Chaotic Advection in a Periodically-perturbed, Three-dimensional Rotating Cylinder

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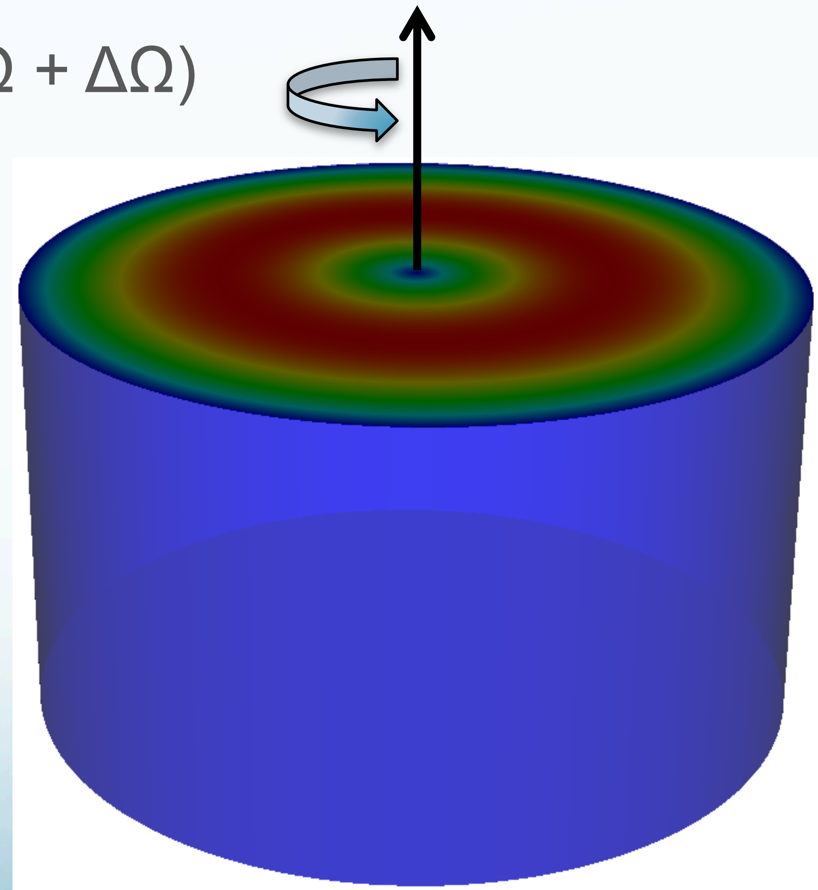
1. Introduction

- "Chaotic Advection" (Aref, 1984)
- Non-integrable system
- Laminar flow; turbulent flow
- Stirring, stretching, folding, mixing

2. Model

- Cylinder rotates at Ω
- Top lid rotates at (Ω + ΔΩ)



- Nek5000 (spectral element)

$$\begin{cases} \dfrac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\nabla P - \dfrac{1}{Ro}\left(2\vec{\Omega} \times \vec{v}\right) + \dfrac{1}{Re}\nabla^2 \vec{v} \\ \nabla \cdot \vec{v} = 0 \end{cases}$$

- Cylinder domain $0 \le r \le 1 \quad 0 \le z \le 1$
- Closed boundary, no-slip, no flux
- Top lid velocity profile

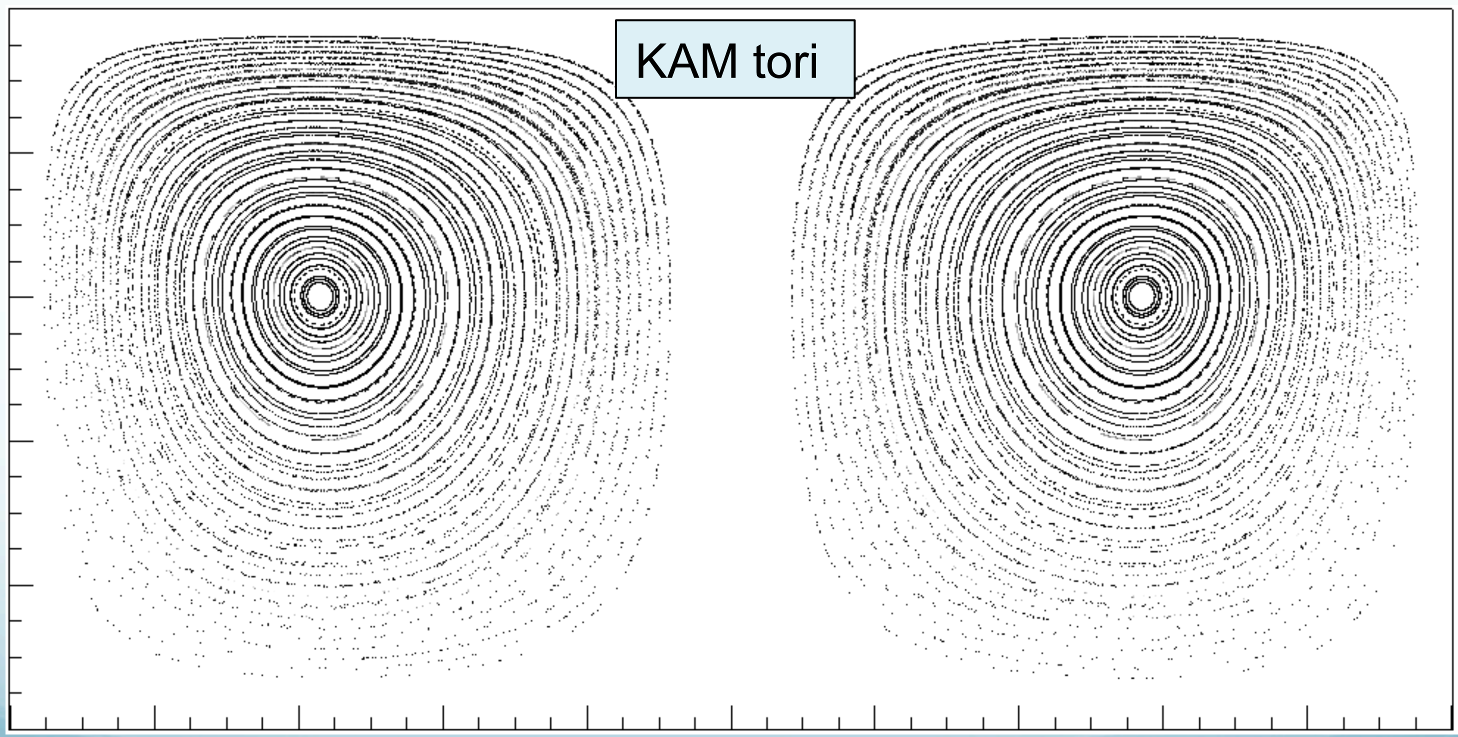
$$u(x,y,1) = -4y(1-r)$$
$$v(x,y,1) = 4(x-x_0)(1-r)$$

x_0 --- Perturbation

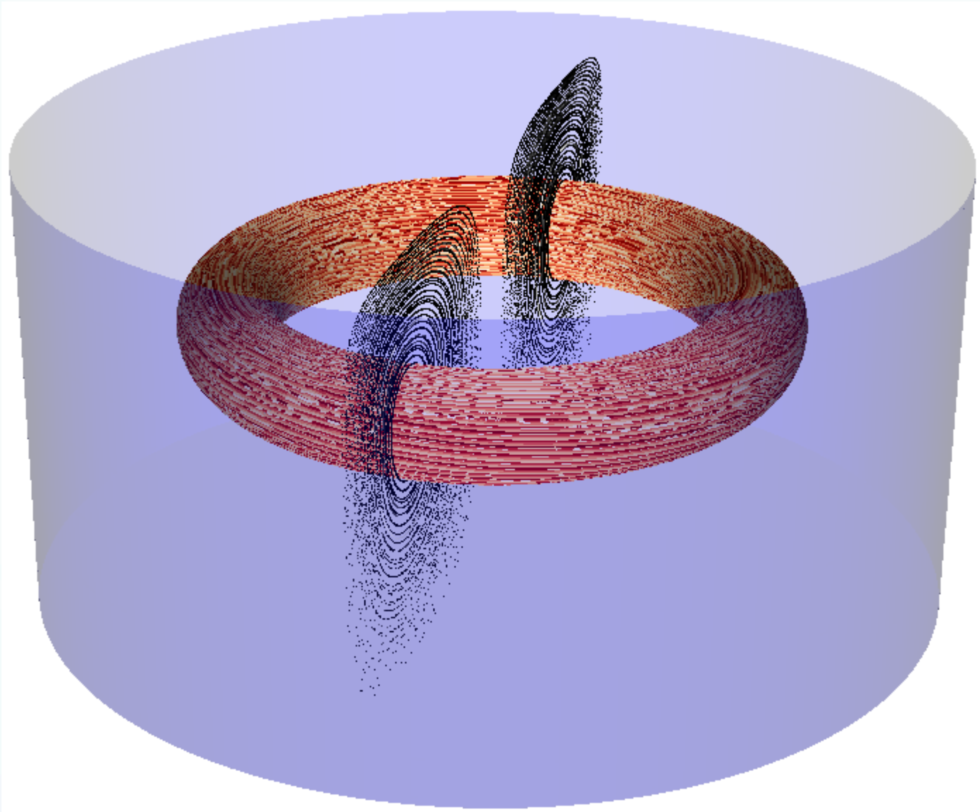
3. Results

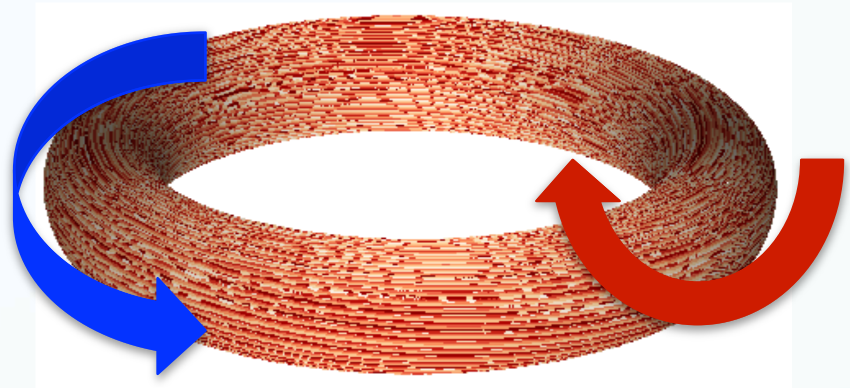
- 3.1 Zero perturbation

$x_0 = 0 \qquad Ro = 0.2 \qquad Re = 1.6 \qquad \frac{H}{R} = 1$



Poincare Section





θ Ω_θ φ Ω_φ

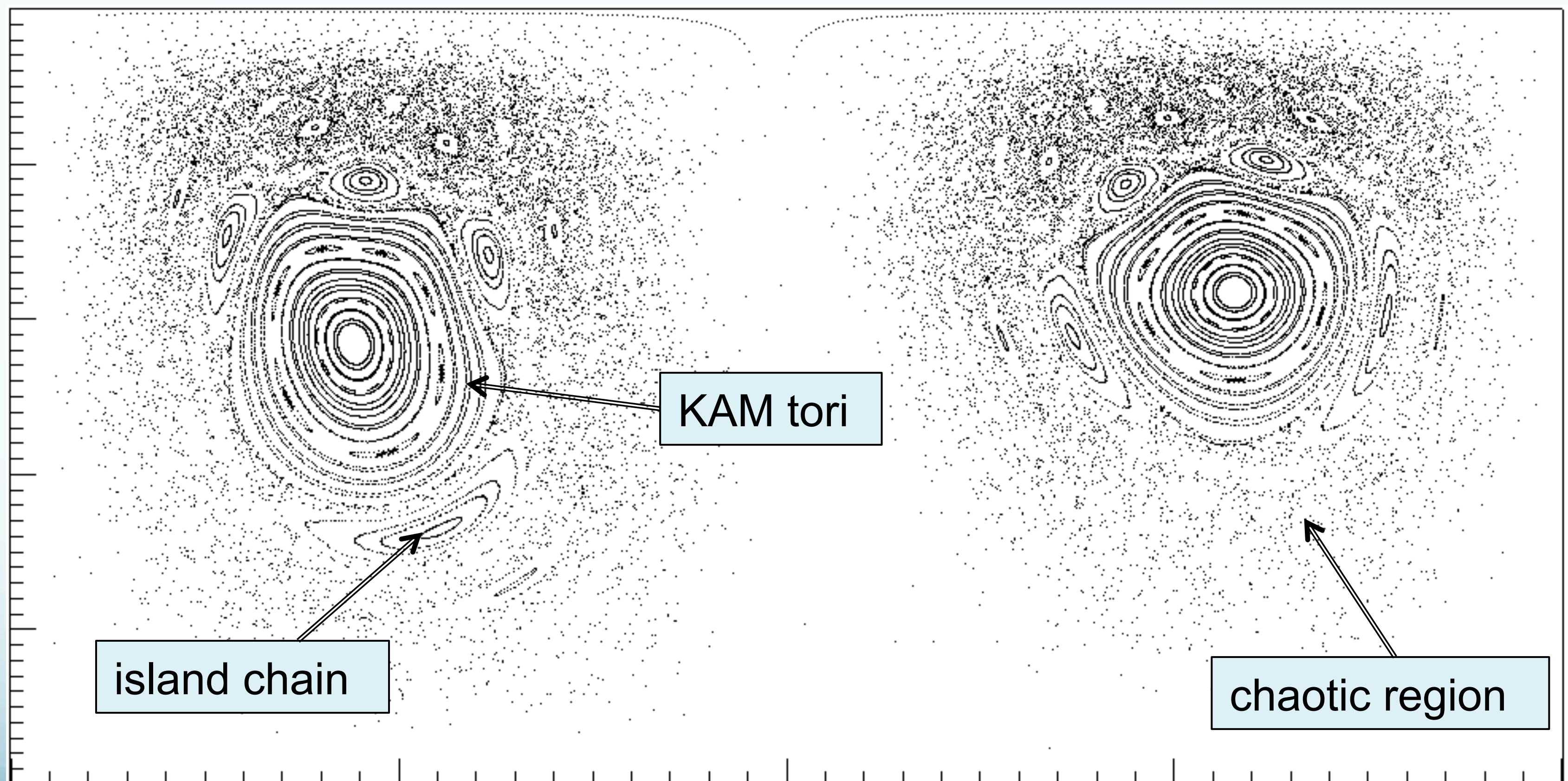
Frequency ratio = $\dfrac{\Omega_\varphi}{\Omega_\theta}$

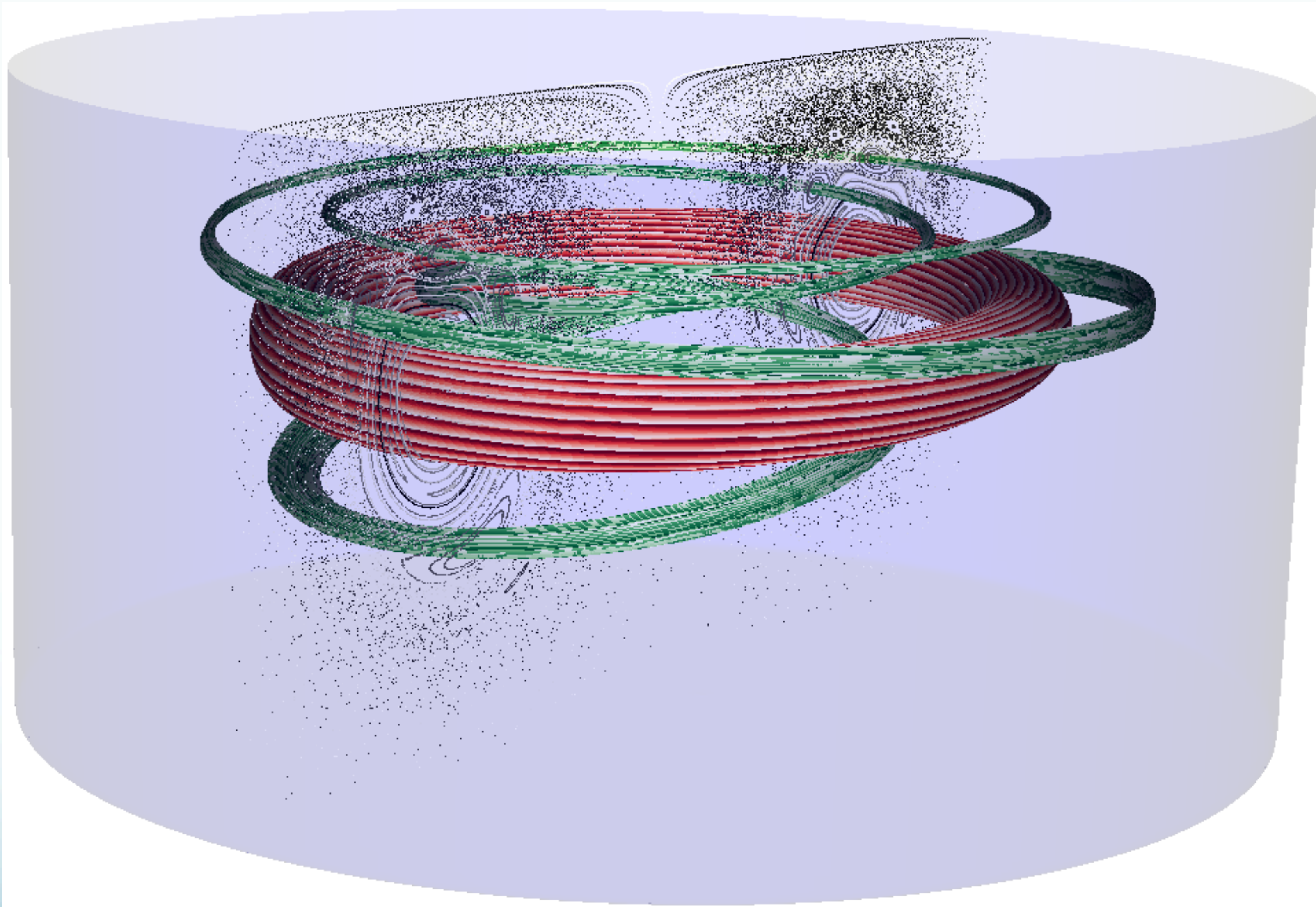
irrational ═ quasi-periodic

rational ═ periodic

3.2 Steady perturbation

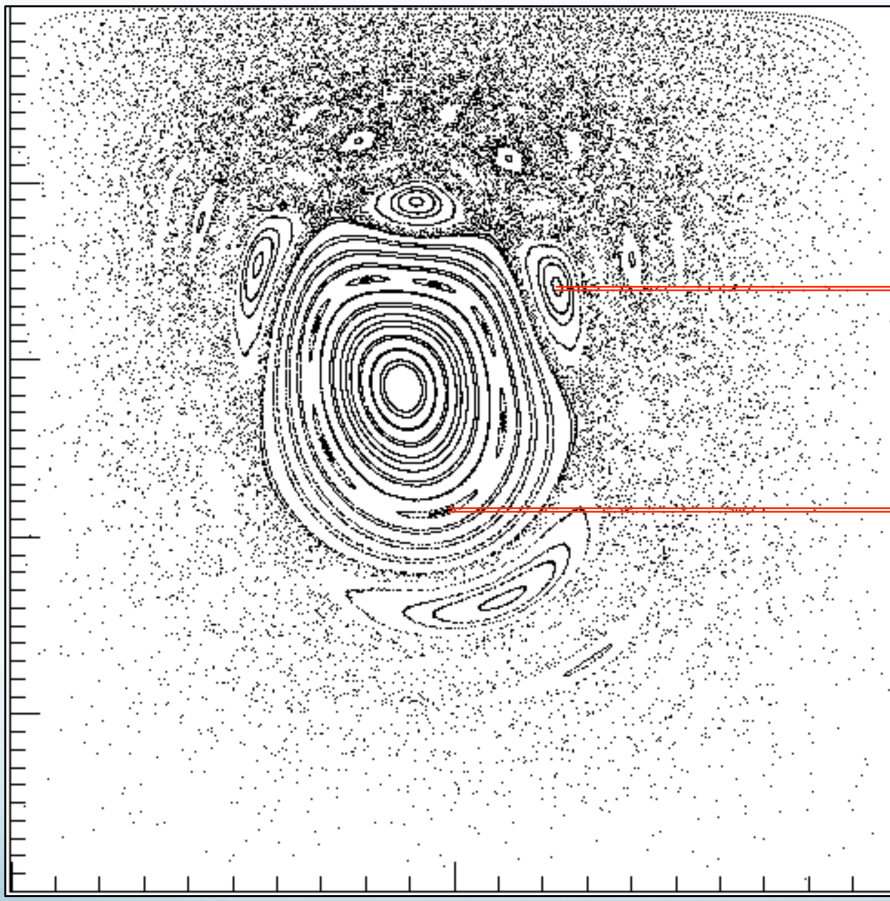
$$x_0 = -0.02$$





Number of Islands

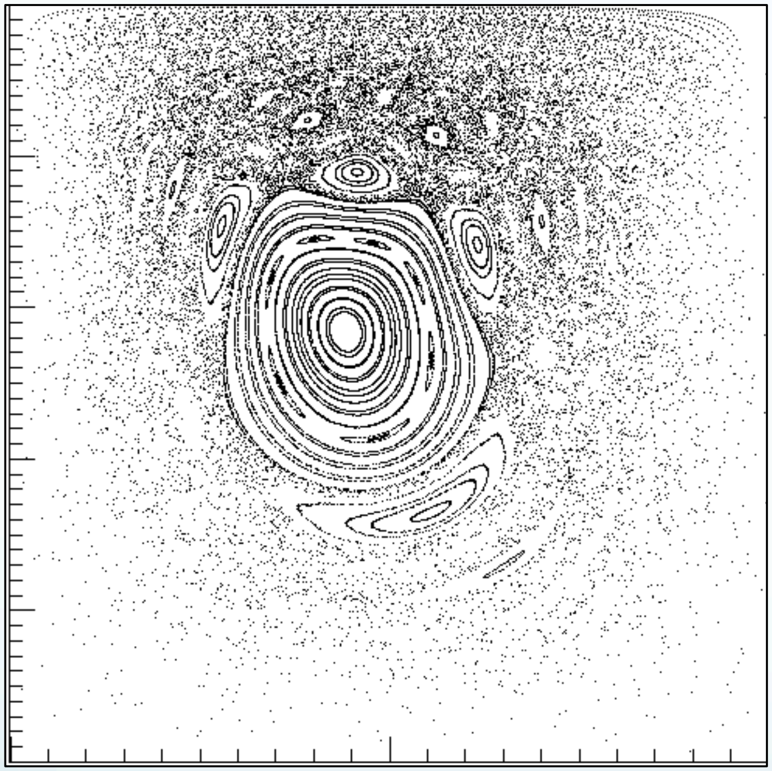
$$\frac{\Omega_\varphi}{\Omega_\theta} = \frac{m}{n}$$ period-**n** island chain on diametral PS

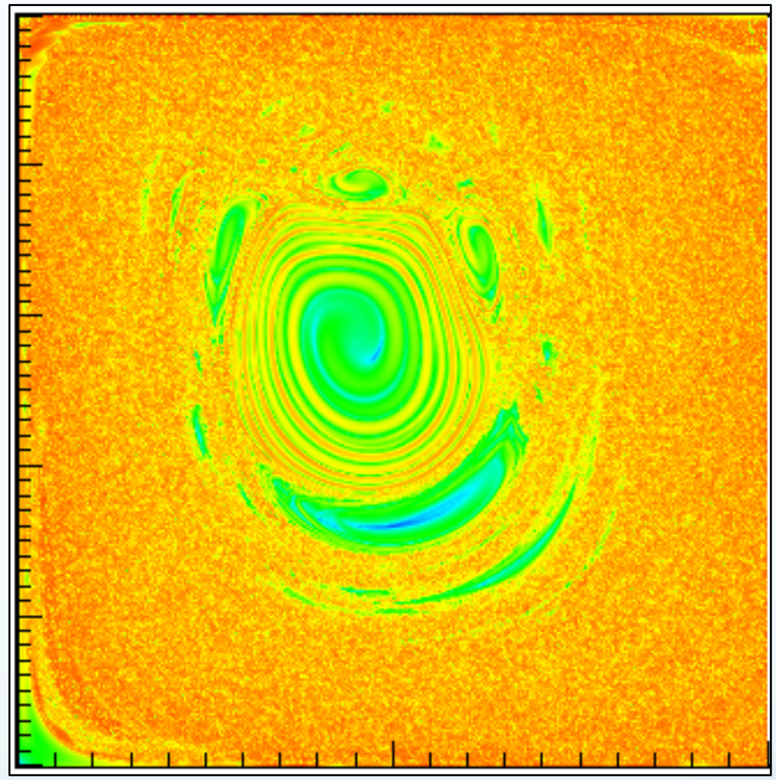


period-4 $\frac{\Omega_\varphi}{\Omega_\theta} = \frac{1}{4}$

period-7 $\frac{\Omega_\varphi}{\Omega_\theta} = \frac{1}{7}$

Poincare Section vs. FTLE (Finite-Time Lyapunov Exponent)





3.3 Periodic perturbation

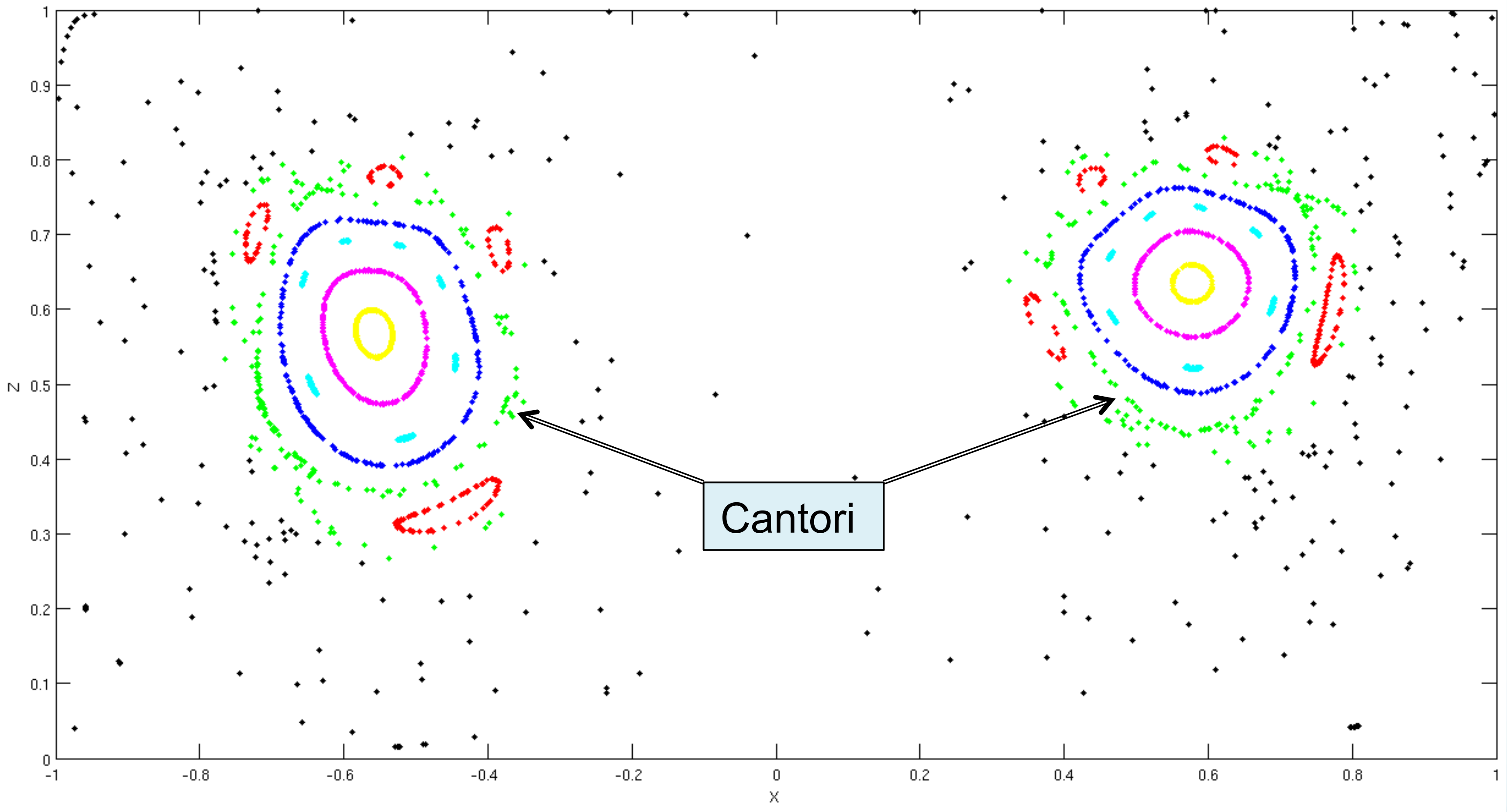
$$x_0 = -0.02 + 0.002\sin\left(2\pi t / T\right)$$

T : period of perturbation

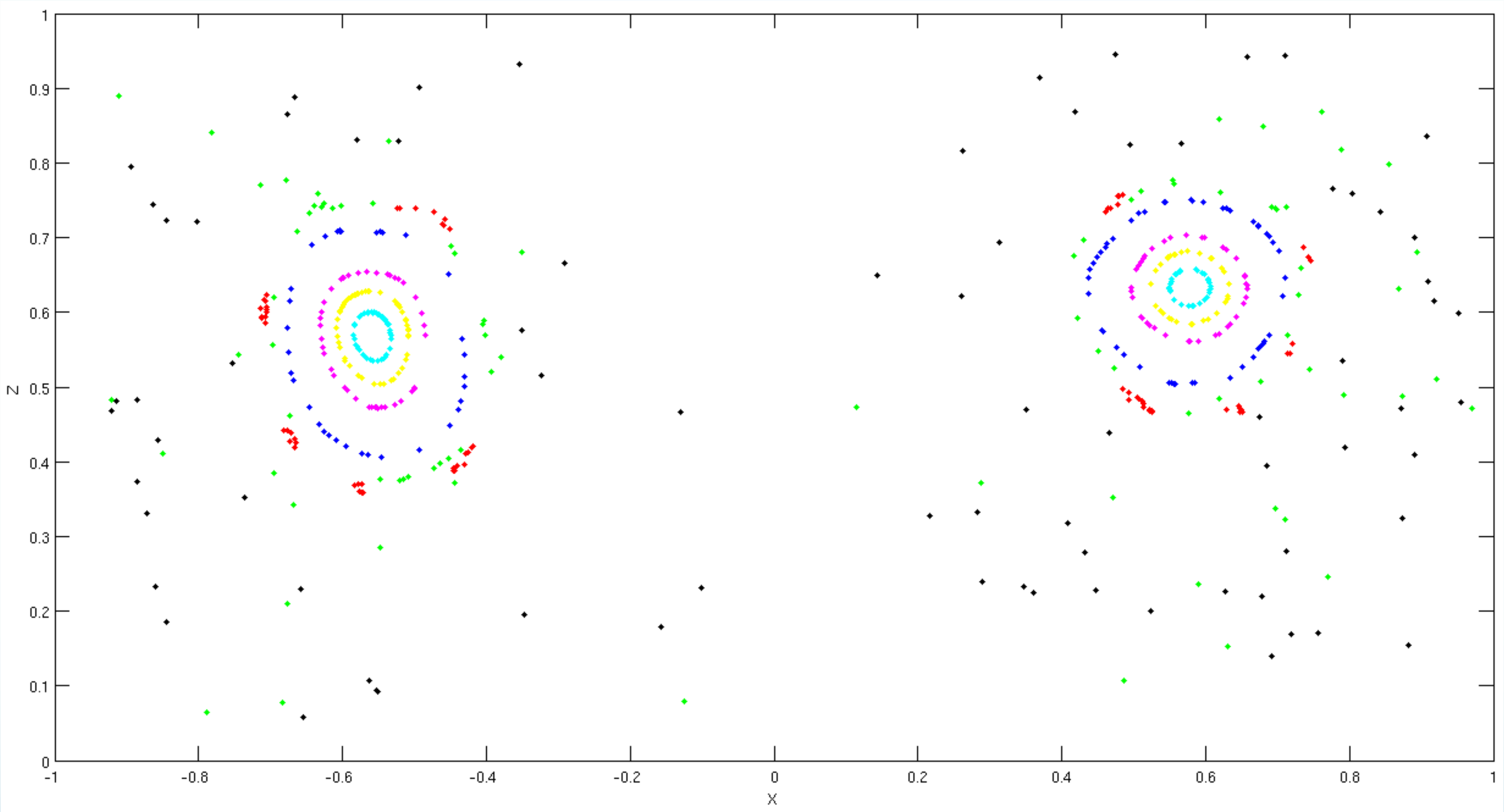
Double Poincare Section

“Period orbits” : time interval = perturbation period

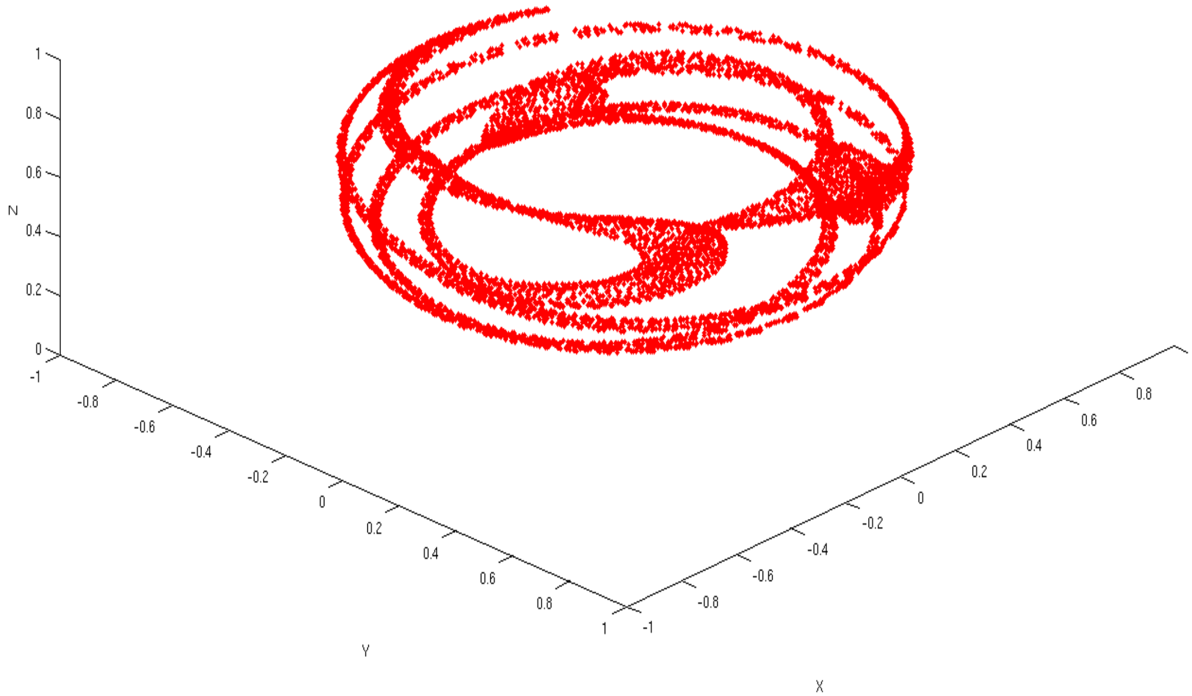
Period T = 1

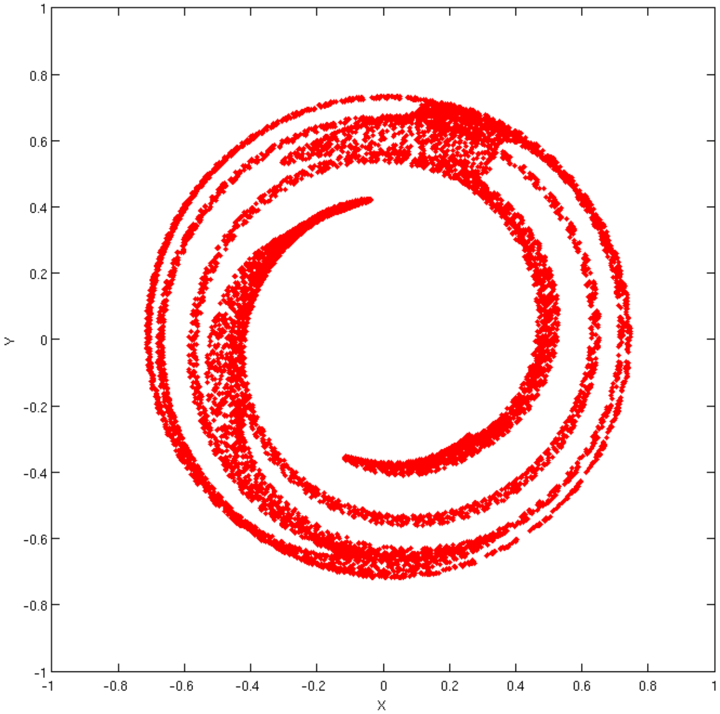


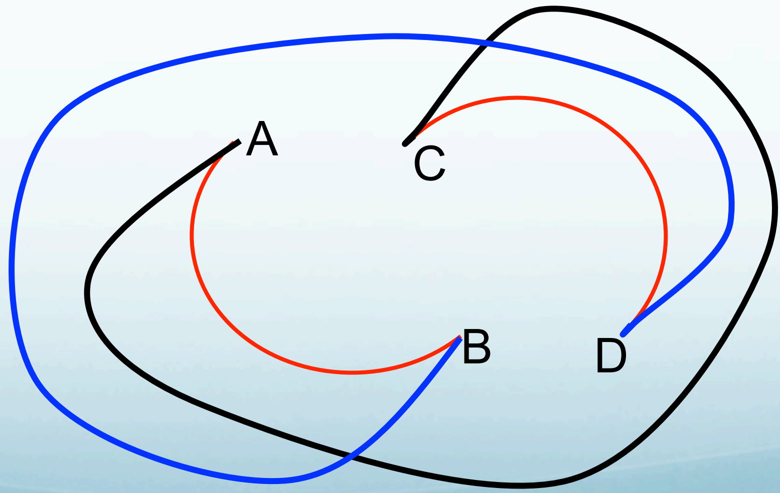
Period T = 6



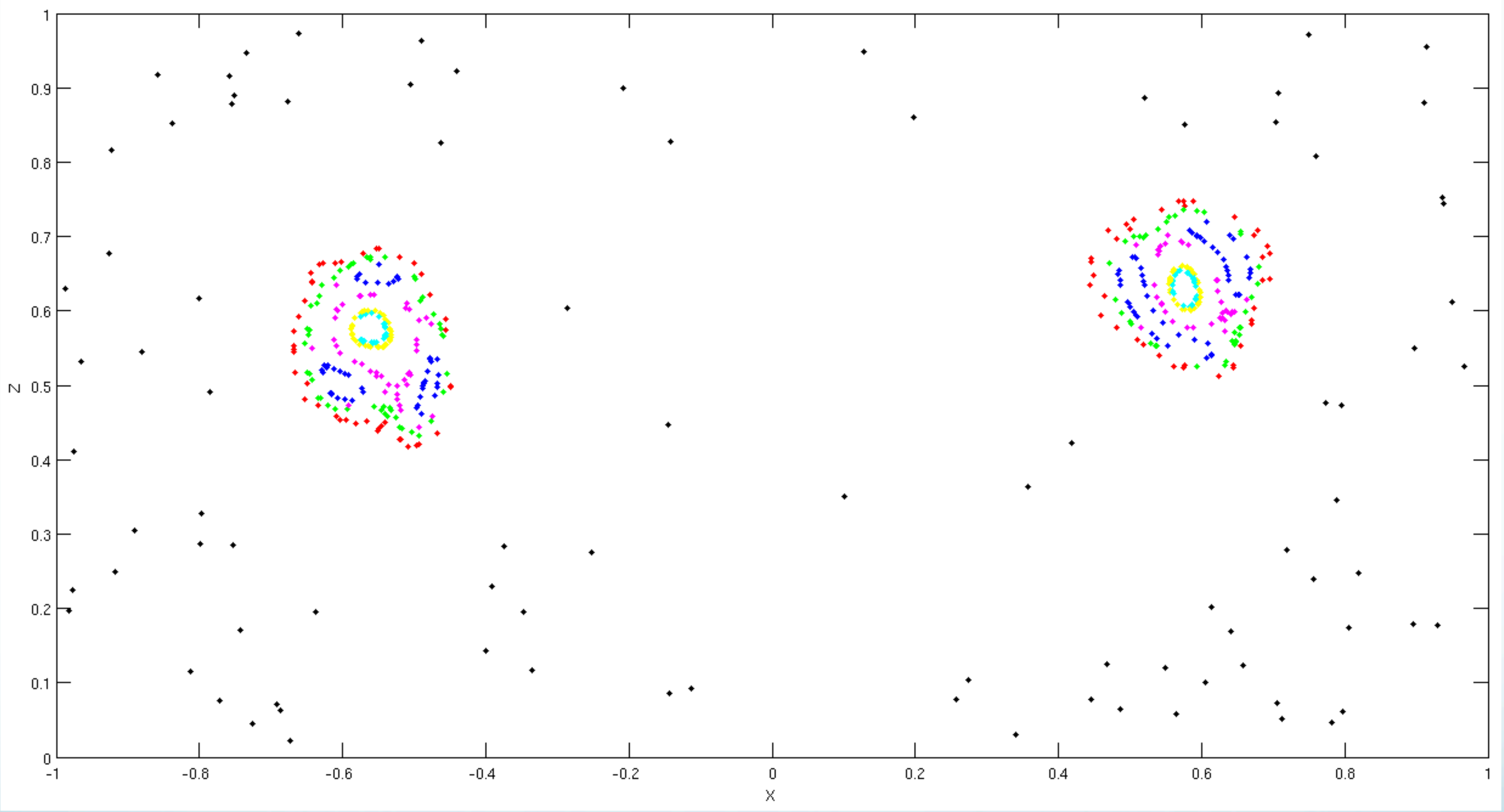
Period Orbits



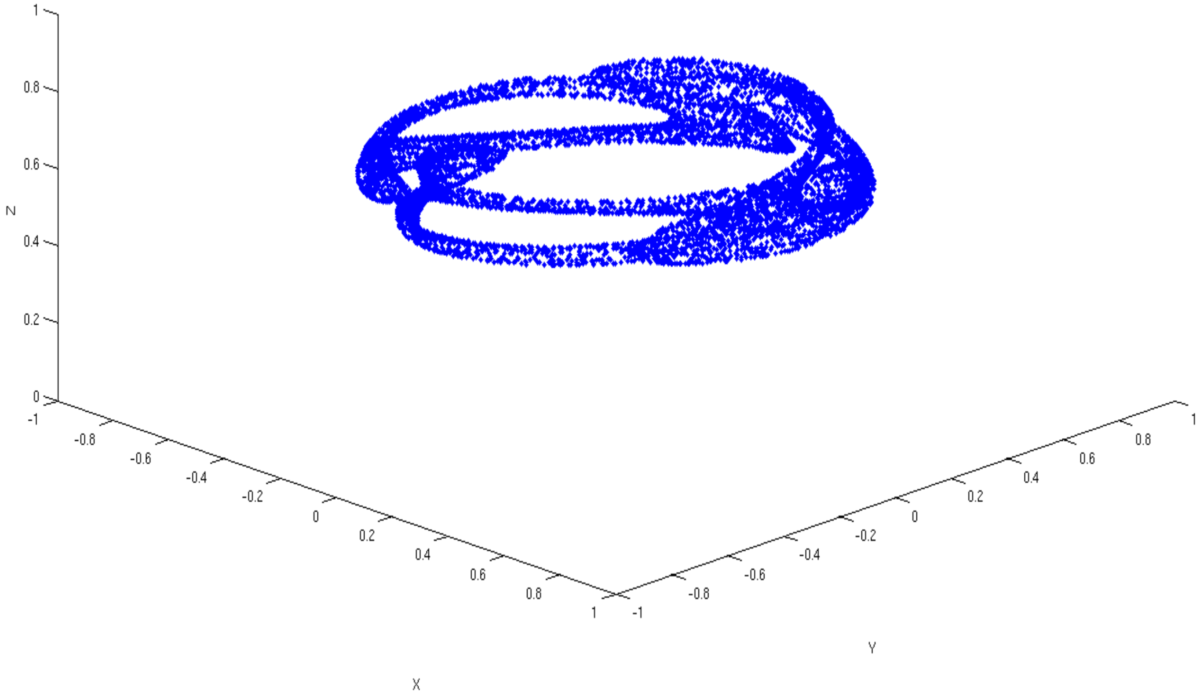


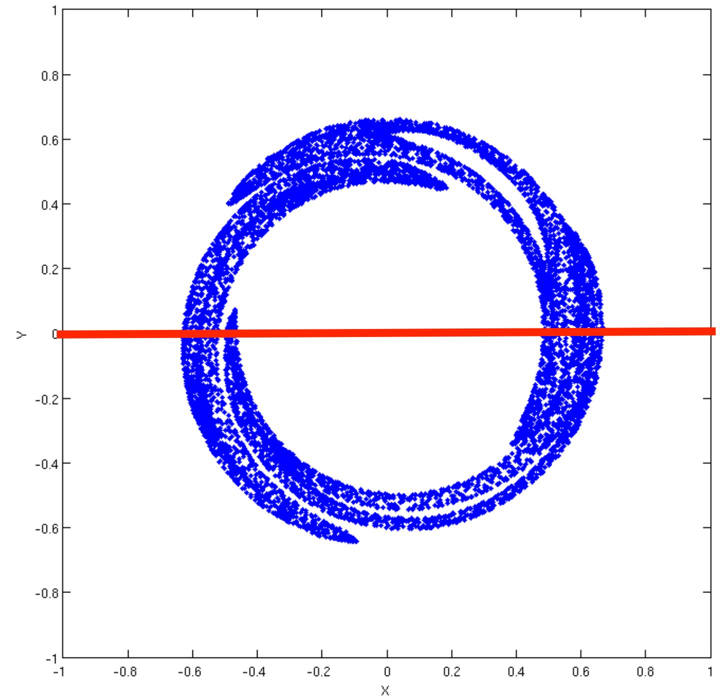


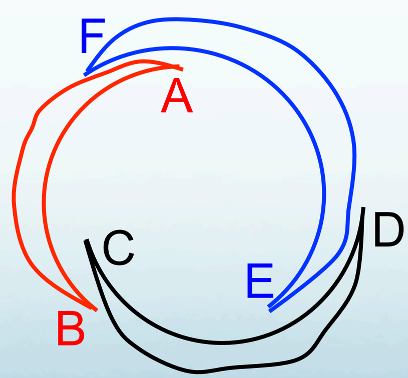
Period T = 11



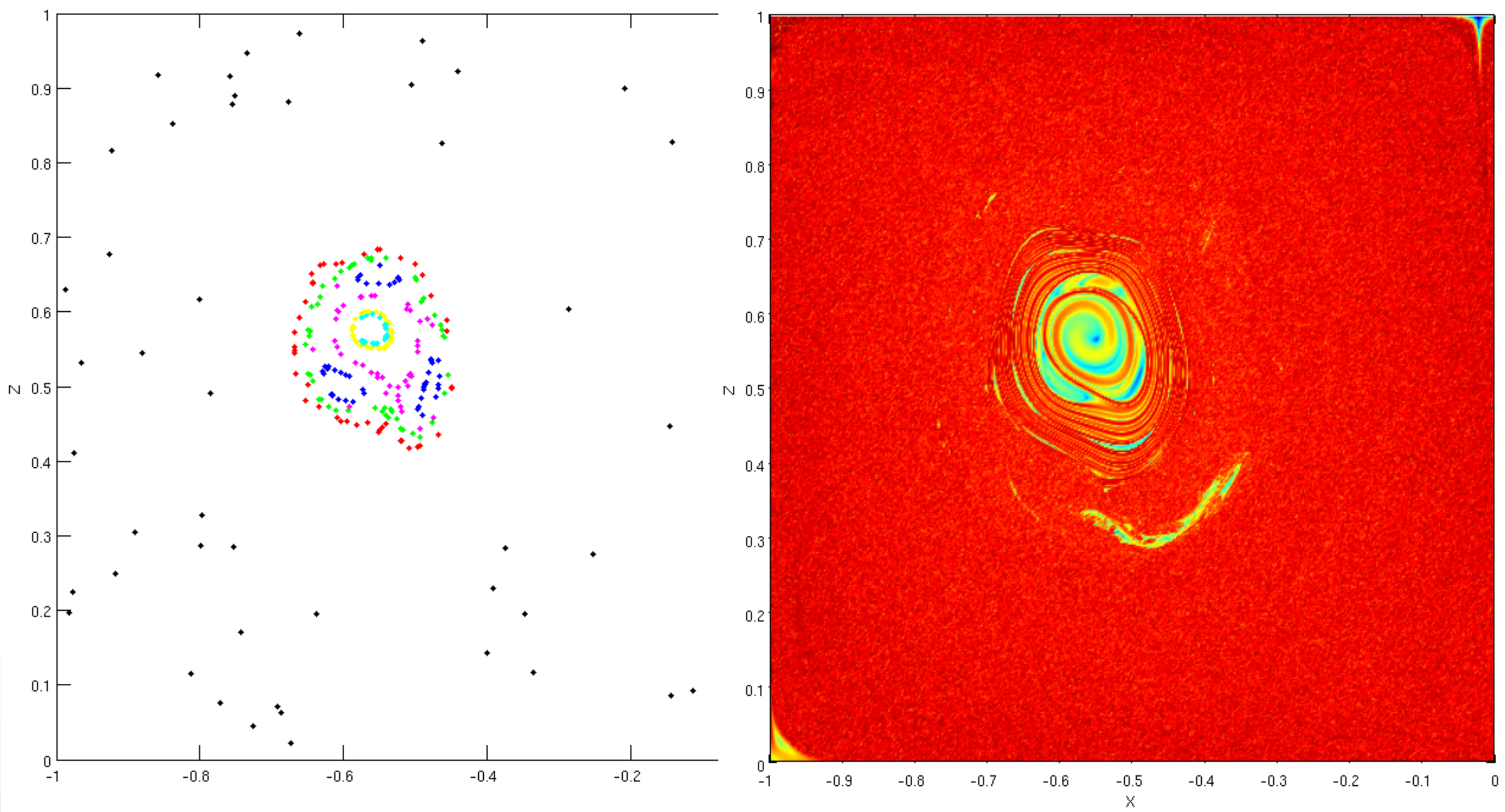
Period Orbits







Double PS vs. FTLE



4. Summary

- Zero perturbation – no resonance, no chaos
- Steady perturbation – resonance, island chain, chaos
- Periodic perturbation – period orbits with tips
- Transport barriers still exist under periodic perturbation