Challenges of Lagrangian Data Assimilation

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Supported by ONR

Augmented system

Append equations for drifters (floats, gliders, AUVs)

Ide, Jones and Kuznetsov (2002)

$$\mathbf{x} = \begin{pmatrix} \mathbf{x}_F \\ \mathbf{x}_D \end{pmatrix}$$ -- augmented state vector

$$\frac{d\mathbf{x}_F^{\mathrm{f}}}{dt} = M_F(\mathbf{x}_F^{\mathrm{f}}, t)$$ -- flow equations

$$\frac{d\mathbf{x}_D^{\mathrm{f}}}{dt} = M_D(\mathbf{x}_D^{\mathrm{f}}, \mathbf{x}_F^{\mathrm{f}}, t)$$ -- advection equation

Apply filtering to augmented system:

1. Ensemble Kalman Filter
2. MCMC (Particle Filter)

Key Issues

Vertical information propagation

Under what conditions does data collected at one layer contain info about another?

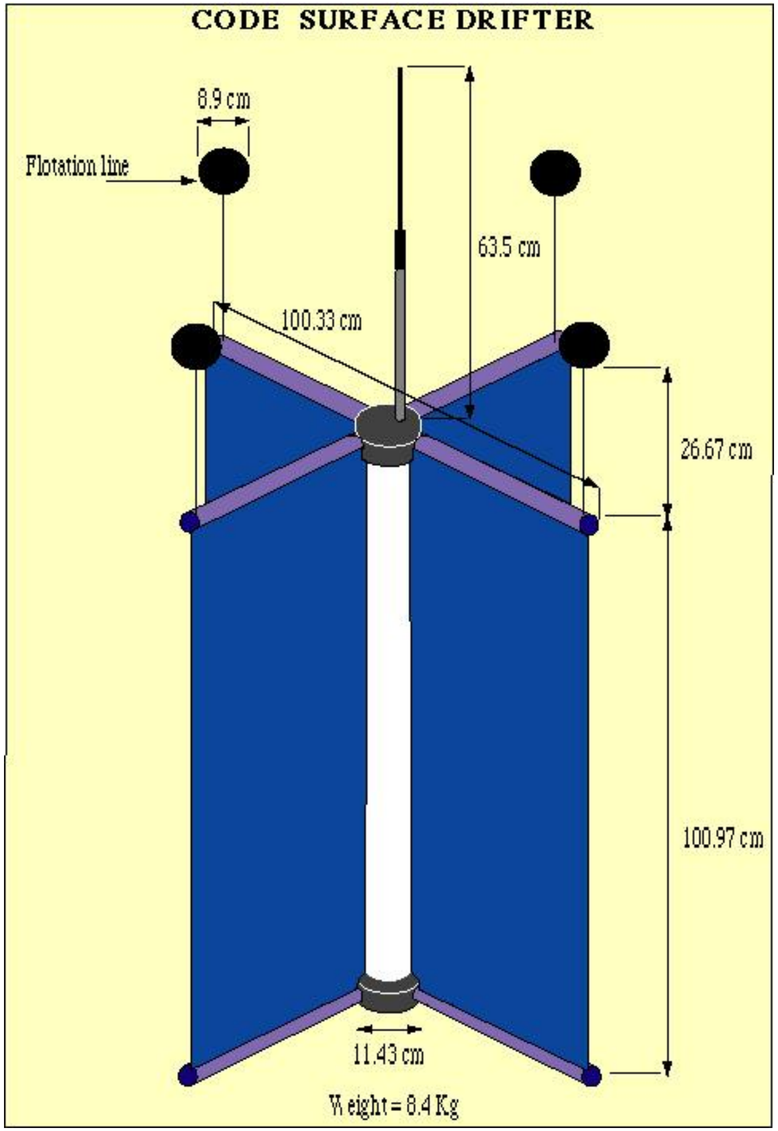
Observations at unknown locations

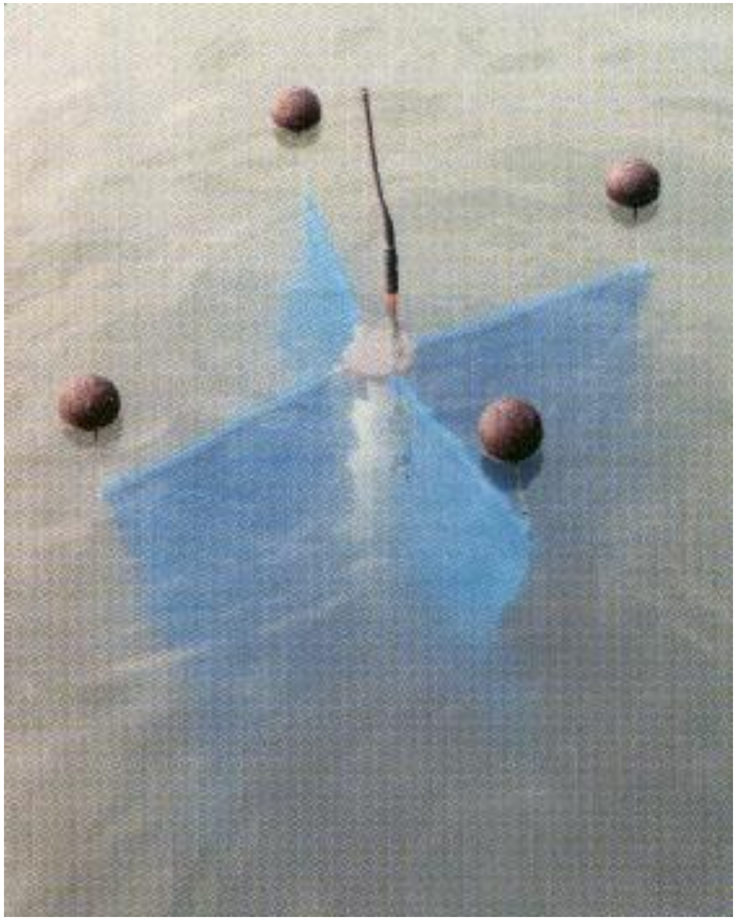
Need to estimate location with system state

Use of autonomous vehicles to collect subsurface data

Mission planning and assimilating data

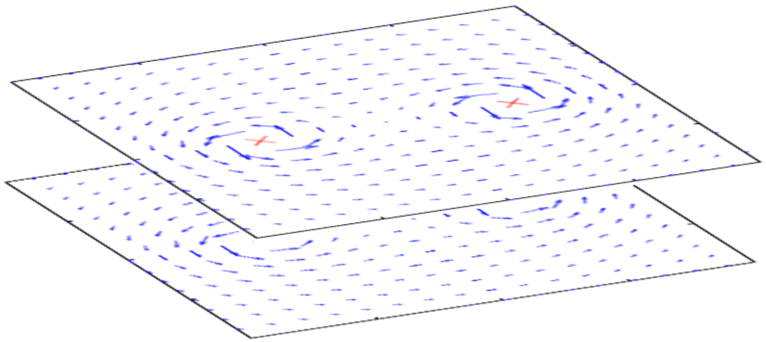
Drifters


CODE SURFACE DRIFTER
8.9 cm
Flotation line
63.5 cm
100.33 cm
26.67 cm
100.97 cm
11.43 cm
Weight = 8.4 Kg

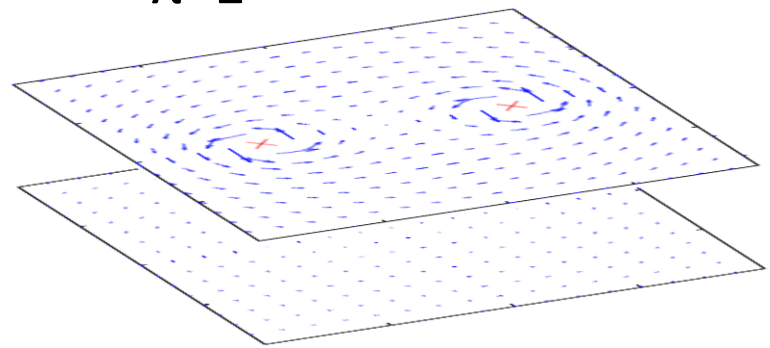


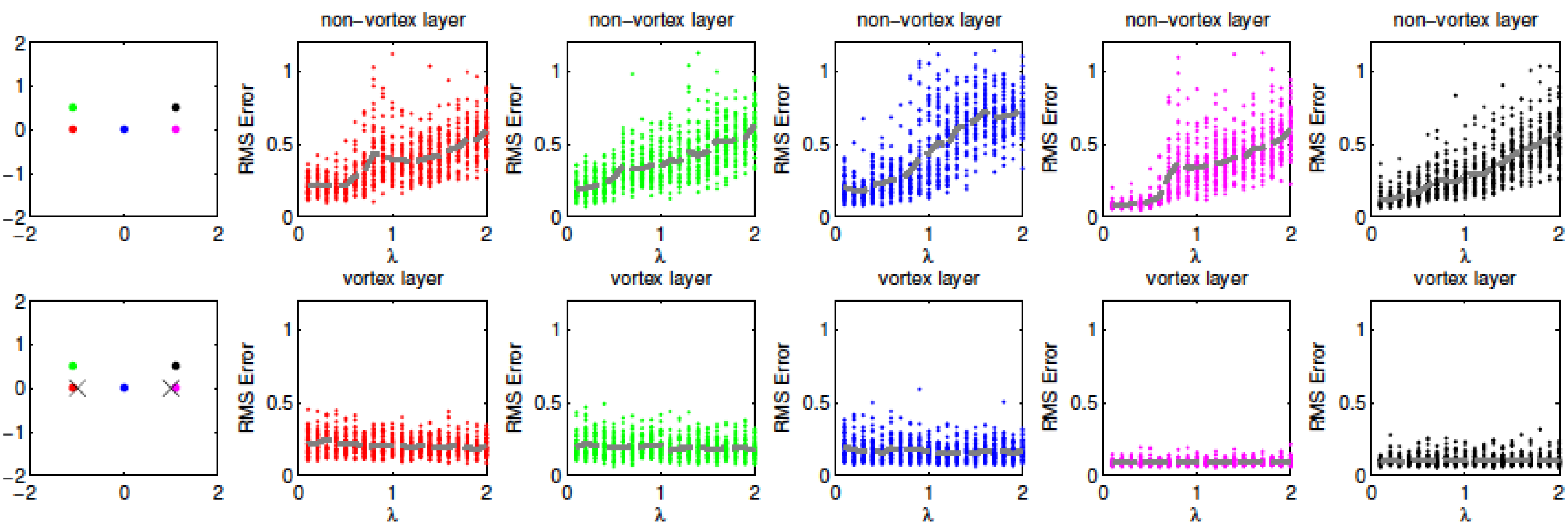
Assimilation of Lagrangian Data across Layers

λ=0.1

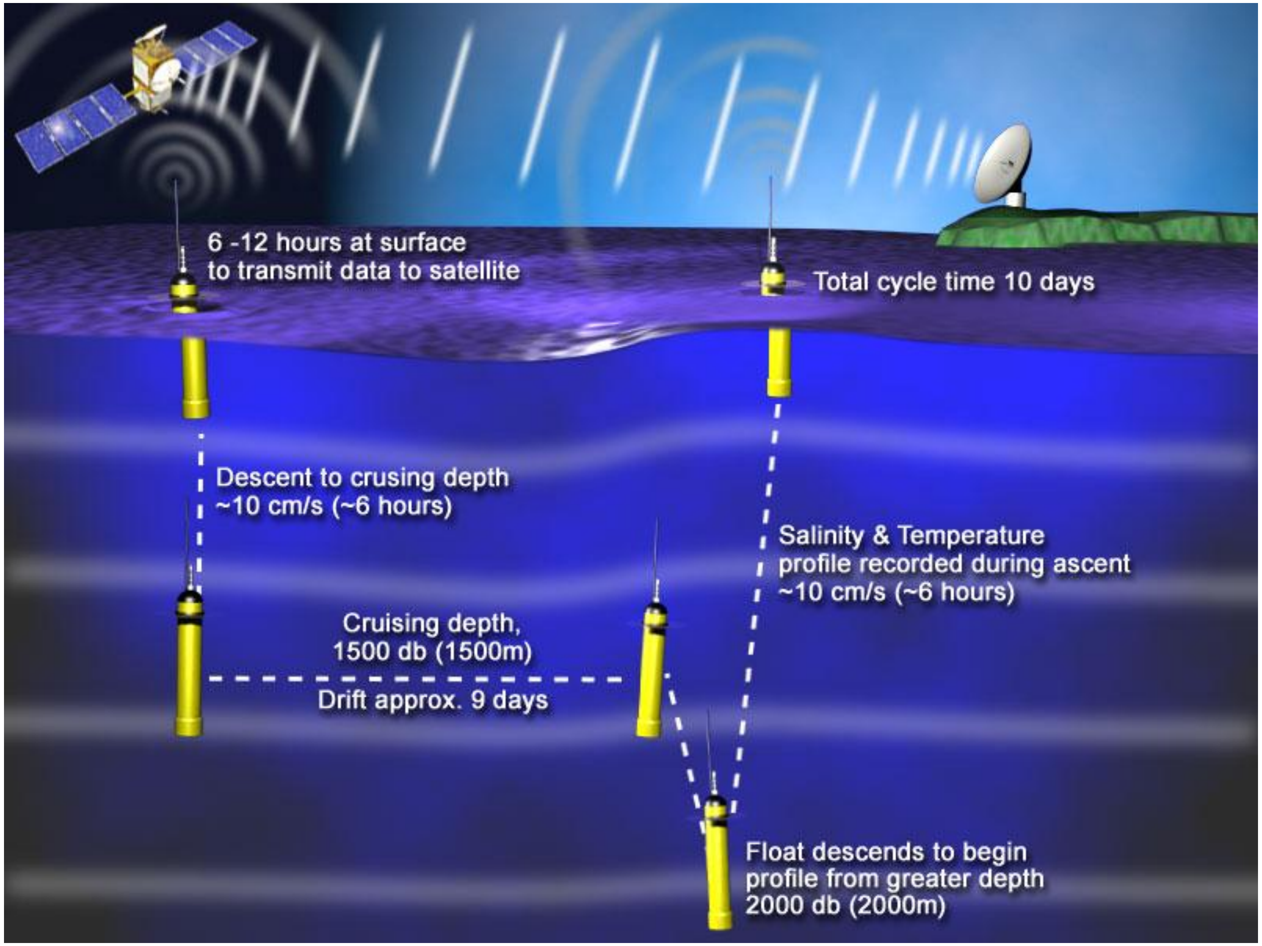


λ=1





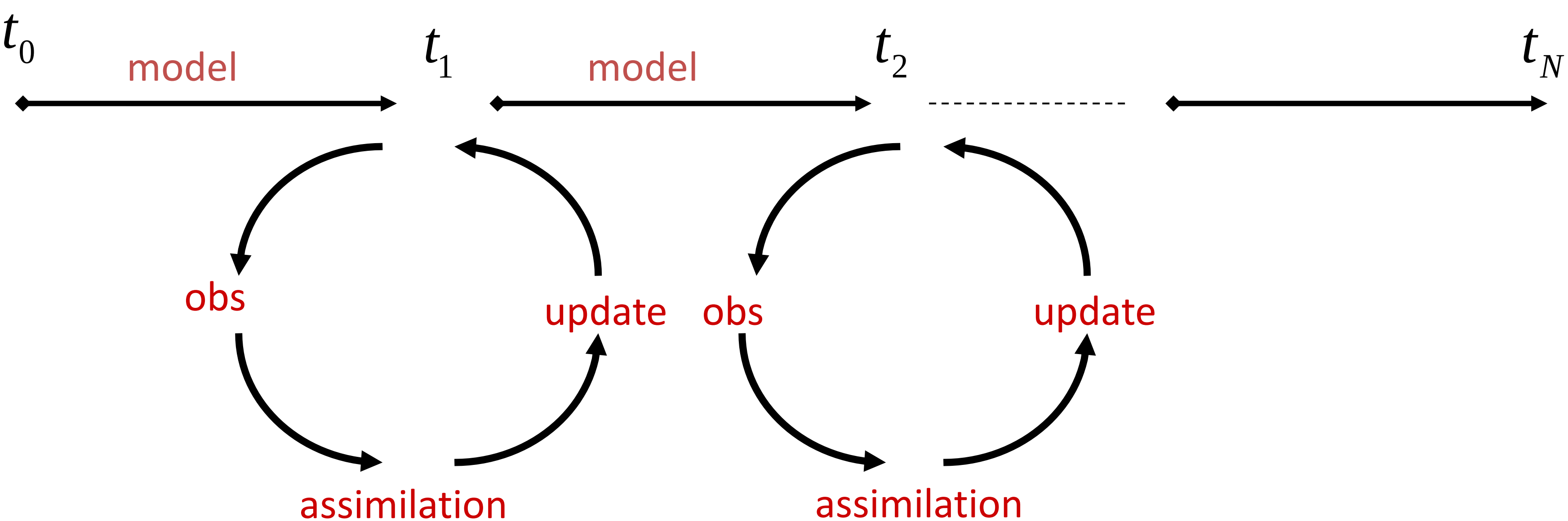
ARGO Floats


6 -12 hours at surface
to transmit data to satellite
Total cycle time 10 days
Descent to crusing depth
~10 cm/s (~6 hours)
Salinity & Temperature
profile recorded during ascent
~10 cm/s (~6 hours)
Cruising depth,
1500 db (1500m)
Drift approx. 9 days
Float descends to begin
profile from greater depth
2000 db (2000m)

Ocean gliders

Data Assimilation in Predictive Mode

Model + observations ⟶ prediction



Assimilation at: $t = t_i$

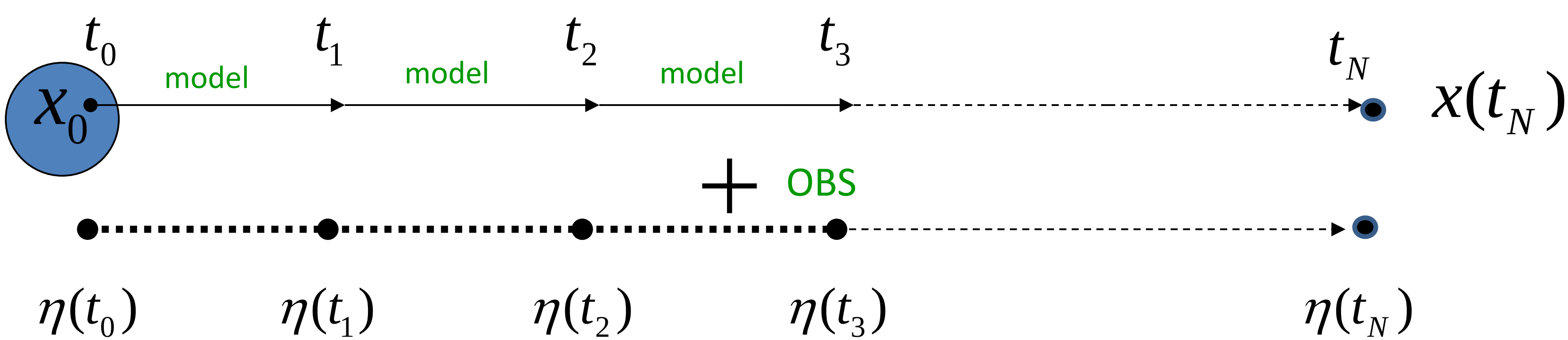
$$x_i^{\,a} = x_i^{\,f} + K_i\left(\eta_i - H(x_i^{\,f})\right)$$

$$P^{\text{posterior}}(x_i \,|\, h_i)\,\mu\, P^{\text{obs}}(h_i \,|\, x_i)\,P^{\text{prior}}\left(x_i\right)$$

Sequential DA/Forward Problem/Filtering

DA in State Estimation Mode

Model run + observations ⟶ state estimate



4DVAR: Minimize the cost function:

$$J(x) = \left\langle x_0 - x_0^*, B^{-1}\left(x_0 - x_0^*\right)\right\rangle + \sum_1^N \left\langle \eta_j - H(x(t_j)), R_j^{-1}\left(\eta_j - H(x(t_j))\right)\right\rangle$$

$x_0^* = \text{estimate}$ $\qquad B = \text{background error covariance matrix}$

$$P(x_0 \mid \eta) \propto \exp\left(-J(x_0)\right) = \exp\left(-\left\langle x_0 - x_0^*, B^{-1}\left(x_0 - x_0^*\right)\right\rangle - \sum_1^N \left\langle \eta_j - H(x(t_j)), R_j^{-1}\left(\eta_j - H(x(t_j))\right)\right\rangle\right)$$

Variational DA/Inverse Problem/Smoothing

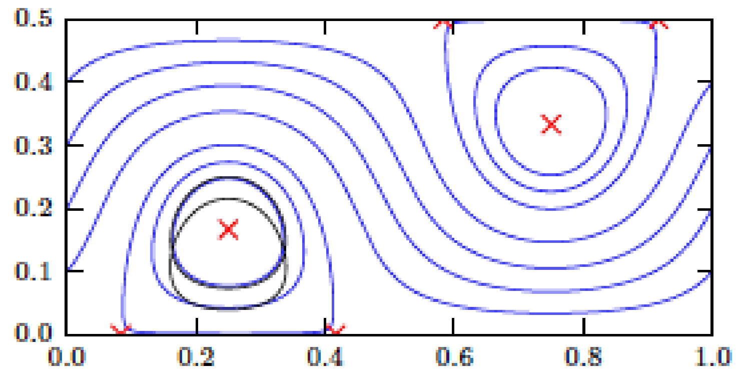
Lagrangian Data in a Traveling Wave

$$\psi(z) = -cy + A\sin 2\pi kx \sin 2\pi y$$

$$z = (x, y),\ v = \left(\frac{\partial \psi}{\partial y}, -\frac{\partial \psi}{\partial x}\right)$$

$$\frac{\partial v}{\partial t} = 0$$

$$\frac{dz}{dt} = v(z)$$



McDougall and J. (in prep)

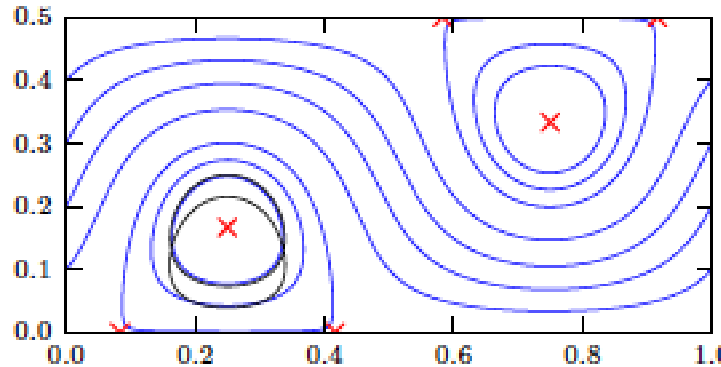
Instrument control to optimize information

$$\frac{\partial v}{\partial t} = 0$$

$$\frac{dz}{dt} = v(z) + f(z)$$

Add *simple* control:

$$f(z) = (V, 0)$$



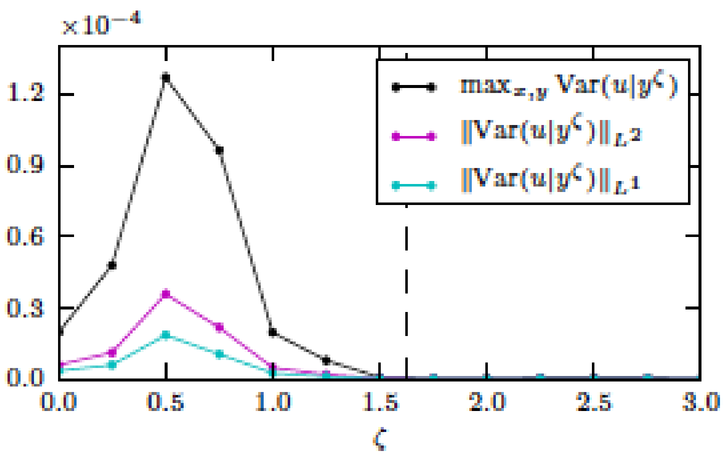
$$\mathbf{y}_k^1 = \mathbf{x}(t_k) + \eta_k, \quad \eta_k \sim \mathcal{N}(0, \sigma^2 I), \quad k = 1, \ldots, K/2$$ no control

$$\mathbf{y}_k^2 = \mathbf{x}(t_k) + \eta_k, \quad \eta_k \sim \mathcal{N}(0, \sigma^2 I), \quad k = K/2, \ldots, K$$ control

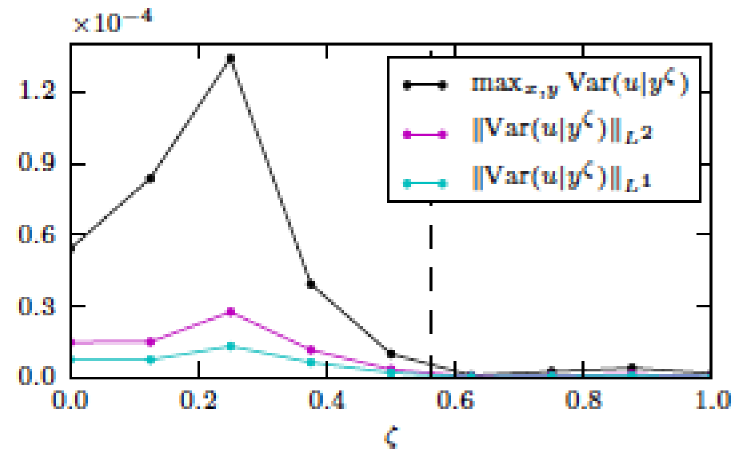
Want: $P(v \mid y^1, y^2)$

$$f(z) = (V, 0)$$

$$f(z) = (V, V)$$



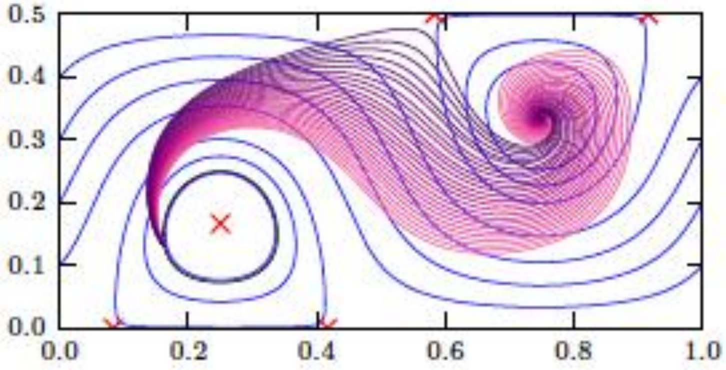
×10⁻⁴
$\max_{x,y} \mathrm{Var}(u|y^{\zeta})$
$\|\mathrm{Var}(u|y^{\zeta})\|_{L^2}$
$\|\mathrm{Var}(u|y^{\zeta})\|_{L^1}$
1.2
0.9
0.6
0.3
0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0
ζ

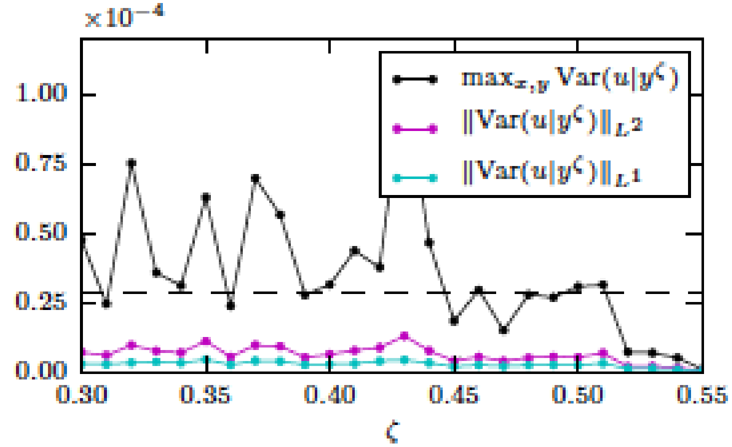


×10⁻⁴
$\max_{x,y} \mathrm{Var}(u|y^{\zeta})$
$\|\mathrm{Var}(u|y^{\zeta})\|_{L^2}$
$\|\mathrm{Var}(u|y^{\zeta})\|_{L^1}$
1.2
0.9
0.6
0.3
0.0
0.0 0.2 0.4 0.6 0.8 1.0
ζ

A Posteriori Control

$$f(z) = \nu \nabla_z \mathrm{E}(u|y^1)$$





Lagrangian DA issues:

- Non-standard data
- Obs location fixing issue
- Prospect of designing optimal obs strategies