LDA by Particle Filtering

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Outline

System: 2-Layer Point Vortices

- Consider a 2-layer PV system with two vortices on one layer and a drifter on the other.
- For $\mathbf{z}^{(\ell)} = (x^{(\ell)}, y^{(\ell)}) \in \mathbb{R}^2$, $\ell \in [1, 2]$,

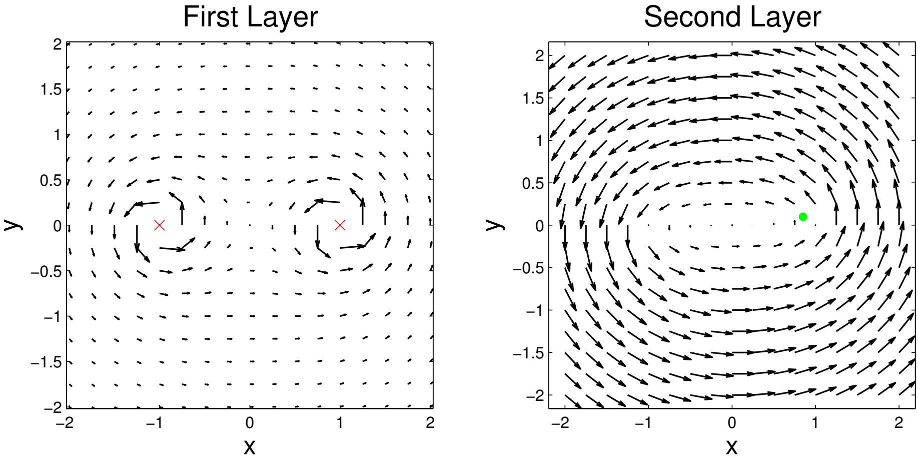
$$\frac{dx^{(\ell)}}{dt} = -\sum_{k=1}^{k=2}\sum_{j=1}^{j=N_k} \frac{\Gamma_j^{(k)}}{4\pi}\left(\frac{y - y_j^{(k)}}{|\mathbf{z} - \mathbf{z}_j^{(k)}|^2}\right) F_j^{(k)}(x^{(\ell)}, y^{(\ell)})$$

$$\frac{dy^{(\ell)}}{dt} = \sum_{k=1}^{k=2}\sum_{j=1}^{j=N_k} \frac{\Gamma_j^{(k)}}{4\pi}\left(\frac{x - x_j^{(k)}}{|\mathbf{z} - \mathbf{z}_j^{(k)}|^2}\right) F_j^{(k)}(x^{(\ell)}, y^{(\ell)})$$

where $k \in [1, 2]$ and N_k is the number of vortices in the $k-$th layer, and

$$F_j^{(k)}(\mathbf{z}) := \left(1 + \theta^{(\ell,k)}\frac{|\mathbf{z} - \mathbf{z}_j^{(k)}|}{\lambda} K_1\left(\frac{|\mathbf{z} - \mathbf{z}_j^{(k)}|}{\lambda}\right)\right)$$

Two-layers point vortex flow



Model and Observation

- The system is the SDE of the 2-layer PV system in a rotating frame; hence, the truth itself is a random walk.
- The system noise is $\theta = 0.02$.
- The state vector $\mathbf{x}_j = x_j^{(1)}, y_j^{(1)}, x_j^{(2)}, y_j^{(2)}, x_j, y_j$
- The model is the same SDE of the 2-layer PV system
- The observation is the trajectory of the drifter$\mathbf{y}_j = (x_j, y_j)$.
- The observation is taken at the integer time, $t_{obs} \in \{1, 2, \ldots, 50\}$
- Observational is with $\theta_{obs} = 0.02$.

Some uncertainty within the system

- 100,000 sample points initially located at $(1, 0)$

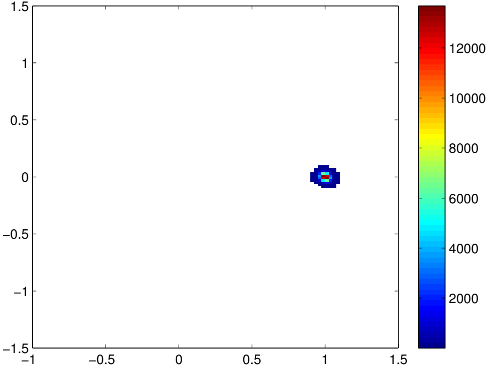


Figure: t=1

Some uncertainty within the system

- 100,000 sample points initially located at $(1, 0)$

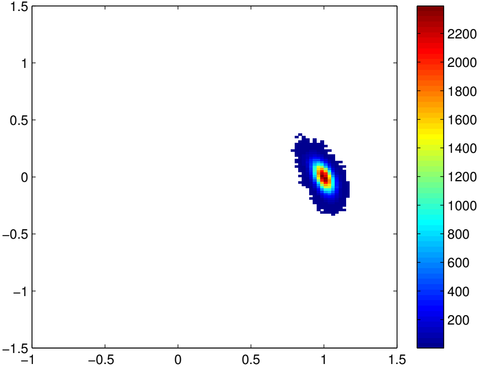


Figure: t=5

Some uncertainty within the system

- 100,000 sample points initially located at $(1, 0)$

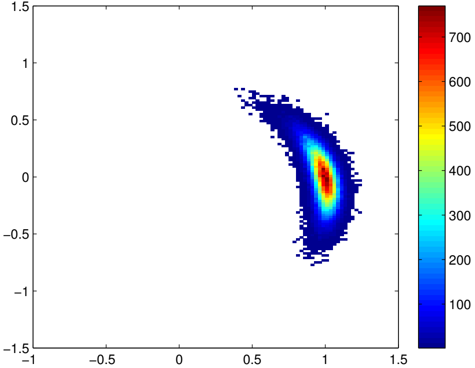


Figure: t=10

Some uncertainty within the system

- 100,000 sample points initially located at $(1, 0)$

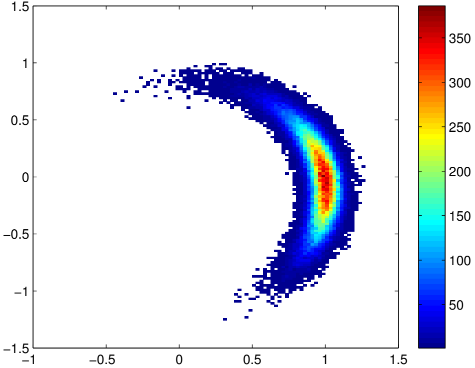


Figure: t=15

Some uncertainty within the system

- 100,000 sample points initially located at $(1, 0)$

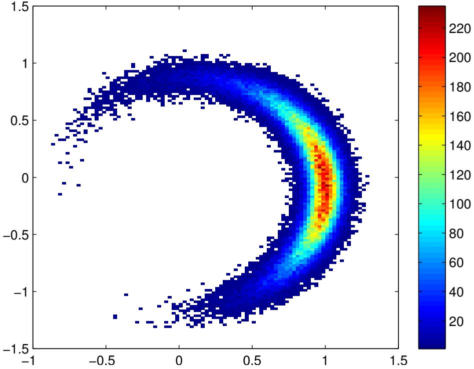


Figure: t=20

Standard (or Simple) PF

- Posterior at step k: $p(x_{0:k}|z_{1:k}) \approx \sum_{i=1}^{N_s} w_k^i \delta(x_{0:k} - x_{0:k}^i)$
- Important Sampling

$$w_k^i \propto \frac{p(x_{0:k}^i|z_{1:k})}{q(x_{0:k}^i|z_{1:k})} \text{ where } x_{0:k}^i \sim q(x_{0:k}|z_{1:k})$$

- Choose $q(x_{0:k}|z_{1:k}) = q(x_k|x_{k-1}, z_k)q(x_{0:k-1}|z_{1:k-1})$
- The weight update equation

$$w_k^i \propto w_{k-1}^i \frac{p(z_k|x_k^i)p(x_k^i|x_{k-1}^i)}{q(x_k^i|x_{k-1}^i, z_k)} \text{ where } x_k^i \sim q(x_k|x_{k-1}, z_k)$$

- If choosing $q(x_k^i|x_{k-1}^i, z_k) = p(x_k|x_{k-1}^i)$,

$$w_k^i \propto w_{k-1}^i p(z_k|x_k^i) \text{ where } x_k^i \sim p(x_k|x_{k-1})$$

StandardPF with resampling

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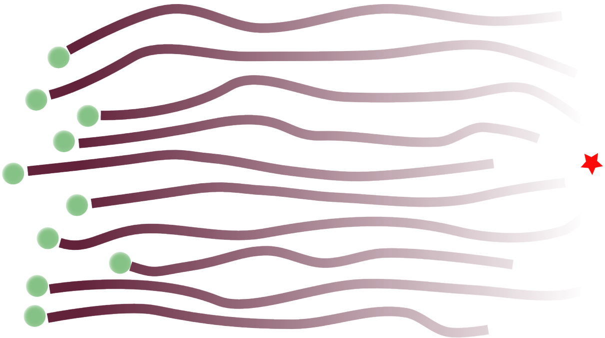
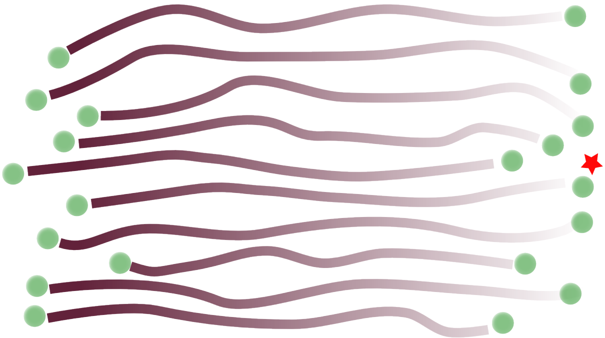
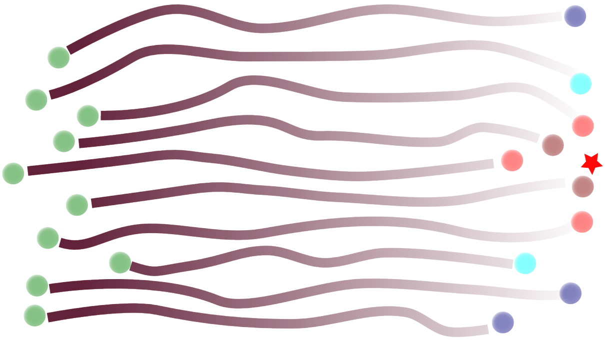


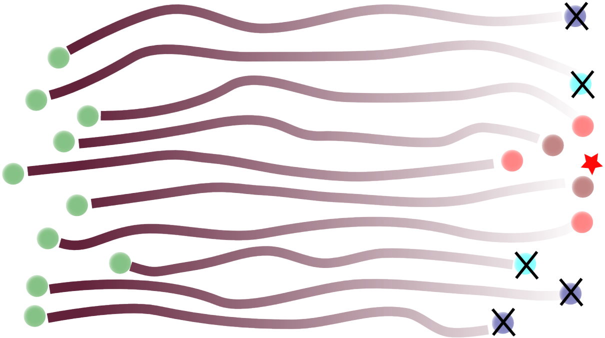
StandardPF with resampling



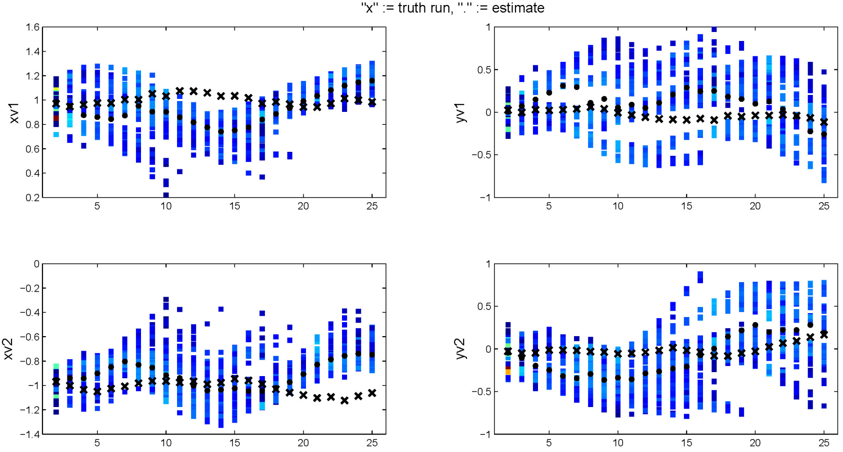
StandardPF with resampling



StandardPF with resampling



Marginal Distribution: Standard PF



Some statistical justification

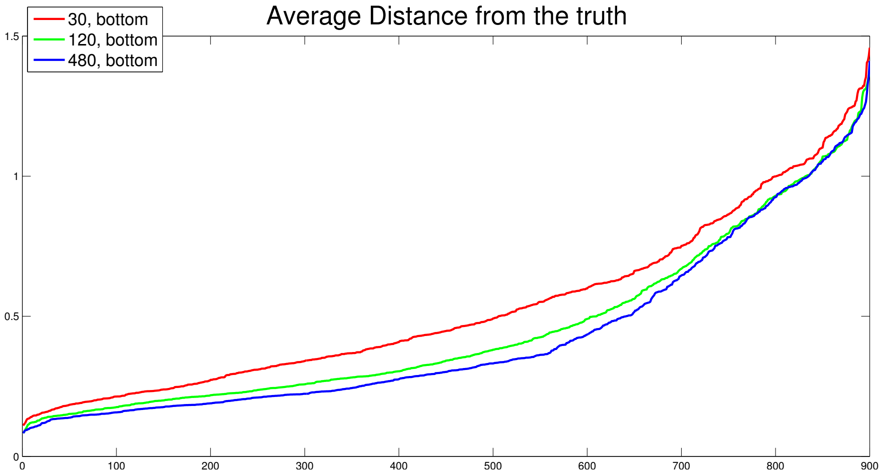
Average distance

$$\overline{D(\widehat{\mathbf{X}}^{(i)}, \mathbf{X}^{(i)})} := \overline{\left((\widehat{X}^{(i)} - X^{(i)})^2 + (\widehat{Y}^{(i)} - Y^{(i)})^2\right)^{1/2}} \qquad i \in \{1, 2\}$$

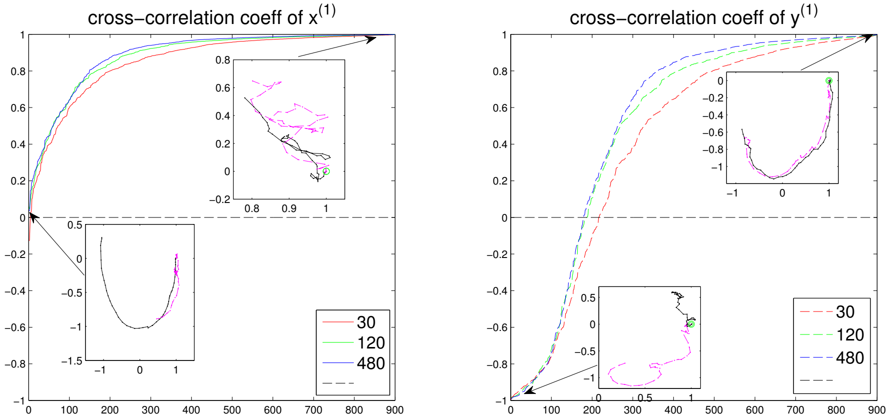
Cross-correlation coefficient

$$Xcorr(\widehat{\mathbf{X}}^{(i)}, \mathbf{X}^{(i)}) = \frac{1}{N} \sum_{k=0}^{N-1} \widehat{X}_k^{(i)} X_k^{(i)} \qquad i \in \{1, 2\}$$

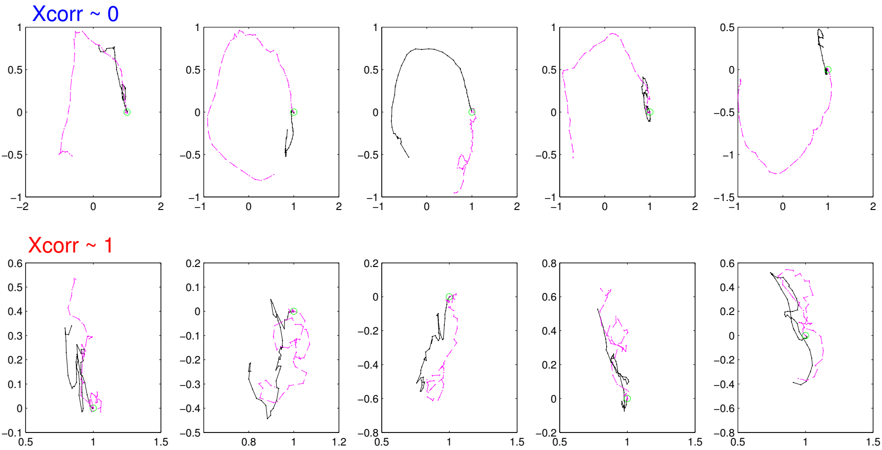
Average distance



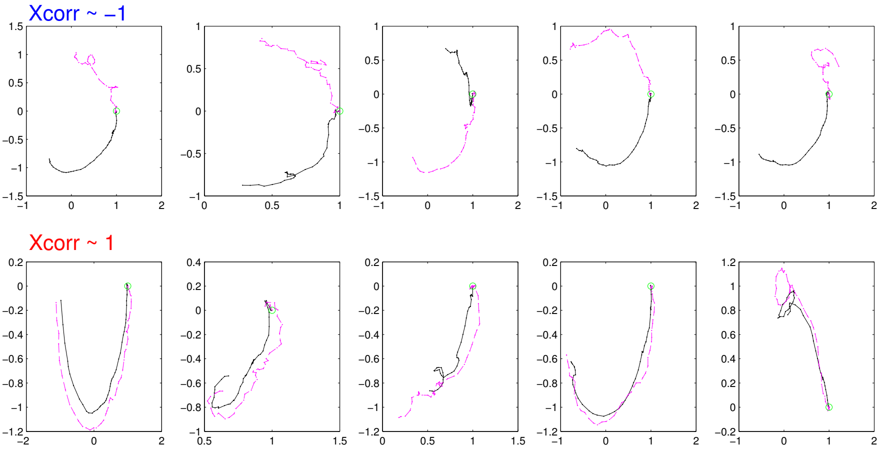
Cross-correlation coefficient



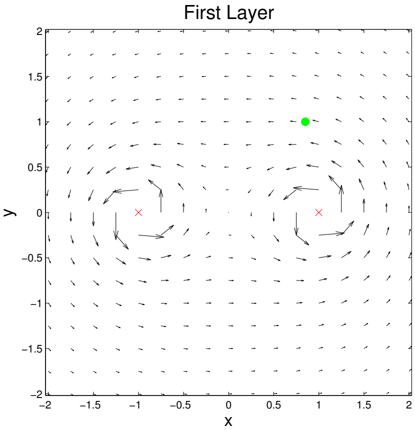
Cross-correlation coefficient, x-coordinate

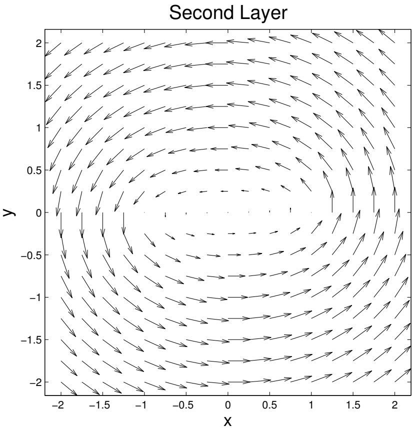


Cross-correlation coefficient, y-coordinate

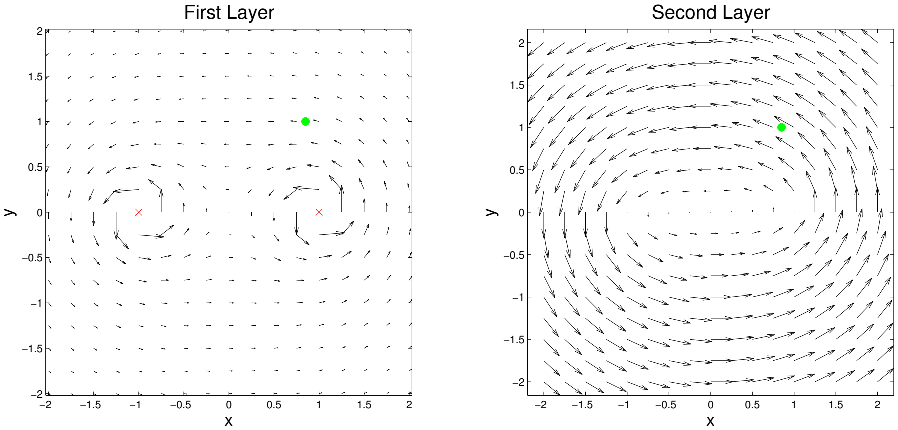


A drifter is on the vortex layer





Two drifters



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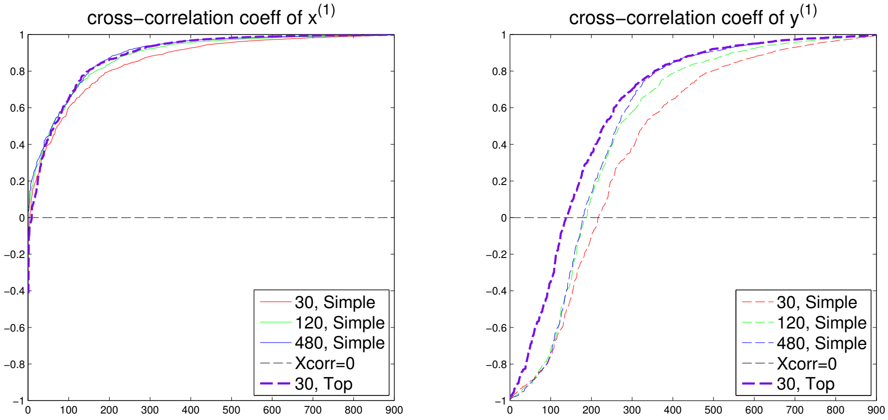

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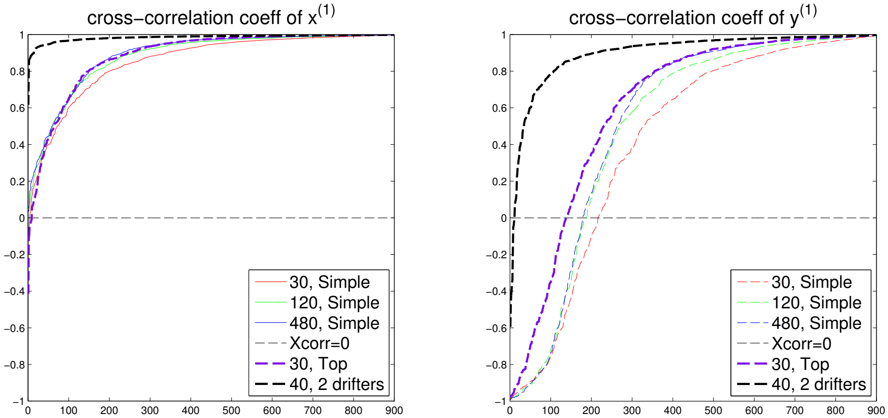

Average distance