3D assimilaiton update: en route Lagrangian data

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Outline

- Motivation
- Test problem
- Observation operator and preliminary results
- Looking forward

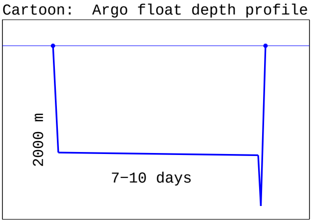
Lagrangian instruments

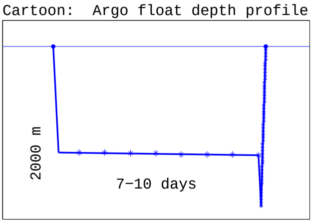
Argo float

glider

- Goal – collect below-surface measurements to better understand 3D dynamics and structures
- Lagrangian instruments collect data en route (temperature, pressure, salinity)
- Observations depend on unknown drifter paths
- What to do with that data?

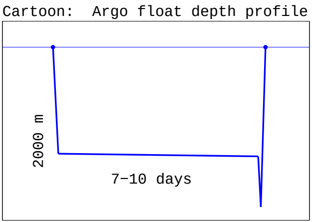
Float depth profile

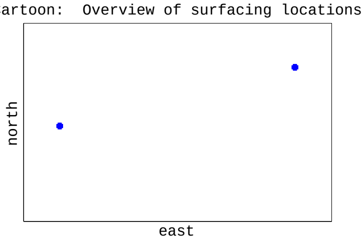




- 7-10 day float results in O(10)-O(100) km traveled
- high frequency data in dive/ascent just before surfacing in water column beneath “surfacing location”
- low frequency en-route measurements at depth, no latitude/longitude information
- en-route measurements averaged, not used in assimilation

Float depth and overview





- Lagrangian DA can help ascertain velocities w/o averaging

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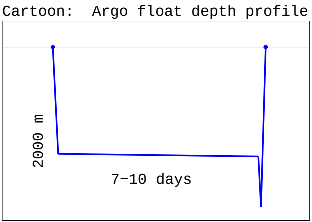
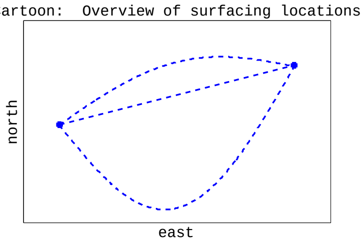
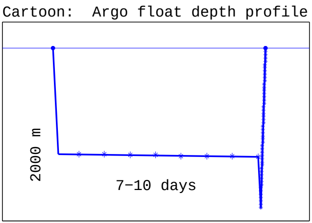
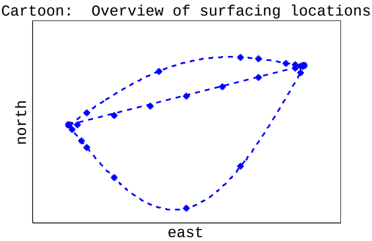




- Some possible Lagrangian paths

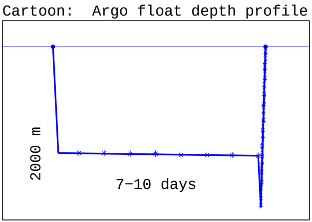
Float depth and overview

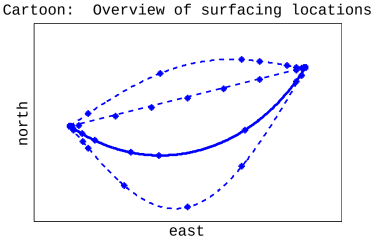




- need path & speed for subsurface observation locations

Float depth and overview



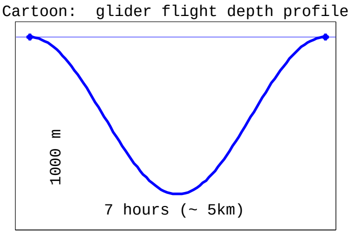


- Can en-route observations help Lagrangian DA?

Assimilated 3-D Lagrangian paths are (possibly) useful for

- aid in resolving Lagrangian structures
- assimilating data into high resolution models
- avoiding averaging via determining en-route data collection locales along paths which cross multiple grid cells

Depth profile for gliders



- roll, pitch with preprogrammed “flight plan”
- paths are semi-Lagrangian
- predict path with estimated velocity field and flight plan

Assimilating glider paths is (possibly) useful for

- figuring out what happened when glider surfaces far from where predicted
- improving local velocity estimates for planning next flight
- describing 3-D transport paths like those theorized to exist in the meridional overturning conveyor belt (Lozier, 2010)
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Can en-route data help Lagrangian DA?

Possibly if gradients are strong

Observations and likelihood

Observations will be related to the state variable by some observation function $y = H(x)$.

(For LaDA $H(x) = x^d$, the instrument's location.)

We can think of observations as random variables distributed as

$$Y_j | (X_j = x_j) \sim g(y|x_j).$$

Or, $Y_j = H(X_j) +$"*noise*".

$g(y|x)$ is the *likelihood* — how likely was an observation given the possible states?

With a whole set of observations $\{Y_j\}$ we can write down the likelihood for the time-series of observations

$$p(y_{1:j}|x_{1:j}) = \prod_{j=1}^{n} g(y_k|x_k)$$

Inference: goal for data assimilation

Given a background distribution of initial conditions, $\mu(x_0)$, and observations, $Y_{1:n}$, we want to infer the distribution of physical states $X_{0:n}$.

- Prior

$$p(x_{0:n}) = \mu(x_o) \prod_{j=1}^{n} m(x_j | x_{j-1})$$

- Likelihood

$$p(y_{1:n} | x_{1:n}) = \prod_{j=1}^{n} g(y = H(x_j) | x_j)$$

- Posterior, obtained by Bayes' rule

$$p(x_{1:n} | y_{1:n}) = \frac{p(y_{1:n} | x_{1:n}) p(x_{0:n})}{p(y_{1:n})}$$

recall, $p(y_{1:n}) = \int p(y_{1:n} | x_{1:n}) p(x_{0:n}) dx_{1:n}$

Breakdown of DA schemes: representation of posterior

- Sample posterior: particle filter or MCMC
  - handles nonlinear/nonGaussianity naturally
  - doesn't scale well as dimension increases
- Approx posterior as Gaussian: Kalman filter (family)
  - relies on Gaussian/linear assumptions
  - ENKF samples to estimate covariance
- Find mode of posterior: variational DA
  - what if posterior is multi-modal w/nearly even masses?

For all cases, including en-route data changes observation function, $H(x)$, and hence likelihood

Test problem: Inviscid linearized Shallow Water Eqns

Non-dimensional velocity fields

$$\frac{\partial u}{\partial t} = \quad v - \frac{\partial h}{\partial x}$$

$$\frac{\partial v}{\partial t} = -u - \frac{\partial h}{\partial y}$$

$$\frac{\partial h}{\partial t} = -\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$$

Lagrangian trajectories

$$\dot{x}(t) = u[x(t), y(t), t]$$

$$\dot{y}(t) = v[x(t), y(t), t]$$

Decomposition into Fourier Modes

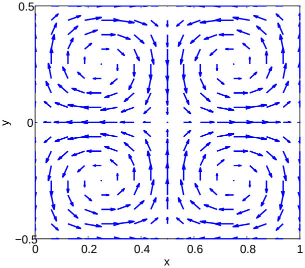
$$u(x, y, t) = -2\pi \sin(2\pi x)\cos(2\pi y)u_o + \cos(2\pi y)u_1(t)$$

$$v(x, y, t) = 2\pi \cos(2\pi x)\sin(2\pi y)u_o + \cos(2\pi y)v_1(t)$$

$$h(x, y, t) = \sin(2\pi x)\sin(2\pi y)u_o + \sin(2\pi y)h_1(t)$$

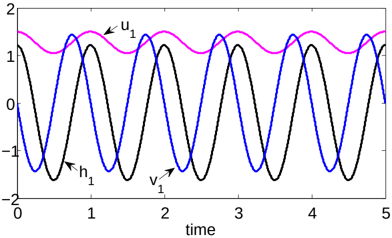
Cellular flow field

If $u_1 = v_1 = h_1 = 0$, flow field is constant & tracers stay w/in cells



Otherwise, $\dot{u}_o = 0, \quad \dot{v}_1 = -u_1 - 2\pi h_1, \quad \dot{u}_1 = v_1, \quad \& \quad \dot{h}_1 = 2\pi v_1$

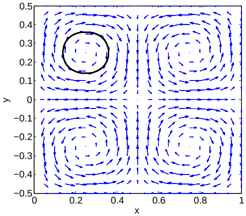
with initial conditions $[u_o(0), u_1(0), v_1(0), h_1(0)]$

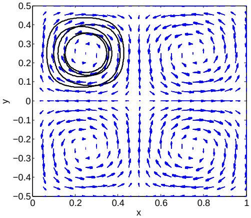


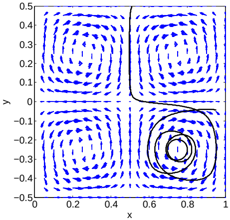
Left: $u_1(0) = v_1(0) = h_1(0) = 0, x(0) = .2,\ y(0) = .3$

Middle: $u_1(0) = v_1(0) = h_1(0) = 0.5, x(0) = .2,\ y(0) = .3$

Right: $u_1(0) = 0.2,\ v_1(0) = 1.3,\ h_1(0) = 1.4,\ x(0) = .51,\ y(0) = .498$







Particle filter for standard LADA

Test problem:

- $u_1(0) = v_1(0) = h_1(0) = 0.5,\ x(0) = .2,\ y(0) = .3$
- broad priors on (u_1, v_1, h_1), tight on (x, y) at $t = 0$
- run to $t = T$ (1 period of coefficients)
- 5 noisy observations of drifter

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- learn about $u_1(0), v_1(0), h_1(0)$ from Lagrangian observations

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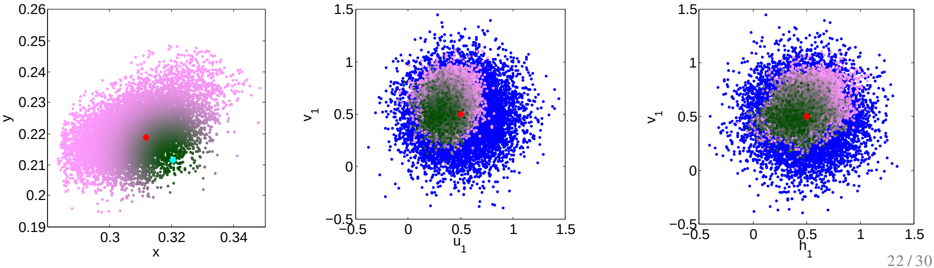

En route Lagrangian data – a test problem

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- Changes the observation space, so now ($z = \{x^d, y^d, u_1, v_1, h_1\}$ whole state)

$$H(z) = \begin{cases} (x^d(t), y^d(t)) & \text{for } t = jT_{obs} \\ \hat{h}(t) & \text{for } t = t_k, \;\; (j-1)T_{obs} < t_k < jT_{obs} \end{cases}$$

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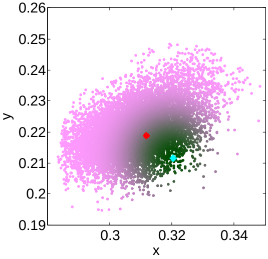
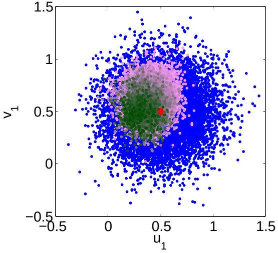
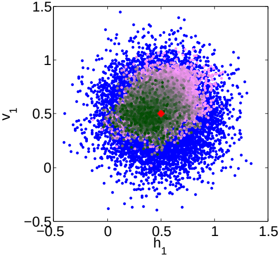
$$-\log(g) = \frac{(x^d - x^o)^2 + (y^d - y^o)^2}{2\sigma_d^2} + \frac{1}{N_h}\sum_{N_h}(h(z_k) - \hat{h}_k^o)^2/2\sigma_h^2$$

Particle filter w/en route observations

"traditional" LADA:







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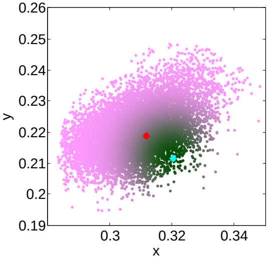

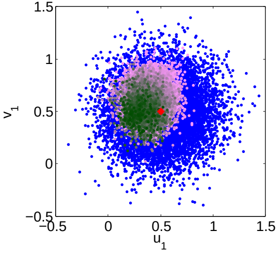

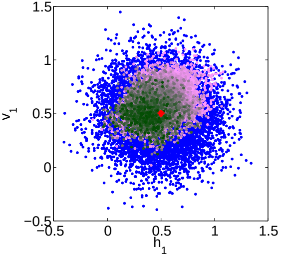

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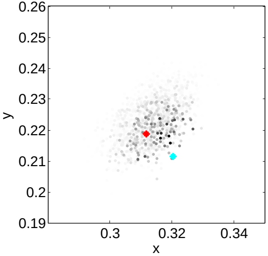

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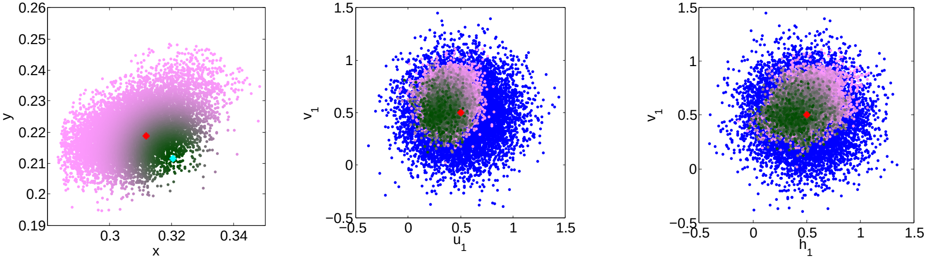

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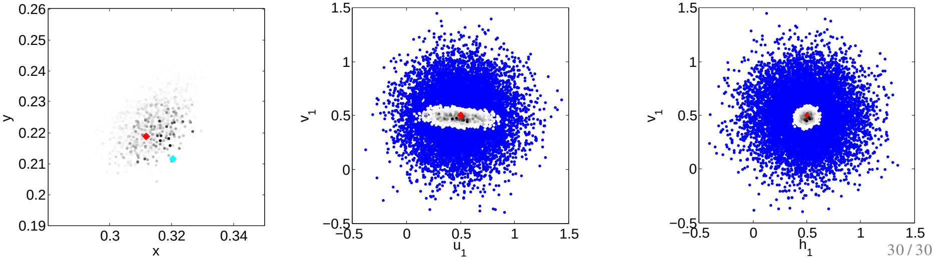
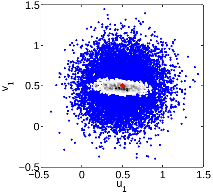
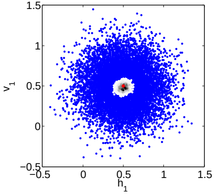


Improvement with en-route observations





Characterizing improvement: compare covariance matrices of prior and posterior distribution

	standard LaDA	w/height obs
ratio of traces	$\frac{0.046}{0.19} = 0.25$	$\frac{0.015}{0.19} = 0.08$
ratio determinants	$\frac{3.5\times10^{-6}}{2.4\times10^{-4}} = 0.015$	$\frac{4.5\times10^{-8}}{2.4\times10^{-4}} = 0.00019$

- robust over numerical experiments
- similar improvement for saddle case

Looking forward

Future directions:

- 3D model problem, depth one of observed en-route variables
- include flight plan control in glider problem
- assimilate likely paths between surfacing locations
  - endpoints pinned
  - assimilate for most likely paths w/Brownian bridge
- suggestions welcome