Geometry of 3D Dipole Interactions: Painting
HYCOM

MURI-4D-DS Workshop

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Overview: ’Paradigm’ 3D Ocean Structures?

- **2D Coherent Structures:**
  - Classical phase space pictures:
  - Hetero-clinic orbits: Cat’s Eye
    - Meandering Jet
    - Eddy-eddy interaction
    - Dipoles
  - Homoclinic Orbits:
    - Eddy-pinchoff
    - Eddy-jet interaction
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- 3D Coherent Structures?
  - 2D + 1
    - z-dependent 2D structures
    \[ \mathbf{u} = (u(x, y, z), v(x, y, z)) \]
  - 3D + symmetry
    \[ \mathbf{u} = (u(r, \theta), v(r, \theta), w(r, \theta)) \]
  - Fully 3D? Role of \( w \) component?
    - Isopycnal advection.
    - Diagnose \( w \) in cartesian coordinates.
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Goal: Tool for quickly visualizing 3D advective pathways in available model data sets.
Eulerian Approach to LCS

- Trouble: Proxy measures rely on differenting gridded trajectories w.r.t. initial conditions.
- Time dependent 3D structures → many particles.
- Look instead at a differentiable scalar field:

\[
\frac{\partial \phi}{\partial t} + (\mathbf{u} \cdot \nabla) \phi = \kappa \nabla^2 \phi + S(\mathbf{x}, t)
\]

\[
\phi(\mathbf{x}, 0) = \phi_0(\mathbf{x})
\]

\(\kappa\) and \(S\) prescribed
\(\mathbf{u}(\mathbf{x}, t)\) given \((I(\mathbf{u}_{ijkl}))\)

- ‘Judicious’ choice of \(S\) and/or \(\phi_0(\mathbf{x})\) ....
- Computationally minimal \(\kappa\) ensures differentiable \(\phi\).
- Backwards in time evolution with \(\mathbf{u} \rightarrow -\mathbf{u}\).
Advection-Diffusion + HYCOM

\[ \frac{\partial \phi}{\partial t} + (\mathbf{u} \cdot \nabla) \phi = \kappa \nabla^2 \phi + S(\mathbf{x}, t) \]

\[ \phi(\mathbf{x}, 0) = \phi_0(\mathbf{x}) \]

- Standard conservative, explicit 2nd order finite-differences.
- 2-pass MPDATA for advection.
  - Upwind + Anti-Diffusive.
  - Stable w/no explicit diffusion.
- Dufort-Frankel diffusion.
- Interpolation:
  - cubic in space.
  - linear in time.

- Fancy CAF Code (F2008)
- User-defined grid

\[ \Delta x < \Delta x_{\text{Model}} \]

- Split diffusion:

\[ \kappa \nabla^2_h \phi + \alpha \frac{\partial^2 \phi}{\partial z^2} \]
Advection-Diffusion: HYCOM Results

HYCOM - GOM30.1

- HYCOM GOM: 1/25 Degree, 2010 archive.
- Daily output, Cartesian grid.
- \( w \) available.

Case 1: Plane Source

- Loop Current Dynamics:
- \( S(x, t) = \text{constant on } x - z \) plane at inflow
- \( \Delta x = 0.75 \Delta x_{HYCOM} \)
- Regrid: \( \Delta z = 25 \text{m} \)
- \( \sim 5 \) minutes for 50 days
  \( (300 \times 300 \times 20\text{layers}) \)
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- \( w \neq 0, \alpha \neq 0 \)

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Case 2: Isolated Structures

- Deep-water, western Gulf.
- One (of many) multi-pole pairs.
- $\phi_0(\mathbf{x}) = \text{constant in } z \text{ dependent cycolone-anticyclone.}$
- Strong vertical component.
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Advection-Diffusion: HYCOM Results

Center of Mass:

\[ M_i(t) = \iiint x_i \phi(x, t) dV \]
To Do List:

- Compare scalar/LCS proxies
- Raw HYCOM output:
  - $\Delta t \sim 1$ hour
  - Isopycnal coordinates:
    
    $$\frac{\partial h\phi}{\partial t} + (\mathbf{u} \cdot \nabla) h\phi = \nabla \cdot \kappa h \nabla \phi + \tilde{S}(\mathbf{x}, t)$$

- Extend to other OGCM data bases.