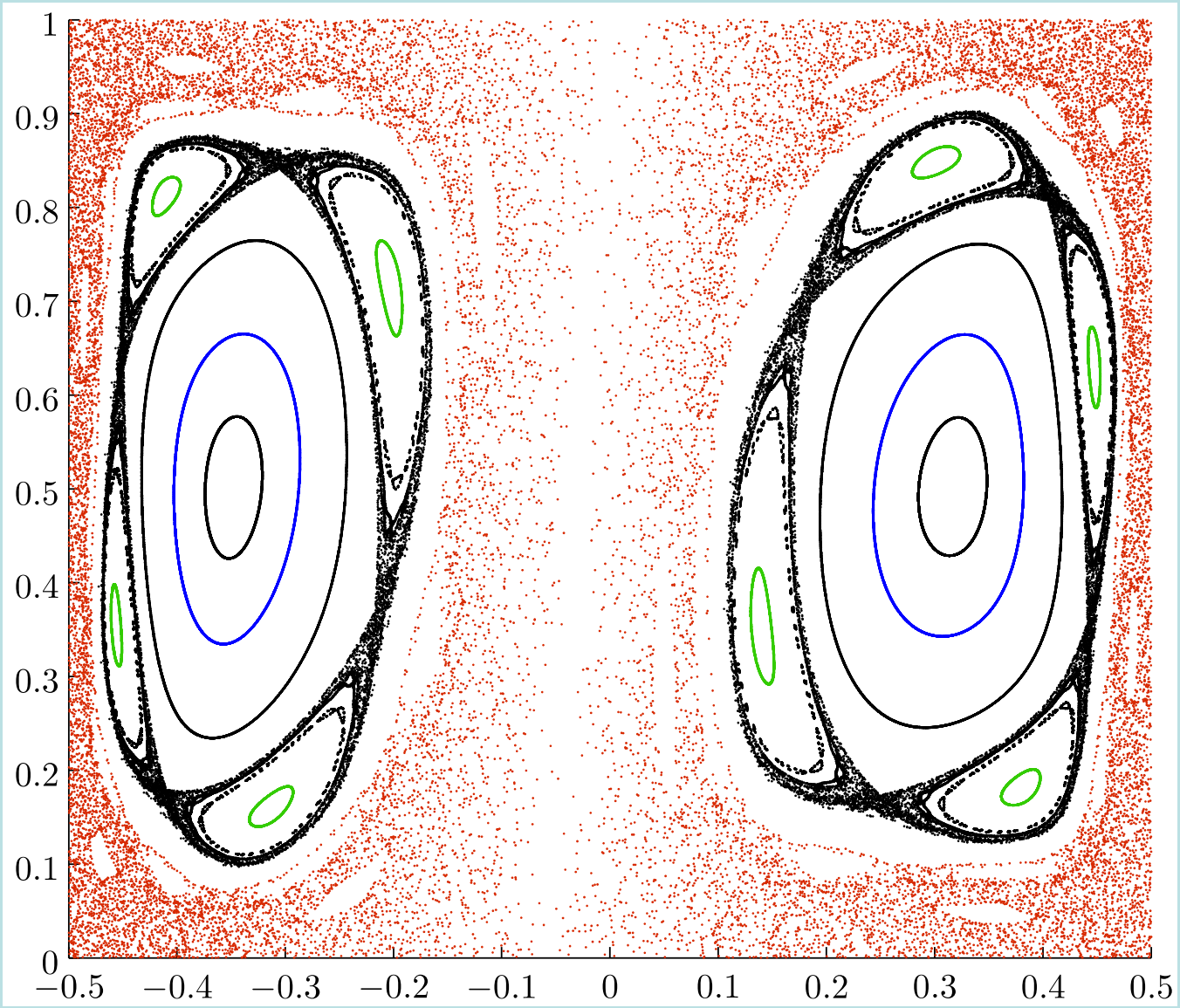



a
1.0
0.5
0.0
Z
-1.0
-0.5
0.0
0.5
1.0
X

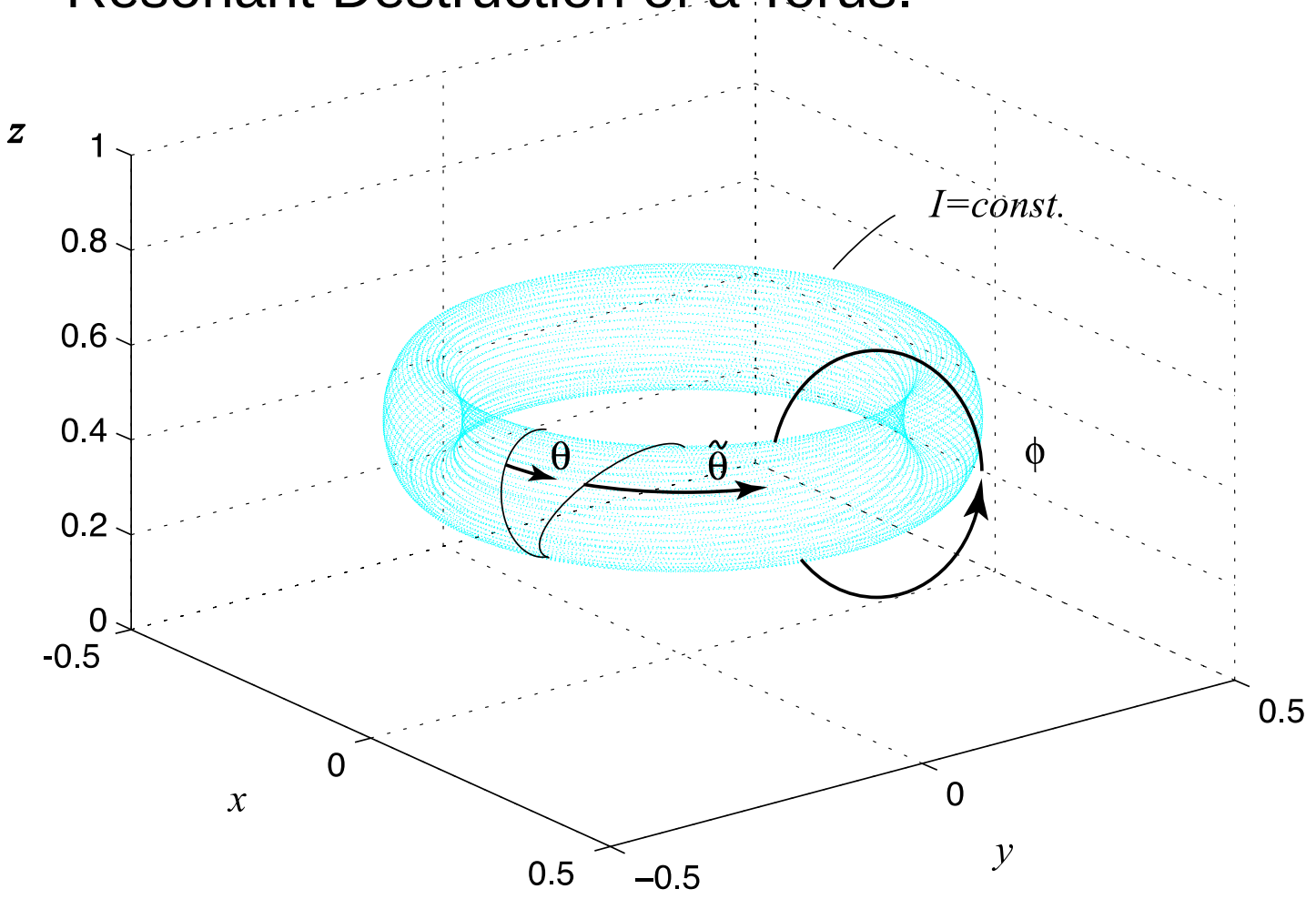

b
1.0
0.5
0.0
Z
-1.0
-0.5
0.0
0.5
1.0
X


c
1.0
0.5
0.0
Z
-1.0
-0.5
0.0
0.5
1.0
X

New work: resonant layers in time-dependent, 3D flow.



Resonant Destruction of a Torus.



$$T_\phi = \frac{2\pi}{\Omega_\phi}$$

$$T_\theta = \frac{2\pi}{\Omega_\theta}$$

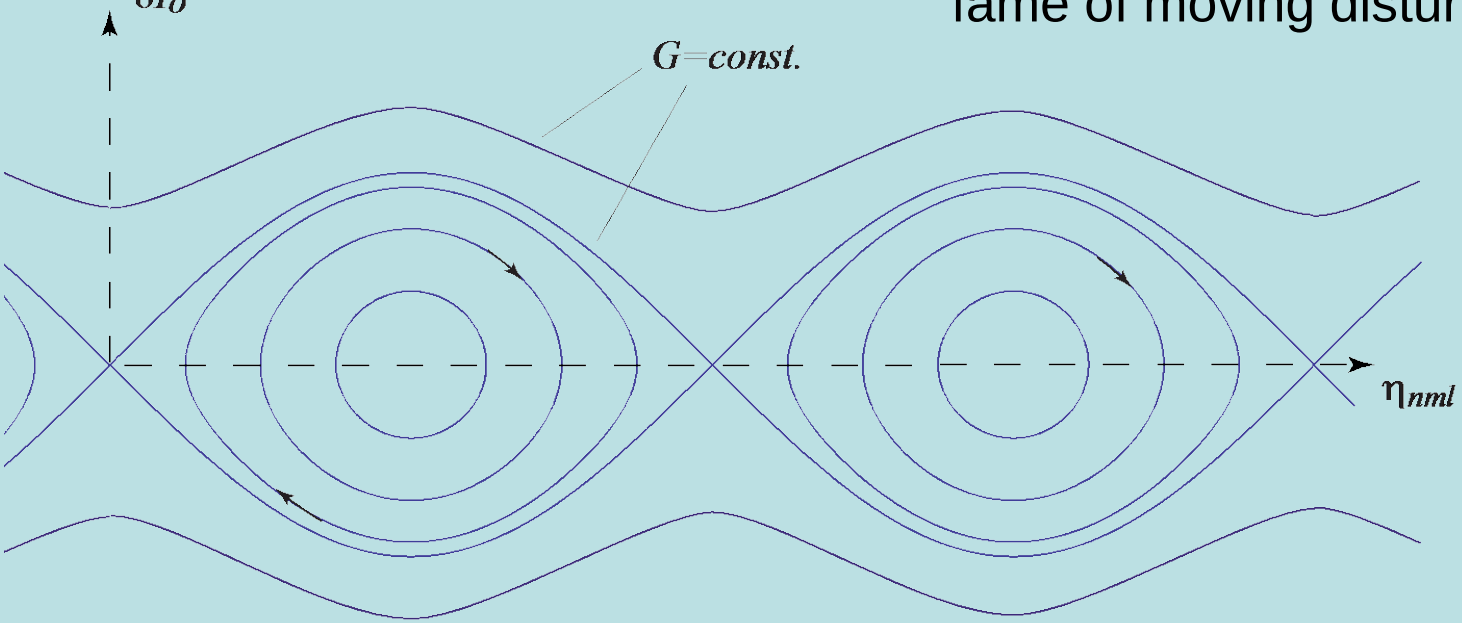
resonance condition: $n\Omega_\phi + m\Omega_\theta - \vec{l}\cdot\vec{\sigma} = 0$

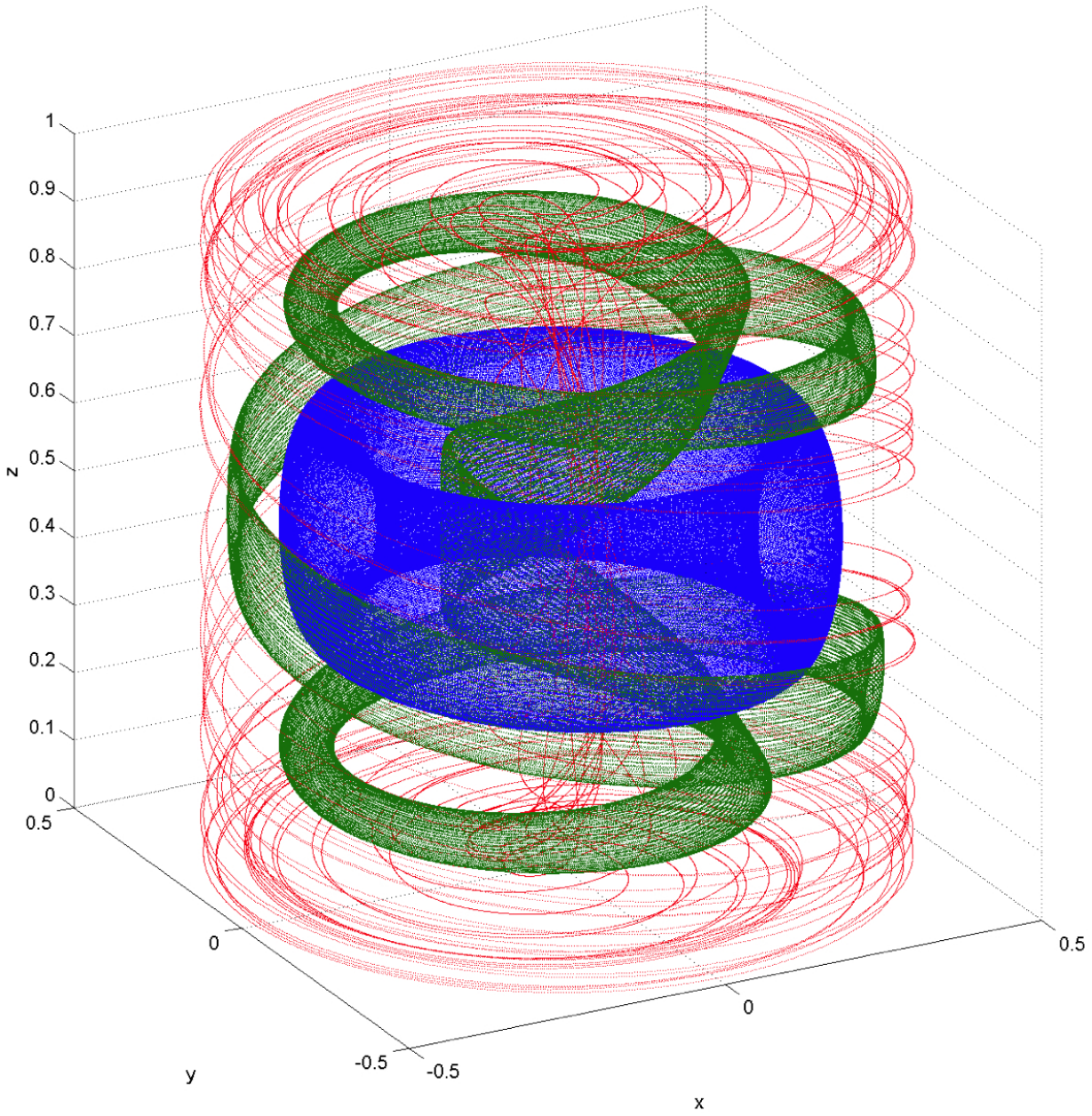
Weakly nonlinear theory for resonant tori leads to integral of motion:

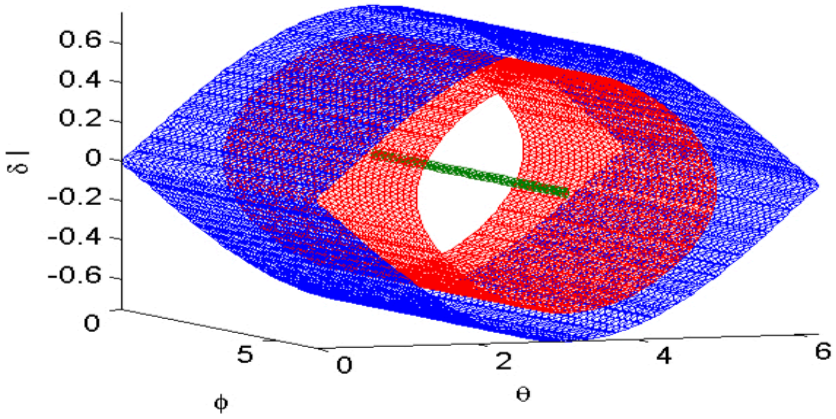
$$G(\delta I_o, \tilde{\phi}_o, \tilde{\theta}_o) = \frac{\delta I_o^2}{2} + \sum_{mn\vec{l}}^{\text{resonant}} \frac{F^{(0)}_{mn\vec{l}}(I_o)\cos[n\tilde{\phi}_o(\tau) + m\tilde{\theta}_o(\tau) + \alpha_{mn\vec{l}}]}{\Omega'_{mn}(I_o)}$$

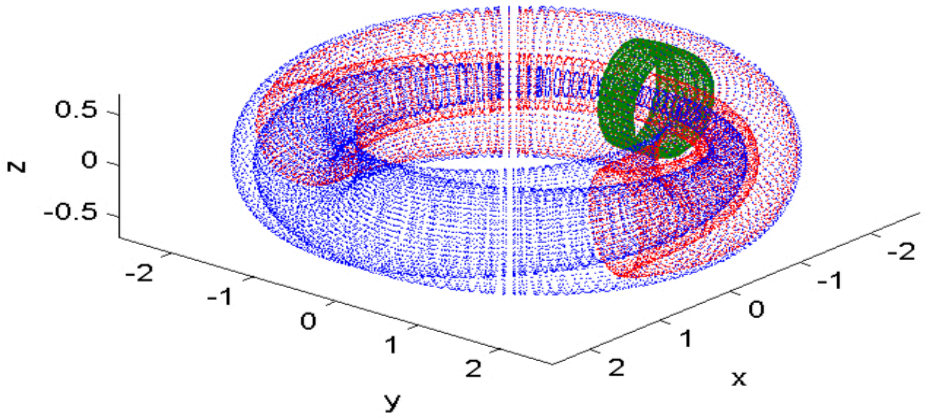
$$\tau = \varepsilon^{-1/2} t$$

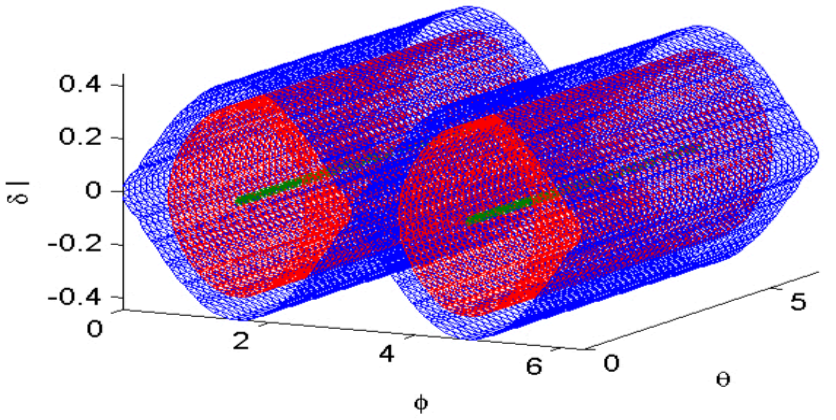
as seen in reference fame of moving disturbance

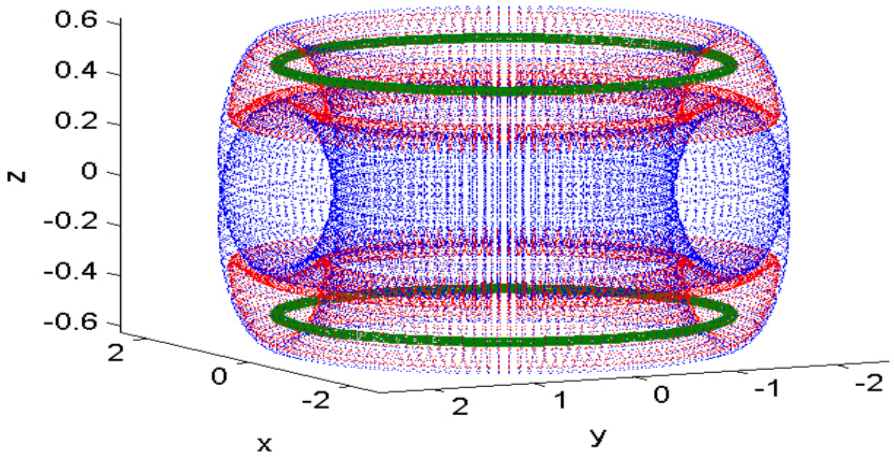


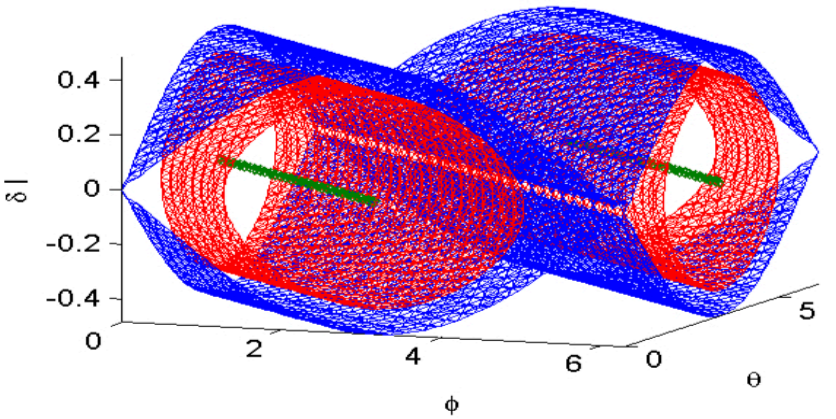

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
z
0.5
0
-0.5
y
-0.5
0
0.5
x

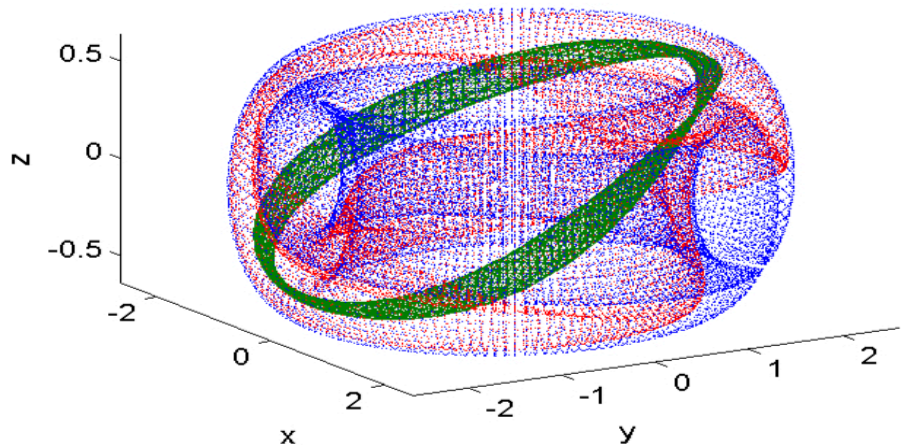

δ I
ϕ
θ

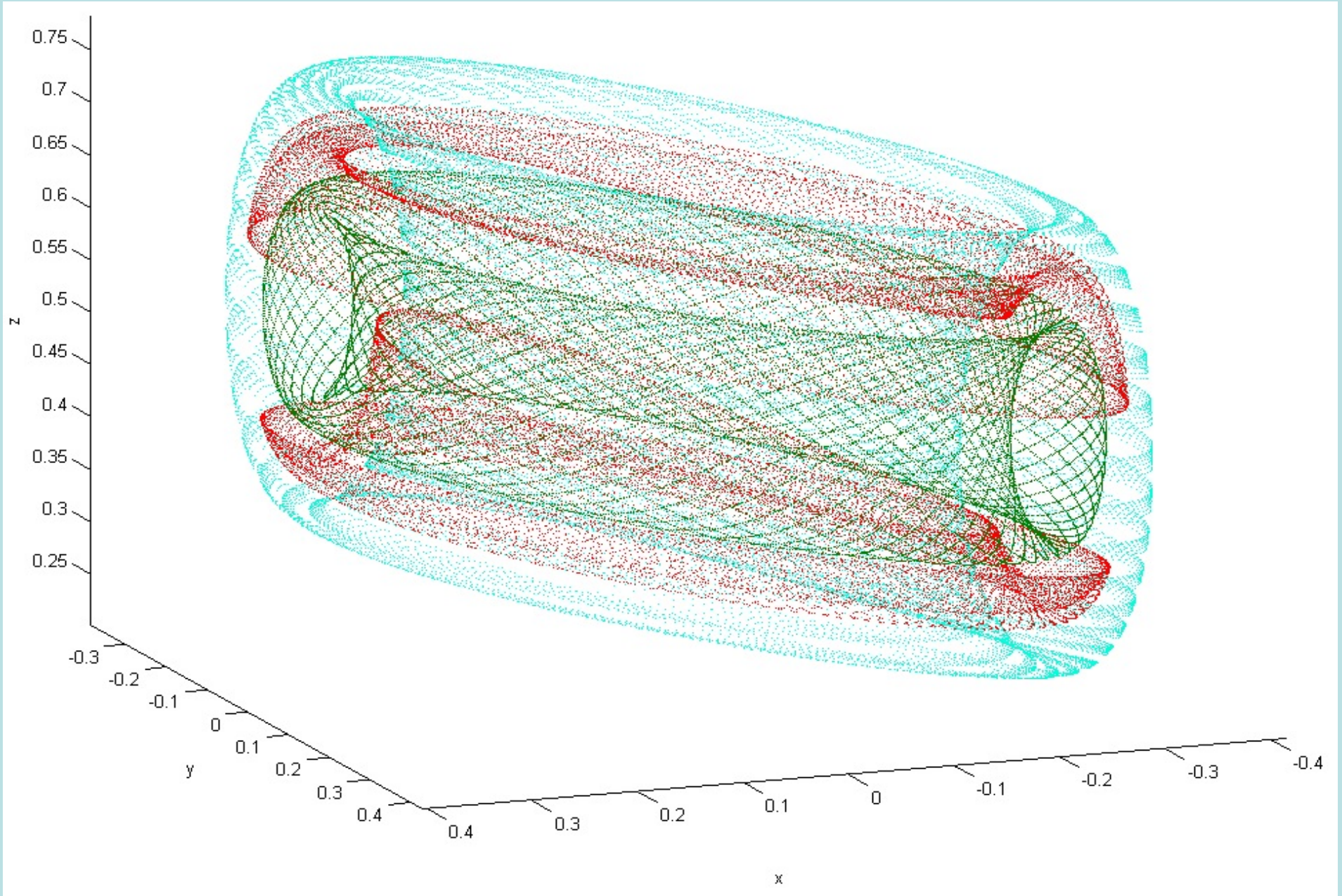

z
y
x


δ I
ϕ
θ

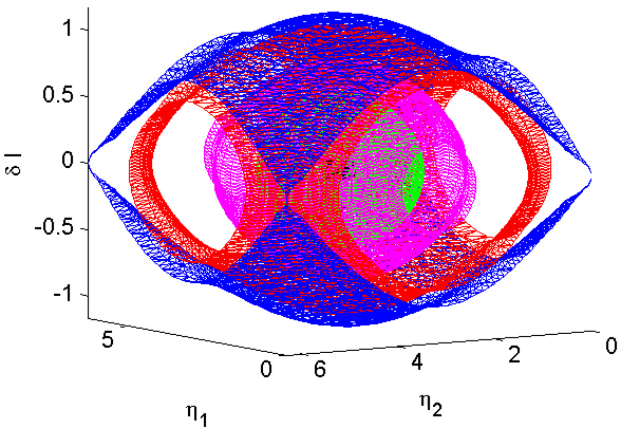

z
x
y

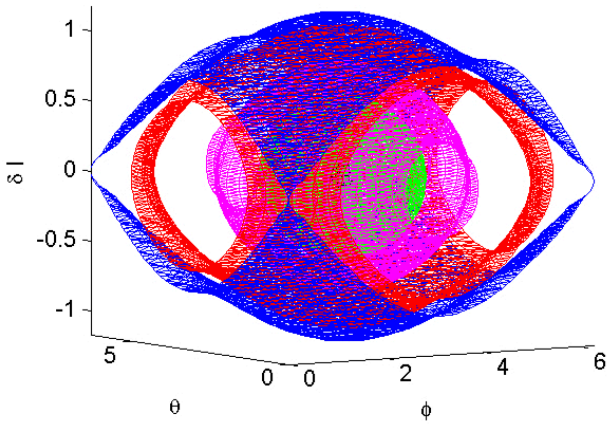

δ I
ϕ
θ

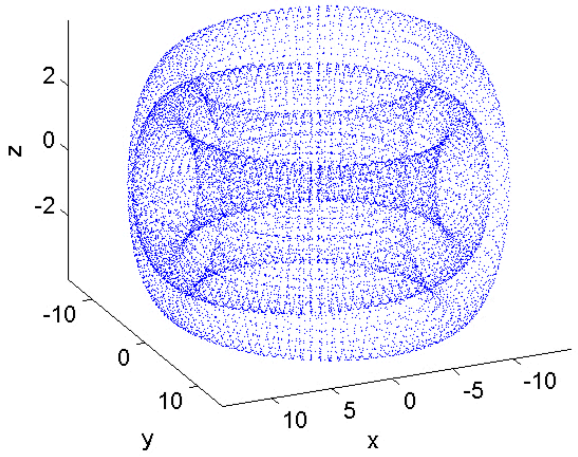

z
x
y

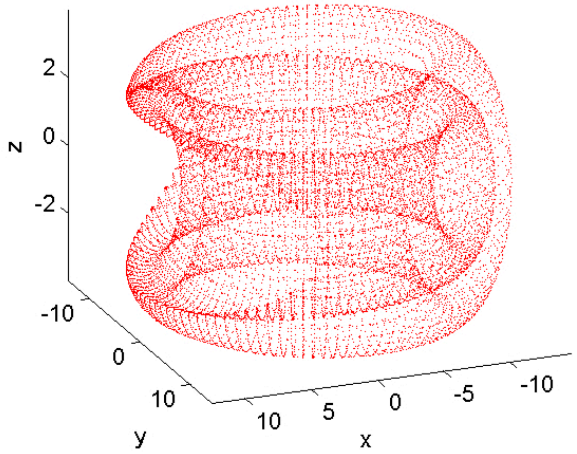

z
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
y
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
x
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4

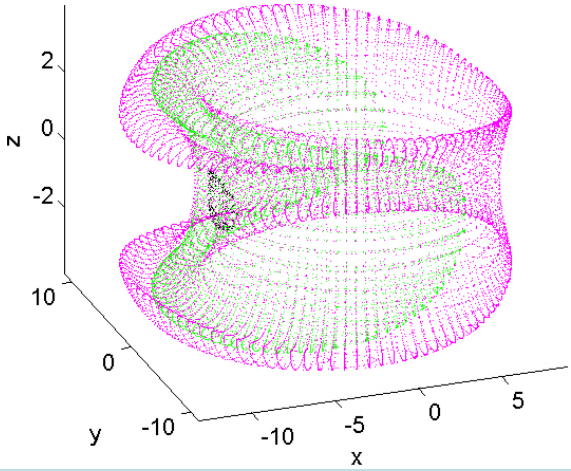
$m_1=1,\ n_1=0$
$m_2=0,\ n_2=1$

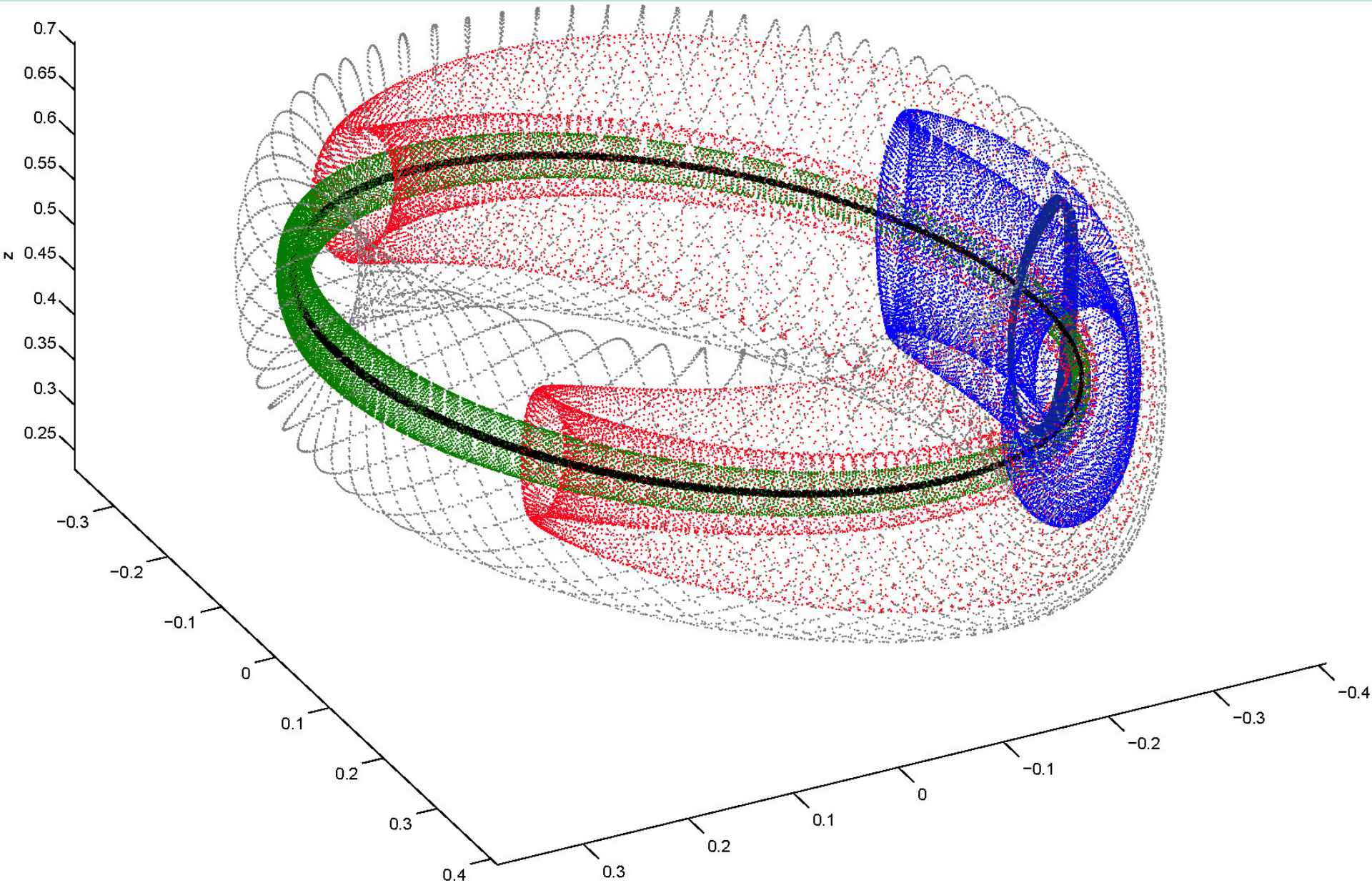


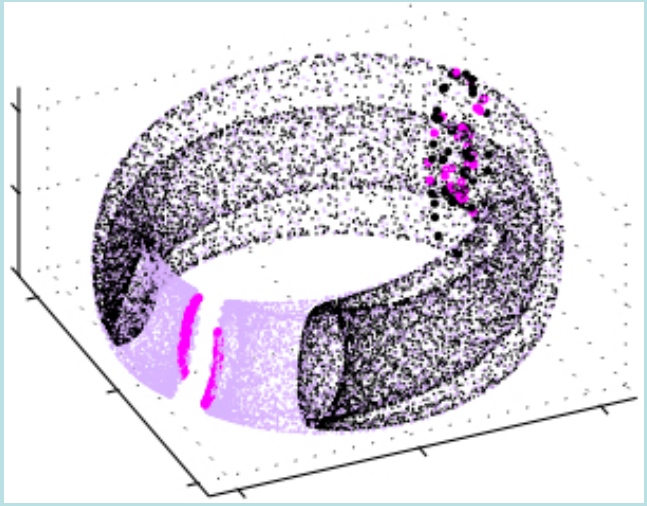






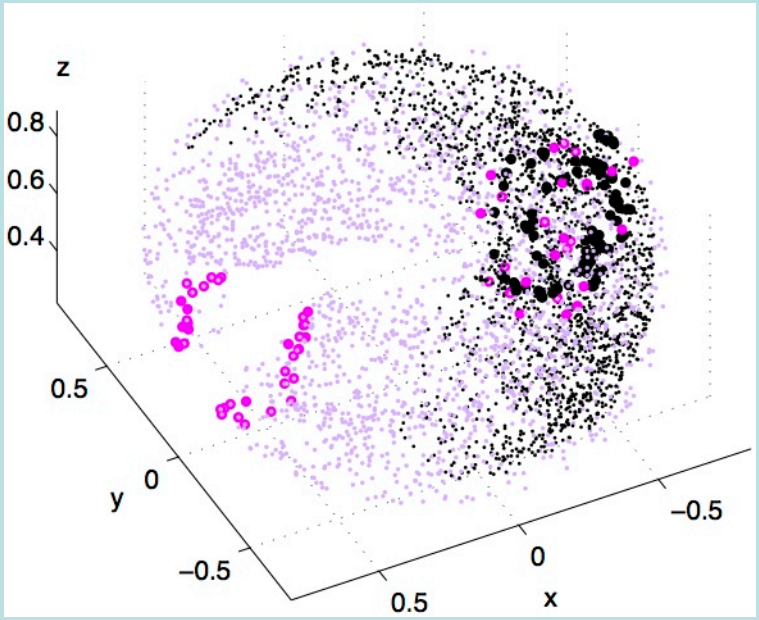



z
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
−0.3
−0.2
−0.1
0
0.1
0.2
0.3
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4

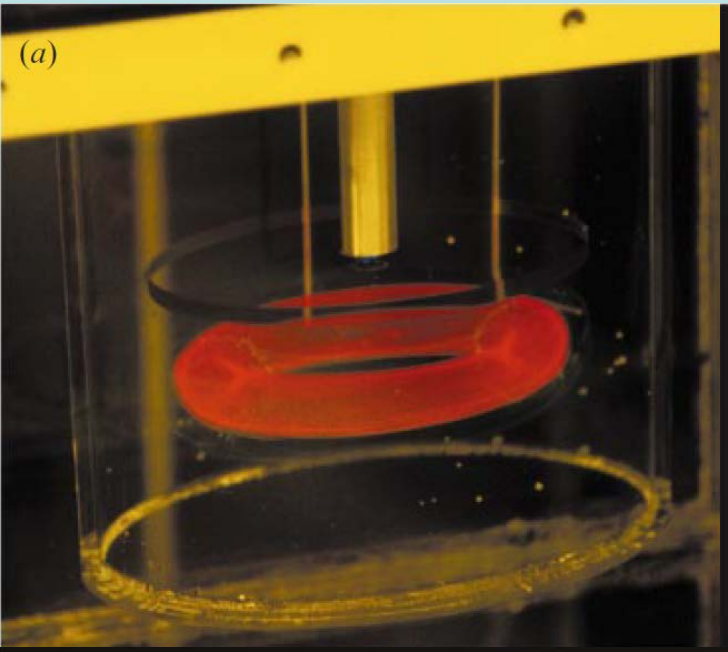


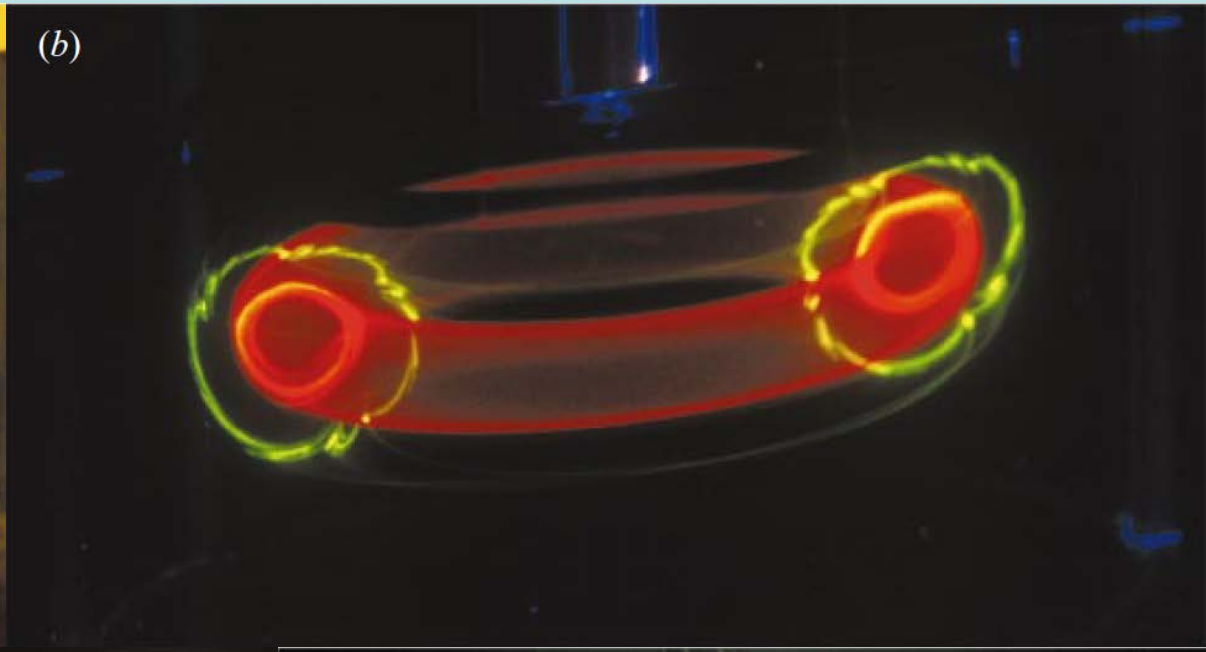
Expected

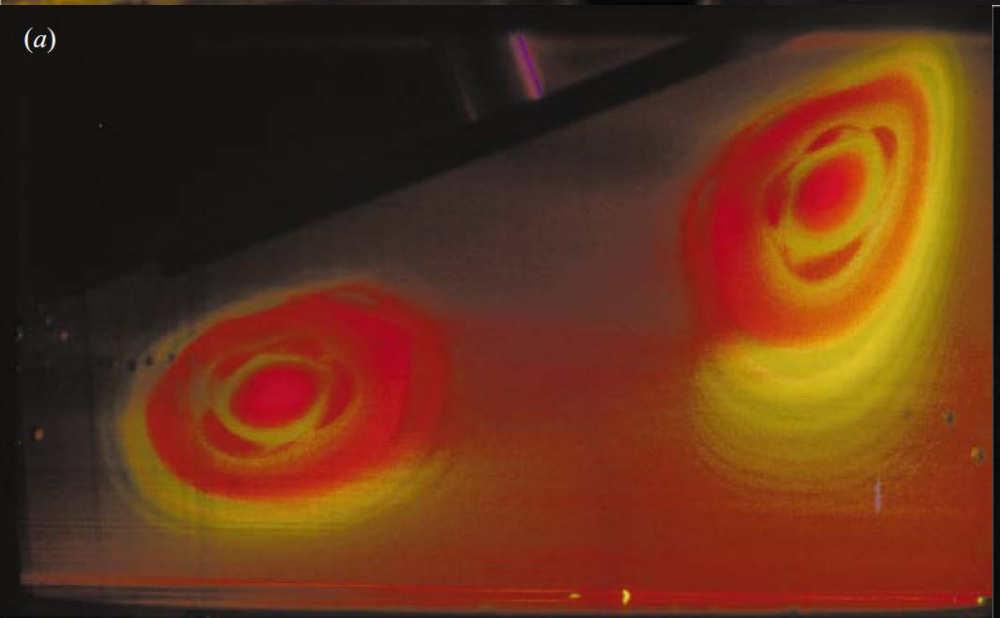
Navier Stokes simulation

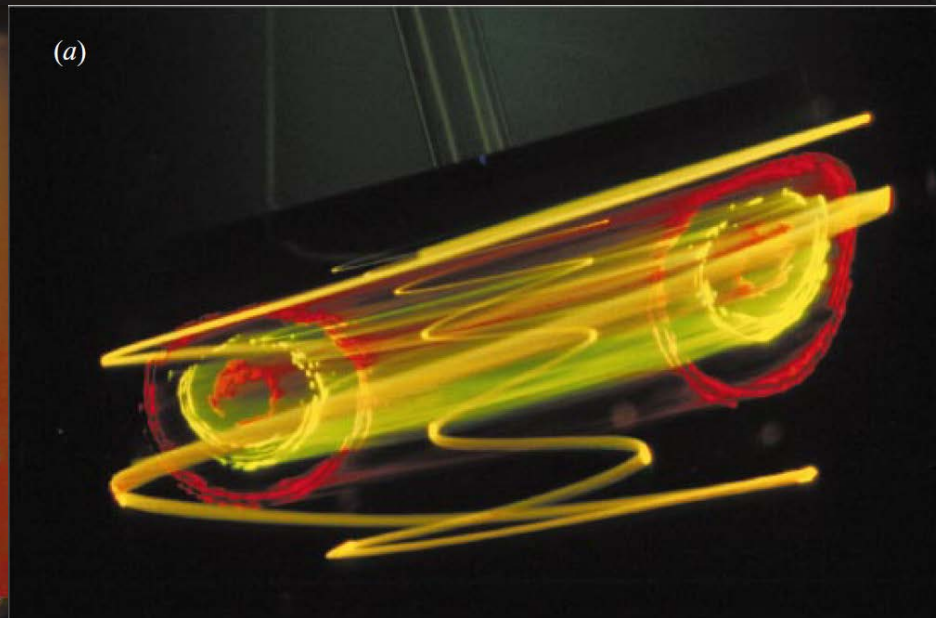


Laboratory observations (Fountain et al. 2000)









What about background turbulence?

see Jay Brett thesis work

$$\text{eddy diffusivity} = \frac{K_o}{1 + k^2 \gamma^{-2} [c_w - U(z)]^2}$$

Ferrari and Nikurashin (2010)

$$\text{in our 'eddy'} = \frac{K_o}{1 + \gamma^{-2} [\vec{l} \cdot \vec{\sigma} - n\Omega_\phi + m\Omega_\theta]^2}$$

Challenges

- How does one observe Lagrangian barriers in 3D+1?
- How to parameterize stirring/mixing due to chaotic motion in 3D+1?
- Establish extent to which LCS are relevant in in turbulent flow fields?

2014 Boston Museum of Science Exhibit w. artist Anastasia Azure

“Beauty will save the world.”

-Dostoyevsky