Spiral Inertial Waves Emitted From Geophysical Vortices

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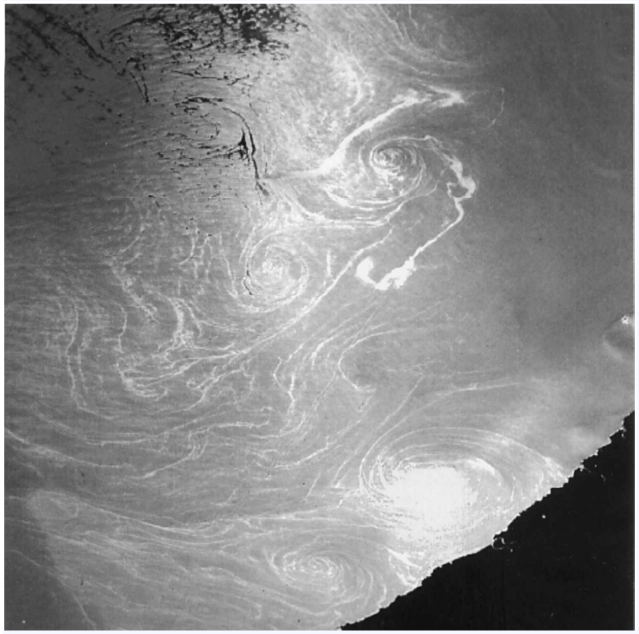
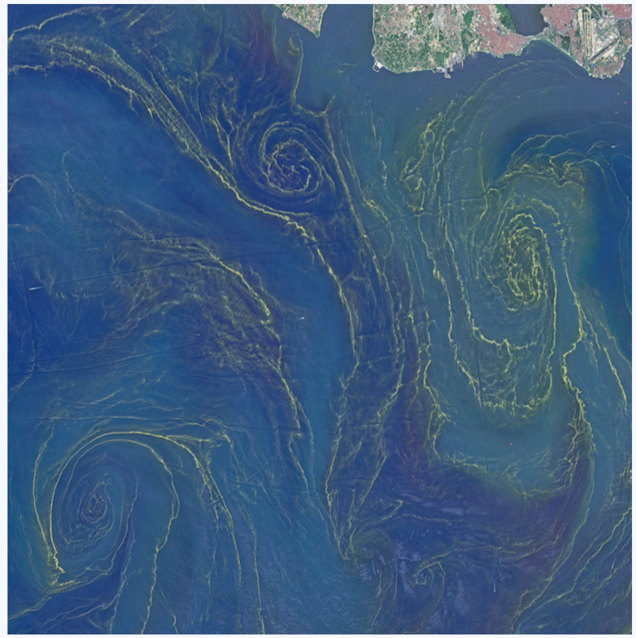
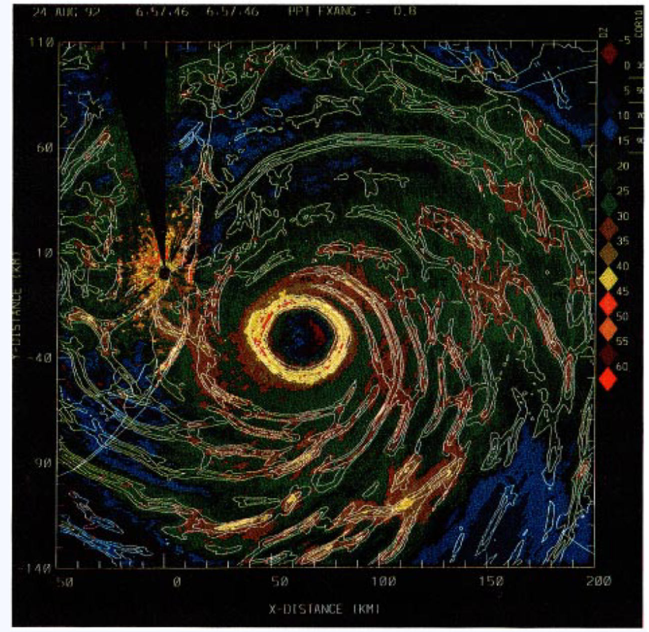
Spiral features in geophysical vortices



Spiral sea eddies
(Munk et al. 2000)



Plankton bloom in
Sea of Marmara

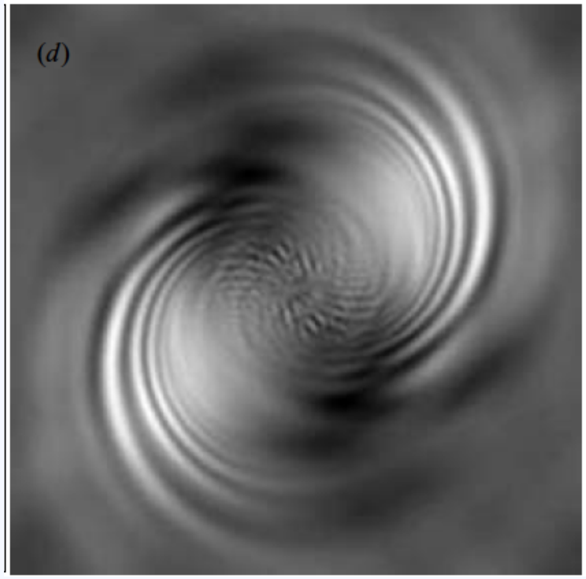


Spiral hurricane rainbands
(Gall et al. 1998)

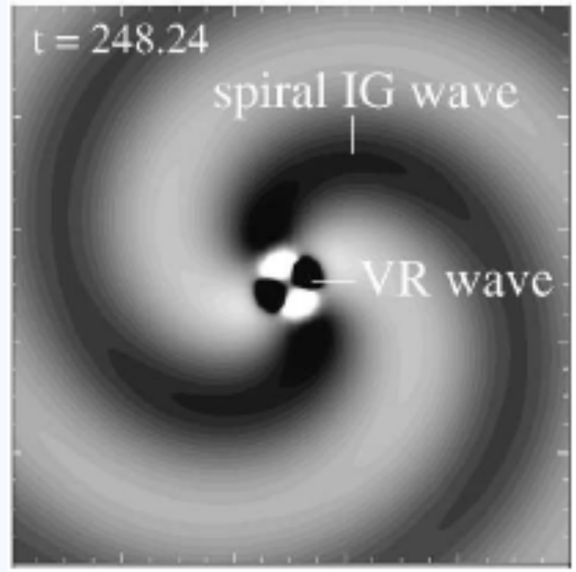
Debates on the formation mechanism:

- Spiral eddies --- 1) frontal or baroclinic instability (Munk et al. 2000)
  --- 2) inertial instability, not involving frontogenesis (Shen & Evans 2002)
- Spiral rainbands --- 1) gravity wave (Willoughby 1978)
  --- 2) vortex Rossby wave (Montgomergy & Kallenbach 1997)

- Spiral inertia-gravity waves



Viudez 2006



Schecter, Montgomery 2006

- Simulations were done in stratified fluids.
- What about density-homogeneous fluids, like upper ocean mixed layers?
  Will a vortex in homogeneous fluids also emit spiral waves?

Numerical Solution

- Numerical solver --- NEK5000;
  Spectral element method; high order of accuracy

- Non-dimensional equations of motion:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p - \frac{1}{Ro}\hat{\mathbf{k}} \times \mathbf{u} + \frac{1}{Re}\nabla^2 \mathbf{u} \ ,$$

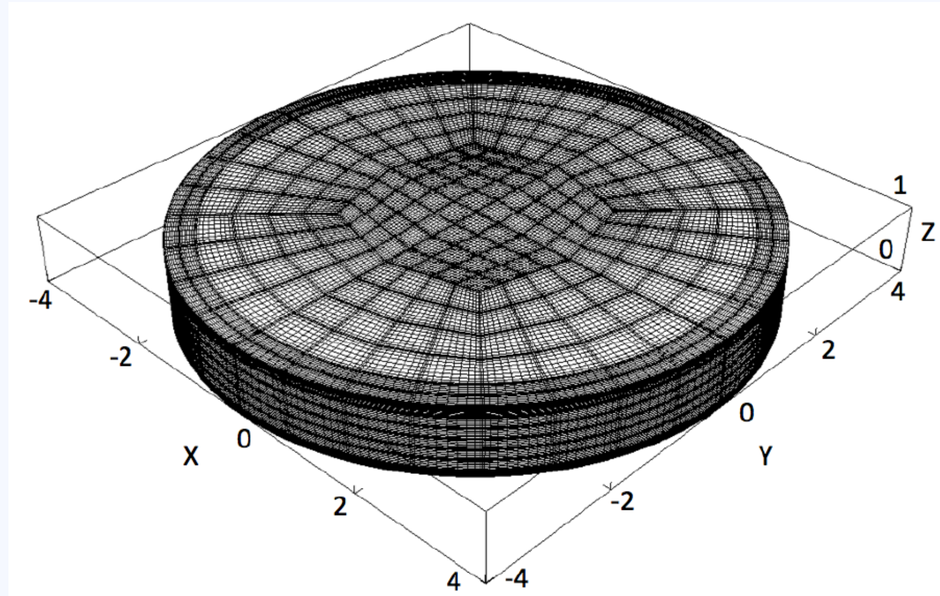
$$\nabla \cdot \mathbf{u} = 0 \ .$$

Re = 10^4

- Non-dimensional advection-diffusion equation for passive tracer:

$$\frac{\partial C}{\partial t} + (\mathbf{u} \cdot \nabla)C = \frac{1}{Pe}\nabla^2 C$$

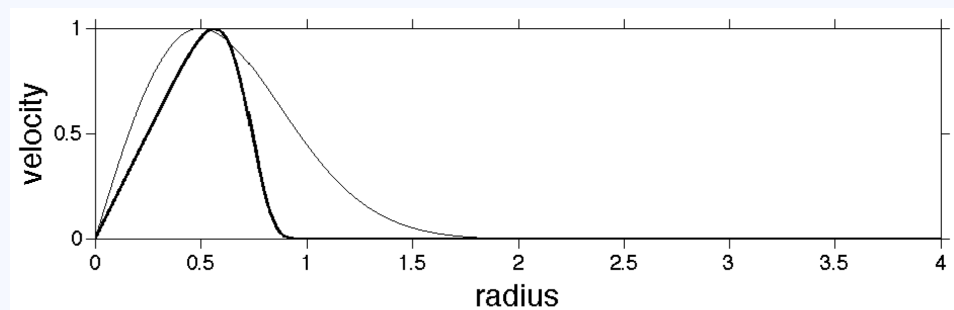
- Cylinder with D = 8, H = 1;
  Filled with homogeneous fluid;
  Background rotation



- Boundary conditions:
  --- free-slip on the sidewall;
  --- no-slip on the bottom;
  --- spinning lid on the top

- Initialize a barotropic vortex in the domain center;
  Two types of azimuthal velocity profile:

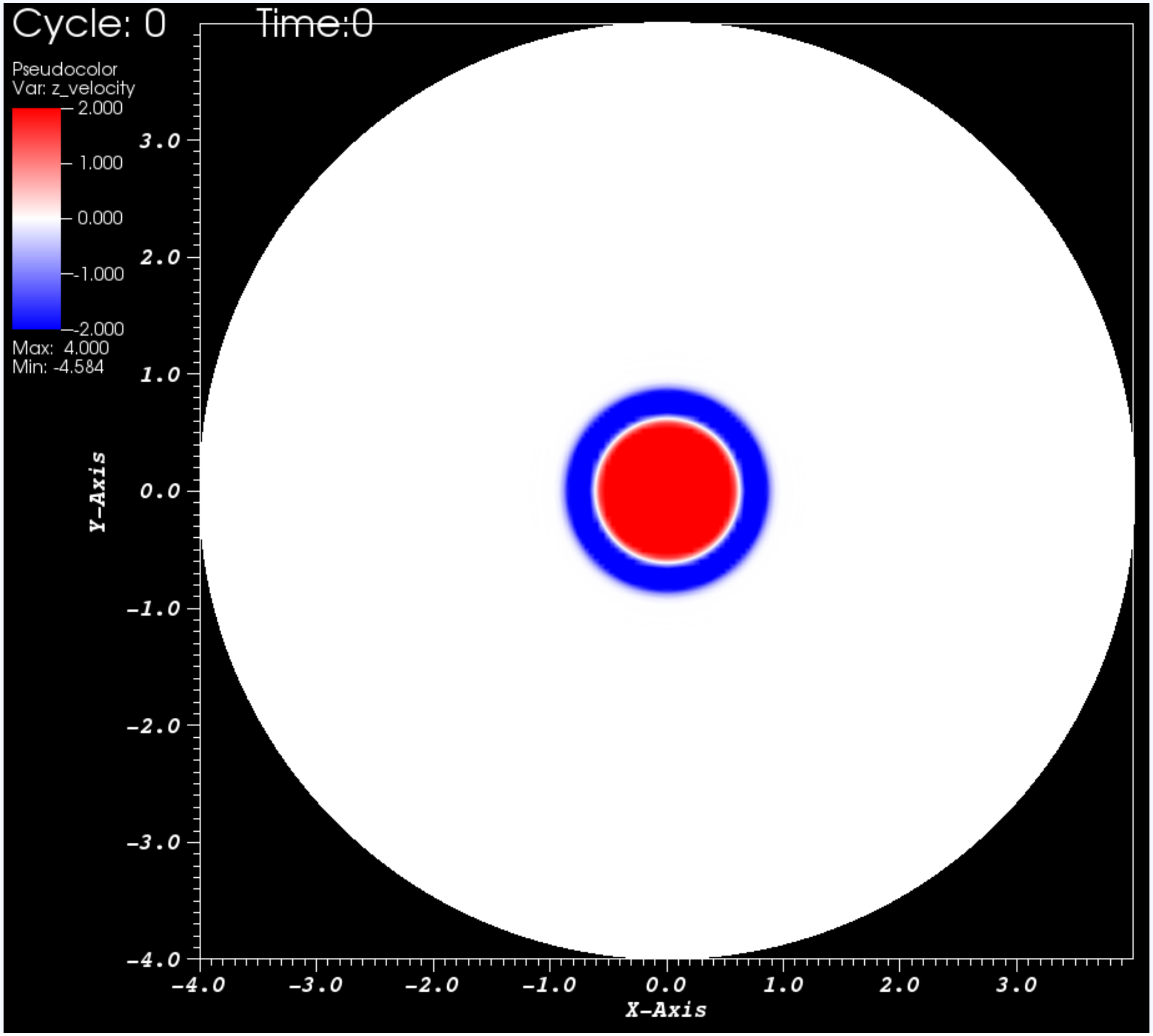
- Type 1. --- $V(r) = 2\,r\exp(-12\,r^8)$
  Unstable; barotropic instability
  "Target experiment"



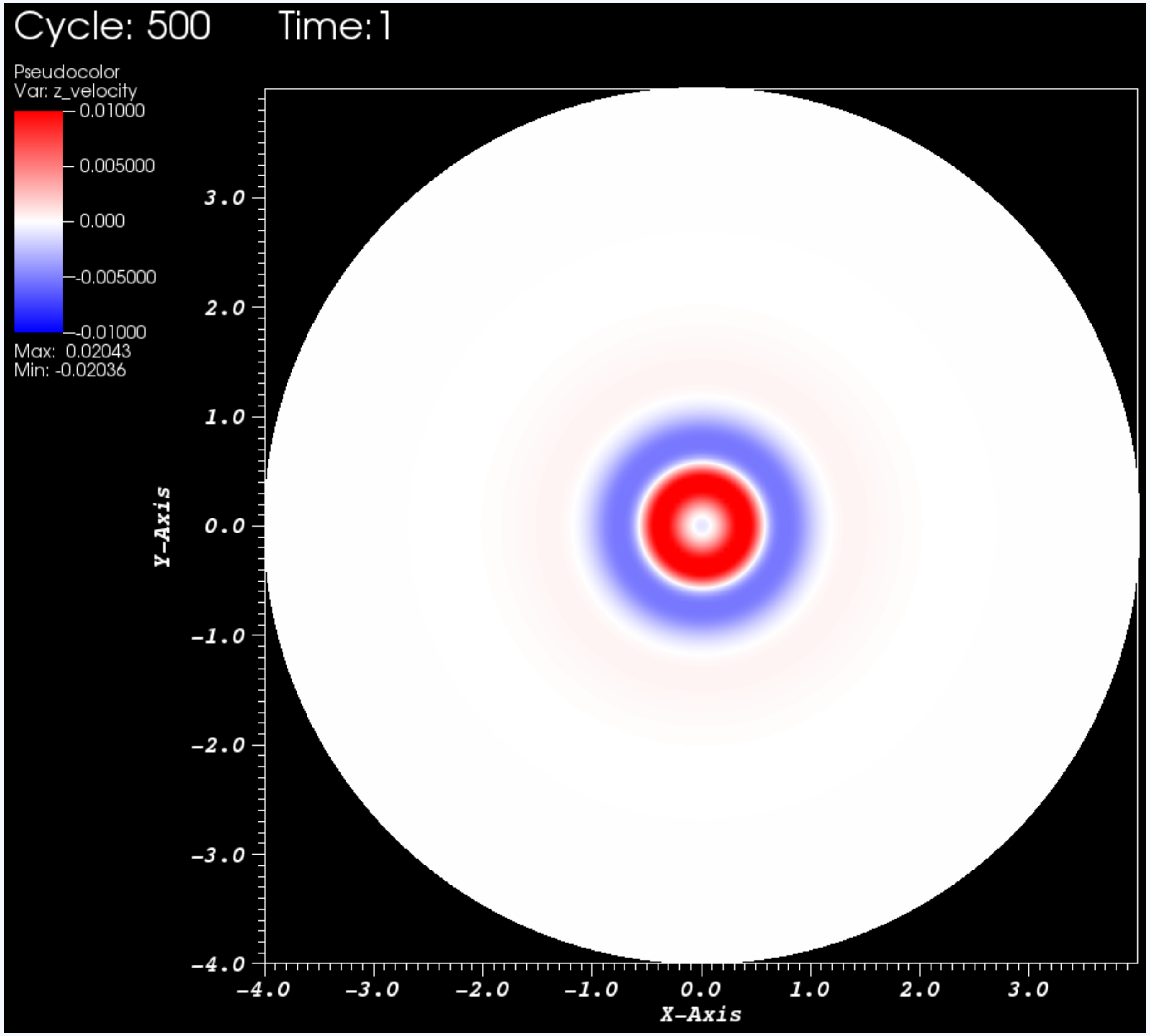
- Type 2. --- $V(r) = 3.3\,r\exp(-2\,r^2)$
  Stable; steady state
  "Control experiment"

- Simulation for a non-dimensional time of 300, about 5 eddy-turnover time;
- Target experiment with Ro = 0.2; cyclonic vortex

Vertical vorticity on mid-depth horizontal plane



- Vertical velocity on mid-depth horizontal plane



- Inertial waves
  Dispersion relation expressed in cylindrical coordinates (Greenspan 1969):

$$\lambda_{nmk} = 2\left(1 + \frac{\xi_{nmk}^2}{n^2\pi^2a^2}\right)^{-\frac{1}{2}}$$

$$\xi\frac{\mathrm{d}}{\mathrm{d}\xi}J_{|k|}(\xi) + k\left(1 + \frac{\xi^2}{n^2\pi^2a^2}\right)^{\frac{1}{2}} J_{|k|}(\xi) = 0$$

  λ is frequency; a is ratio of radius to height;
  (n,m,k) denote the axial, radial and azimuthal wavenumbers

- Wavenumbers (n,m,k) = (1,11,10);
  Theoretical frequency $\lambda = 0.545$;
  Experiment frequency $\omega = 0.616$

- Dispersion relation expressed in Cartesian coordinates:

  Inertial waves: $\omega^2 = \left(f^2m^2\right)/\mathrm{K}^2$

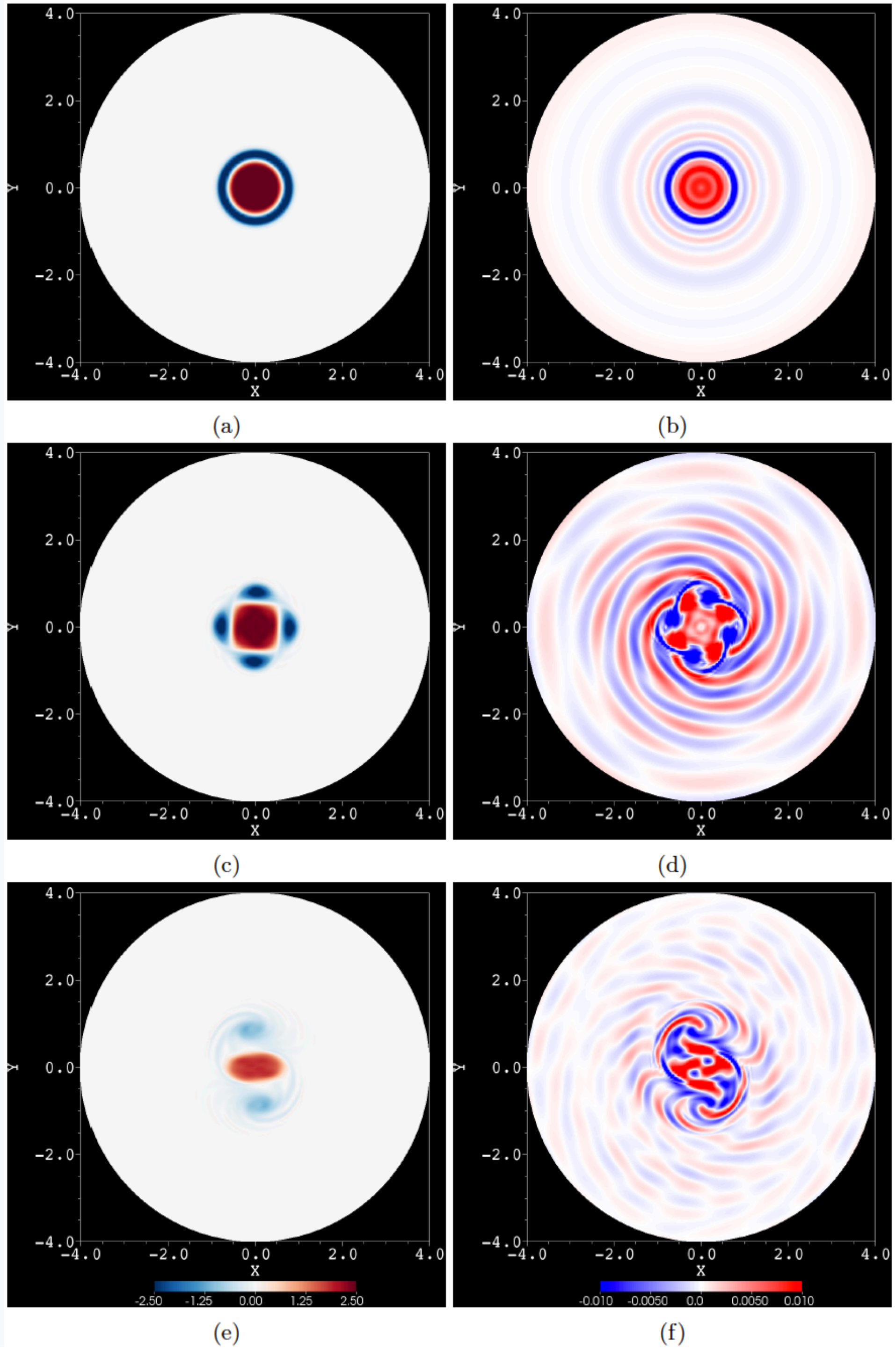
  Inertia-gravity waves: $\omega^2 = \left(f^2m^2 + N^2\mathrm{K}_H^2\right)/\mathrm{K}^2$

  An extreme of inertia-gravity wave with buoyancy frequency N = 0 (Gill 1982);
  $\omega < f$, sub-inertial signals in the ocean and atmosphere

Emission of
spiral inertial waves
(SIWs)

Left: vertical vorticity

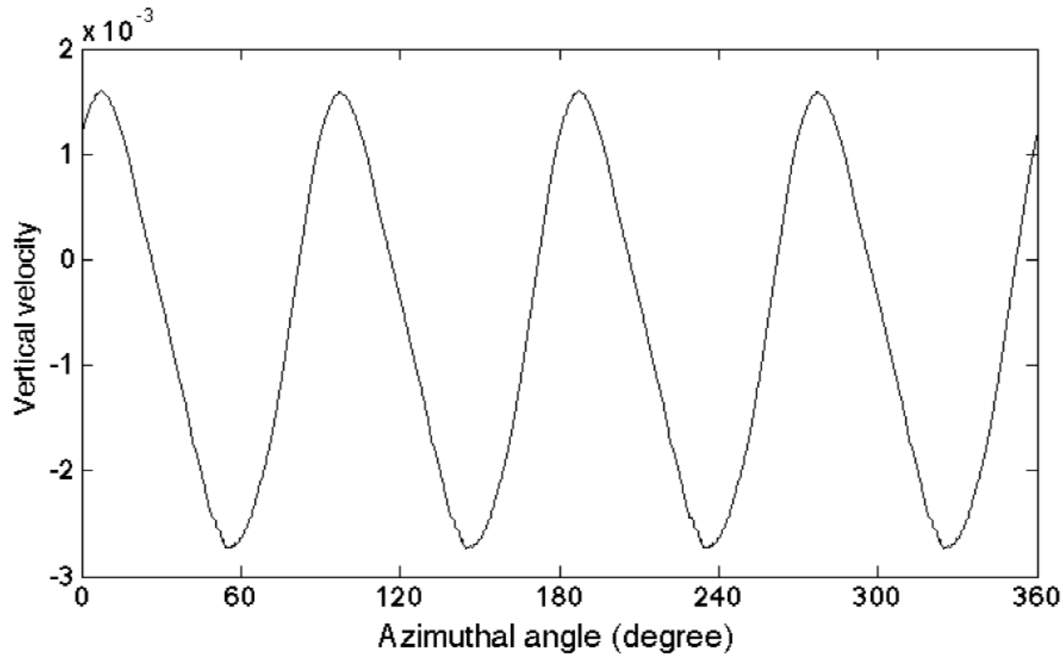
Right: vertical velocity



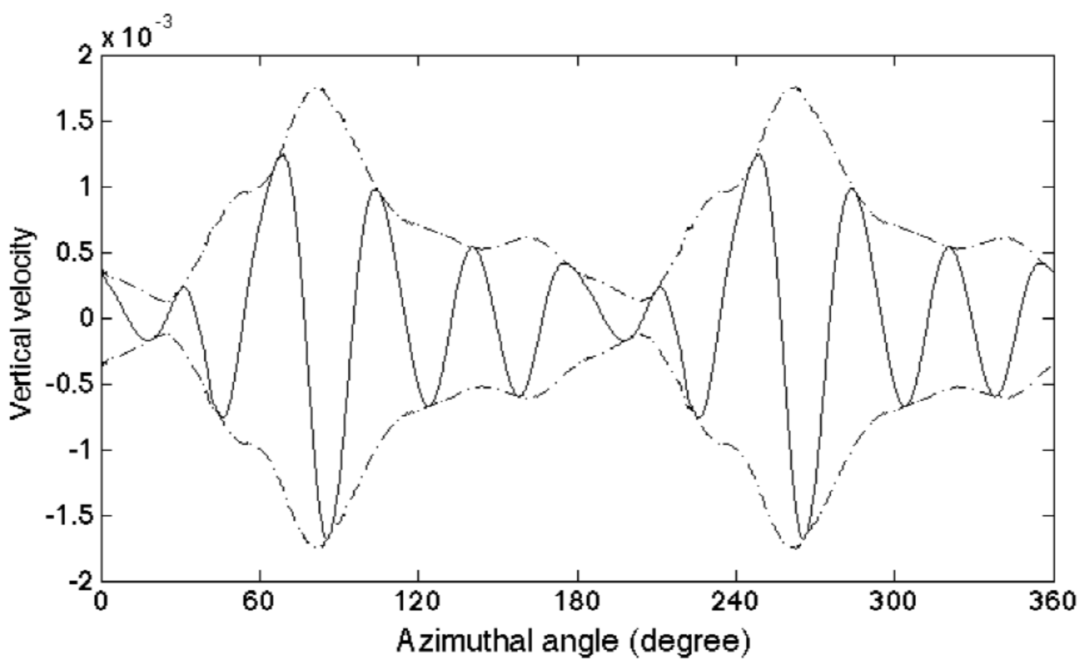
Time = 6

Time = 30
Phase SIW-4

Time = 204
Phase SIW-2



(a)

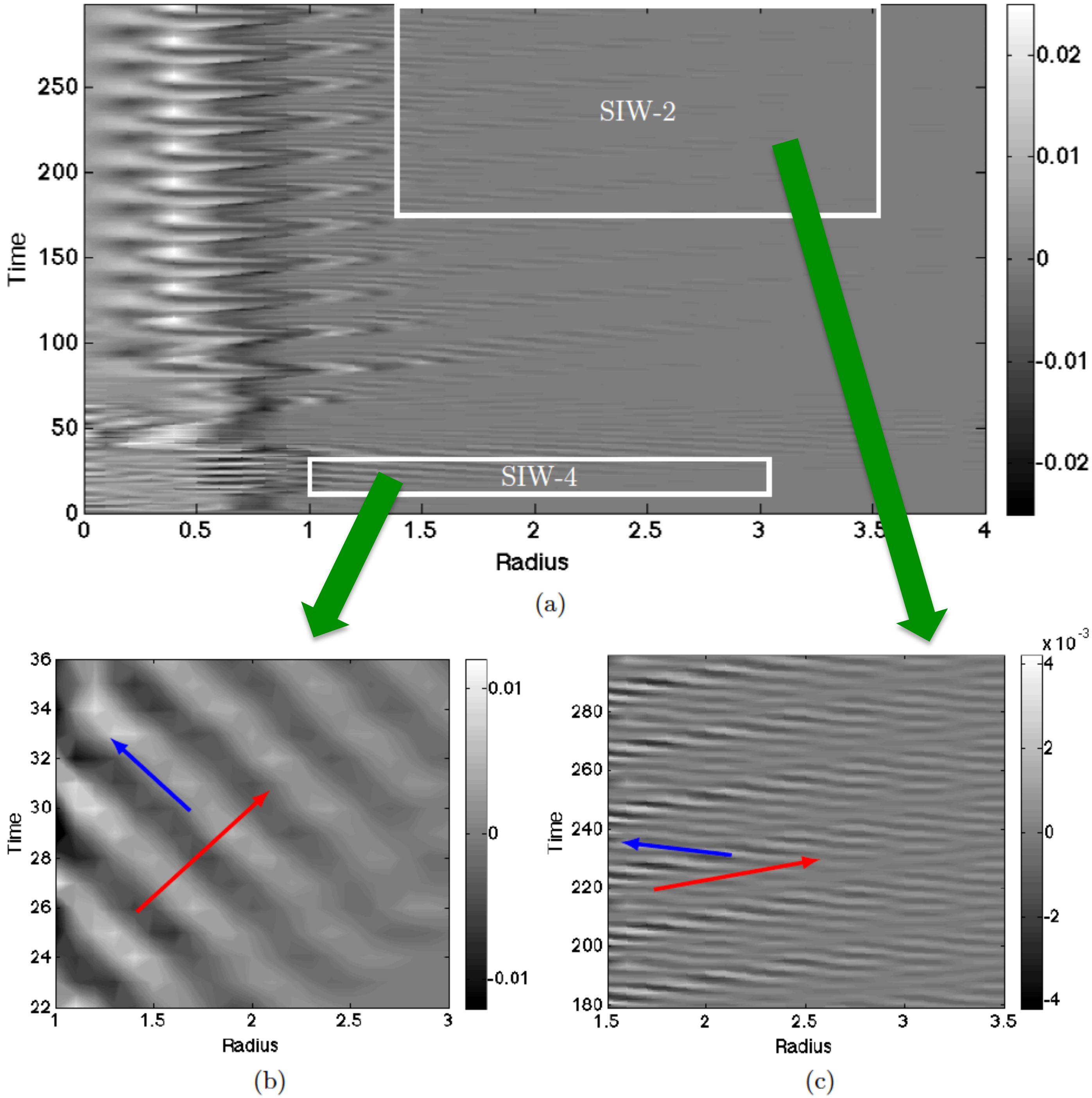


(b)

Vertical velocity sampled along a circle of radius = 2.5

Phase SIW-4

Phase SIW-2



Hovmöller diagram in radius-time section

Radially-inward phase propagation

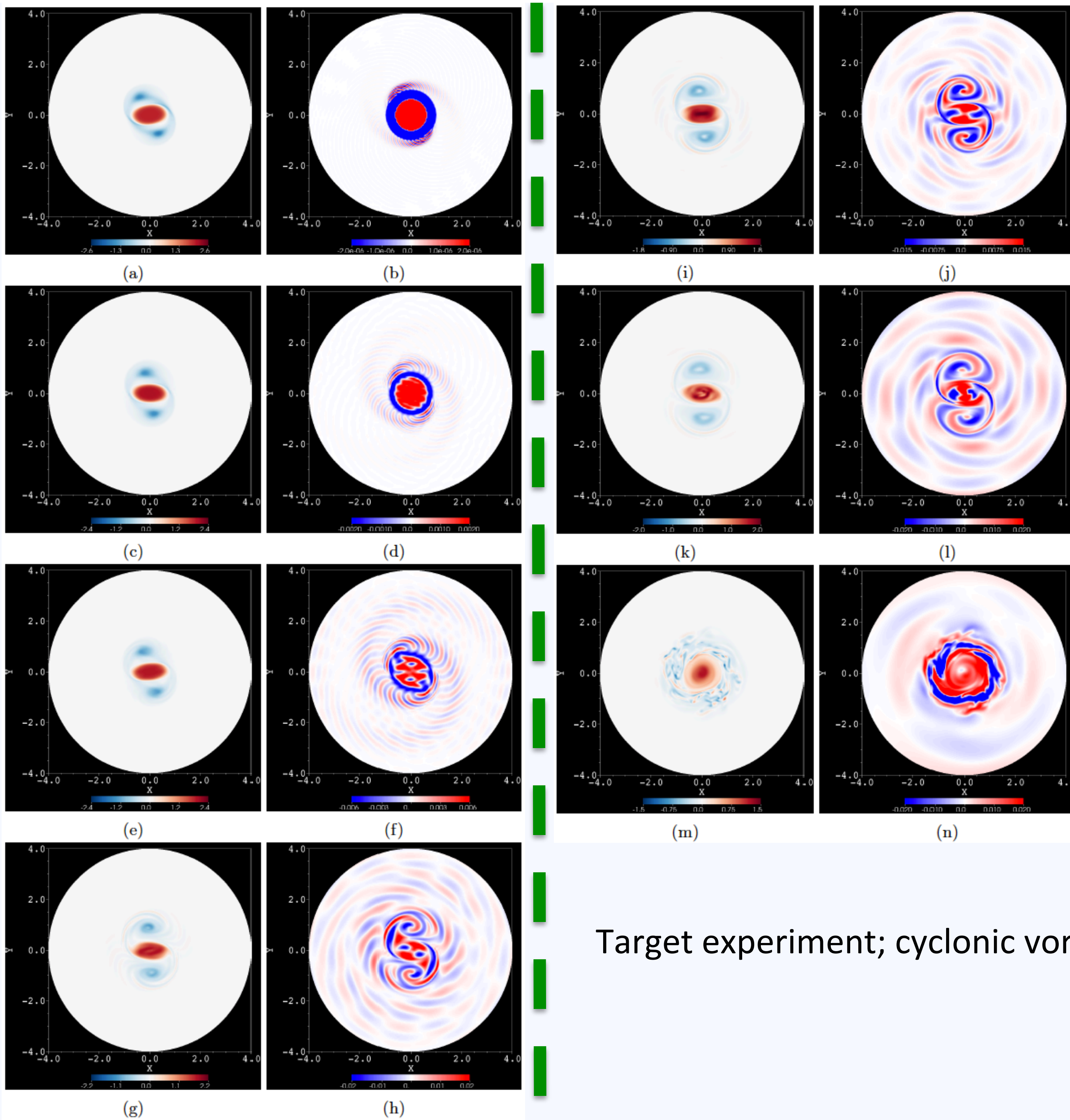
Radially-outward energy propagation

Ro = 0.01

Ro = 0.04

Ro = 0.1

Ro = 0.4



Ro = 0.6

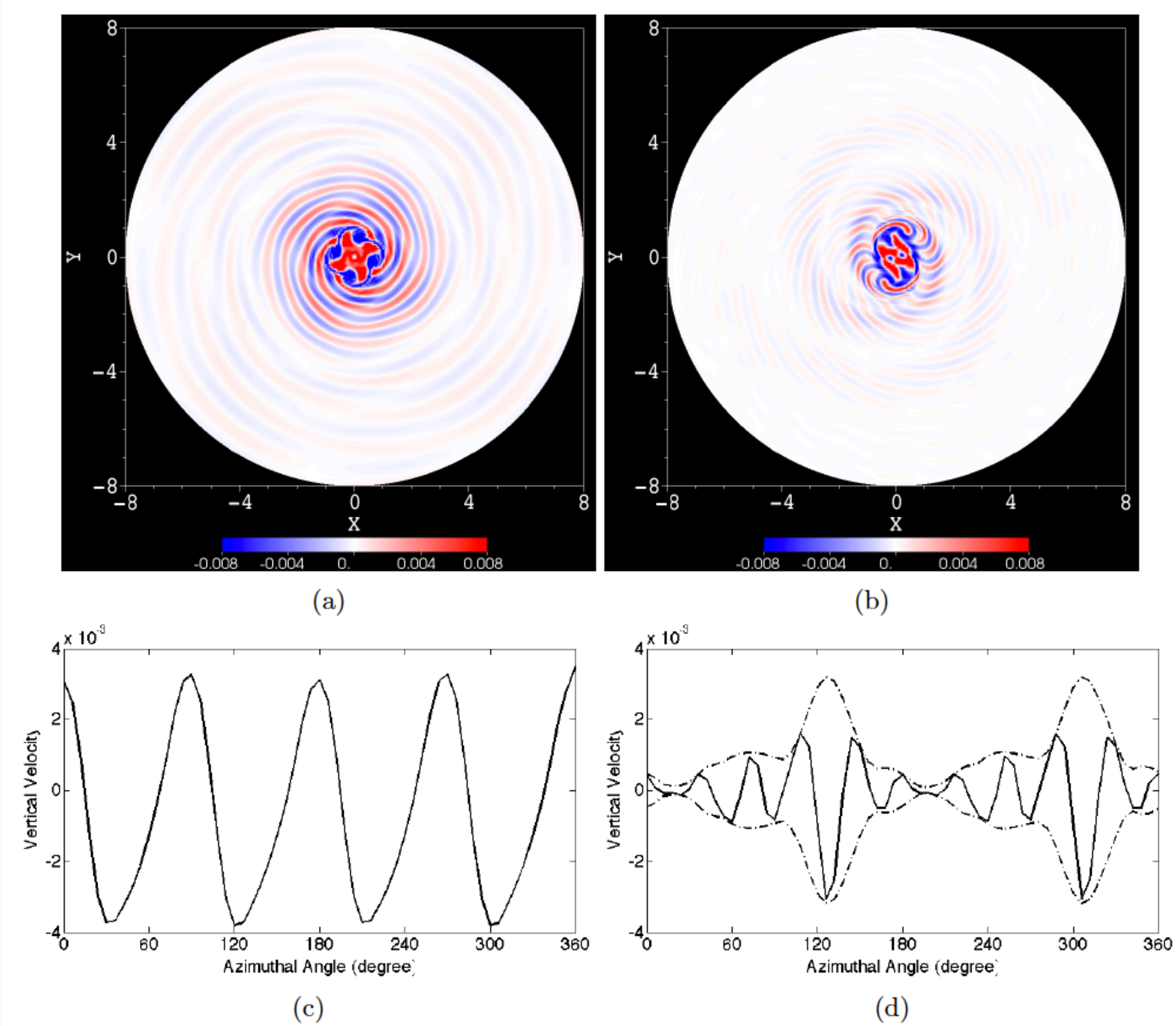
Ro = 1

Ro = 5

Target experiment; cyclonic vortex

Comparison of wave frequency between theory and experiment

Ro	Mode (n, m, k)	Eigenfrequency	
		Theory	Experiment
0.04	(1,26,20)	0.234	0.231
0.1	(1,14,12)	0.431	0.462
0.2	(1,11,10)	0.545	0.616
0.4	(1,11, 6)	0.624	0.667
0.6	(1, 7, 6)	0.881	0.923
1	(1, 7, 4)	0.966	0.898



Phase SIW-4

Phase SIW-2

Double domain radius;

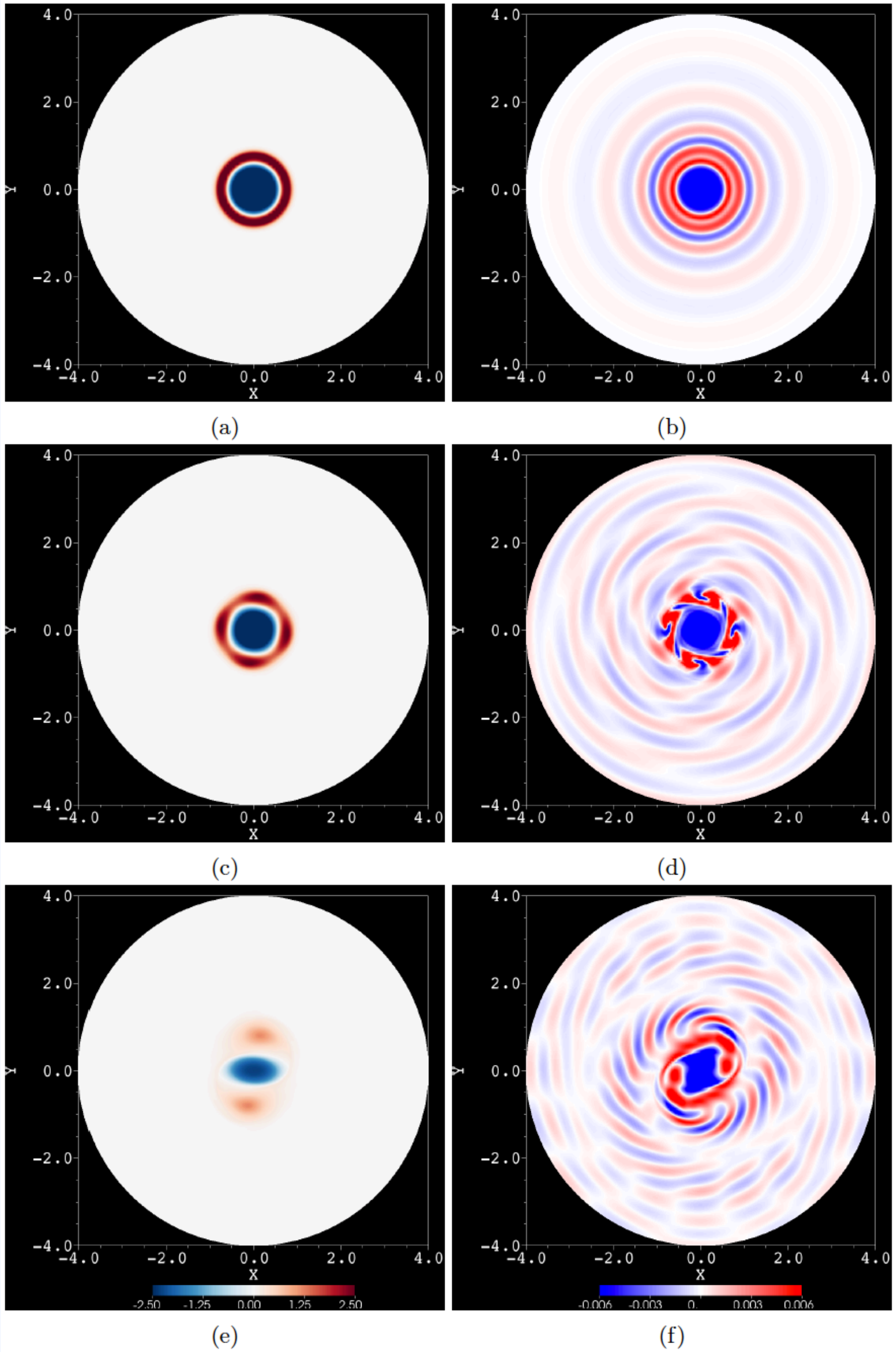
Target experiment with Ro = 0.2;

Cyclonic vortex

- Anti-cyclonic vortex
  Target experiment
  with Ro = 0.2

Left: vertical vorticity

Right: vertical velocity



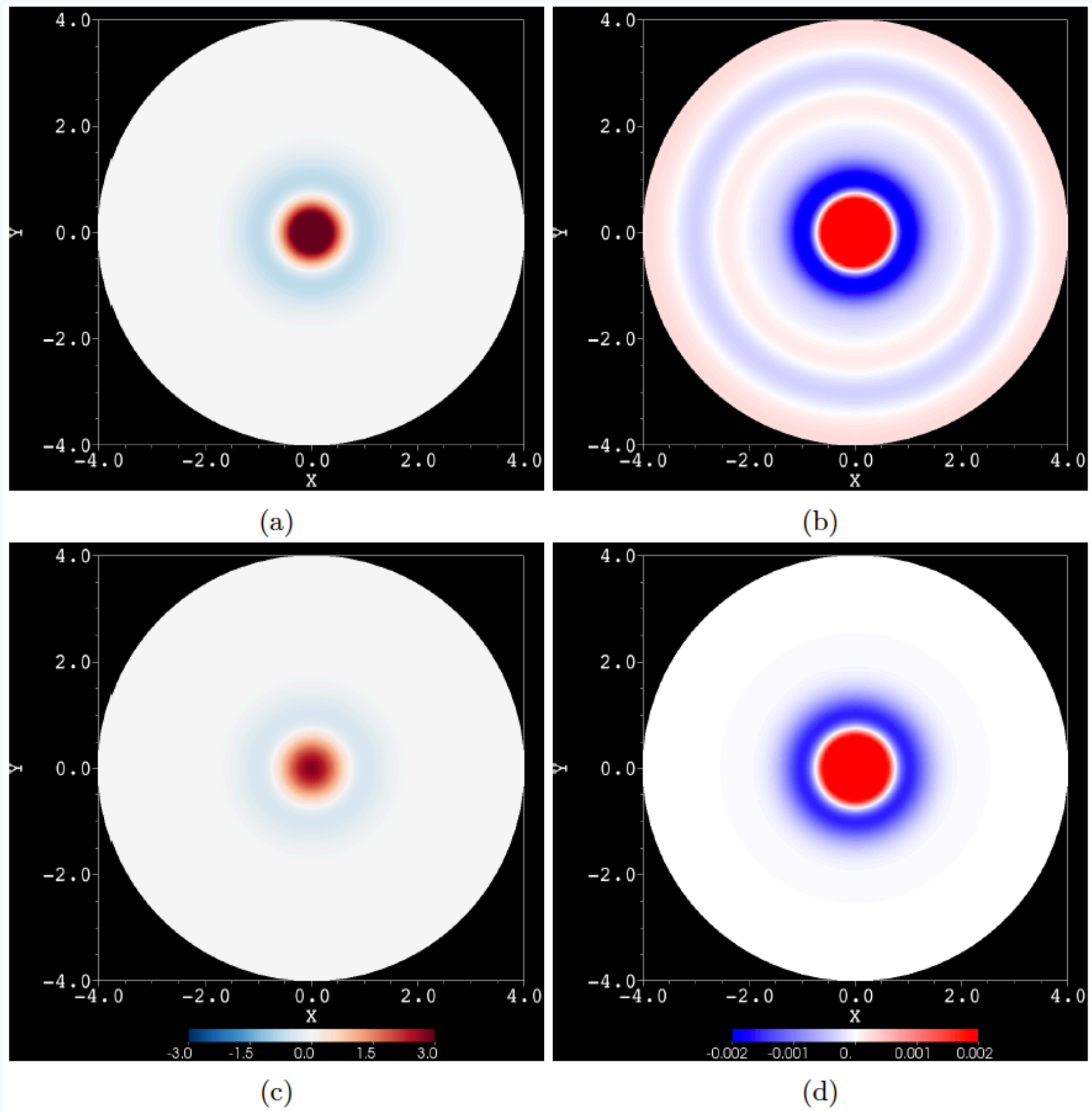
(a) (b)
(c) (d)
(e) (f)

Time = 6

Time = 26

Time = 205

- Can SIWs always appear?
- Control experiment with Ro = 0.2; cyclonic but stable vortex



Time = 10

Left: vertical vorticity

Right: vertical velocity

Time = 296

- Mechanisms of wave emission

- Early stage of vortex evolution:
  Geostrophic adjustment --- unbalanced flows (e.g. not in geostrophic balance);
  Inertial waves observed in both target and control experiments --- unbalanced initial vortex

- Later stage of vortex evolution:
  Spontaneous emission --- initially balanced but unsteady flows;
  Inertial waves observed only in target experiment --- unsteady vortices

- Lighthill-Ford radiation theory
  Spontaneous emission of inertia-gravity waves in unsteady, stratified vortices;
  By analogy, but in homogeneous fluids

- Source term of inertial-wave equation

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-fv=-\frac{1}{\rho}\frac{\partial p}{\partial x}\,,$$
$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+fu=-\frac{1}{\rho}\frac{\partial p}{\partial y}\,,$$
$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}\,,$$
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0\,,$$

(Inviscid fluid)

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)w+f^2\frac{\partial^2 w}{\partial z^2}=\boxed{S},$$

where

$$\boxed{S=\sum_{i=1}^{2}\sum_{j=1}^{3}\frac{\partial^2}{\partial x_i\partial x_j}T_{ij}},$$

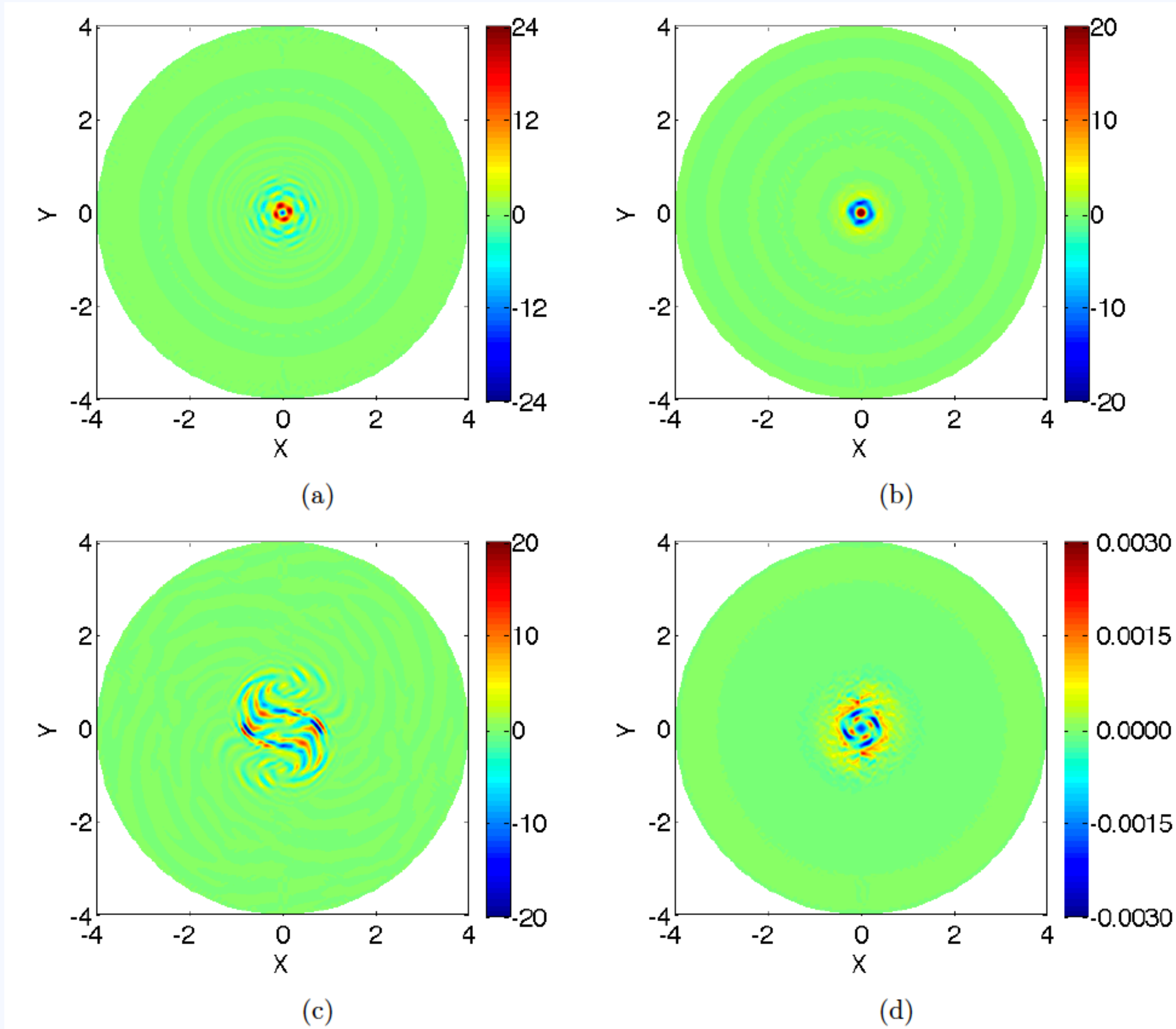
and

$$T_{ij}=\frac{\partial}{\partial t}\left[\frac{\partial}{\partial x_3}(u_iu_j)\right]+f\sum_{m=1}^{2}\frac{\partial}{\partial x_3}(\varepsilon_{im}u_mu_j)-\frac{\partial}{\partial t}\left[\sum_{n=1}^{3}\frac{\partial}{\partial x_n}(\delta_{ij}u_nu_3)\right]$$

Fields of
wave source “S”

Time = 10
Early stage

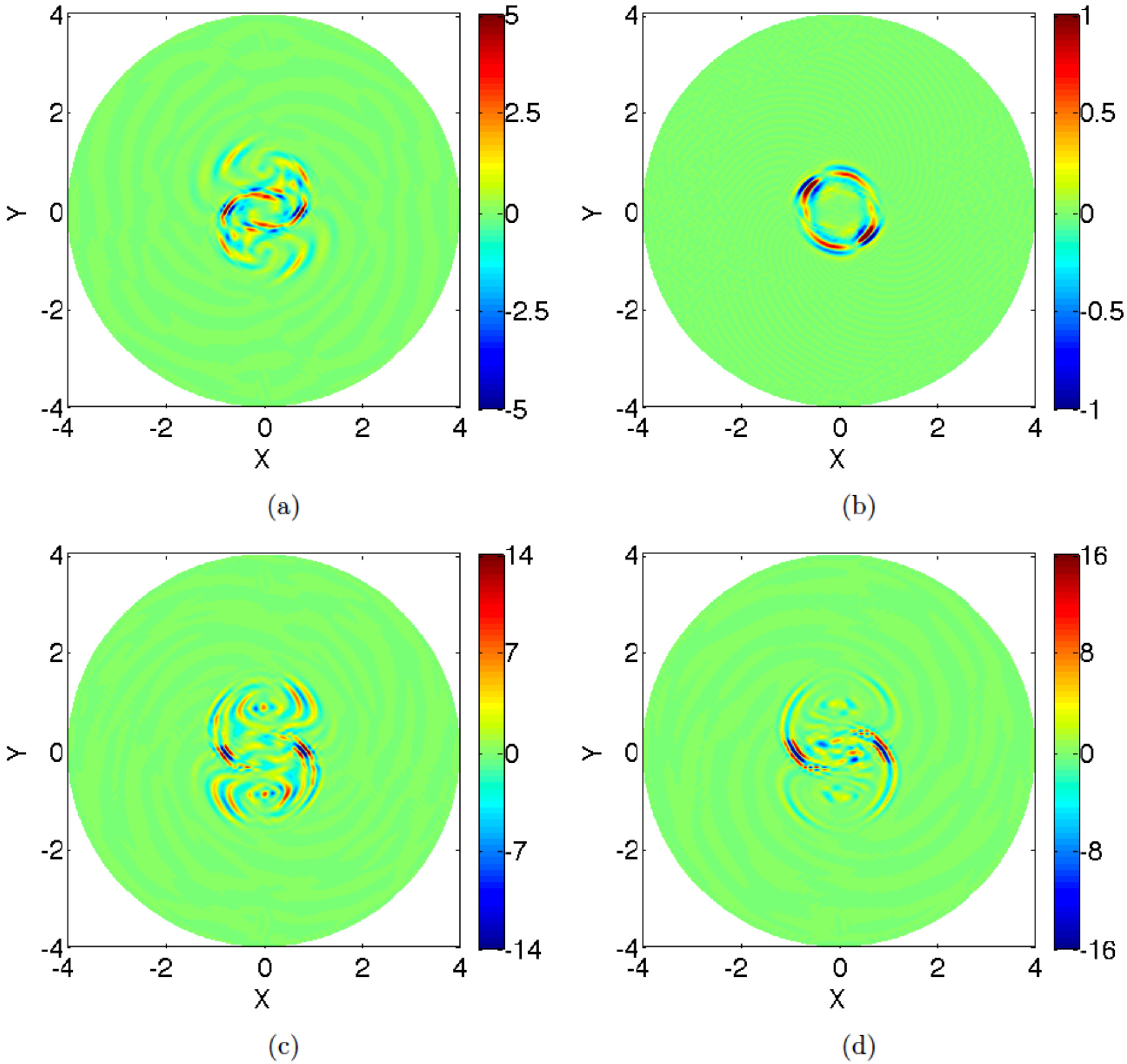
Time = 296
Later stage



Ro = 0.2

Target experiment

Control experiment



Fields of
wave source "S"

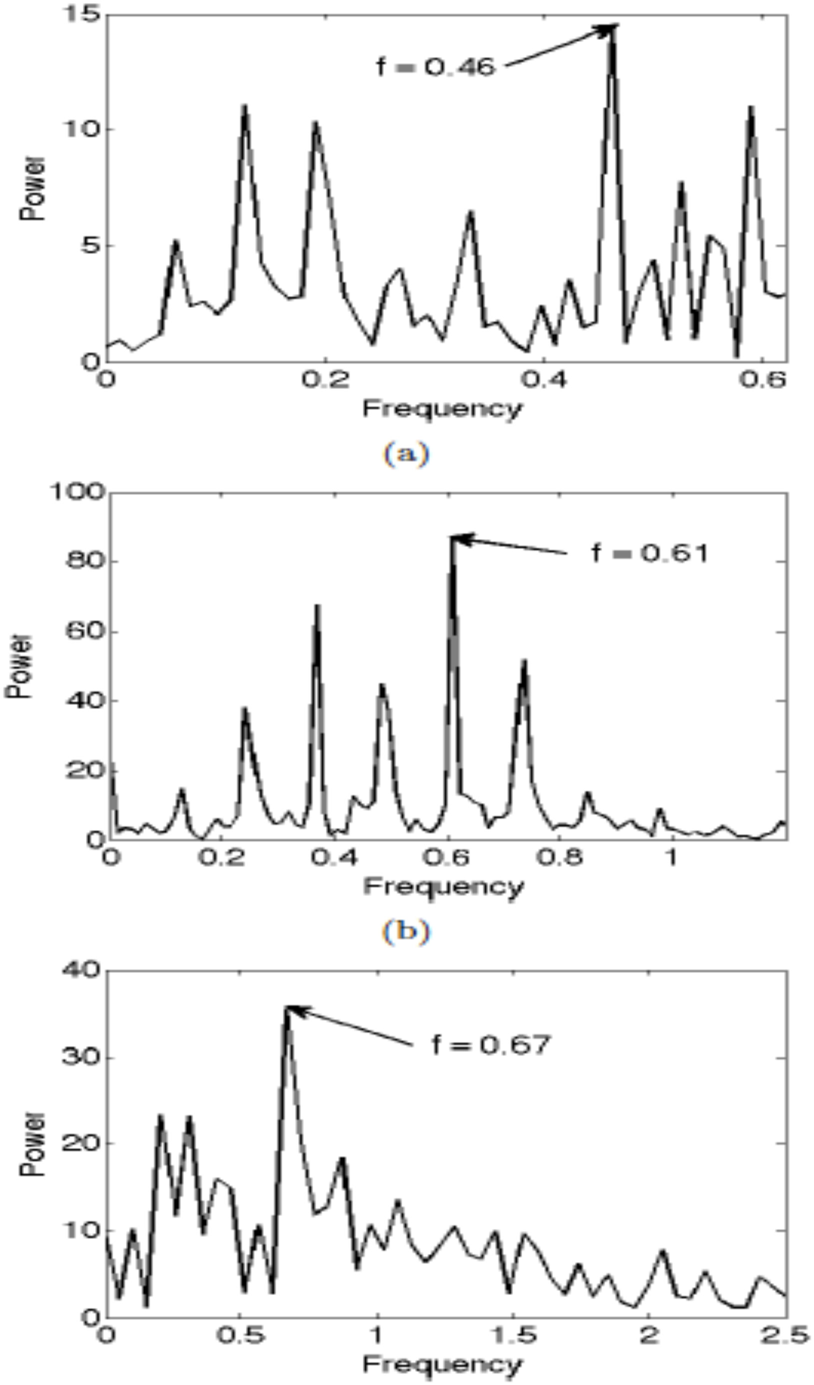
Target
experiments

Frequency spectra of wave source "S" inside vortices

Ro = 0.1

Ro = 0.2

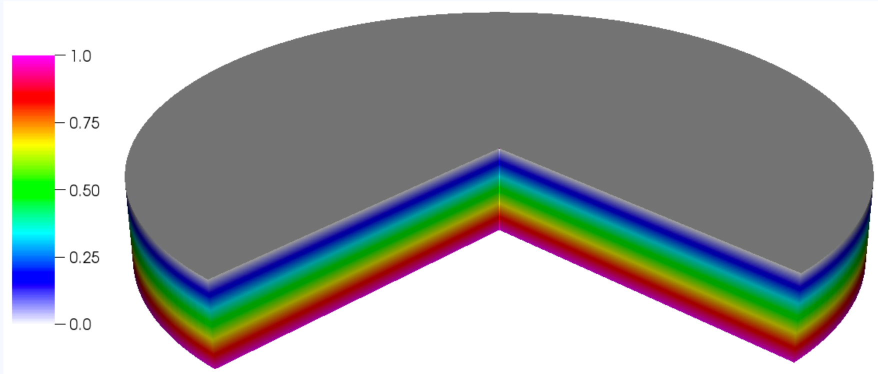
Ro = 0.4



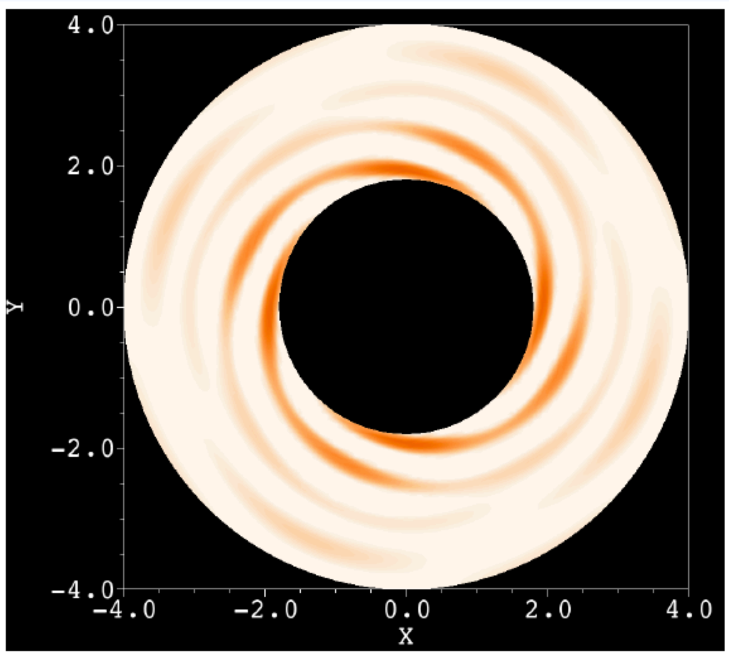
(a)

(b)

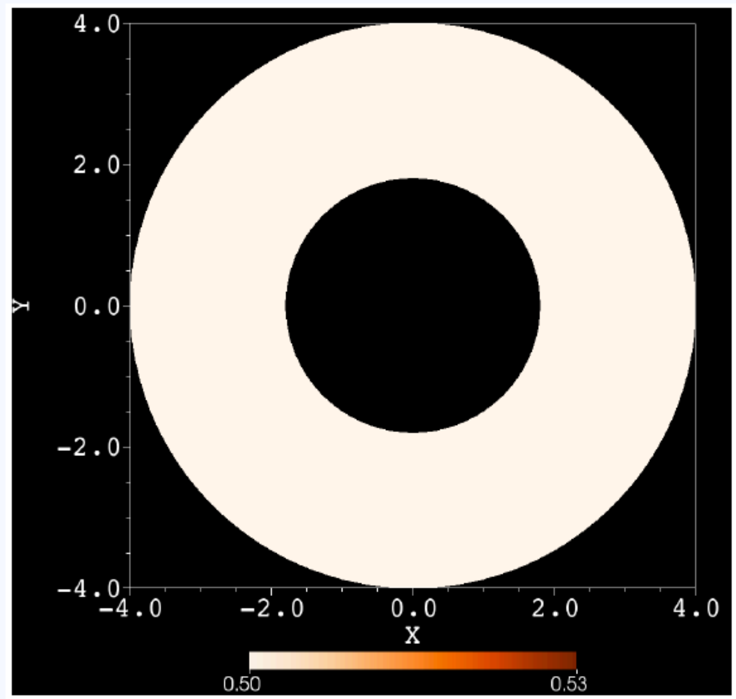
(c)



Initial tracer field;
Horizontally uniform;
Vertically linear distribution



(a)



(b)

Target experiment

Control experiment

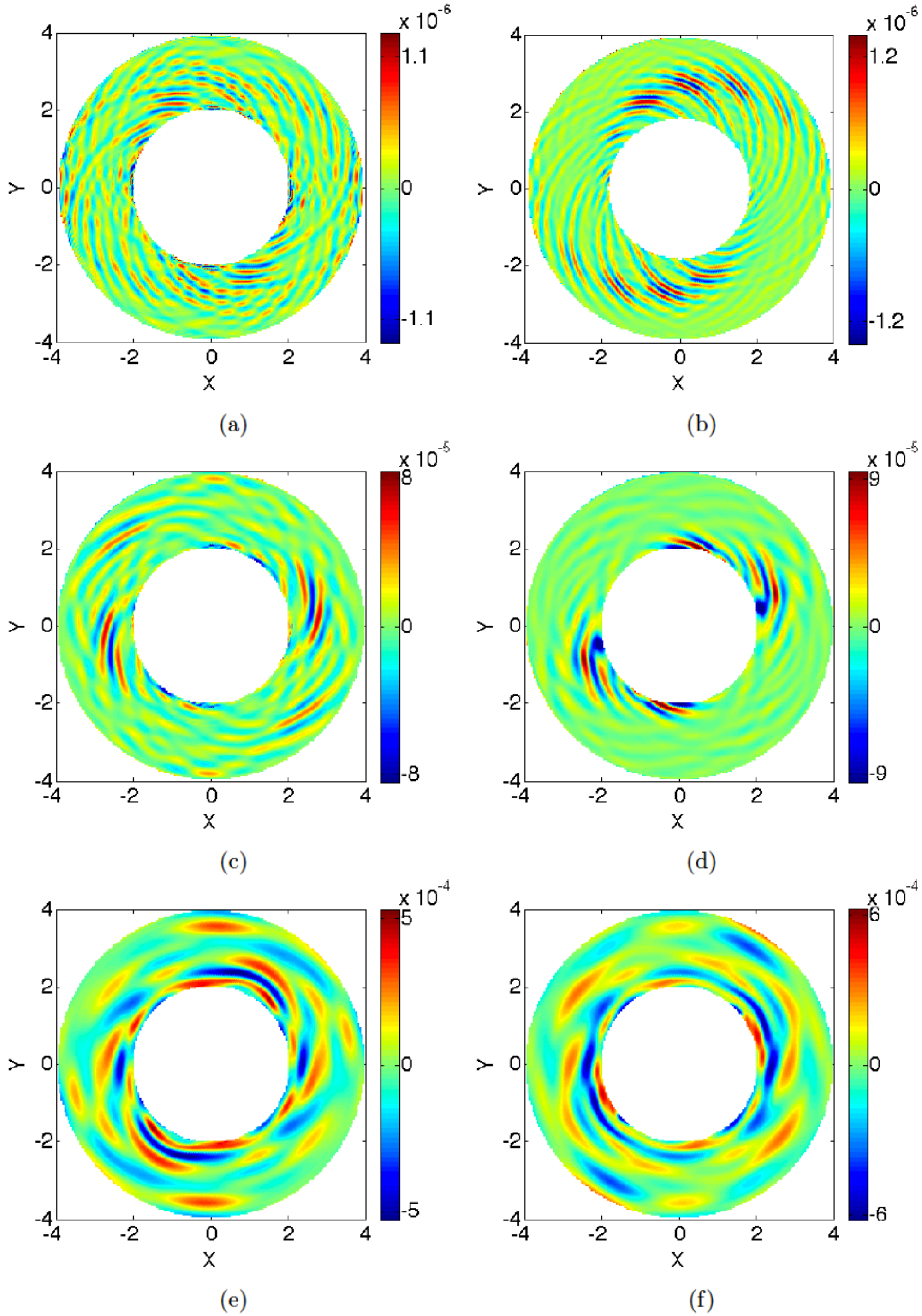
on mid-depth horizontal plane; Ro = 0.2

Target experiments

Ro = 0.04

Ro = 0.2

Ro = 1

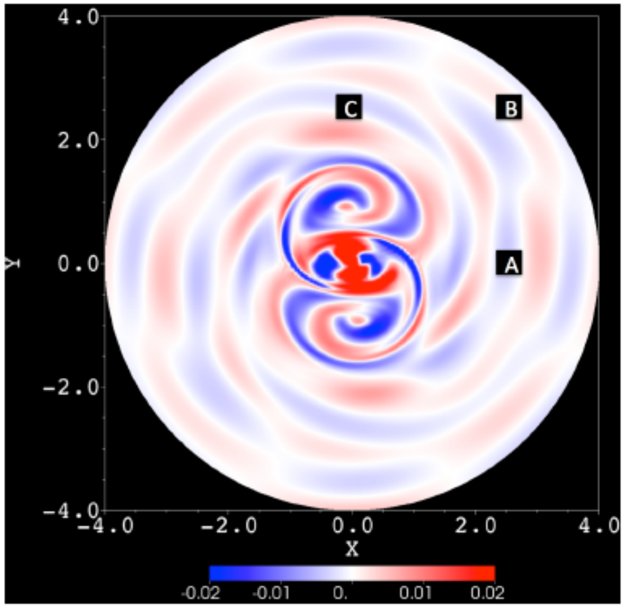


$\partial(wC)/\partial z$

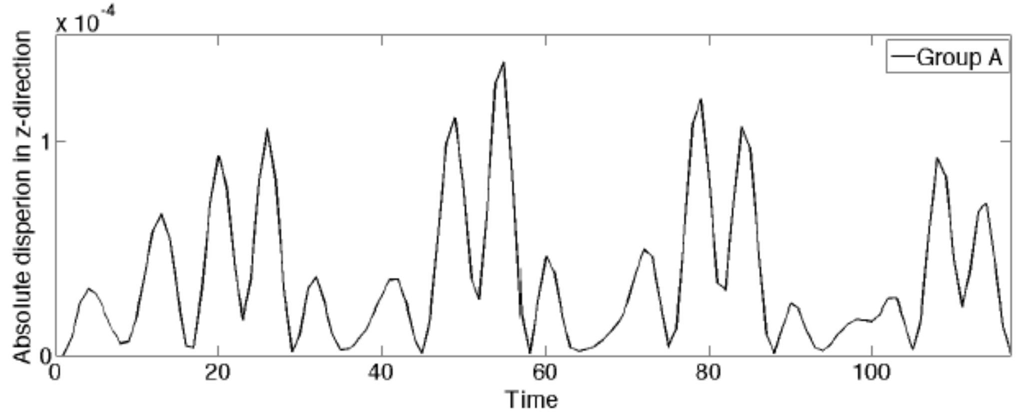
on mid-depth horizontal plane;
averaged over one eddy-turnover time

Left: Pe = 10^4

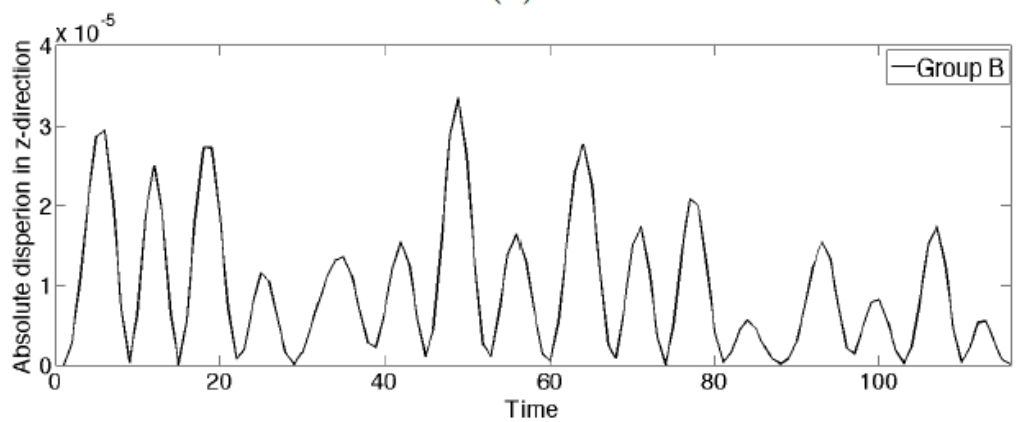
Right: Pe = 10^5



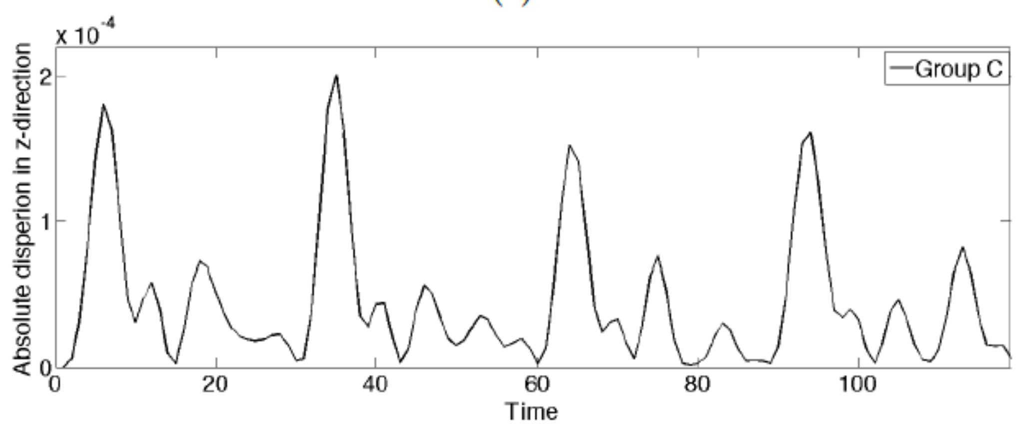
(a)



(b)



(c)



(d)

Target experiment with Ro = 1

Vertical absolute dispersion:

$$A_z^2(t, t_0) = \frac{1}{M} \sum_{i=1}^{M} |z_i(t) - z_i(t_0)|^2$$

Summary

- Inertial waves emitted in a spiral manner from geophysical vortices
- Well-organized SIWs for $0.01 < Ro < 1$; suitable for ocean meso- and submeso-scale eddies
- By analogy with Lighthill-Ford theory, source term for inertial waves is derived; parameterized into GCM
- Distribute tracer into spirals; enhance tracer vertical exchange