Investigating the connection between complexity of isolated trajectories and Lagrangian coherent structures

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outline

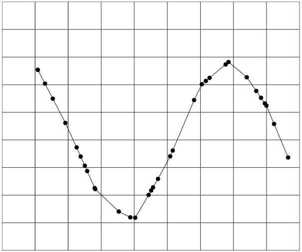
- 2 complexity measures and the relationship between them
- New complexity-measure-based diagnostic for locating LCSs
- Duffing Oscillator
- Bickley Jet
- Realistic numerically-generated oceanic flow
- Applications to sparse and non-uniform data
- Applications to 3d flows

Introduction

- For 2d flows, trajectories satisfy $\mathrm{d}\vec{x}/\mathrm{d}t = \vec{u}\,(\vec{x}, t)$
- Very different behavior of trajectories – from stationary trajectories to very complex trajectories that densely cover areas of the domain
- These differences are quantified by estimating trajectory complexities

Correlation dimension c: measure of area covered by a trajectory

- cover domain with
  adjacent squares of length s
- count the number of points,
  N_j, inside each square
- compute
  the distribution function
  $F(s) = \frac{1}{N^2} \sum_j [N_j(s)]^2$
- use $F(s) \propto s^c$
  for small s to estimate c



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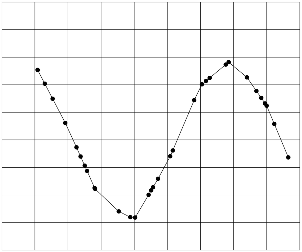
- we use c in the "finite-time" sense, similar to FTLEs



Ergodicity defect d: "time average - space average"

$$d(s;\vec{x_0},t_0) = \sum_{j=1}^{s^{-2}}[\frac{N_j(s)}{N} - s^2]^2$$

- Ergodic (or most complex) trajectory spends equal time in each square so its time average = its space average and $d = 0$ for all s
- Stationary (or least complex) trajectory spends all time in one square so $d = 1 - s^2 \xrightarrow[s\to 0]{} 1$
- Comments about the finite-time nature still apply

Connection between c and d

$$d(s) = \sum_{j=1}^{s^{-2}} [N_j(s)/N - s^2]^2 =$$
$$\frac{1}{N^2} \sum_{j=1}^{s^{-2}} N_j^2(s) - \frac{2s^2}{N} \sum_{j=1}^{s^{-2}} N_j(s) + \sum_{j=1}^{s^{-2}} s^4$$

But $\sum_{j=1}^{s^{-2}} N_j(s) = N$ and $F(s) = \frac{1}{N^2} \sum_j [N_j(s)]^2$ so

$$d(s) = F(s) - s^2.$$

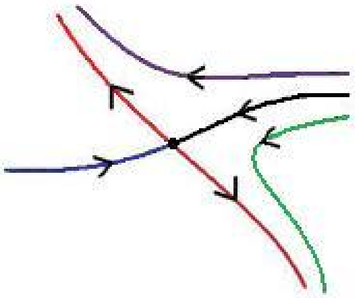
Over the range of scales where the scaling $F(s) \sim s^c$ holds,

$$d(s) + s^2 \sim s^c.$$

d is a better measure than c because the c is only valid over the range of scales where the scaling relationship $F(s) \sim s^c$ holds, while d is valid for any s

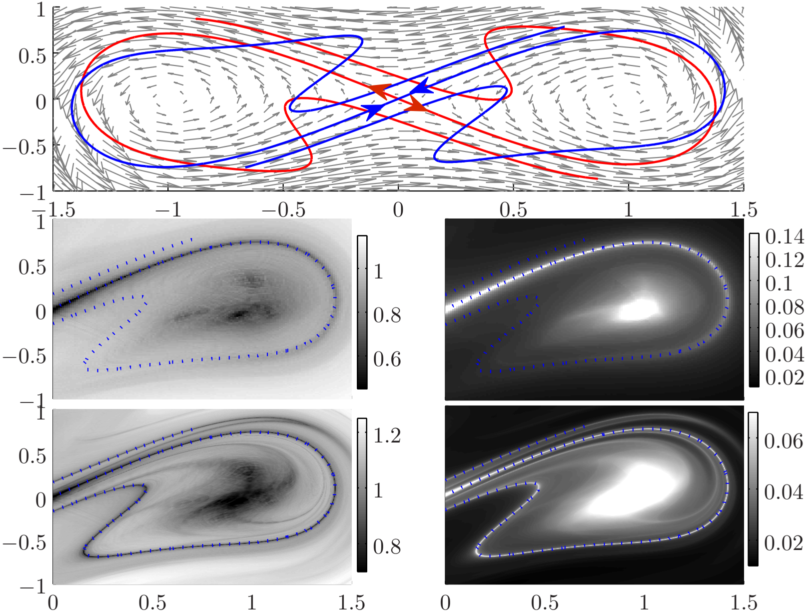
Complexity-measure-based diagnostic for locating LCSs

- Trajectories on a manifold approach the HT, so their complexities are similar to that of the HT
- Trajectories even slightly off the maniofold diverge from the HT, so complexity should change rapidly in the normal direction to the manifold

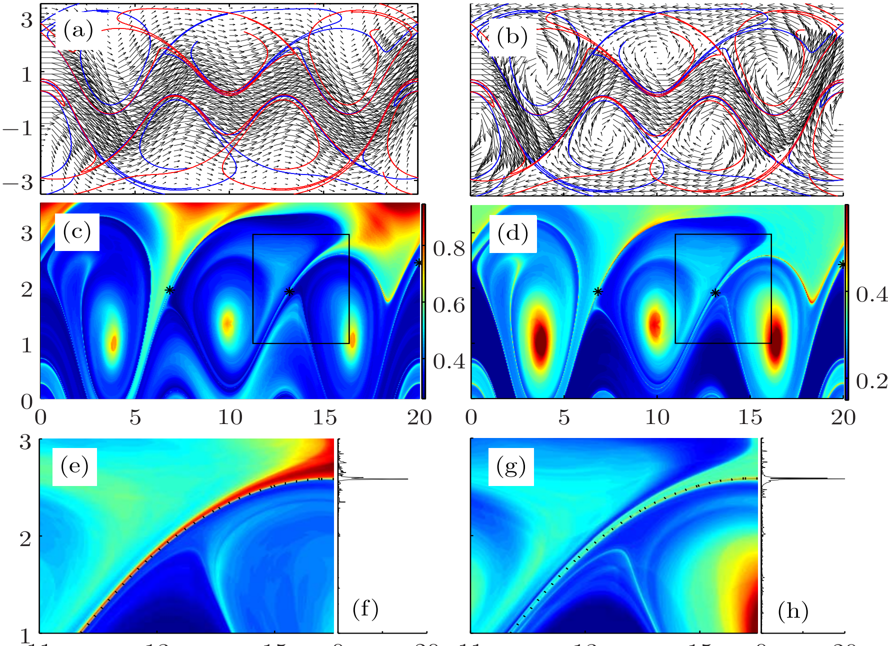


- Thus, manifolds correspond to level sets of c or d, with complexity changing rapidly as one "steps" to either side
- For nearly stationary HTs, these level sets are minimizing [maximizing] ridges of c [d]-fields.
- Since trajectories on opposite sides of a manifold diverge, they often have qualitatively different complexities

Duffing Oscillator



Discussion of frame-independence: Bickley jet



Application to realistic ocean flows: a numerically-simulated mesoscale eddy

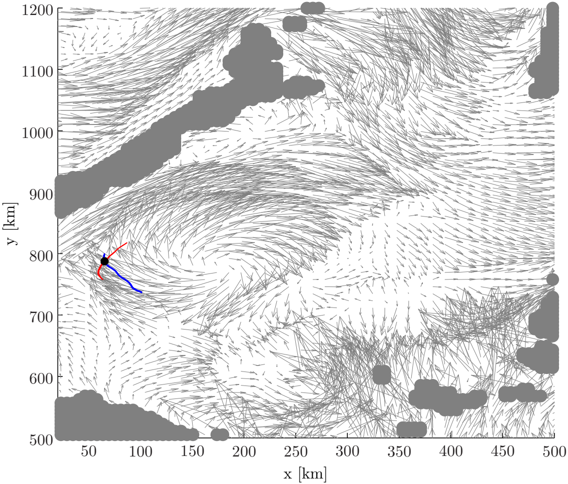


Figure: Velocity and manifolds for the numerically-generated flow field produced by ROMS on 15 June 2007

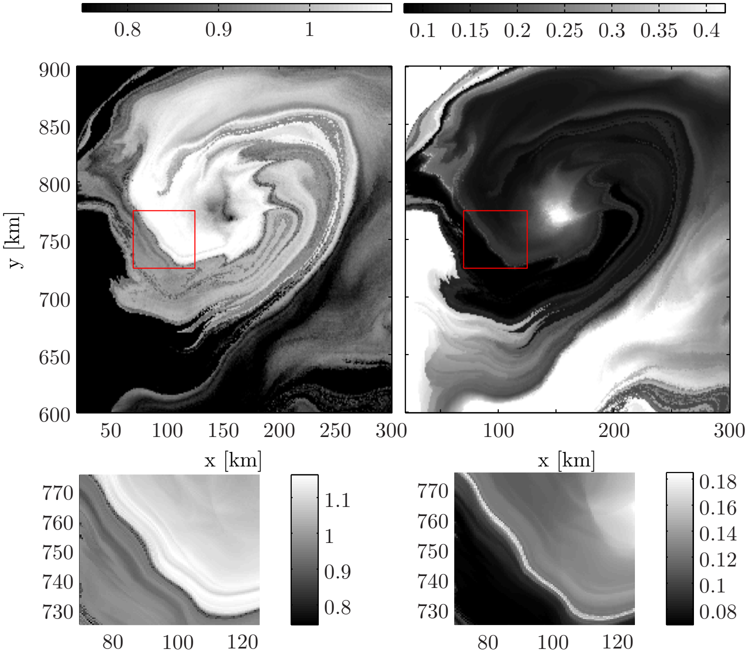


Figure: c and d_{mean} fields for the numerically-generated eddy

Application of CM with sparse and non-uniform data

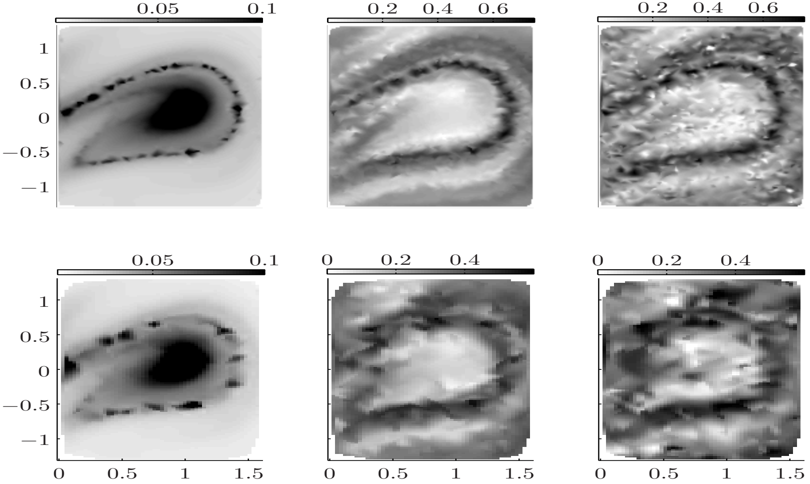


Figure: (left) d_{mean} (middle) FTLEs produced using the Lekien and Ross (2010) method, and (right) conventional FTLEs computed using (top) 2550 and (bottom) 640 randomly distributed drifters

Application of CM to 3d flows

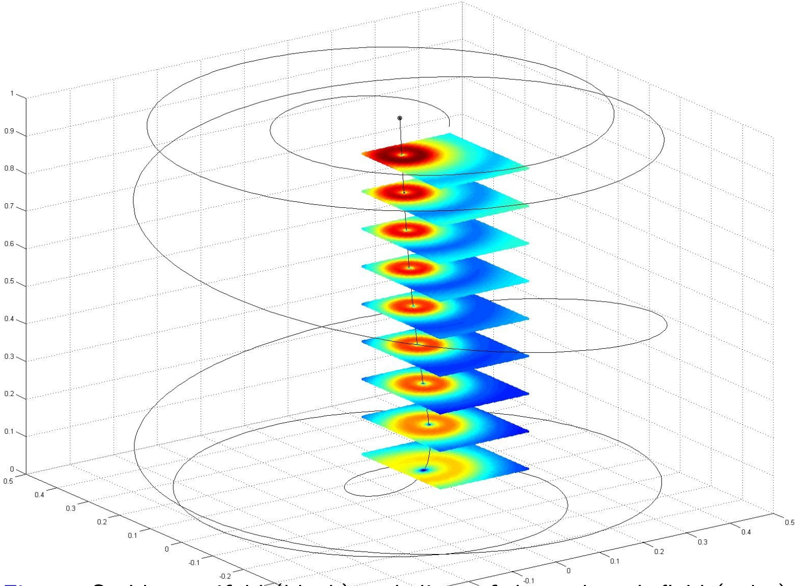


Figure: Stable manifold (black) and slices of the arclength field (color).

Summary

- we introduced 2 measures of complexity
- we investigated the relationship between them
- we proposed to use these complexity measured to identify LCSs
- we tested CM method in several examples and it worked
- CMs appear to have possible advantages for applications to sparse and nonuniformly spaced dats
- Rypina, I.I., S.E. Scott, L.J. Pratt, and M.G. Brown. 2011. Investigating the connection between complexity of isolated trajectories and Lagrangian coherent structures, *Nonlin. Proc. in Geophys.*, 18, 977-987, doi:10.5194/npg-18-977-2011
- CMs show good potential for applications to 3d flows