A world map showing ocean eddy mixing energy. The map uses a color scale from blue (low energy) to red (high energy). High energy regions are visible in the North Atlantic, the western Pacific, and the Indian Ocean. The map is overlaid with a grid of latitude and longitude lines.

Eddy mixing in energetic ocean regions with jets: Float-based estimates and theories

Ru Chen

Scripps Institution of Oceanography

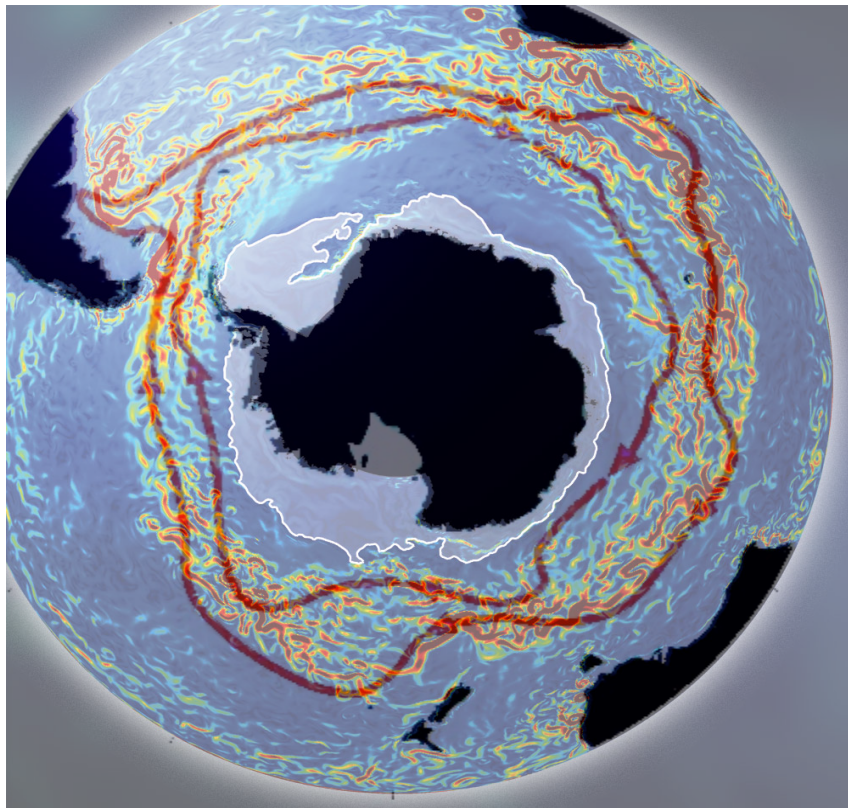
With Sarah Gille, Julie McClean,
Alexa Griesel, Glenn Flierl

*Ocean 3D+1 workshop
Sep. 28-30, 2015*

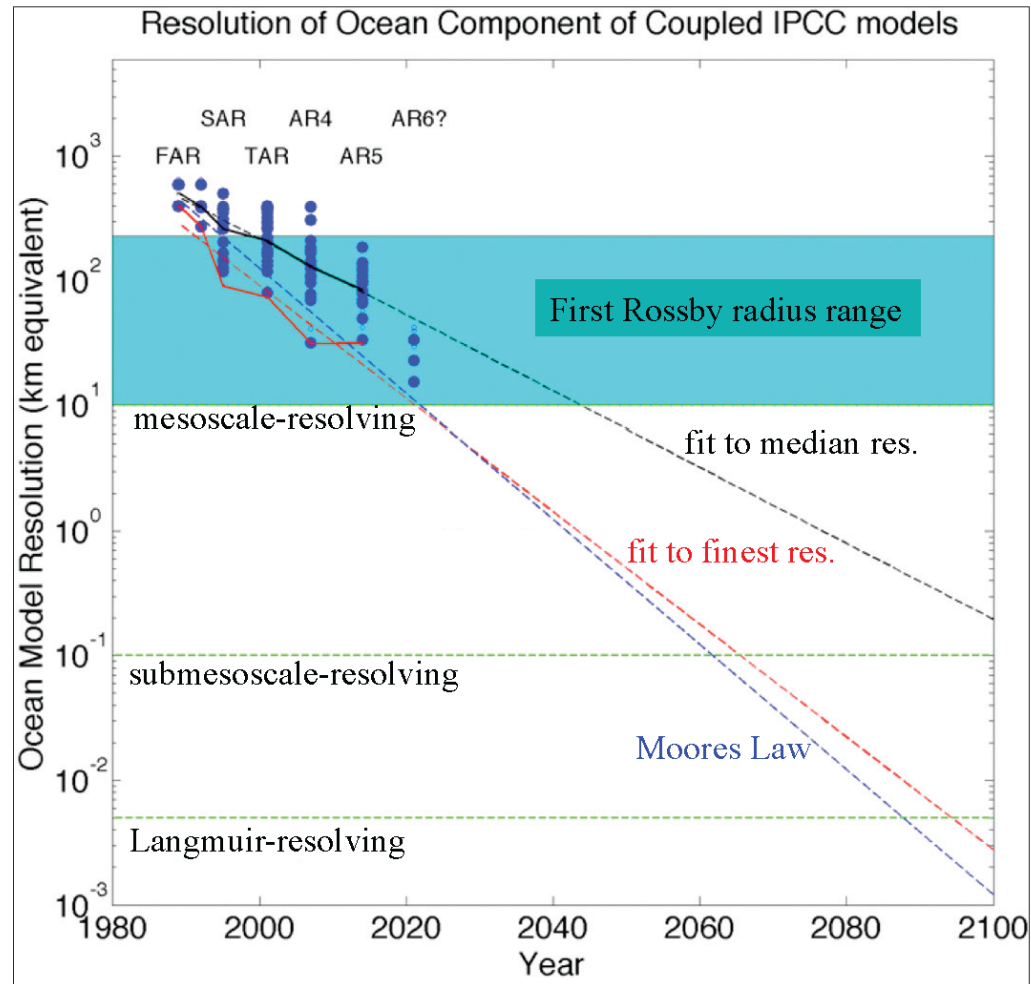
Outline

- (1) Background:** Eddies, climate modeling, and previous mixing theories and estimations.
- (2) Our methodology:** Global eddy model with one million numerical floats.
- (3) Results:** Complex mixing patterns at high spatial resolution.
- (4) Results:** Comparison between new and original mixing theories.
- (5) Summary**

Eddy field and model resolution



Courtesy of Matt Mazloff



From Fox-Kemper et al. (2014)

Eddy parameterization in models

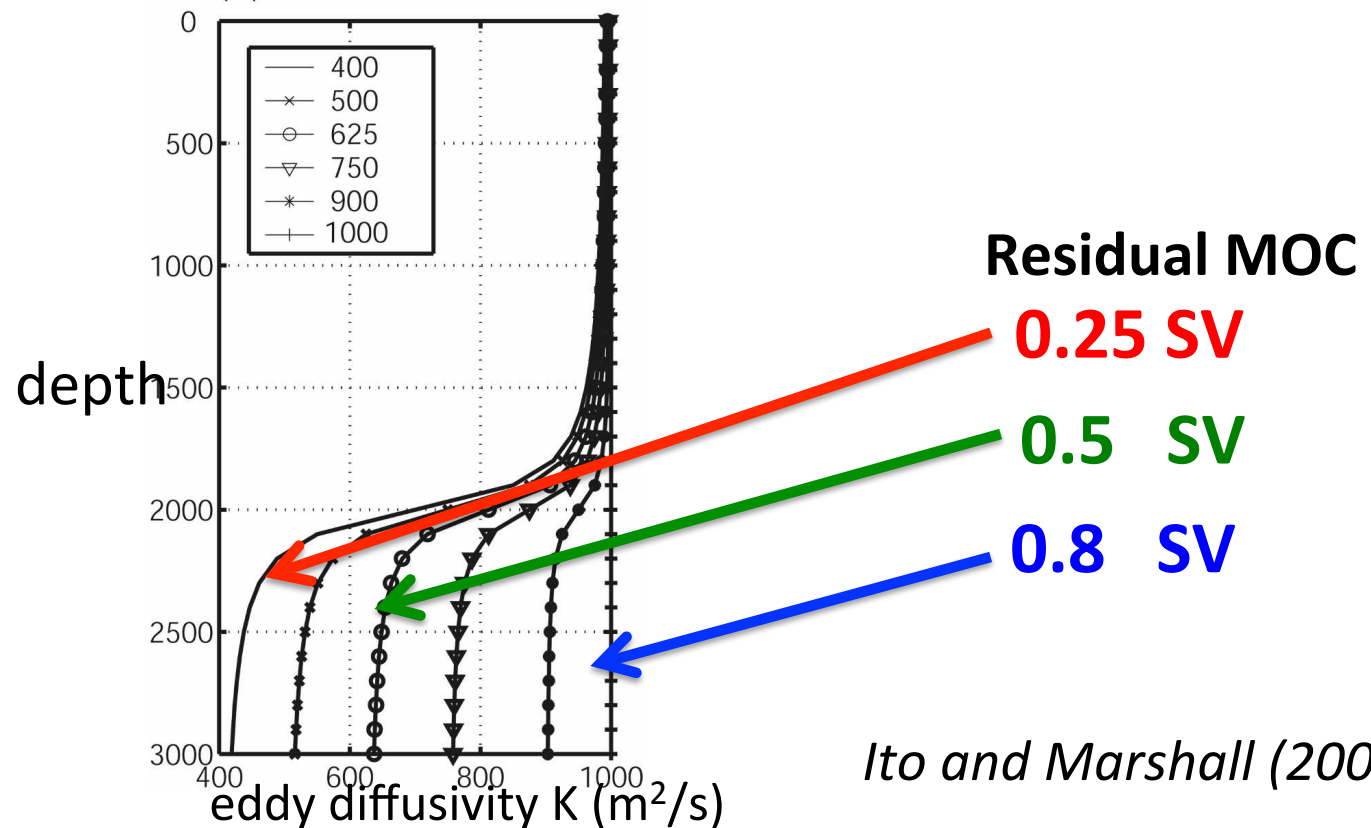
General eddy parameterization problem in climate models

$$\nabla \cdot \overline{\mathbf{v}'c'} = \nabla \cdot (-\mathbf{K}\nabla\bar{c})$$

K is a function of x, y, z, and t!

Eddy diffusivity

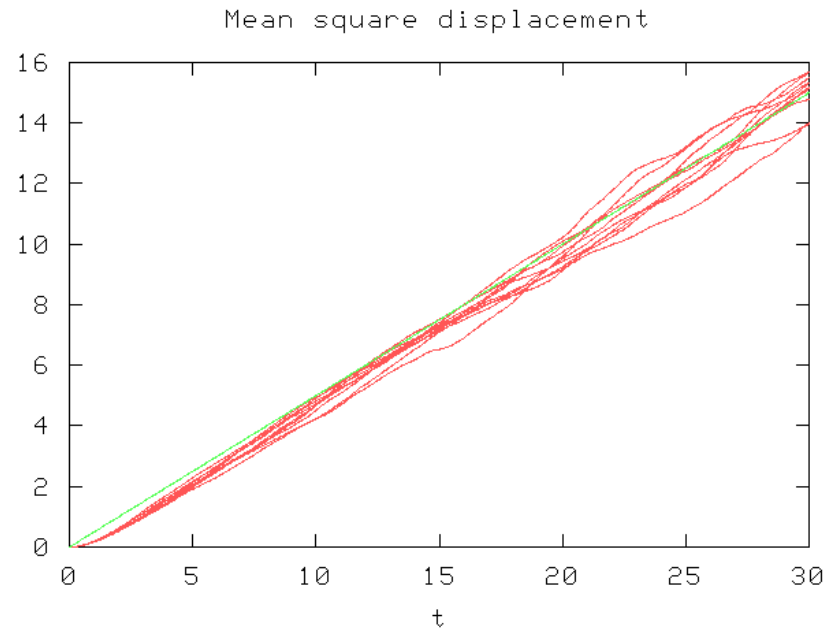
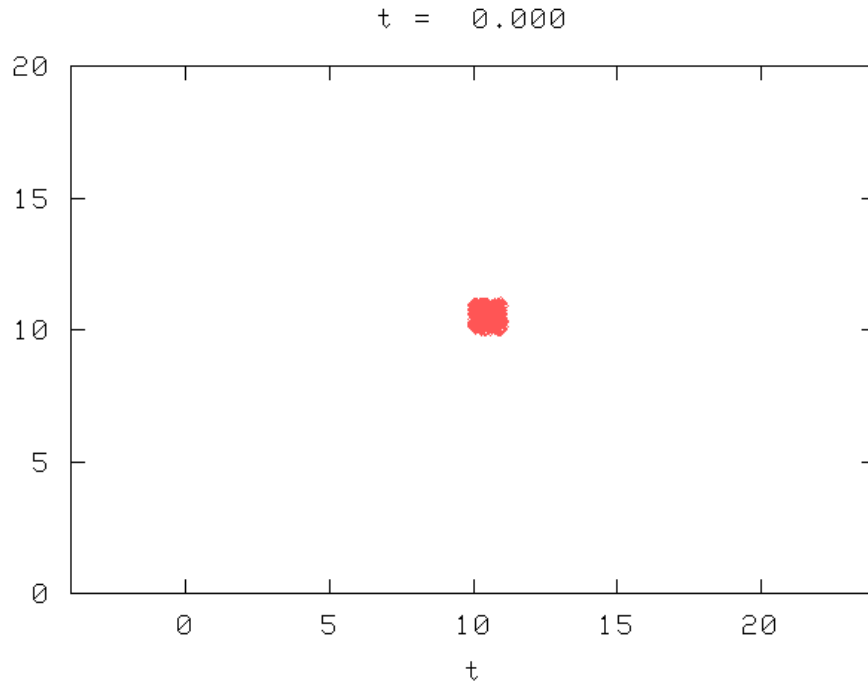
Meridional overturning circulation (MOC) is sensitive to K



Ito and Marshall (2008)

Estimate eddy diffusivity from Lagrangian particles

Pure diffusion case: particles evolution with time through random walk (molecular diffusion).



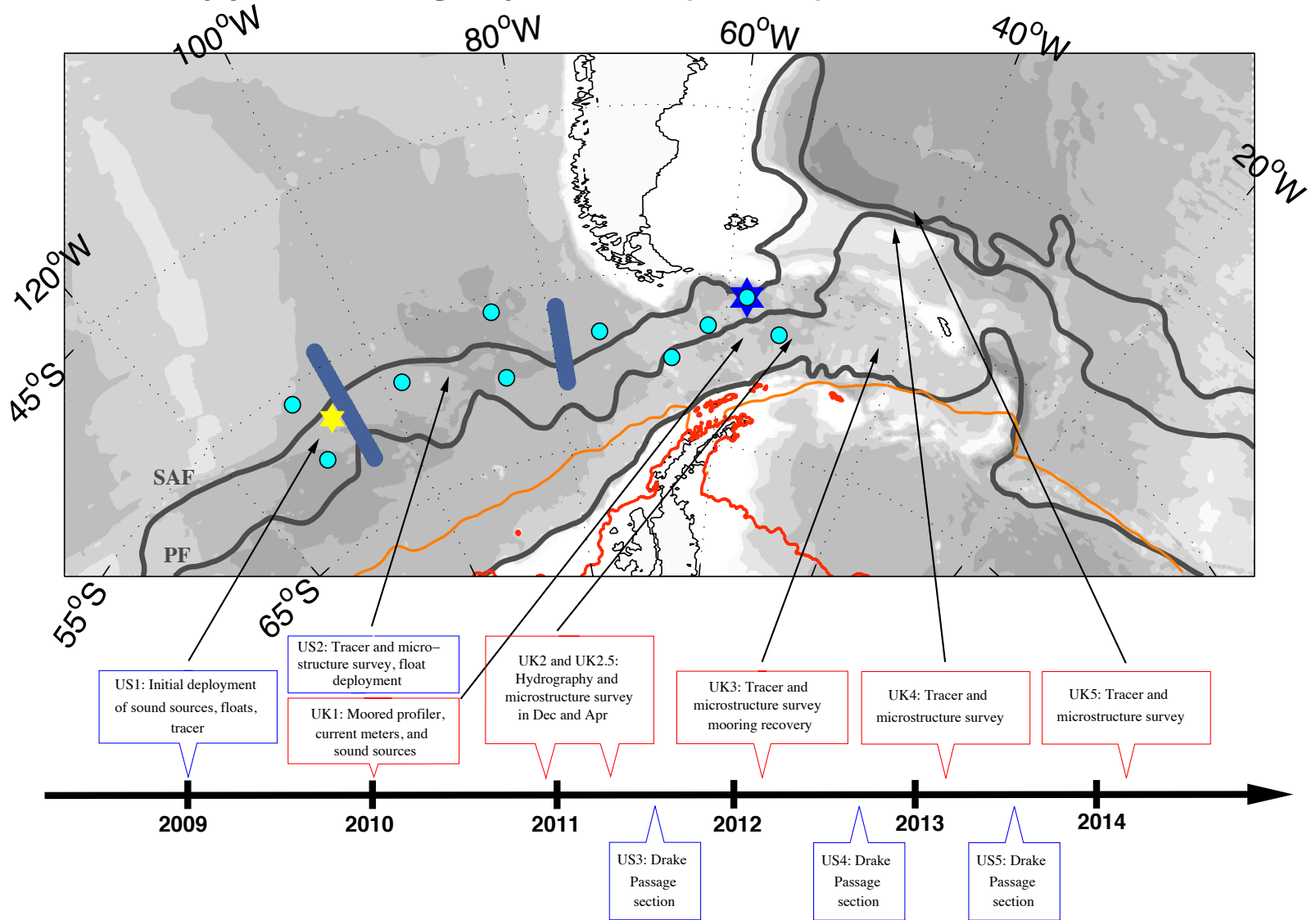
Course 12.820 at MIT

Diffusivity: half of the disperse rate

$$\kappa = \frac{1}{2} d \langle x^2 \rangle / dt$$

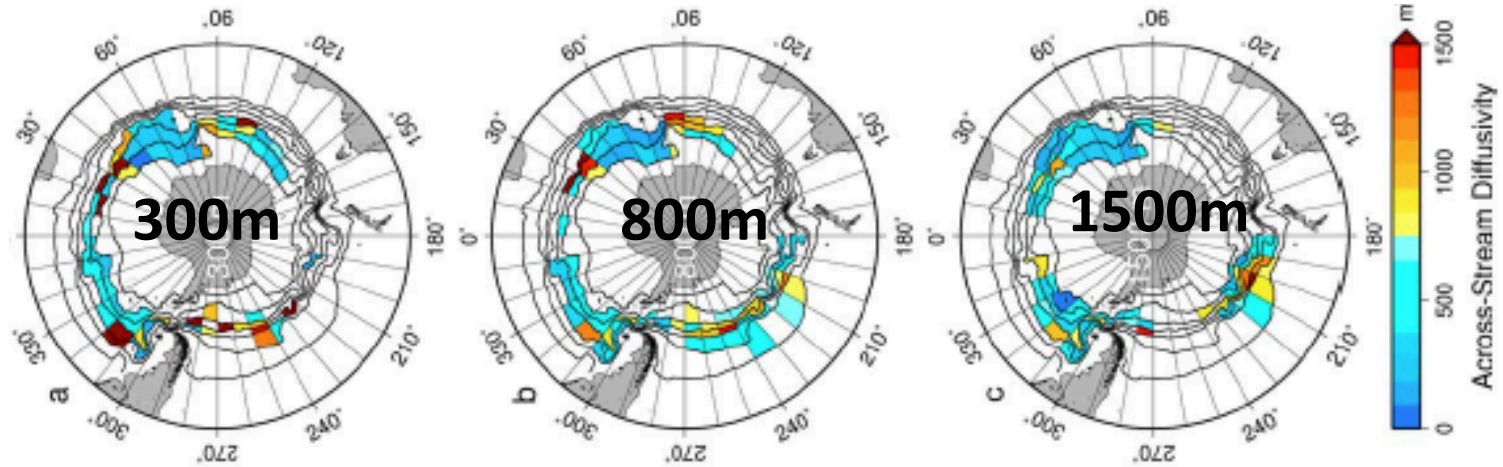
Float-based mixing estimates: observations

Diapycnal and isopycnal mixing experiment (DIMES)



Float-based mixing estimates: numerical models

Cross-stream (mean flow) diffusivities

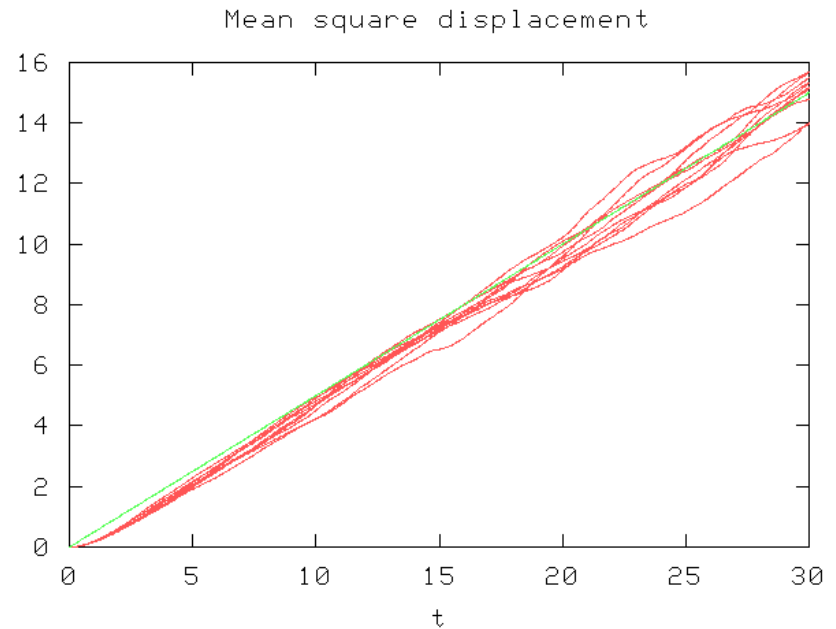
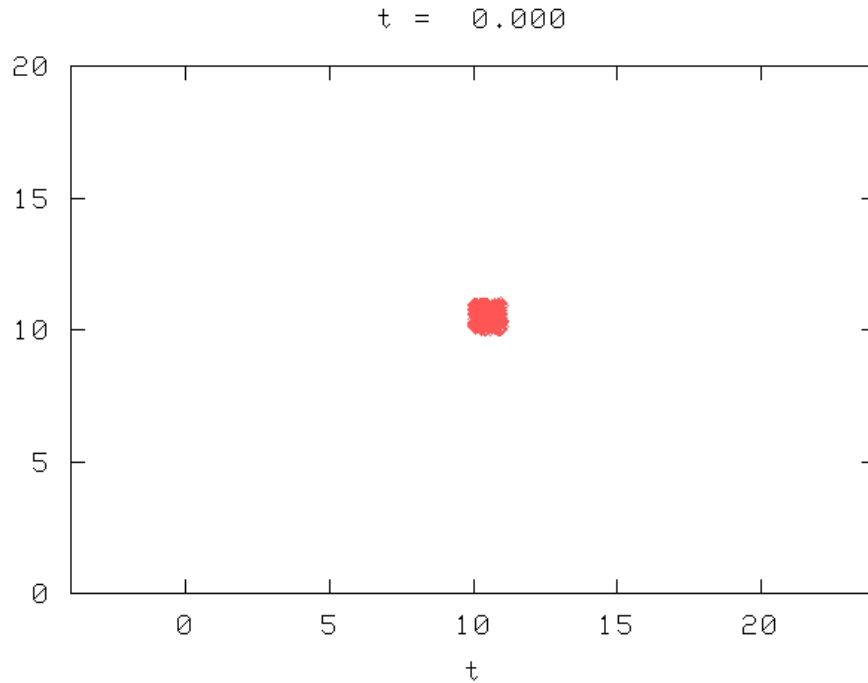


Griesel et al. (2006)

Patchy eddy mixing patterns in the Southern Ocean from a global eddying model

Estimate eddy diffusivity from Lagrangian particles

Pure diffusion case: particles evolution with time through random walk (molecular diffusion).

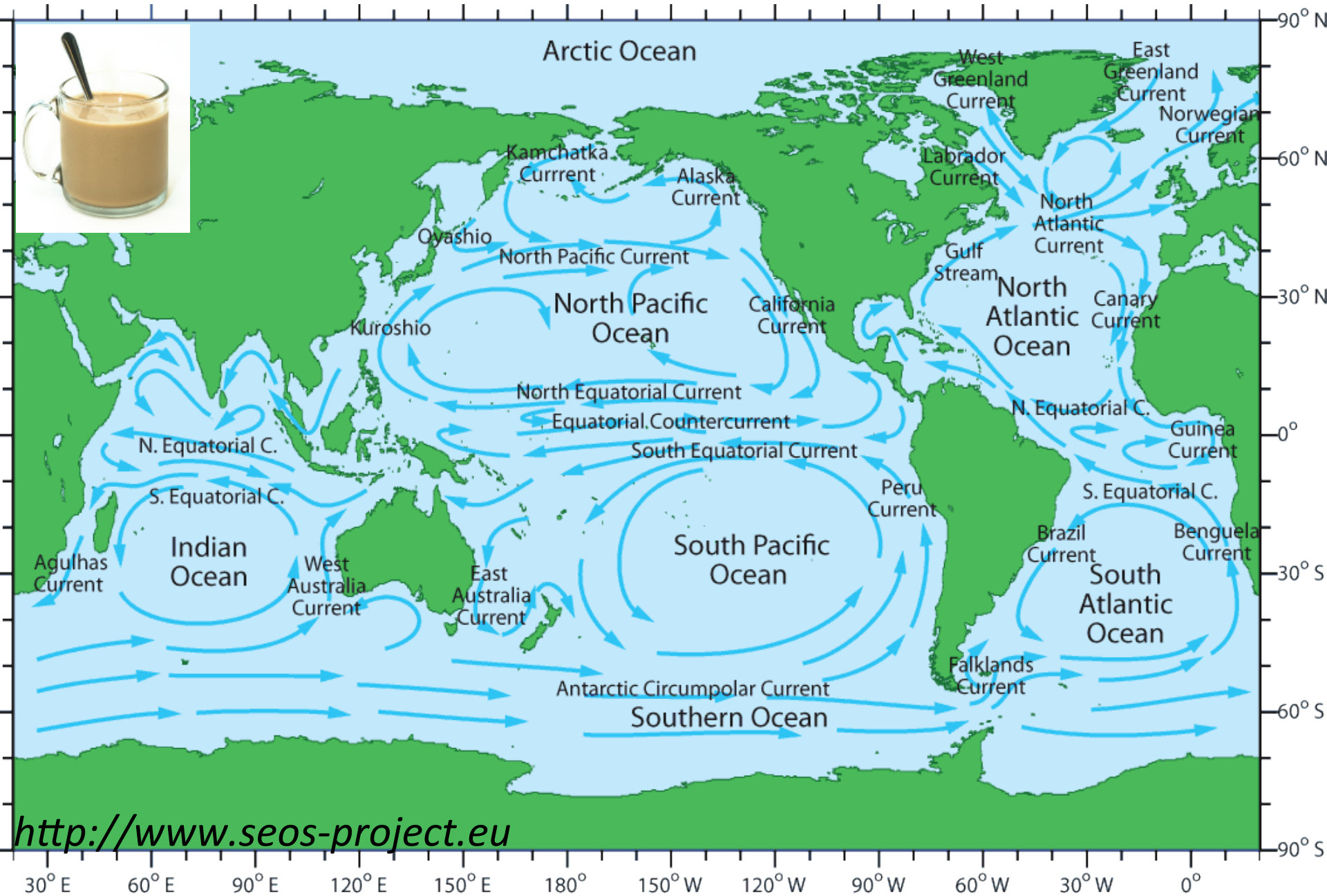


Course 12.820 at MIT

Diffusivity: half of the disperse rate

$$\kappa = \frac{1}{2} d \langle x^2 \rangle / dt$$

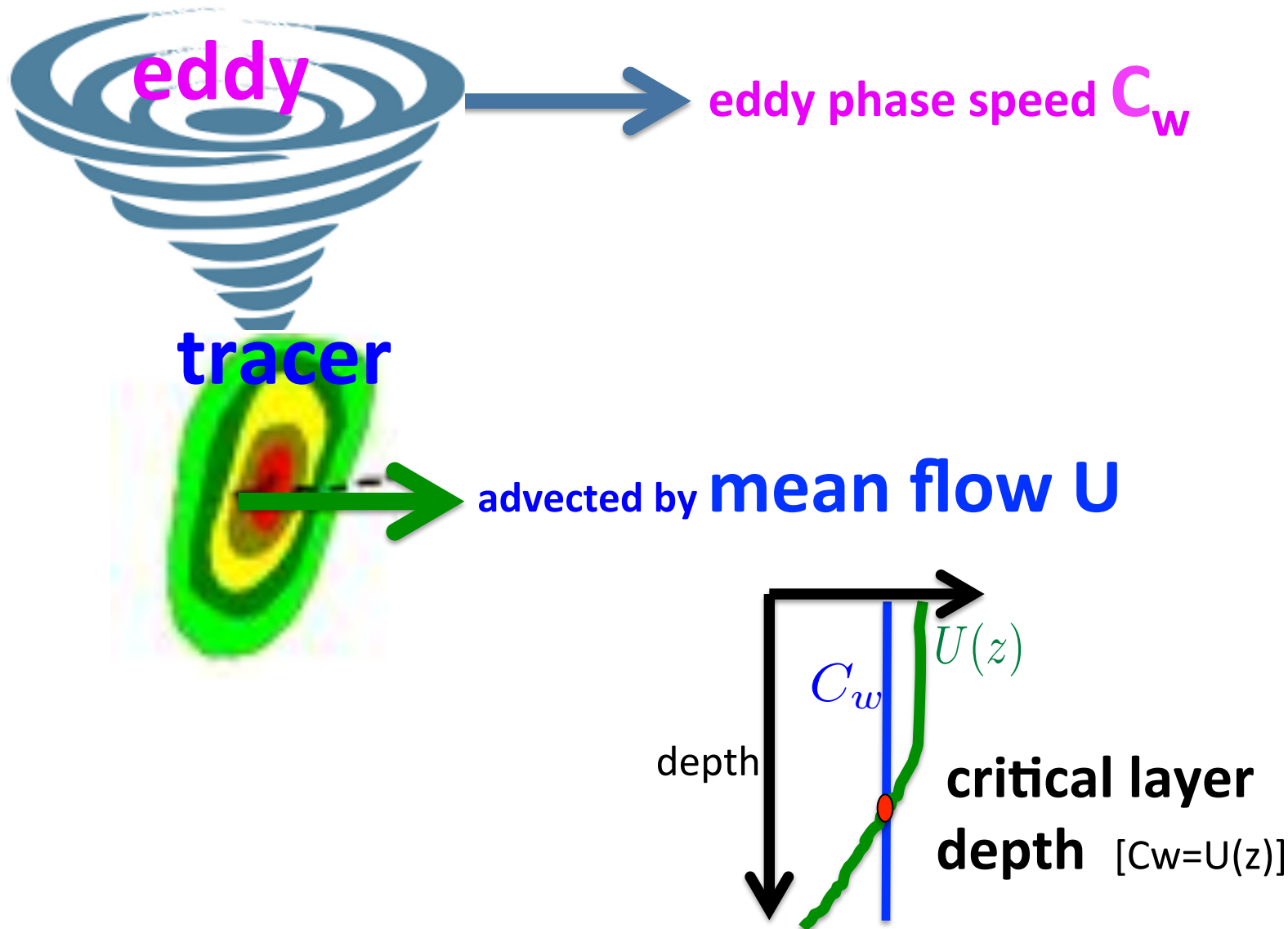
Effect of large-scale mean flow on mixing



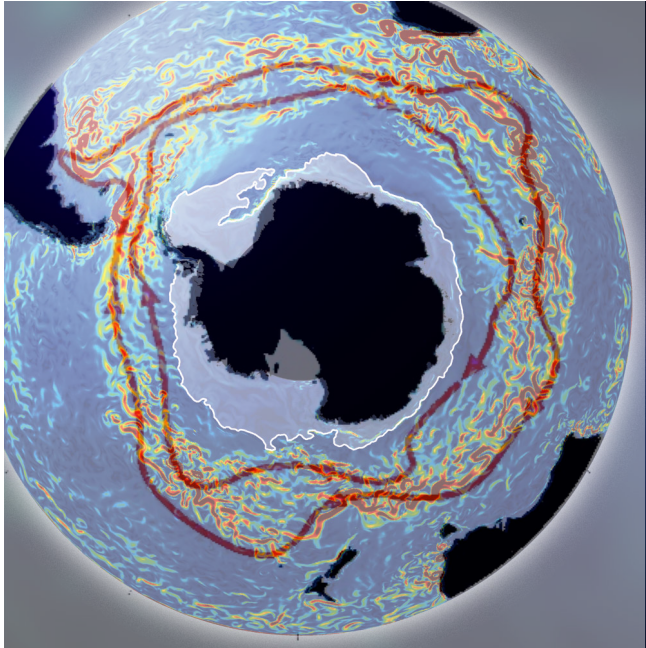
<http://www.seos-project.eu>

Mixing in the mean flow: Critical layer

When $C_w=U$, elevated mixing occurs.



Mixing in the mean flow: Theory



Both eddy properties and mean flow control mixing!

Courtesy of Matt Mazloff

**cross-stream
diffusivities**

1/(eddy decorrelation time)

eddy kinetic energy

$$K_{\perp} = \frac{k^2}{k^2 + l^2} \frac{\gamma}{\gamma^2 + k^2 [C_w - U(z)]^2} EKE$$

eddy wavenumber

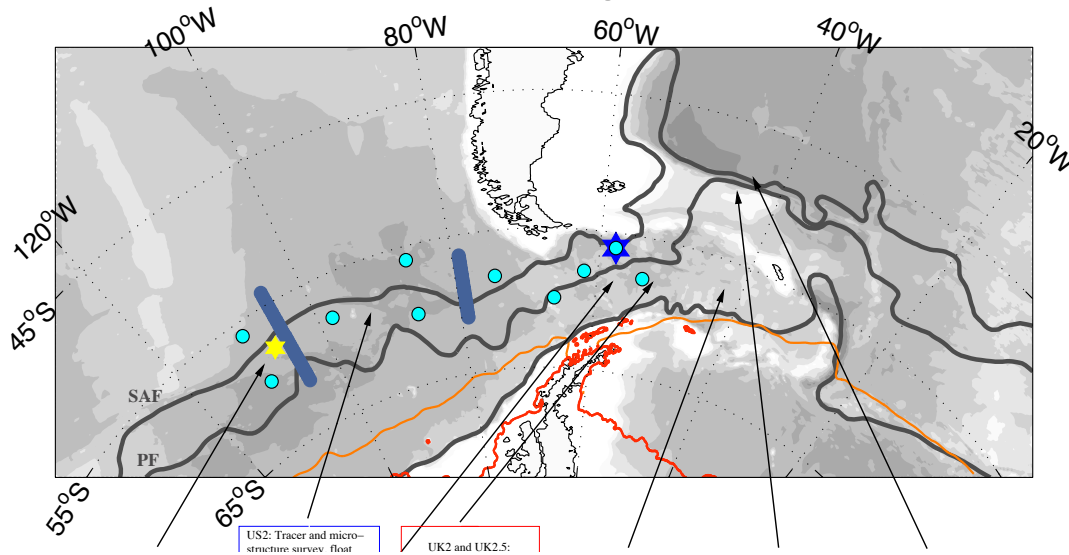
eddy phase speed

mean flow magnitude

(e.g. Green, 1970; Ferrari and Nikurashin, 2010)

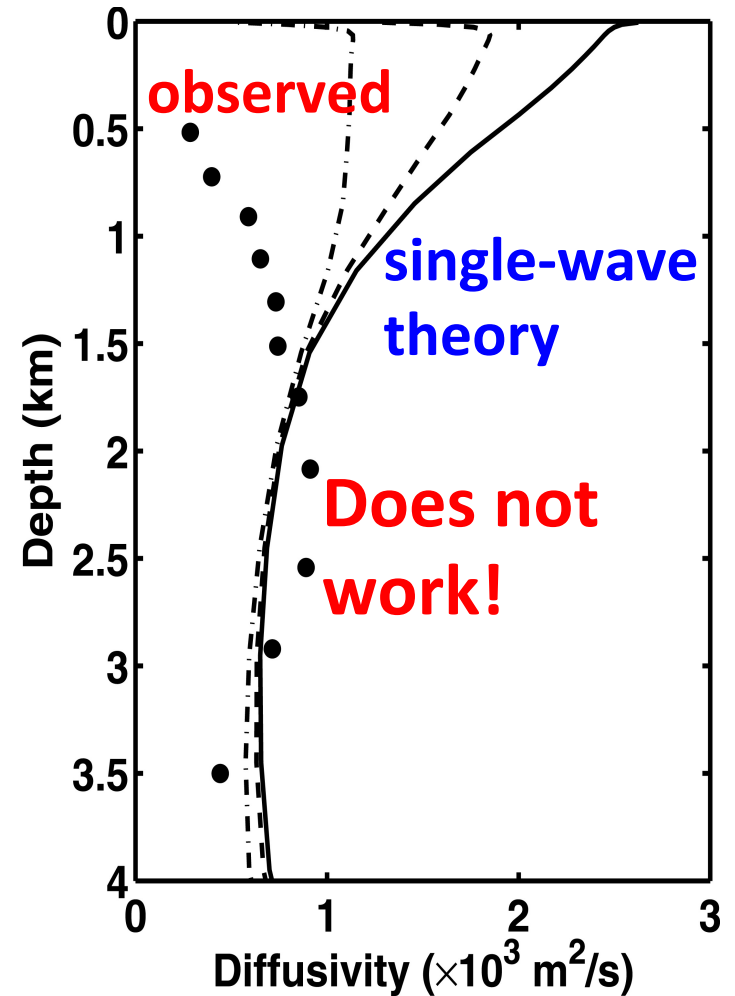
Efforts testing the theory: Observations

DIMES field experiment



<http://dimes.ucsd.edu/en/>

eddy diffusivities



Bates et al. (2014)

Problem of the mixing theory

cross-stream
diffusivities

1/(eddy decorrelation time)

eddy kinetic energy

$$K_{\perp} = \frac{k^2}{k^2 + l^2} \frac{\gamma}{\gamma^2 + k^2 [C_w - U(z)]^2} E K E$$

eddy wavenumber

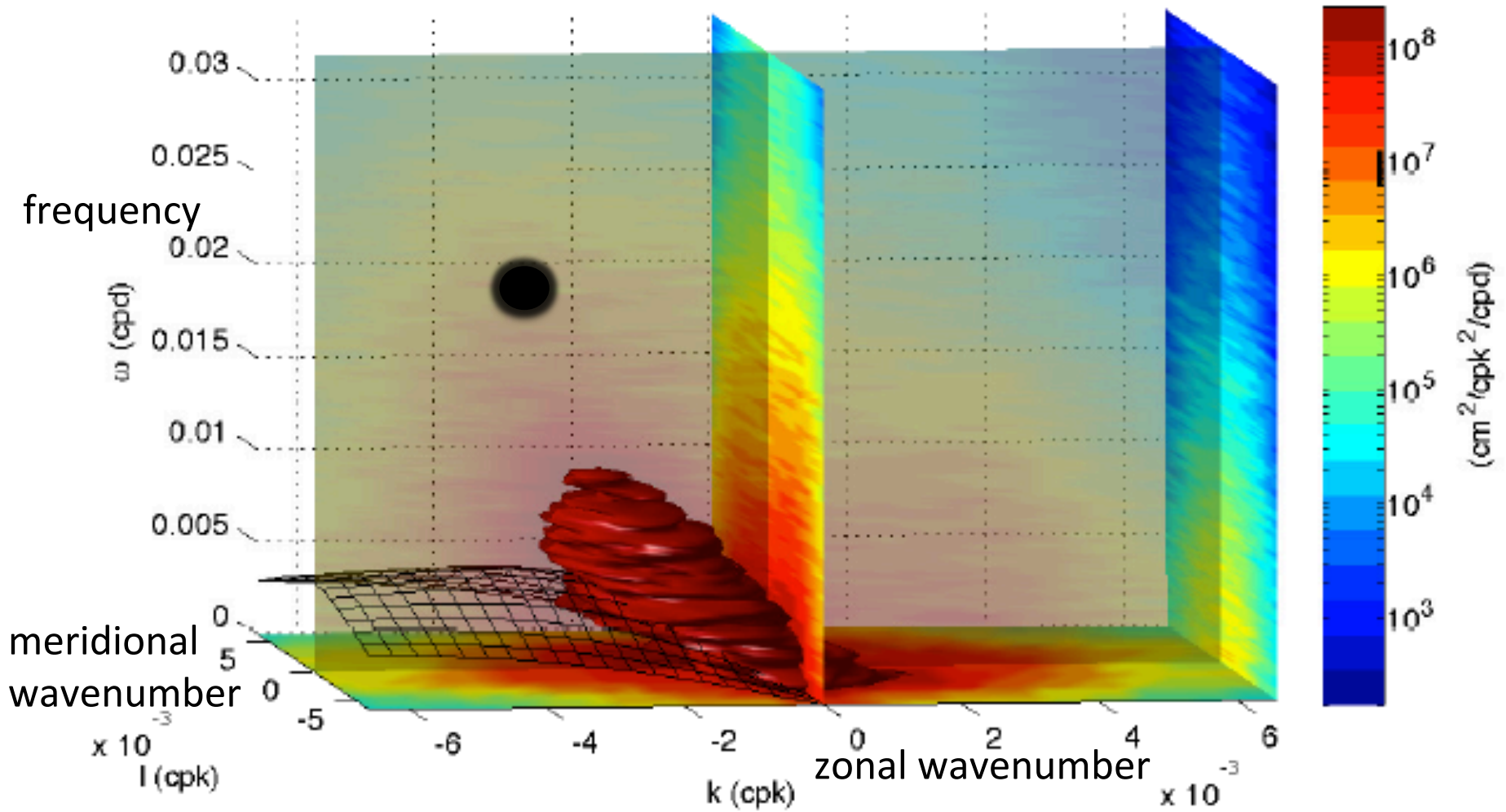
eddy phase speed

mean flow magnitude

- **(1) single wavenumber and phase speed**
- (2) constant mean flow
- (3) linear system

Our hypothesis: Multi-waves matter!

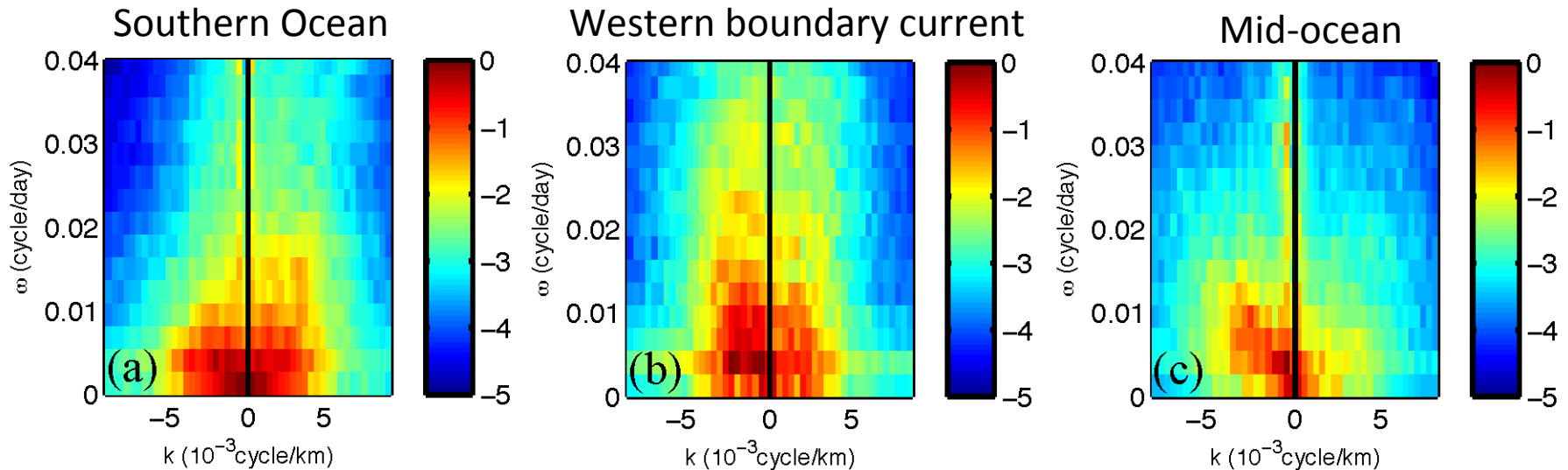
Spectrum of observed sea surface height (30°N, 170°W)



Wortham (2012)

Our hypothesis: Multi-waves matter!

Normalized spectrum of sea surface height



Neither single wavenumber nor single phase speed.

Our goal

Barotropic streamfunction

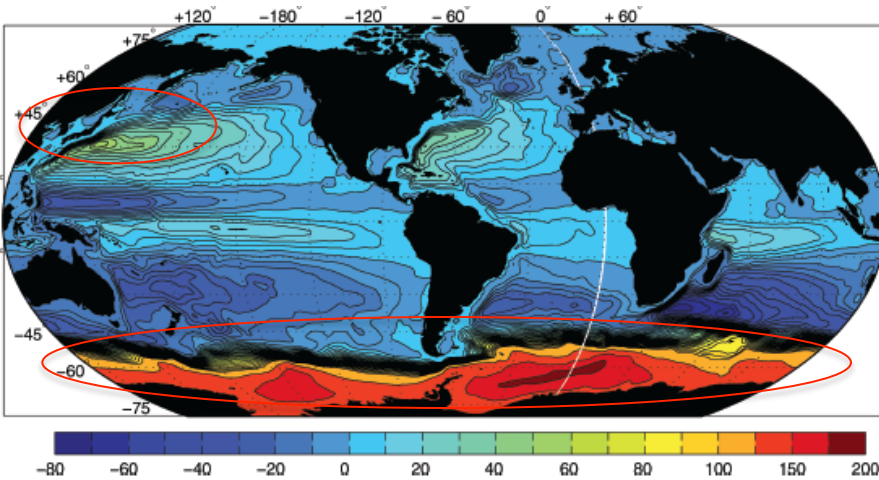


Figure 3. Transport streamfunction in $10^6 \text{ m}^3/\text{s}$ from the 16-year average. Some of the contours in the high latitude Southern Ocean have been omitted. The function is set to zero on the western boundaries (from a code of B. Klinger).

Wunsch (2011)

Surface eddy kinetic energy

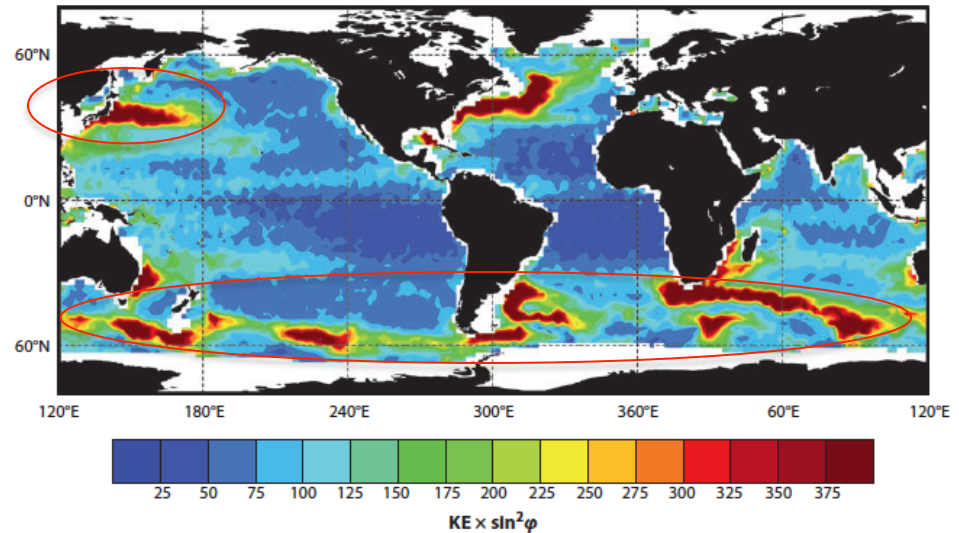


Figure 4

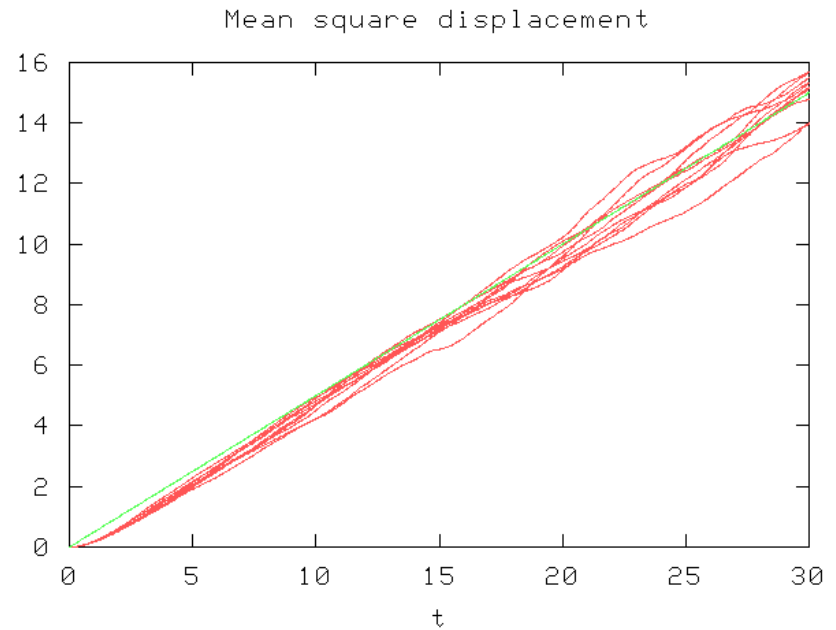
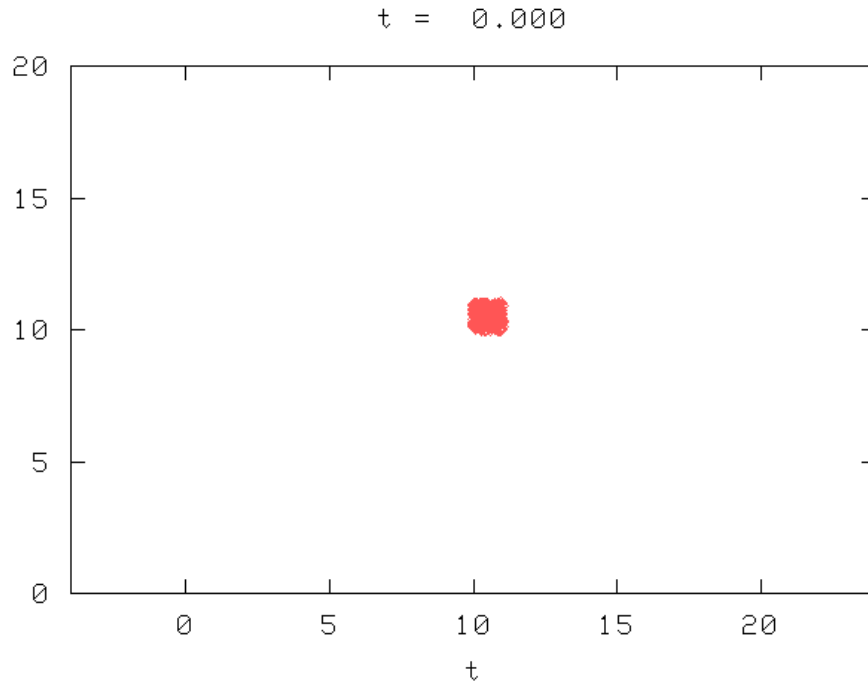
Estimate of the geostrophic kinetic energy (KE) $(\text{cm s}^{-1})^2$ of oceanic variability at the sea surface, here multiplied by $\sin^2 \phi$, where ϕ is the latitude, to avoid the equatorial singularity in noisy data. Note the very large spatial changes of kinetic energy. Figure taken from Wunsch & Sammer 1998.

Ferrari and Wunsch (2009)

1. Provide float-based cross-jet mixing at high spatial resolution.
2. Develop a mixing theory with multi-wavenumbers and test its validity.

Tool: Lagrangian particles

Pure diffusion case: particles evolution with time through random walk (molecular diffusion).



Course 12.820 at MIT

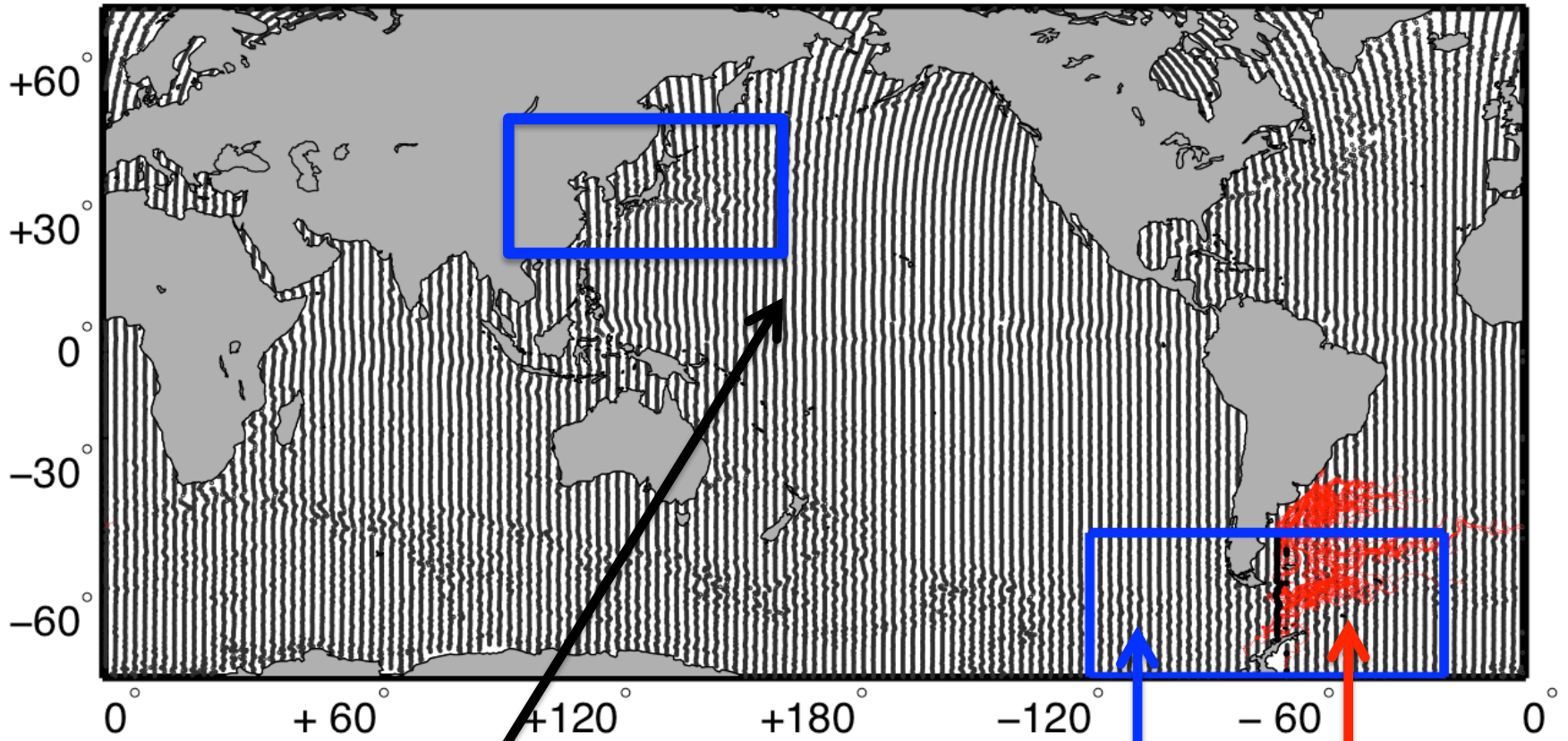
Diffusivity: half of the disperse rate

$$\kappa = \frac{1}{2} d \langle x^2 \rangle / dt$$

Tool: High resolution model with floats

Parallel Ocean Program model with floats (Griesel et al. 2014)

- Global eddying (0.1deg) ocean model with 42 levels
- **One million** floats evenly deployed **globally** in 1994 and advected by velocity fields



Floats deployed globally at 23 vertical levels

Study domain: DIMES region

Examples of float trajectories

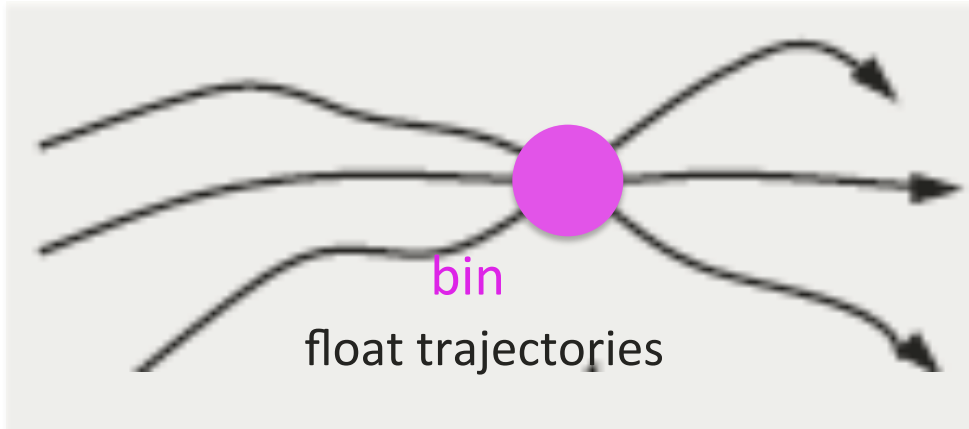
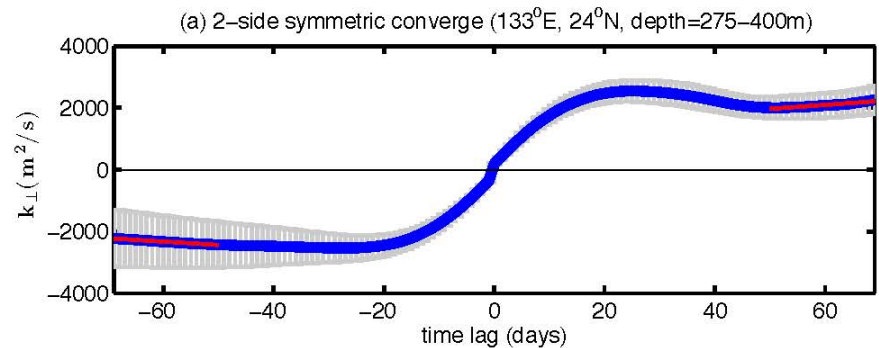
Estimating diffusivity from floats

Calculation method

Average autocorrelation of residual float velocities over all tracks in the defined bin (Davis 1991).

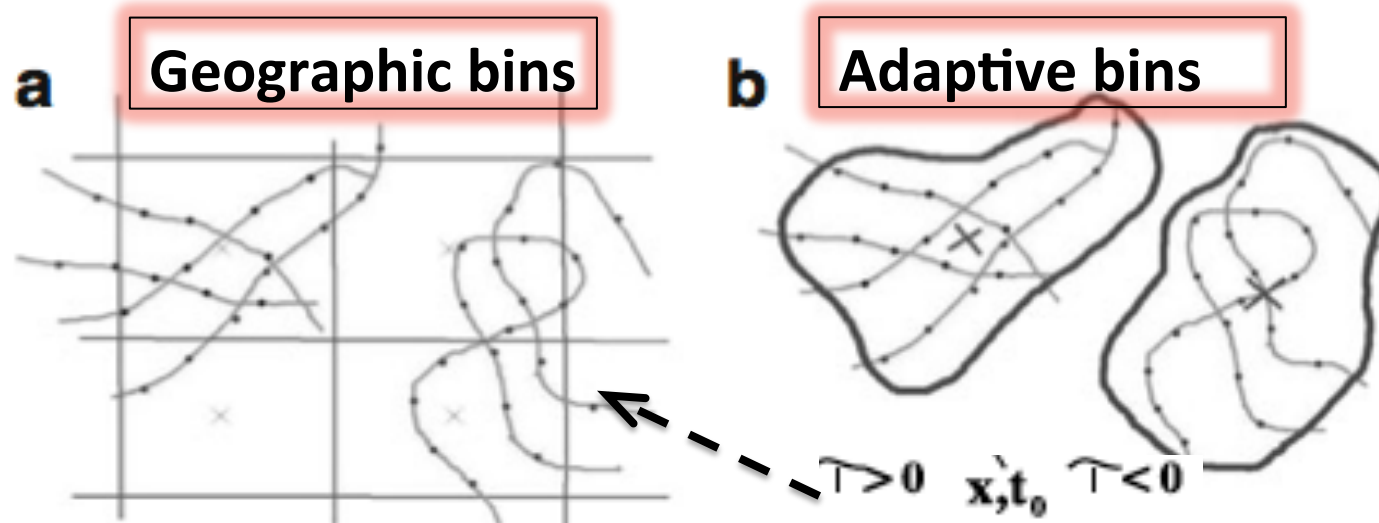
$$\kappa_{ij}(\mathbf{x}, \tau) = \int_{-\tau}^0 d\bar{\tau} \langle u'_i(t_0 | \mathbf{x}, t_0) u'_j(t_0 + \bar{\tau} | \mathbf{x}, t_0) \rangle_L,$$

$$\kappa_{ij}^{\infty} = \lim_{\tau \rightarrow \infty} \kappa_{ij}(\mathbf{x}, \tau),$$



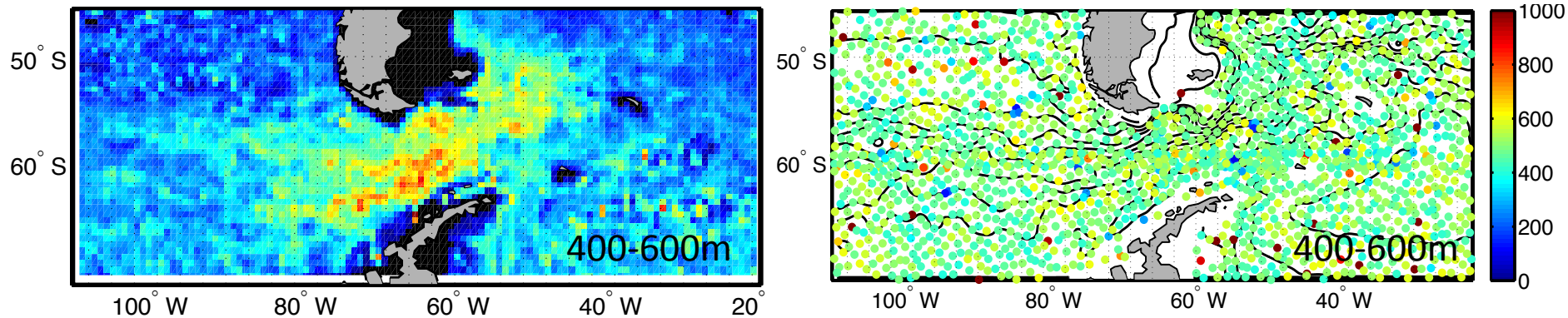
How to divide the study domain into bins?

Estimating diffusivity from floats

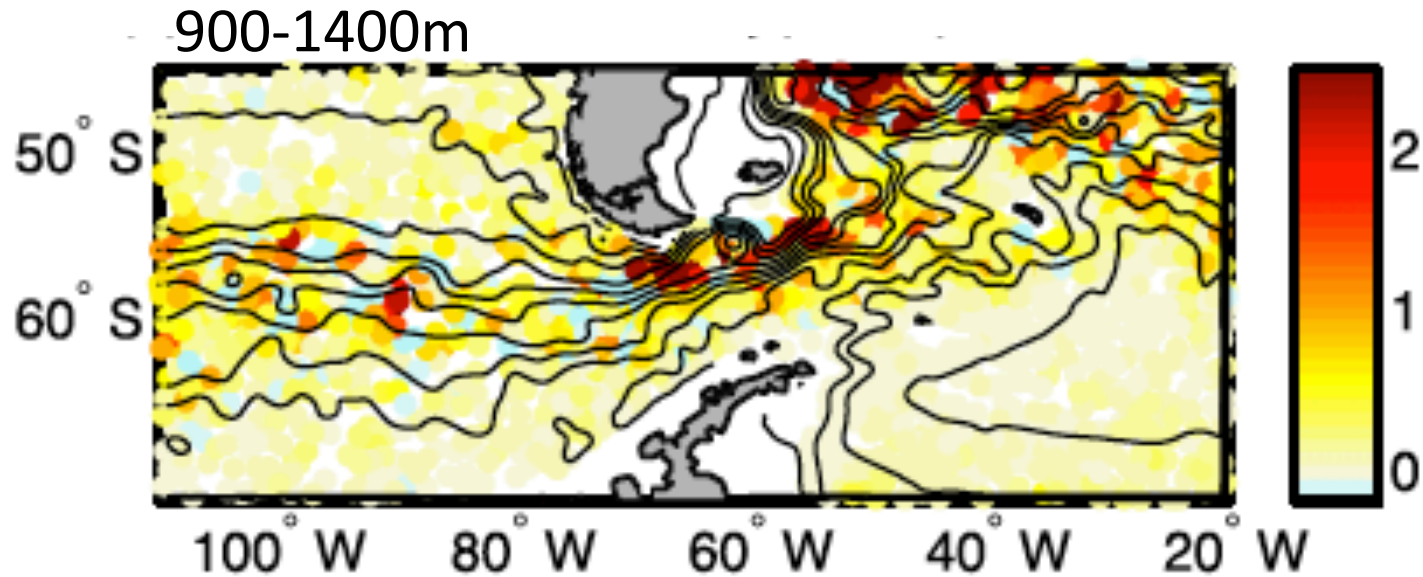
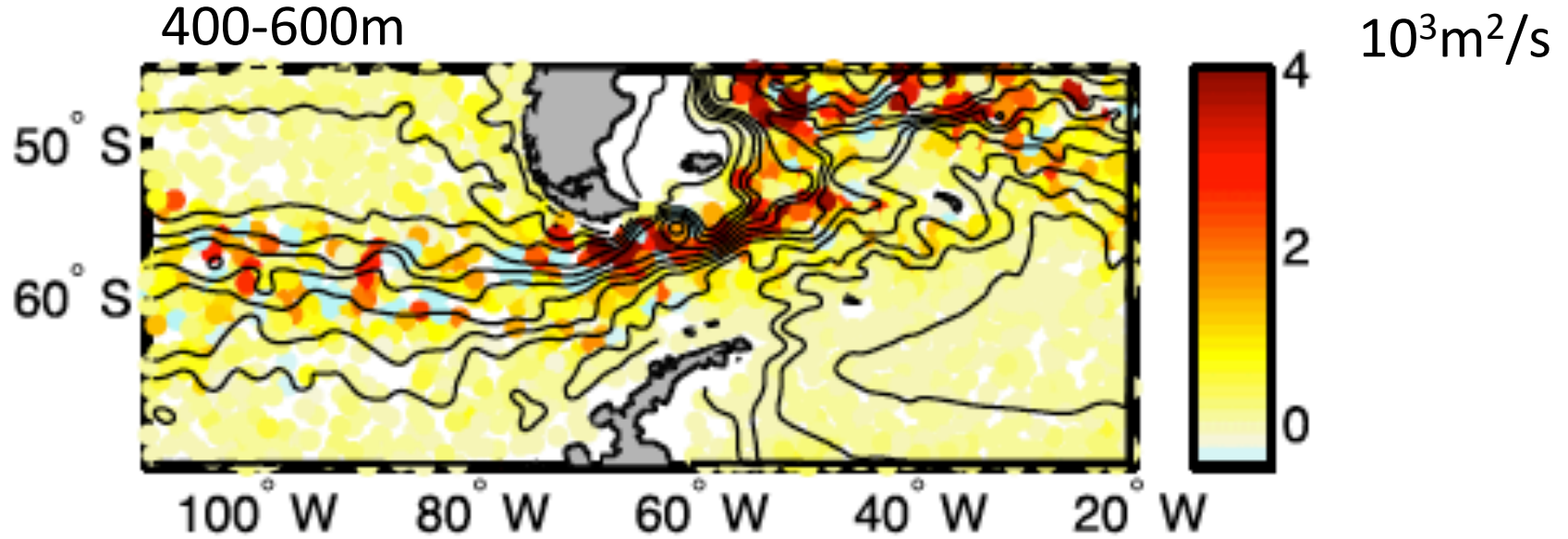


Koszalka and LaCasce (2010)

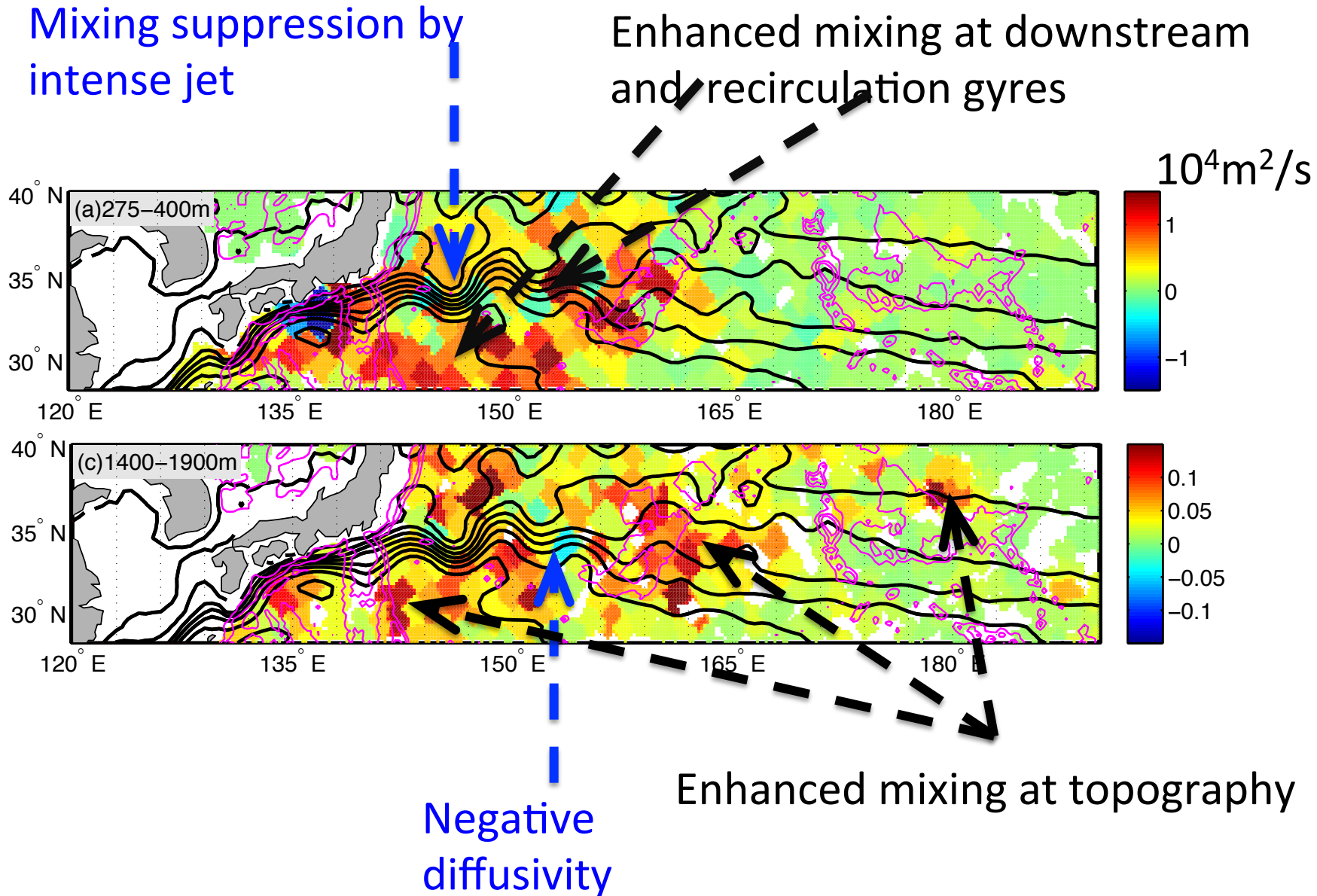
Number of trajectories in each geographic (left) and adaptive bin (right)



Spatial patterns of mixing: Drake Passage



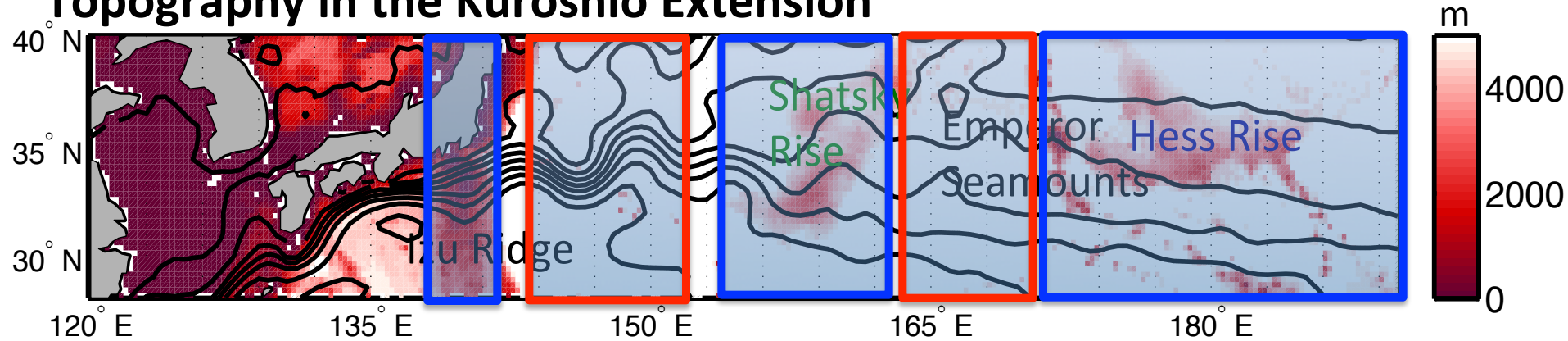
Spatial patterns of mixing: Kuroshio



Analogous to the Southern Ocean in many aspects

Single-wavenumber theory works poorly at topography

Topography in the Kuroshio Extension



Single-wavenumber theory

cross-stream
diffusivities

1/(eddy decorrelation time)

eddy kinetic energy

$$K_{\perp} = \frac{k^2}{k^2 + l^2} \frac{\gamma}{\gamma^2 + k^2 [C_w - U(z)]^2} EKE$$

eddy wavenumber

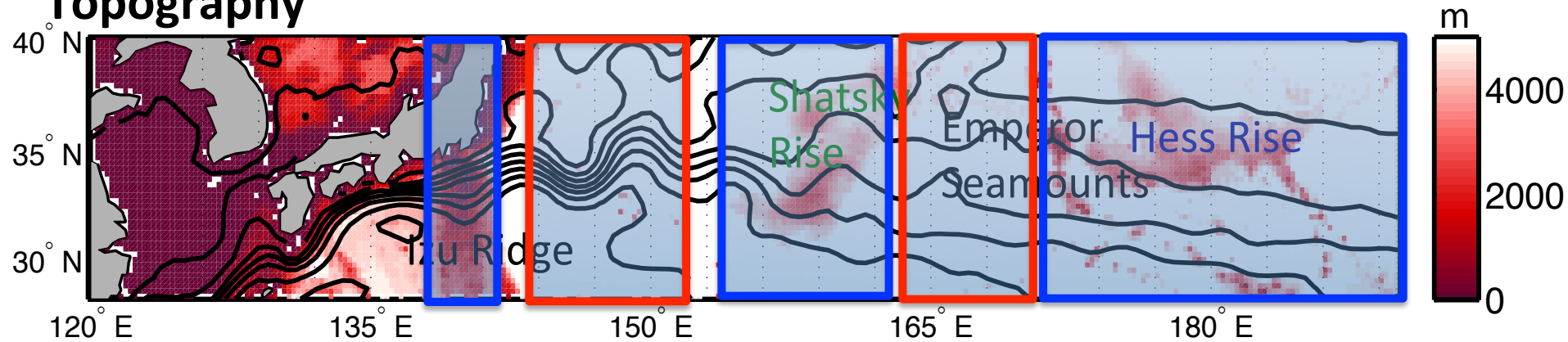
eddy phase speed

mean flow magnitude

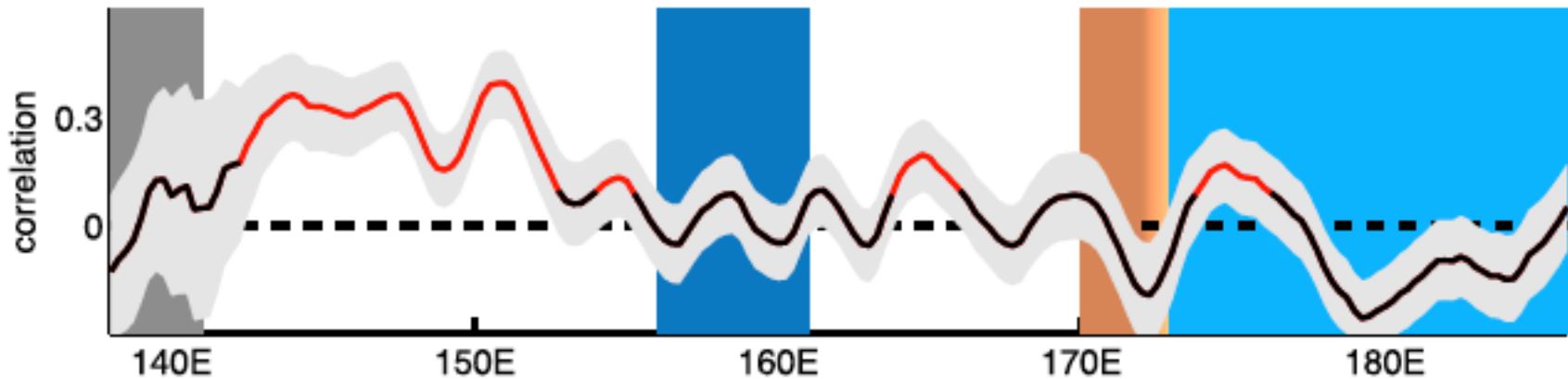
(e.g. Green, 1970; Ferrari and Nikurashin, 2010)

Single-wavenumber theory works poorly at topography

Topography



Correlation between float-based mixing lengths and single-wavenumber theory



Multi-wavenumber theory

cross-stream
diffusivities

$$K_{\perp}(\mathbf{x}) = \frac{1}{2} \int_{-\infty}^{\infty} S_{u'_{\perp}}(k', \omega, \mathbf{x}) \Big|_{\omega=Uk'} dk'$$

wavenumber

frequency

mean flow magnitude

frequency-wavenumber spectrum
of cross-mean flow Eulerian eddy
velocities along a slice aligning
with the mean flow vector

Derivation available in Chen et al. (2015).

In the single-wave limit, single-wave and multi-wave theories are equivalent!

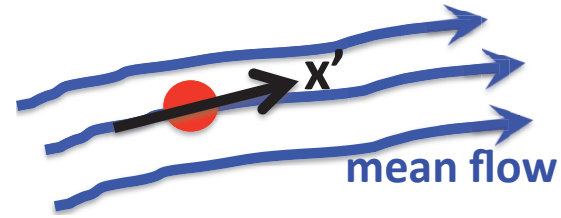
Estimate diffusivity from multi-wavenumber theory

cross-stream
diffusivities

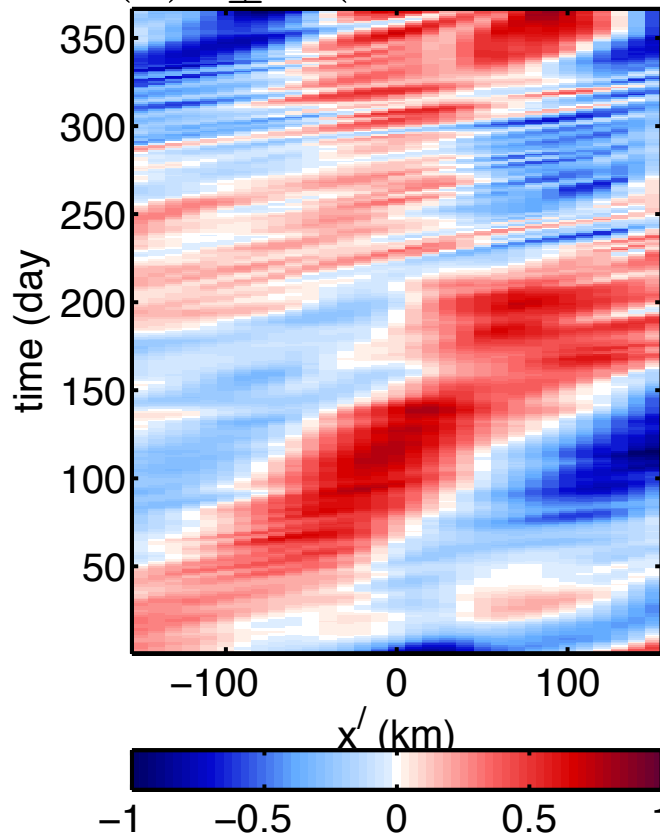
$$K_{\perp}(\mathbf{x}) = \frac{1}{2} \int_{-\infty}^{\infty} S_{u'_{\perp}}(k', \omega, \mathbf{x})|_{\omega=Uk'} dk'$$

frequency-wavenumber spectrum
of cross-mean flow Eulerian eddy
velocities along a slice aligning
with the mean flow vector

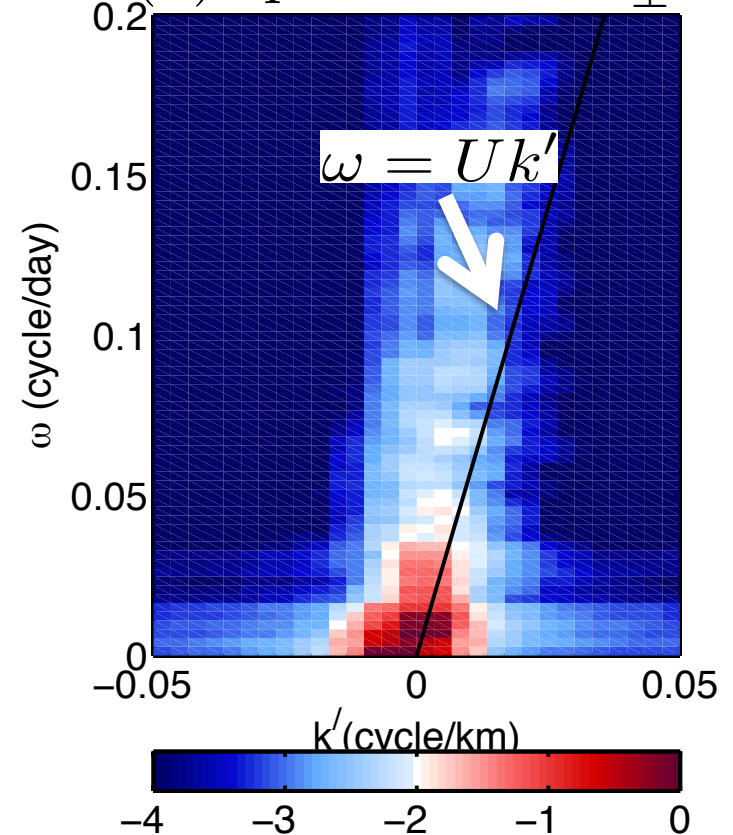
mean flow magnitude



(a) u'_{\perp} at (58°S, 74°W)

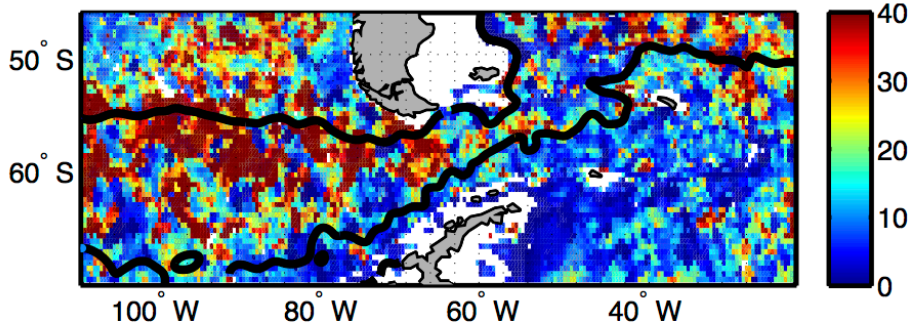


(b) spectrum of u'_{\perp}

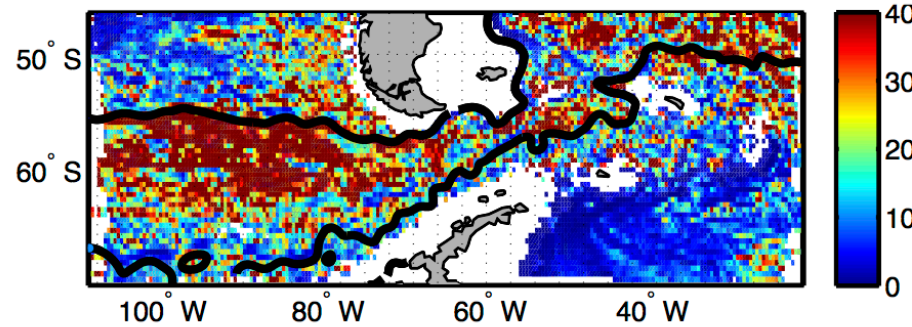


Testing the theory: Mixing lengths

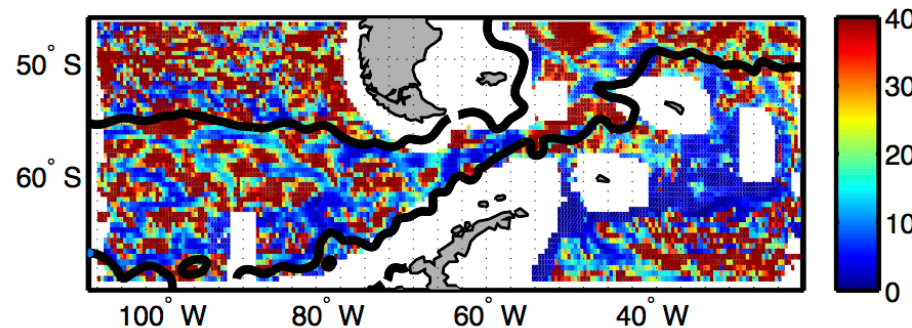
(a) mixing lengths from floats



(b) mixing lengths from multi-wavenumber theory

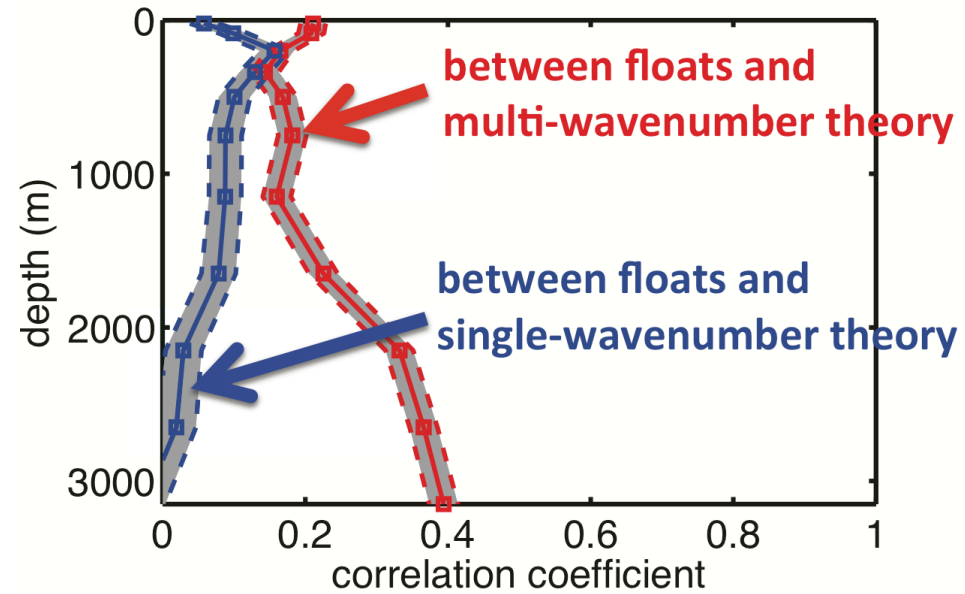


(c) mixing lengths from single-wavenumber theory



1900-2400m

correlation of horizontal structures



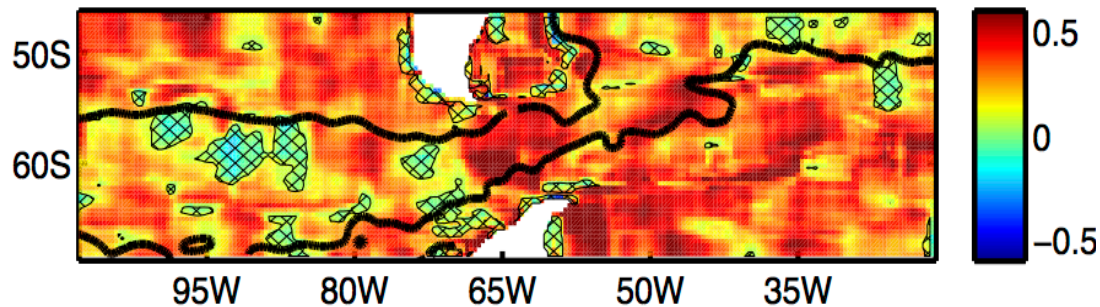
0.4
0

Testing the theory: Diffusivities

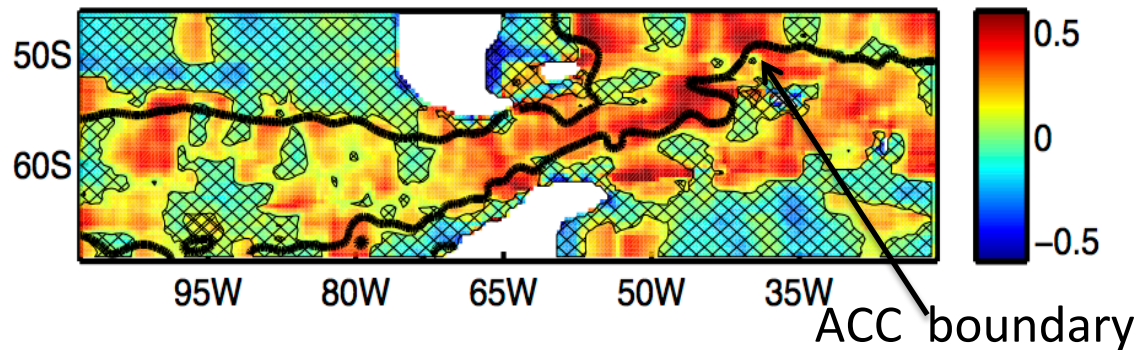
correlation of vertical structures

K varies with z!

(a) floats vs. multi-wavenumber theory



(b) floats vs. single-wavenumber theory



91%

55%

Hatched: negative correlation.
Non-hatched: positive correlation.

mean flow vs. eddies

Summary

- (1) To include the contribution of the entire eddy spectrum to mixing, **we formulated and tested a multi-wavenumber theory.**

equivalent in the single-wavenumber limit



Breaks down in topographic regions
In the Kuroshio Extension and
at the spot of the DIMES experiment

DIMES region from the Southern Ocean
a. Better represents both horizontal and vertical structures of mixing lengths.
b. Better represents the vertical structures of eddy diffusivities.
c. Both theories capture large-scale horizontal structure of eddy diffusivities.

(2) Details are available at

- Chen et al., 2014: "Isopycnal eddy diffusivities and critical layers in the Kuroshio Extension from an eddying ocean model." *JPO*.
- Chen et al., 2015 "A multi-wavenumber theory for eddy diffusivities and its application to the southeast Pacific (DIMES) region." *JPO*.

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Stefan Llewellyn Smith

...