Collaborative Robot Tracking of Geophysical Flows:

How Local Measurements Discover Global Structures

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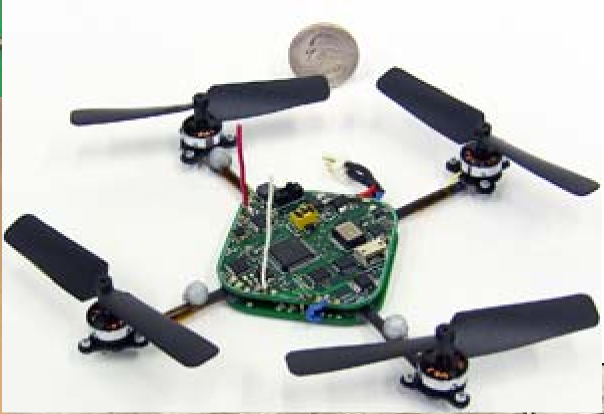
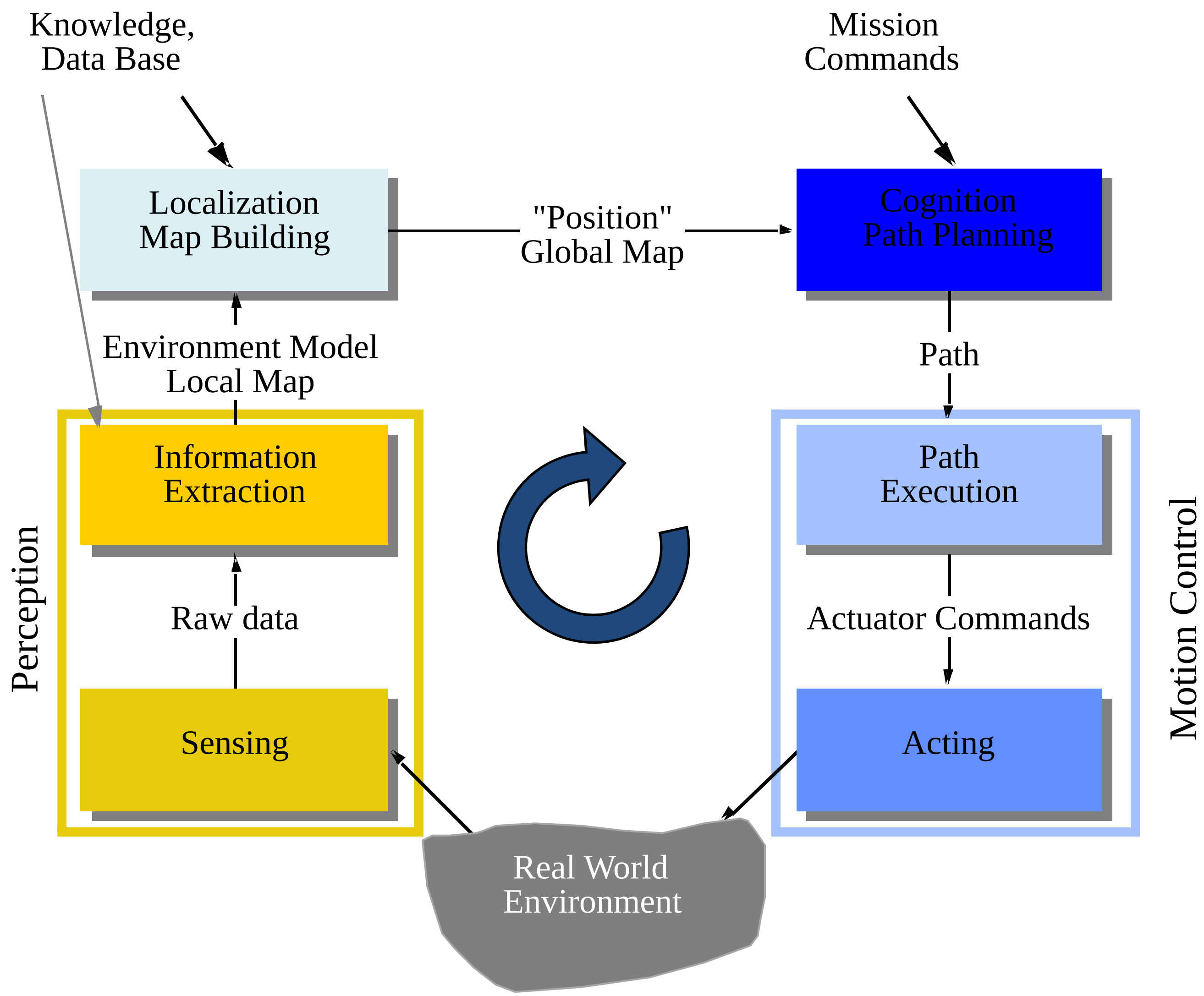
Drexel University

Highlights and Lowlights



Anatomy of a Robot Control Scheme



The Last Robotic Frontier

by: Kai Schumann, CA Dept of Public Health (volunteer)

CBS News

System Design & Control

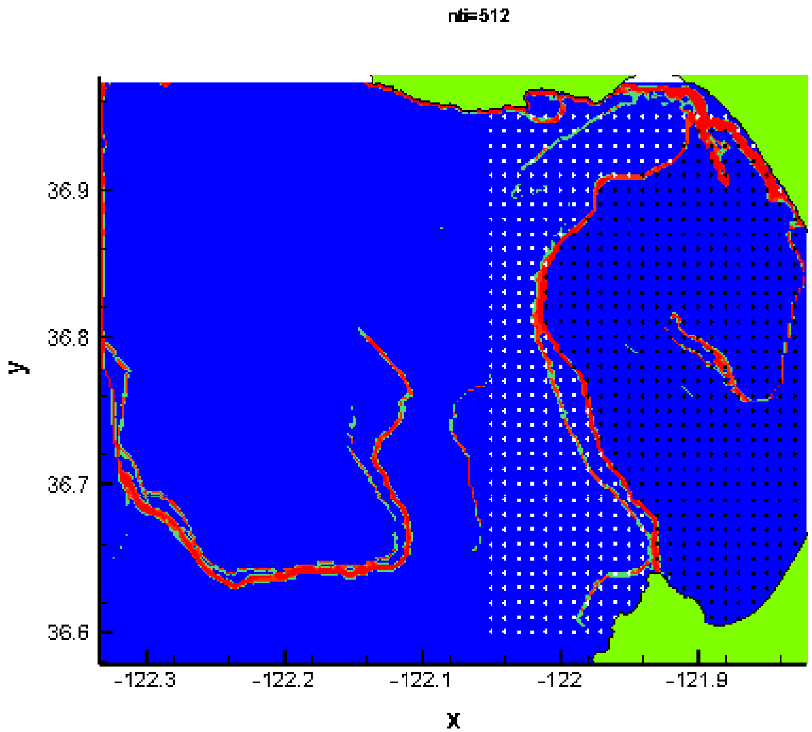
- Maneuverability of ASVs and AUVs
- Perception
  - » Proprioceptive vs Exteroceptive Sensing
  - » Biology vs Engineered Systems
- Communication
  - » Low Bandwidth
  - » Lossy

System Design & Control

- Geophysical Fluid Dyanmics

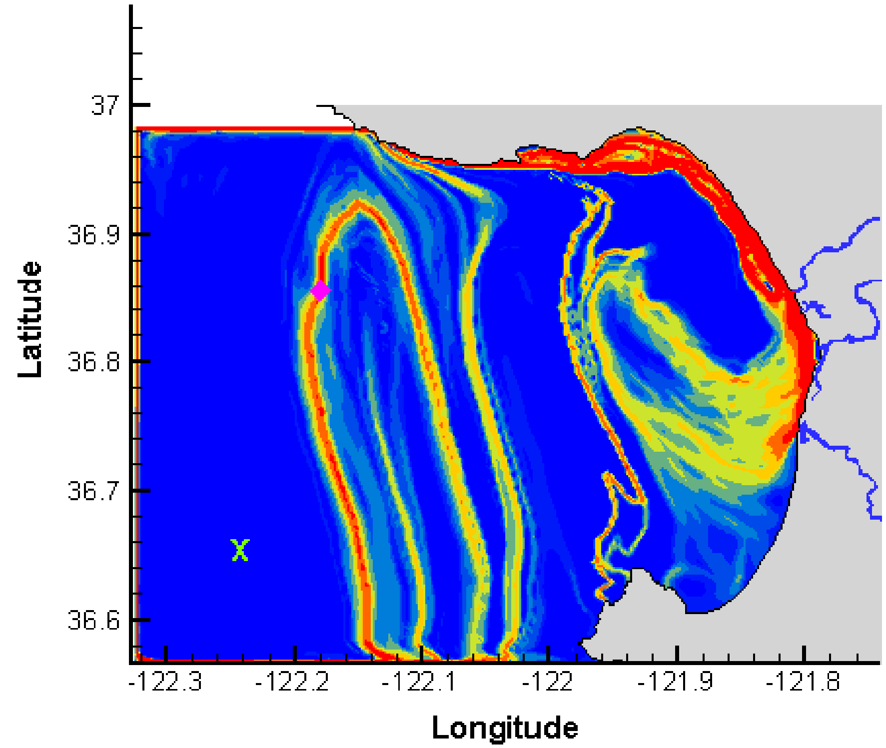
Significance of LCS for Robotics

For Understanding Dynamics



Courtesy of Shadden (http://mmae.iit.edu/shadden/LCS-tutorial/)

For Autonomy



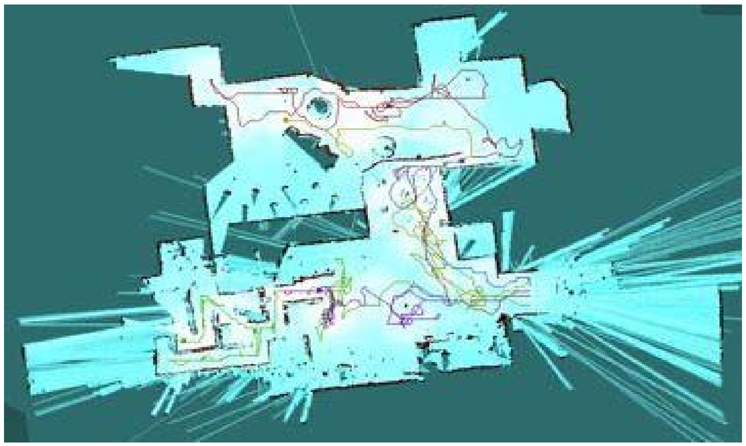
by: Inanc, Shadden, and Marsden (ACC 2005)

Unfortunately, for robotics

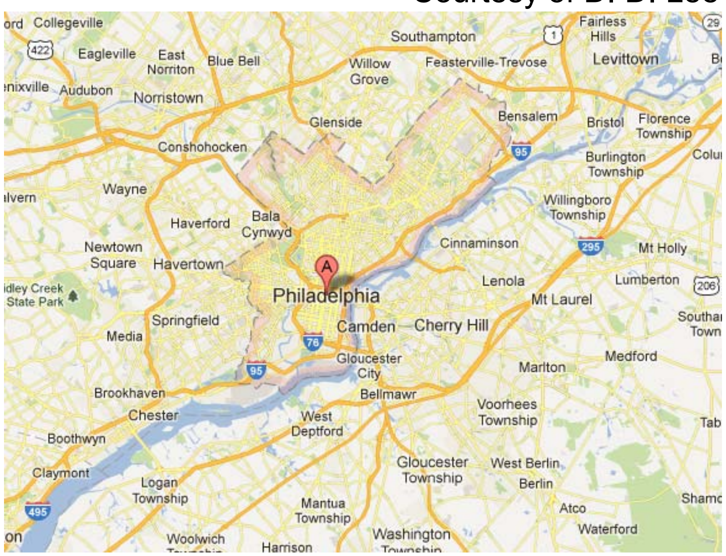
1. Global structures
2. Low spatio-temporal resolution of data

Changing our View of the Ocean

Air & Ground Navigation

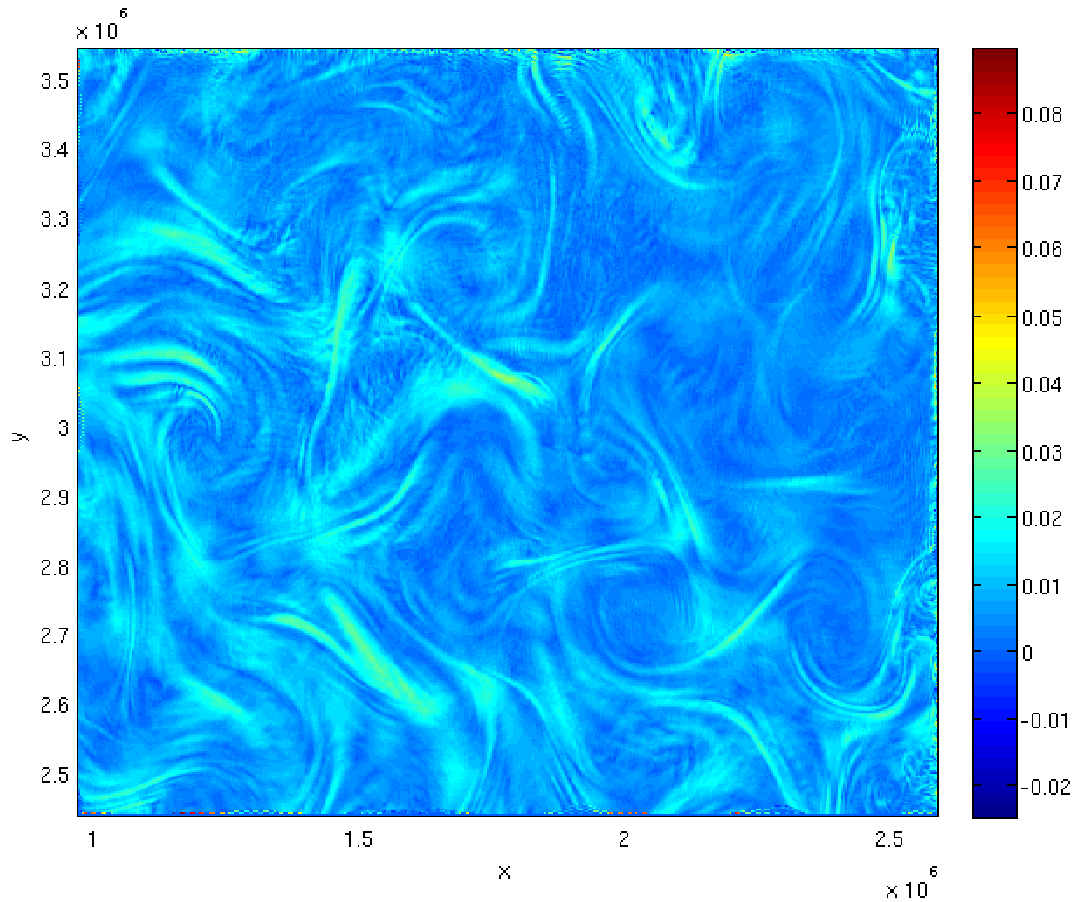


Courtesy of D. D. Lee



© Google Maps

The Oceanic Super Highway



Courtesy of by: Eric Forgoston (Montclair State)

A Story in Two Acts

» Tracking Coherent Structures

Development of *distributed control and coordination* strategies for autonomous robots to *quantify key transport phenomena in flows*

» Distributed Sensing and Sampling

Development of *distributed strategies* to control the spatial distribution of autonomous robots *given the flow dynamics*

Existing Strategies

- AUV and ASV Planning, Control, & Coordination
  - » Planning & Control: Whitcomb et al. , Eustice et al. (UMich), Leonard et al. (MIT), Sukhatme et al. (USC), Leonard et al. (Princeton), Paley et al. (UMD), Gupta et al. (UMD), Lermusiaux et al. (MIT), Smith et al. (QUT), Rhoads et al. (UCSB), Inanc et al. (CalTech)
  - » Perception: Eustice et al. (UMich), Leonard et al. (MIT), Sukhatme et al. (USC), Zhang et al. (GATech), Lynch et al. (Northwestern), Farques et al. (NPS), Horner et al. (NPS)
  - » Coordination: Bishop et al. (Naval Academy), Esposito (Naval Academy), Sukhatme et al. (USC), Bullo et al. (UCSB), Zhang et al. (GATech), Paley et al. (UMD), Chung et al. (NPS)
- Resource Allocation
  - » Distributed Algorithms: Diaz (CMU), Mataric (USC), Parker (UTenn), Veloso (CM), Shen (USC)
  - » Macroscopic Approaches: Berman et al. (Harvard/ASU), Hsieh et al. (Drexel), Lerman et al. (UCSB), Martinoli et al. (EPFL), Milutinovic et al. (UCSC)

Act I

Tracking Coherent Structures

» Objective:

- Track material lines that separate regions of flow with distinct dynamics using a team of N robots

» Approach:

- Take advantage of the fluid dynamics
- Control strategy based Proper Interior Maximum (PIM) Triple Procedure *(Nusse and Yorke, 1989)*

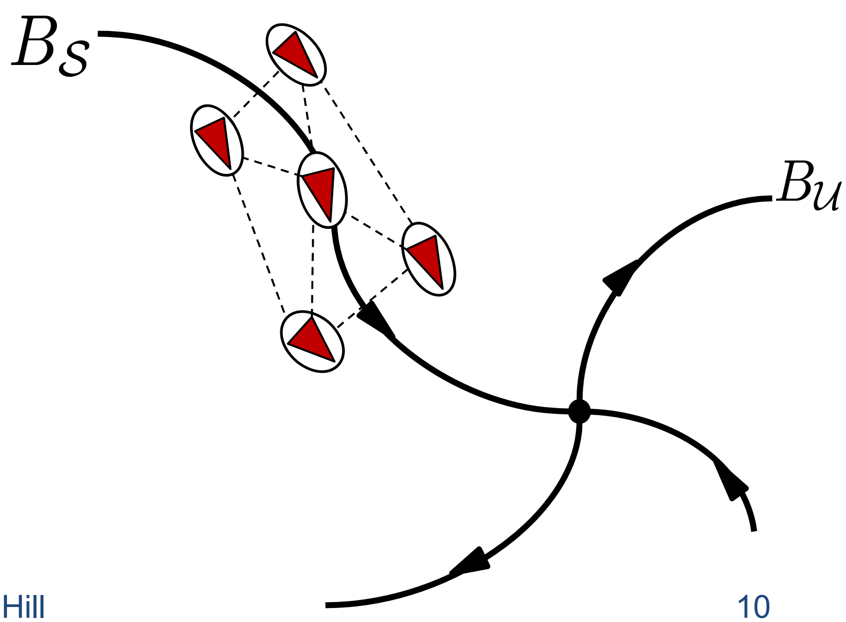
Tracking Coherent Structures

Tracking Coherent Structures

- N robot team in 2D w/ 2D vehicle kinematics

$$\dot{x}_i = V_i \cos\theta_i + u_i$$
$$\dot{y}_i = V_i \sin\theta_i + v_i$$

- Flow modeled by 2D planar conservative vector $\dot{\mathbf{x}} = F(\mathbf{x})$ field w/ $F : \mathbb{R}^n \to \mathbb{R}^n$
- B_S and B_U are 1-D curves

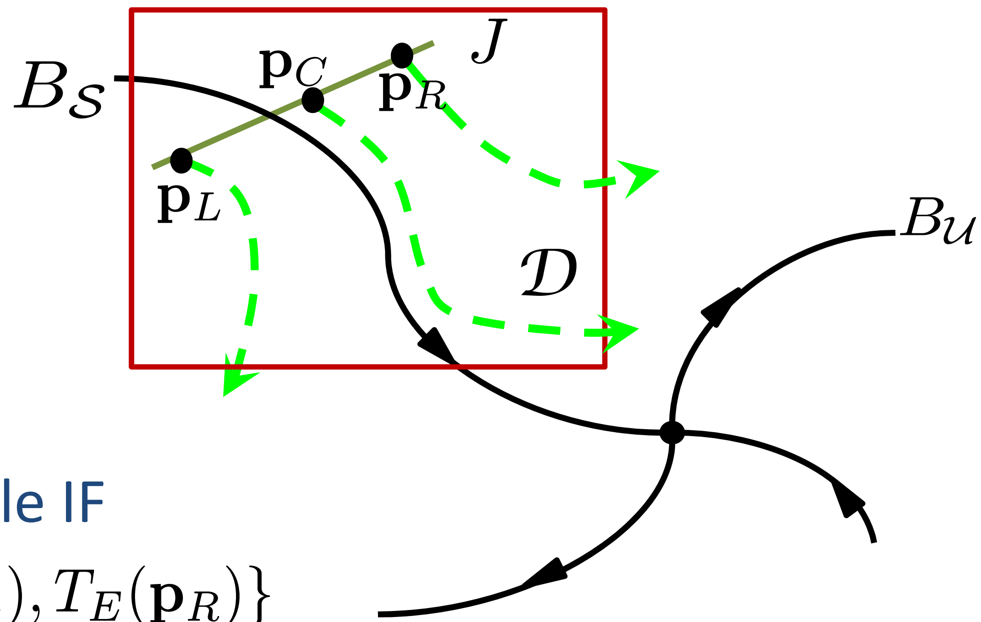


- Additional Assumptions
  - $\rho_{min}(B_S)$, $\rho_{min}(B_U) > r$
  - Min vehicle turning radius r

Proper Interior Maximum Triples

PIM Triples

- » $\mathcal{D}$ is a closed and bounded set w/ no attractors
- » Escape time $T_E(\mathbf{p})$ of point **p** from $\mathcal{D}$
- » J is line segment that crosses B_S



Nusse and Yorke (1989)

- » [$\mathbf{p}_L$, $\mathbf{p}_C$, $\mathbf{p}_R$] is a PIM Triple IF

$$T_E(\mathbf{p}_C) > \max\{T_E(\mathbf{p}_L), T_E(\mathbf{p}_R)\}$$

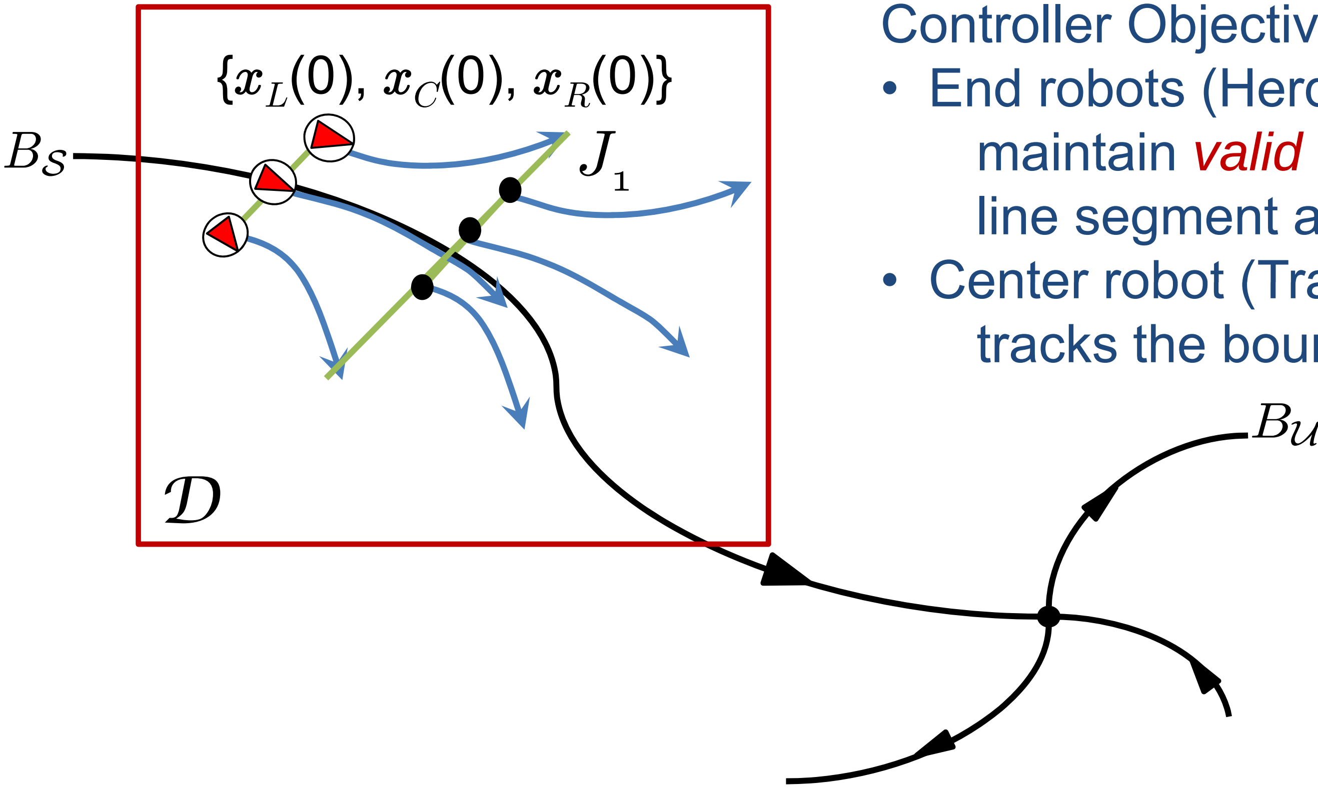
and $\{\mathbf{p}_L, \mathbf{p}_C, \mathbf{p}_R\} \subset J$

3-Robot PIM Triple Inspired Control

Given $\dot{\mathbf{x}} = F(\mathbf{x})$

Initial positions lie on J_0, a saddle straddle line segment

Mapping out the boundary



Controller Objectives:

- End robots (Herders):
  maintain *valid* saddle straddle line segment at all times
- Center robot (Tracker):
  tracks the boundary B_S or B_U

Analysis

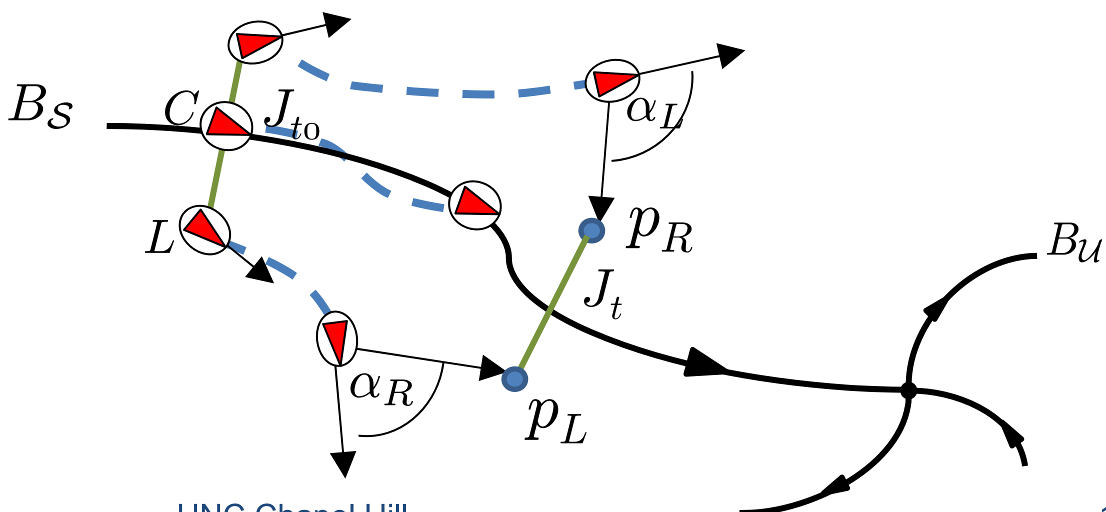
Theorem: *(Hsieh et. al., ICRA 2012)*

Given a team of 3 robots with kinematics given by

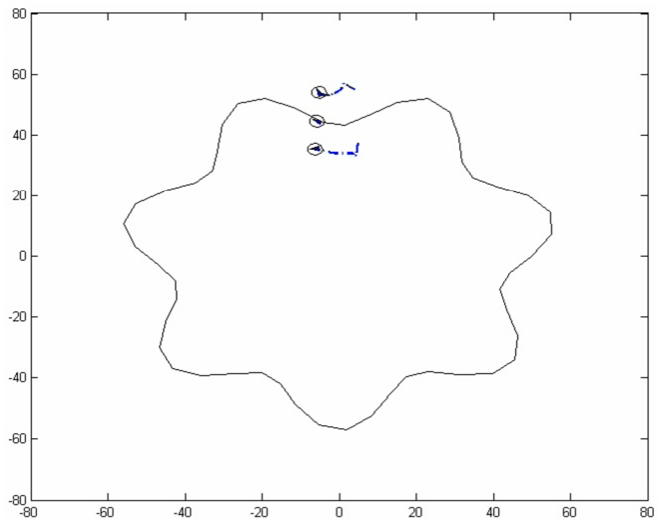
$$\begin{aligned} \dot{x}_i &= V_i \cos\theta_i + u_i \\ \dot{y}_i &= V_i \sin\theta_i + v_i \end{aligned}$$

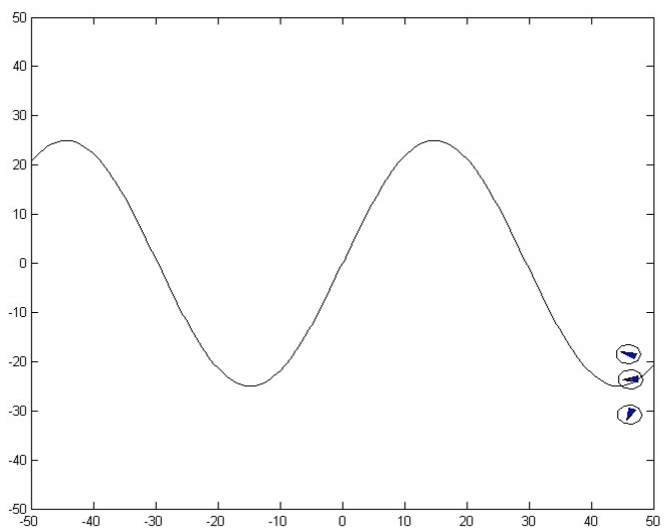
where u_i and v_i are given by a 2D planar conservative field, the feedback control strategy maintains a valid saddle straddle line segment in the time interval [t, t+Δt] if the initial positions of the robots **x**(0) is a valid PIM triple.

Sketch of Proof:



Tracking in Stationary Flows





Some Remarks

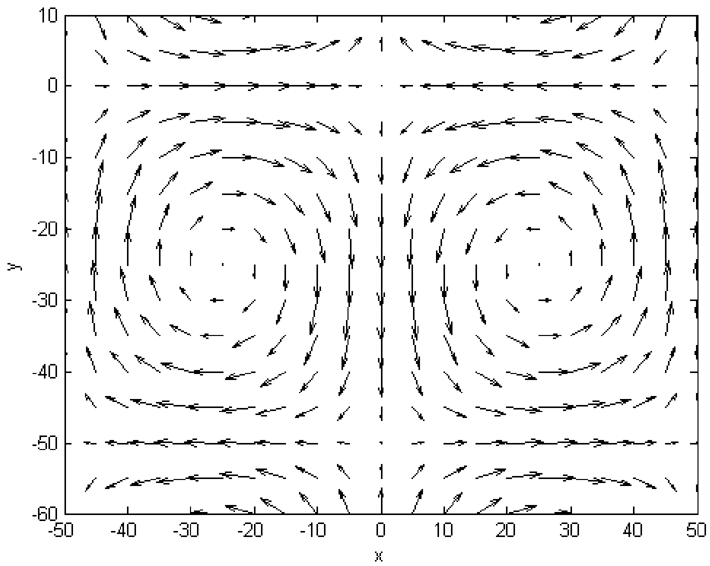
- PIM Triple inspired approach reveals *global* structures through *local* sensing alone
- Requires initial location of the unstable ridge

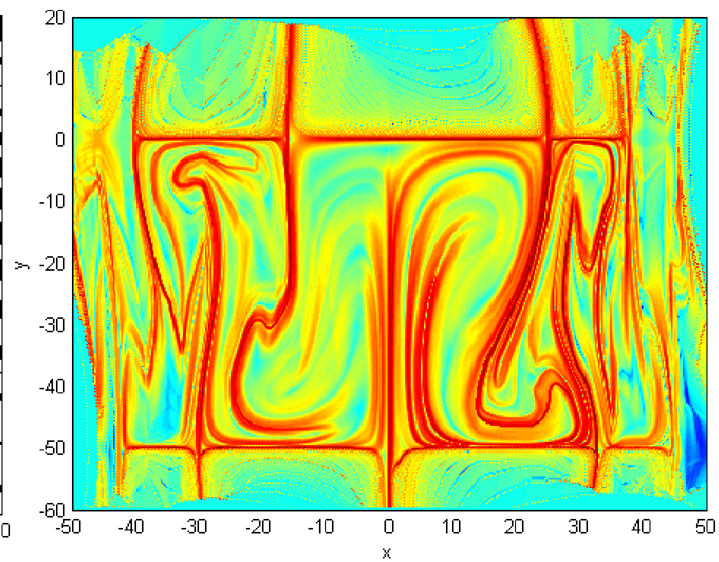
Proposition:

» Given a team of 3 robots w/ assumed 2-D kinematics in a 2-D conservative flow, the PIM Triple inspired control strategy in an estimate of B_S, or $\hat{B}_{\mathcal{S}}$, such that $\langle B_S, \hat{B}_{\mathcal{S}} \rangle_{L_2} < W$ for some $W > 0$.

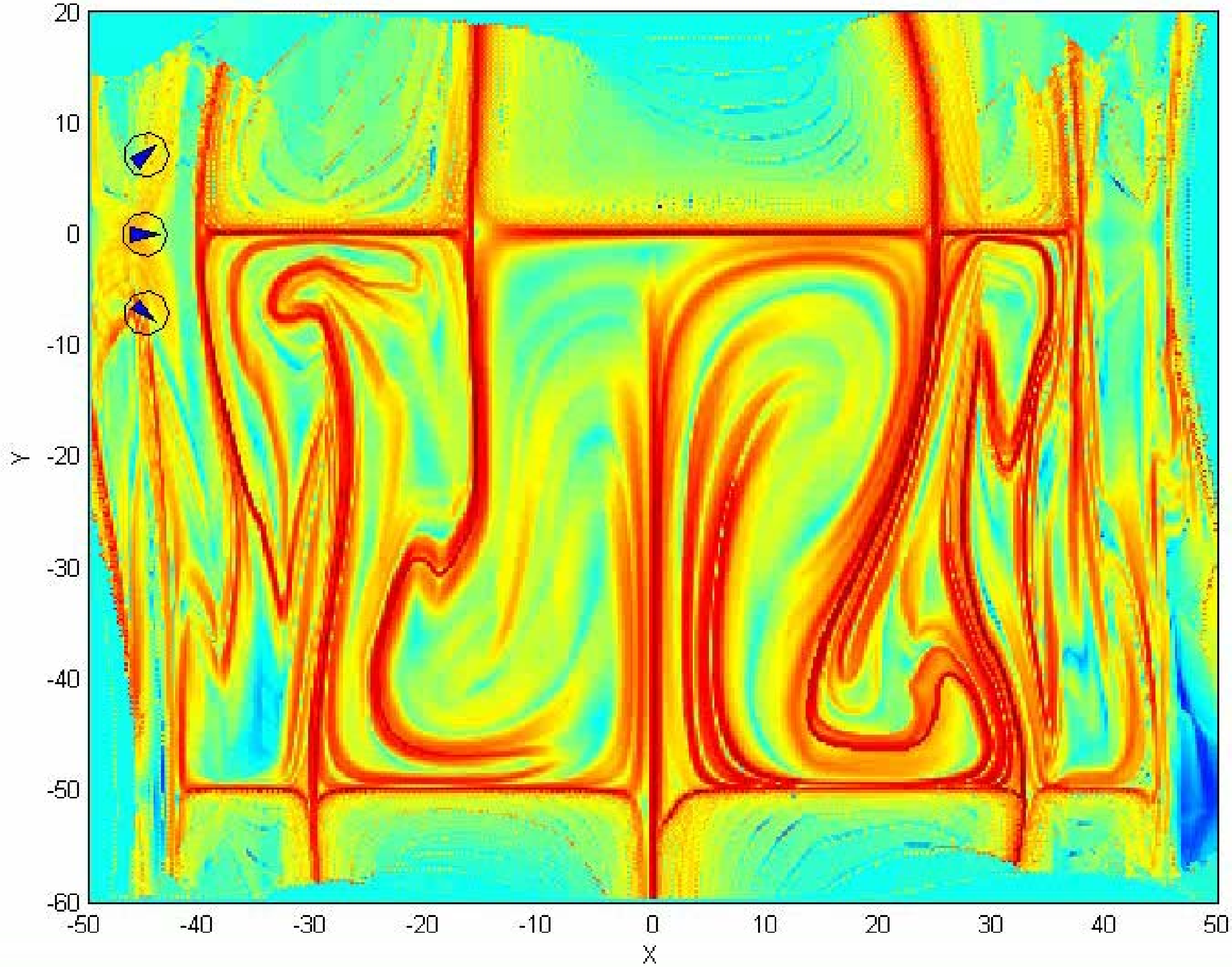
Manifolds Static vs. LCS

- Lagrangian Coherent Structures:
  - » Time-dependent versions of stable/unstable manifolds of a saddle point
  - » In 2-D, analogous to ridges defined by maximum local instability quantified by local FTLEs
- Driven double-gyre model w/ noise

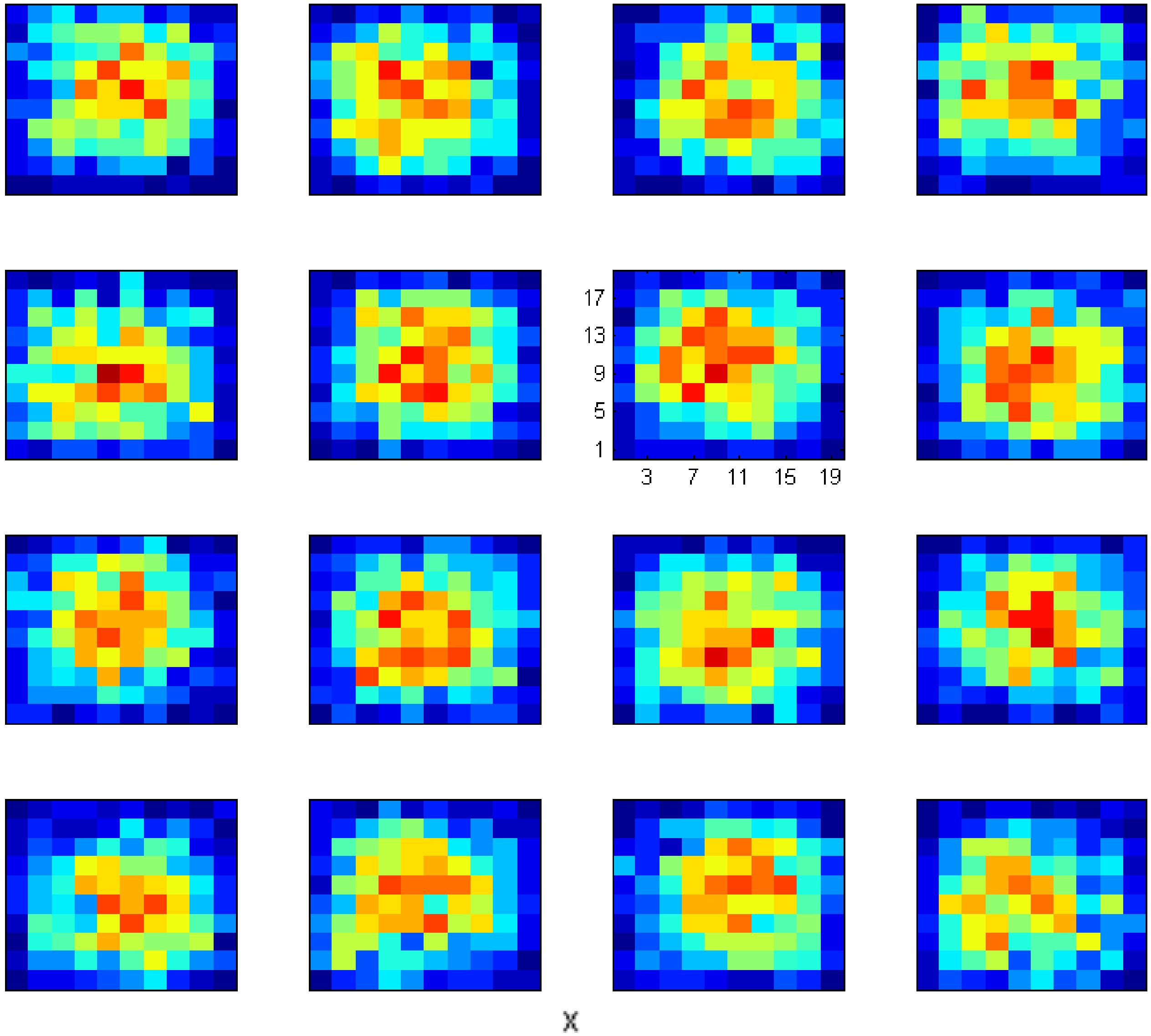




Tracking LCS in Periodic Flows

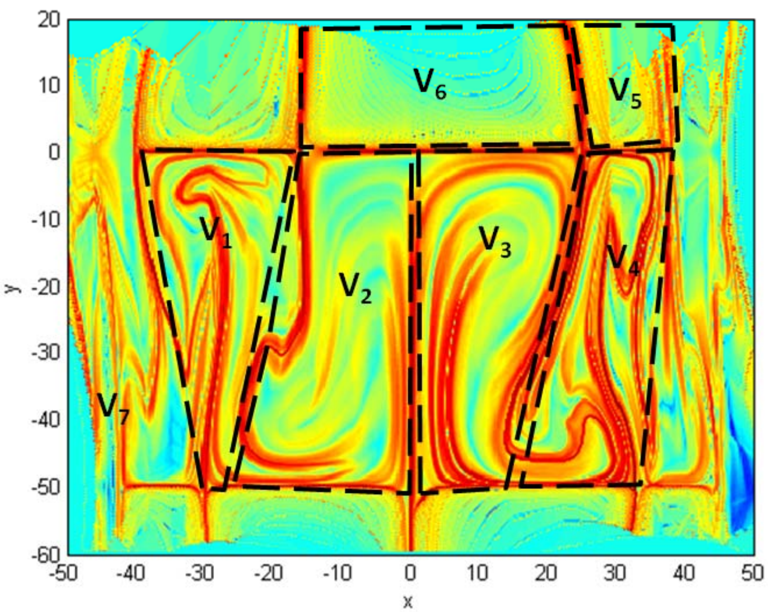


LCS and the Presence of Noise



Distributed Spatial Allocation of Autonomous Sensing Resources

- » N robots w/ 2-D kinematics $\mathbf{q}_k = \mathbf{u}_k + F(\mathbf{q}_k)$
- » $\dot{\mathbf{x}} = F(\mathbf{x})$ is a 2-D planar vector field
- » Leverage the environmental dynamics: Flow dynamics + Noise
- » $\{V_1, \ldots, V_M\}$ s.t. $V_i \subset \mathcal{W}$ and $\cup V_i = \mathcal{W}$
- » Robots have ability to localize

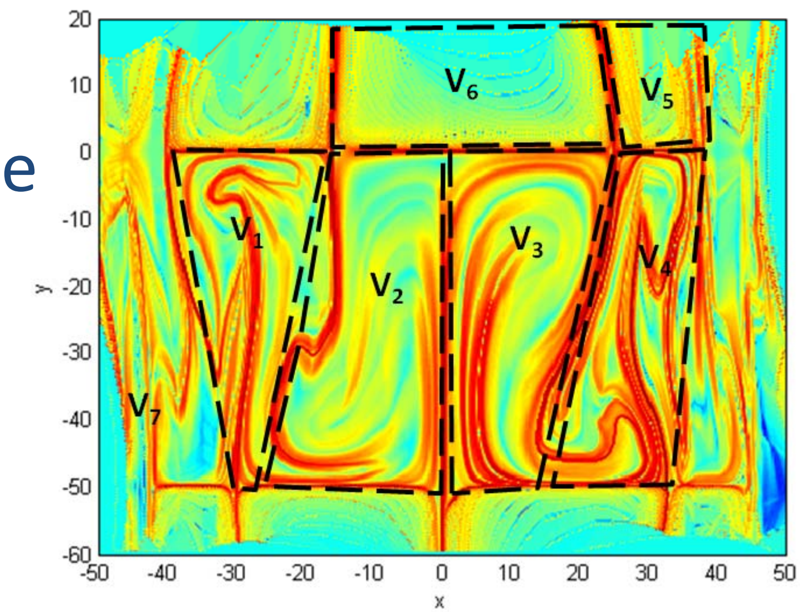


Assumptions

- N robots w/ 2-D kinematics

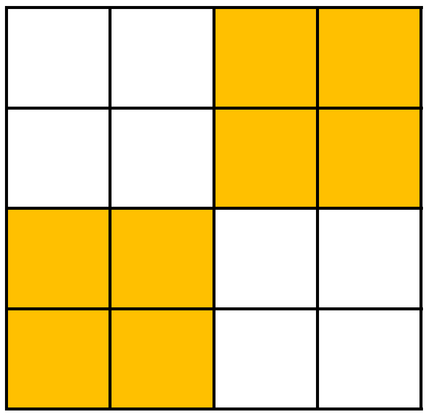
$$\mathbf{q}_k = \mathbf{u}_k + F(\mathbf{q}_k)$$

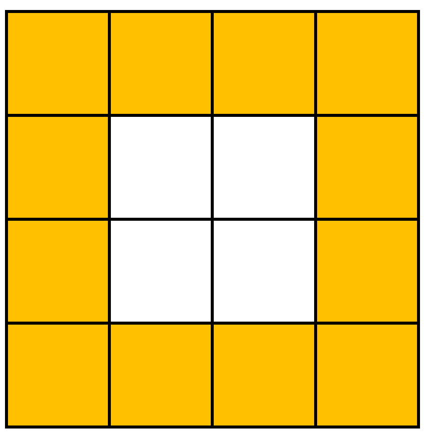
- Workspace dynamics w/ dynamics given by 2-D planar vector field $\dot{\mathbf{x}} = F(\mathbf{x})$
- $\{V_1, \ldots, V_M\}$ s.t. $V_i \subset \mathcal{W}$ and $\cup V_i = \mathcal{W}$
- V_i's are dynamically distinct
- Robots have ability to localize

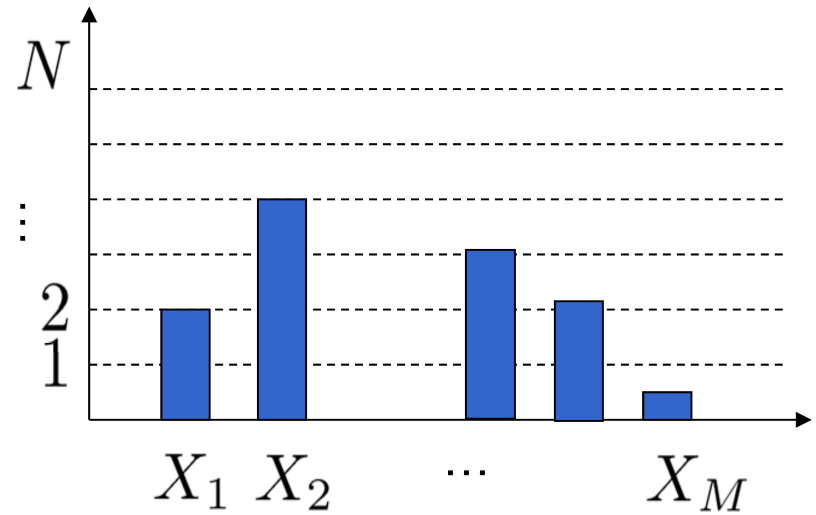


Distributed Allocation Strategy

Example Desired Allocations





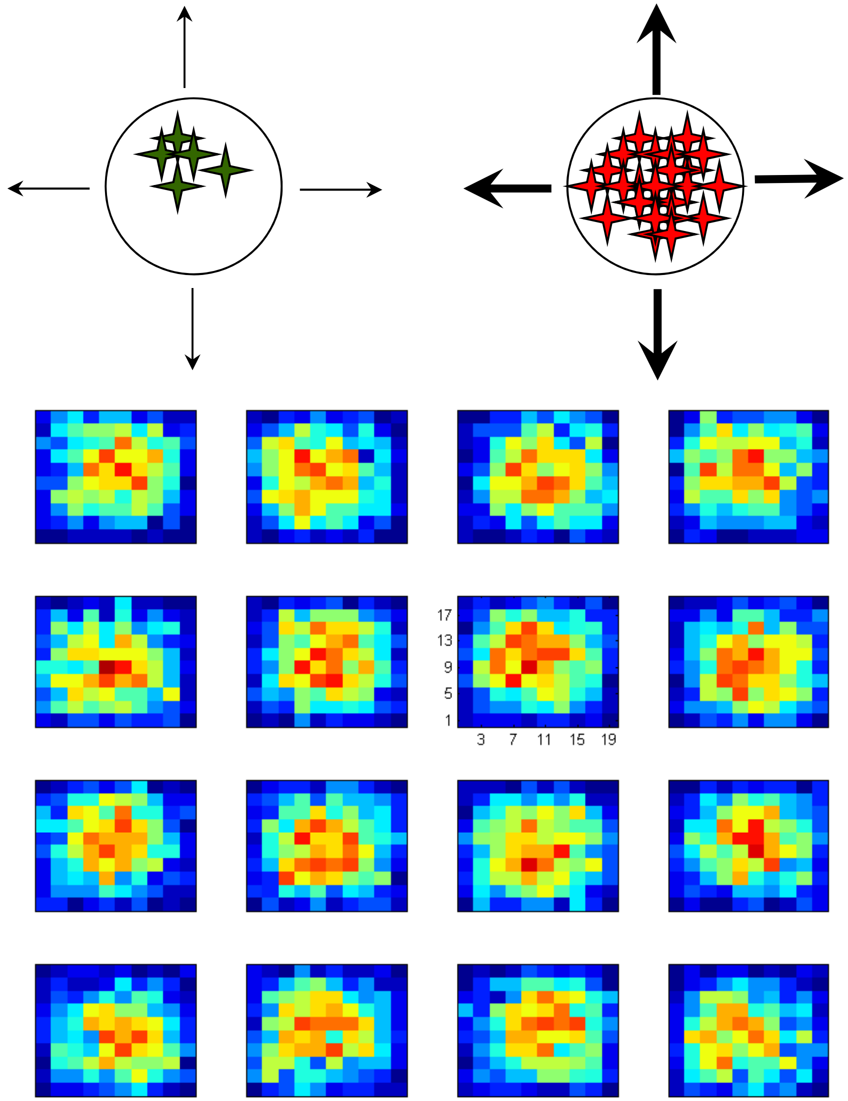


Assumptions

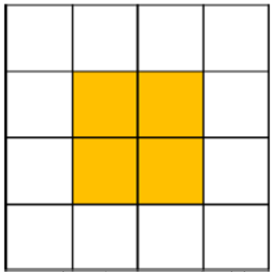
- Robots know $\{\bar{X}_1, \ldots, \bar{X}_M\}$ and $\{X_1(t), \ldots, X_M(t)\}$
- Prioritization based on escape likelihoods

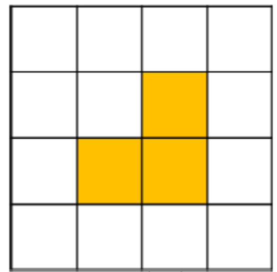
Two Step Process

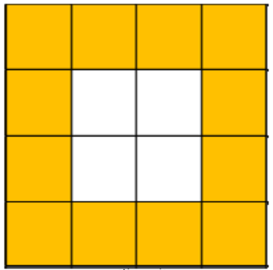
- I: Assignment Phase
  - » Escape Likelihoods
- II: Actuation Phase
  - » Leave
  - » Active Stay
    - o w/ Actuation
  - » Passive Stay
    - o w/o Actuation

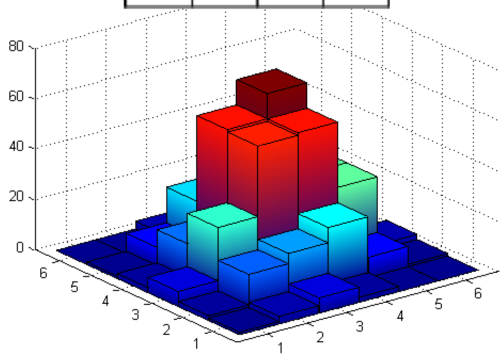


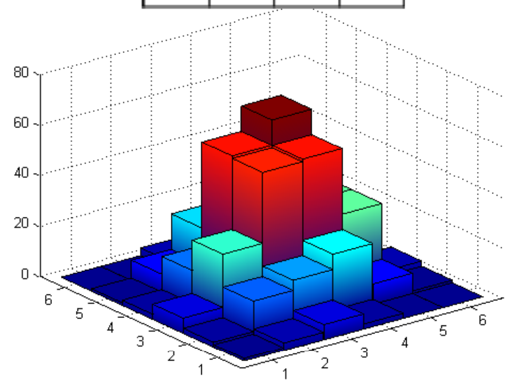
With & Without Controls

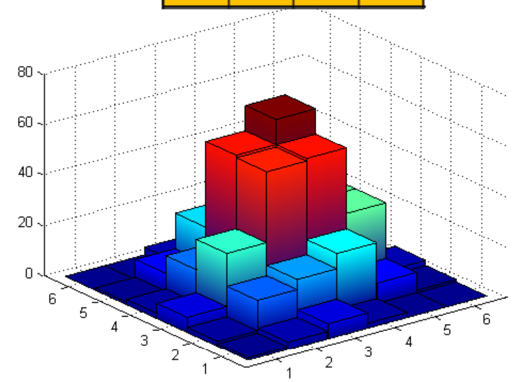




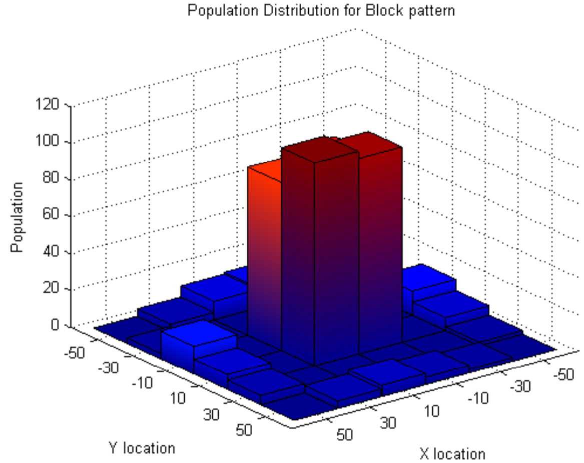




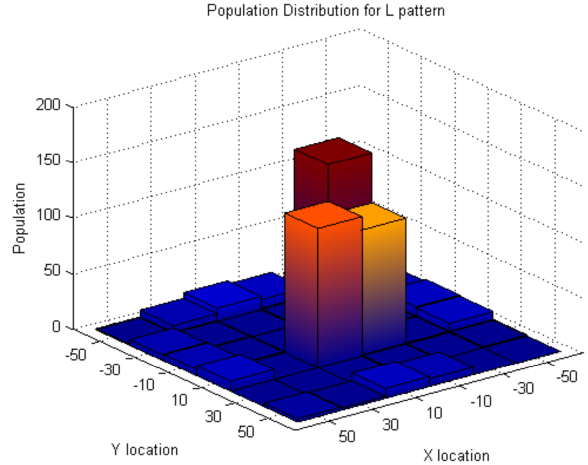




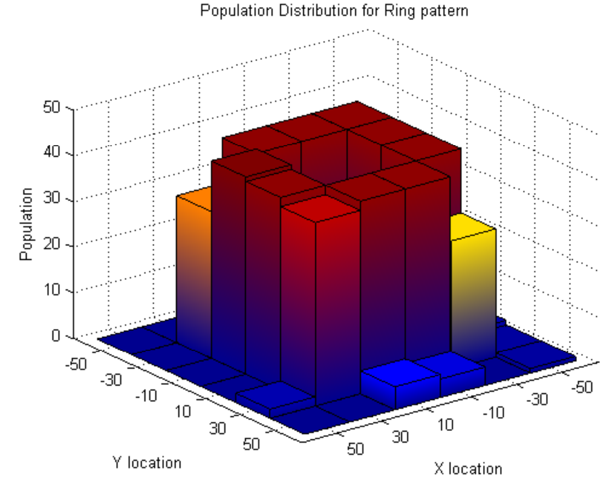
Population Distribution for Block pattern



Population Distribution for L pattern

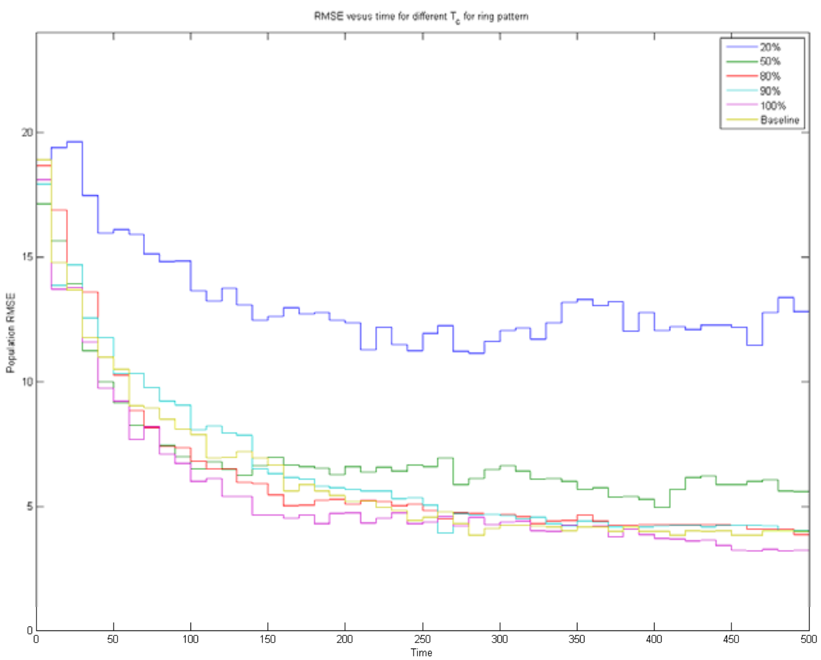


Population Distribution for Ring pattern

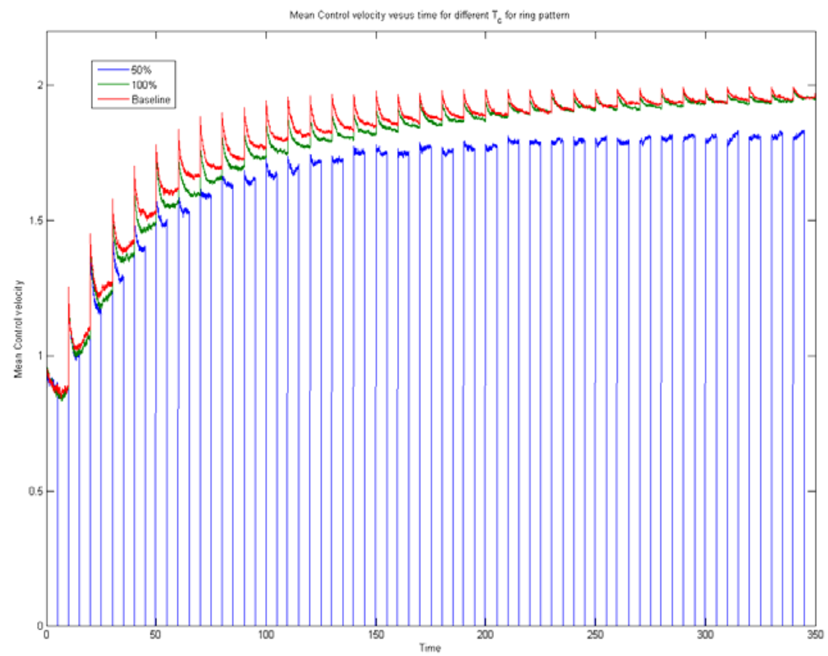


What is Gained

Convergence Rate

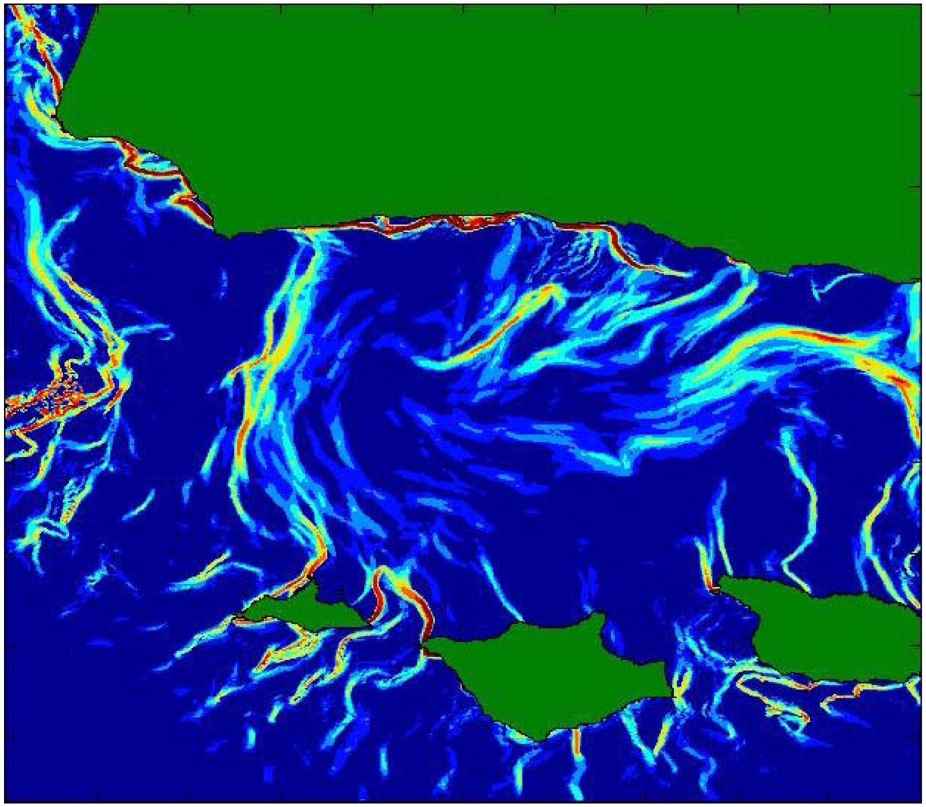


Control Effort Expenditure



What Next?

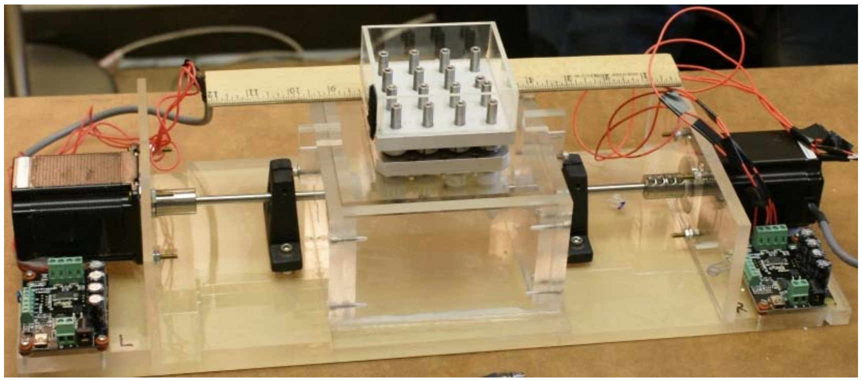
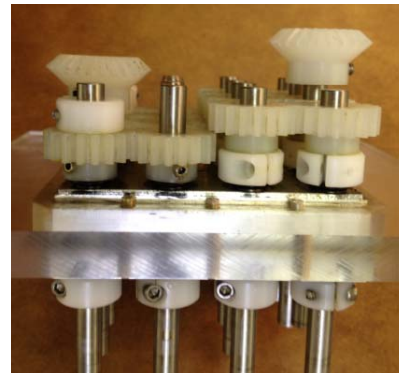
Tracking and control in real ocean flows



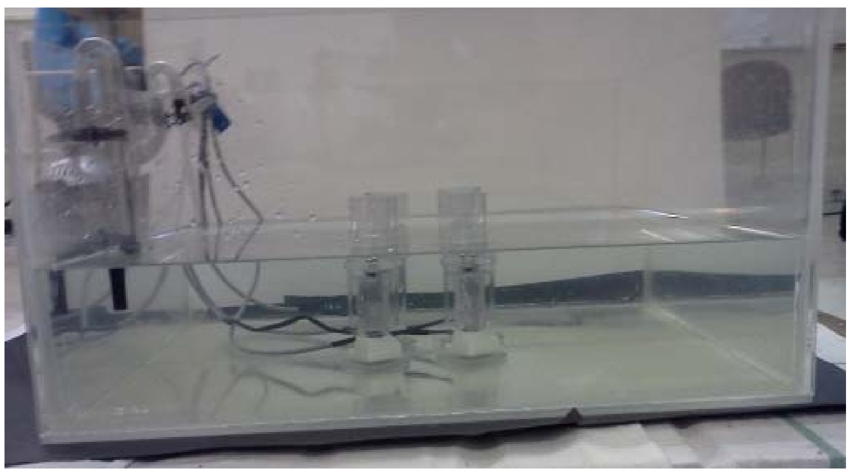
Santa Barbara, CA

mCoSTe

multi-Robot **Co**herent **S**tructure **Te**stbed

LoRe Tank: Re ~ O(1) – O(10^4)

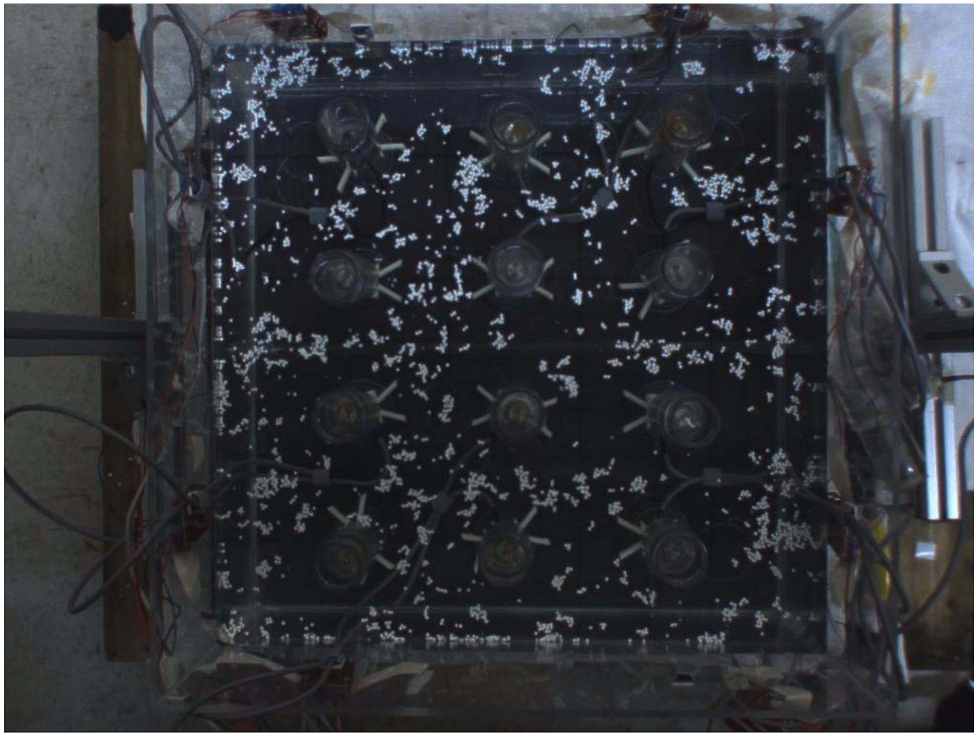


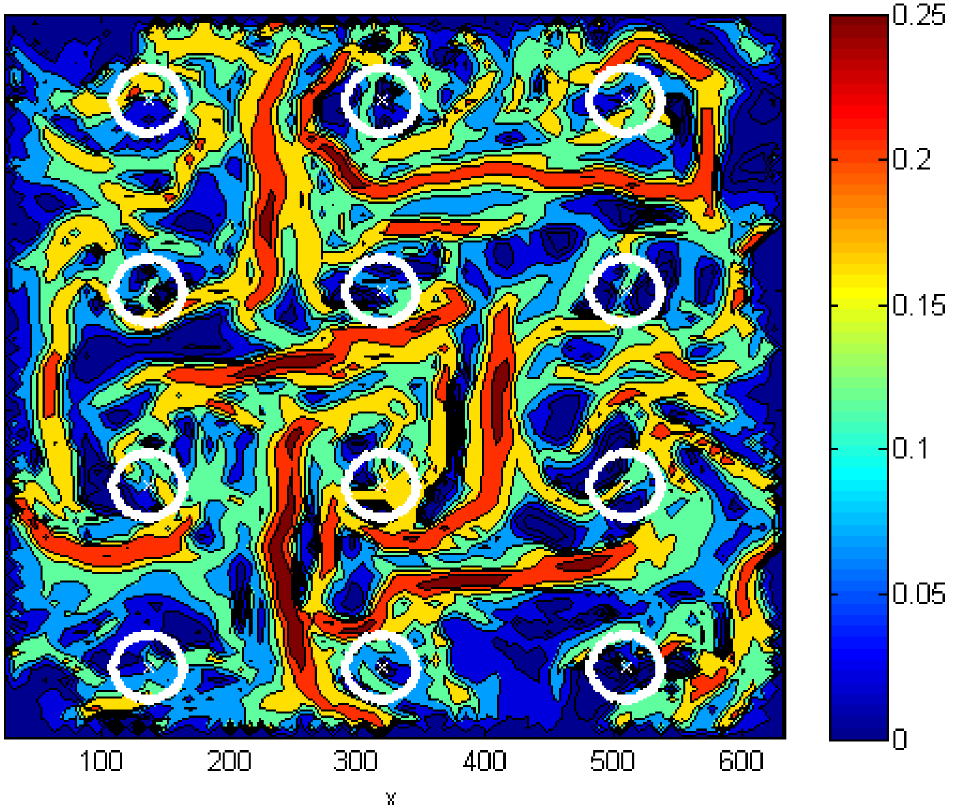

HiRe Tank: Re ≥ O(10^4)

Prelim Results in HiRem Tank

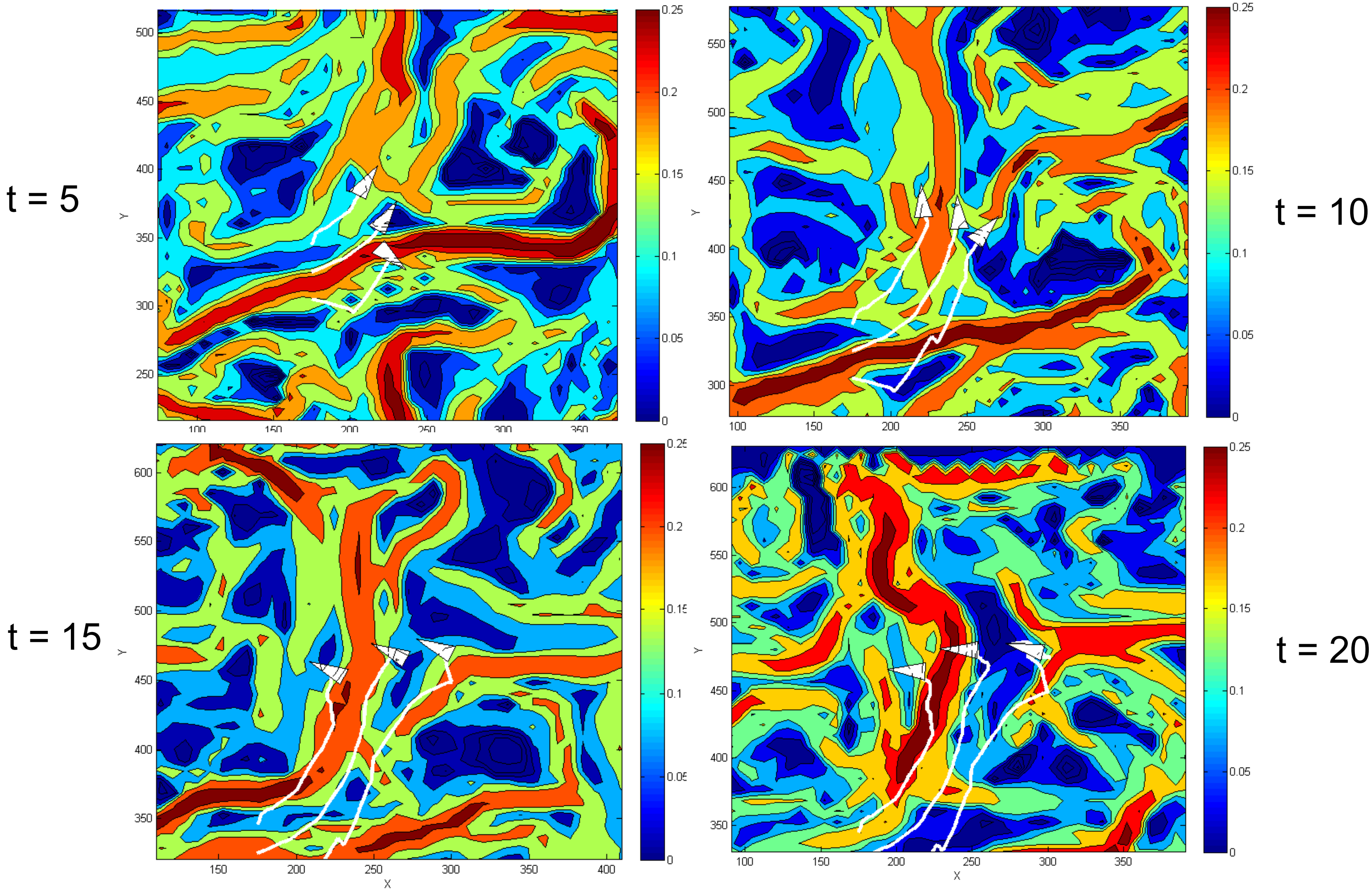
Flow Tank

Simulated LCS Tracking



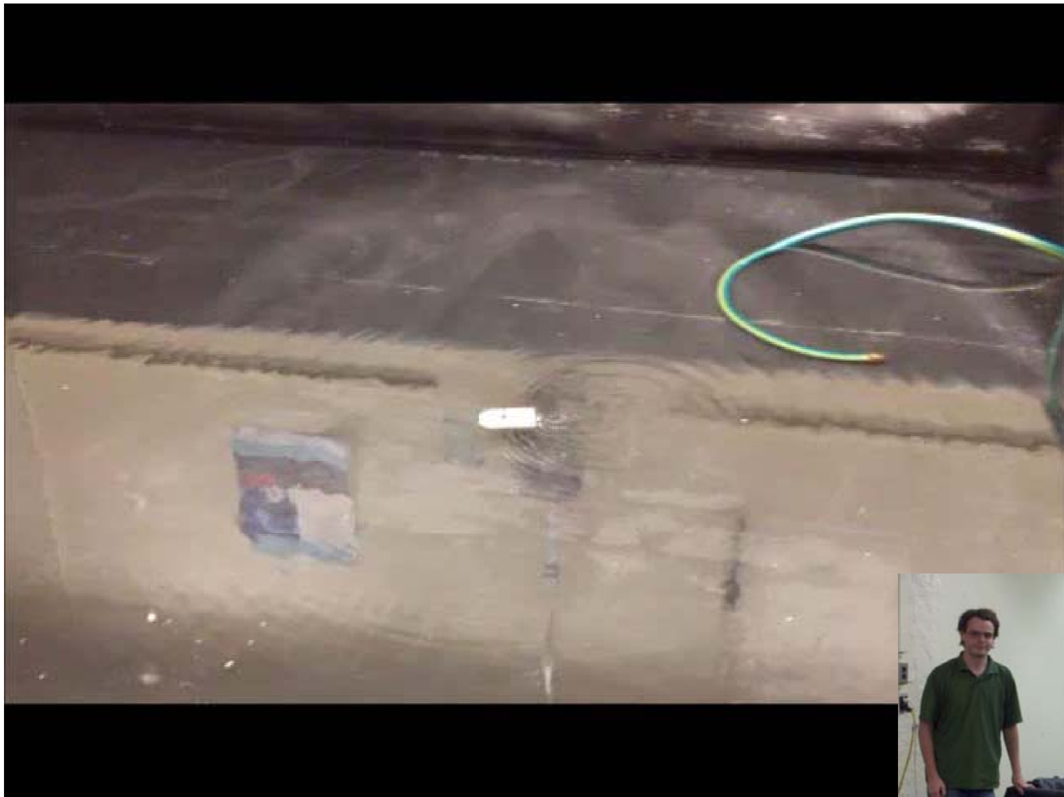


Cooperative Tracking w/ Tank Data

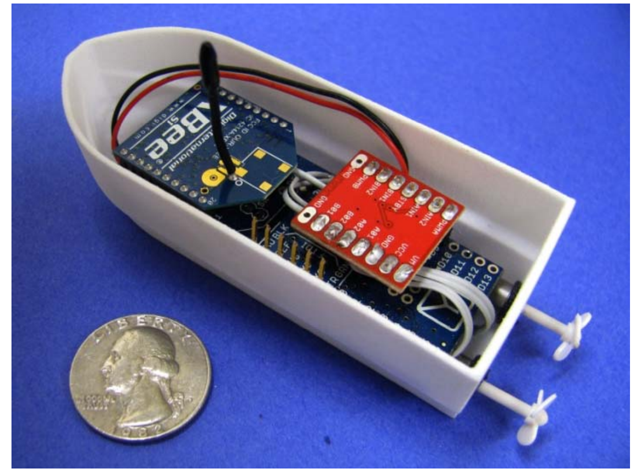


Scaling Up – MR Tank

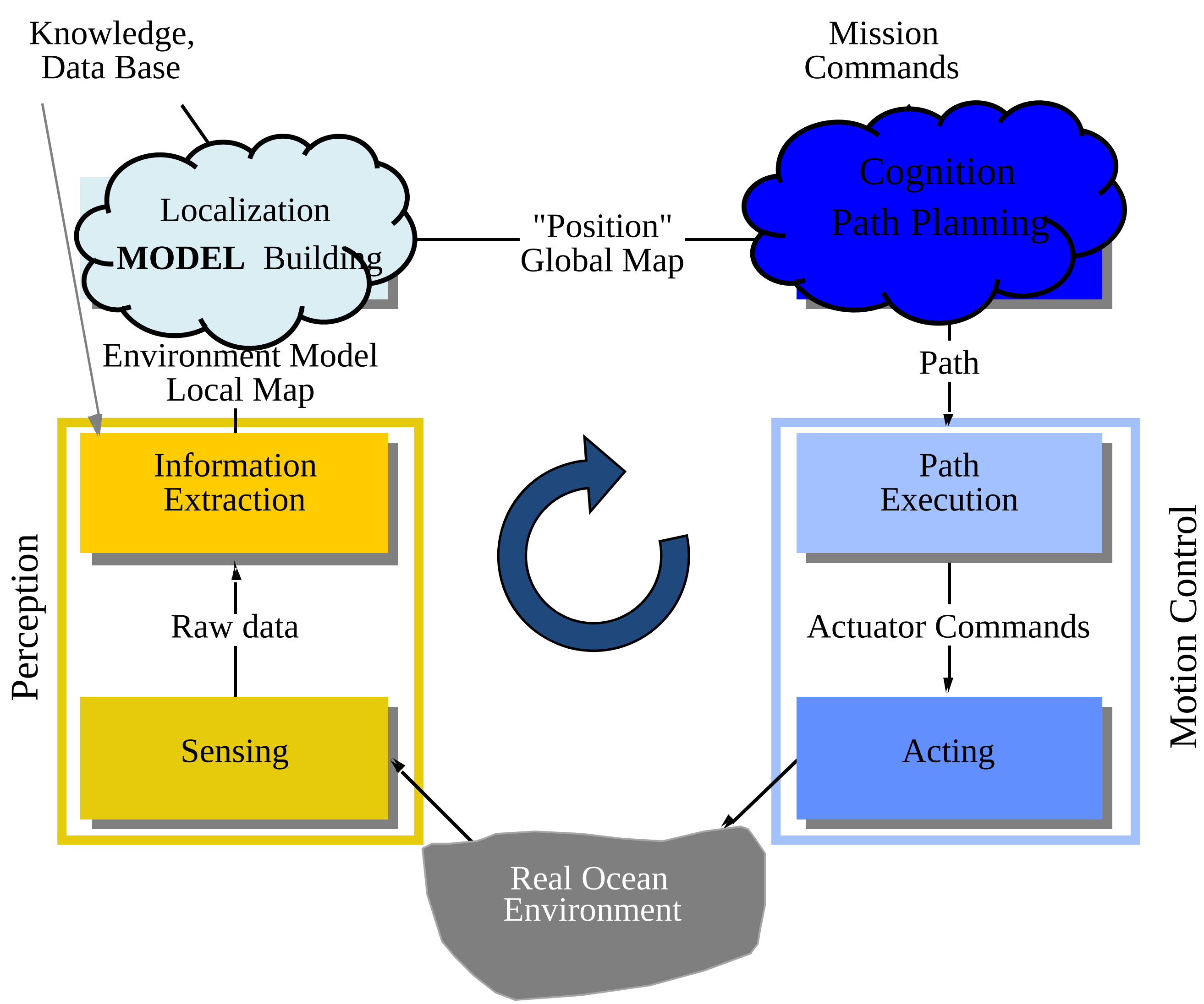
MR Tank



mASVs



A Grand Vision



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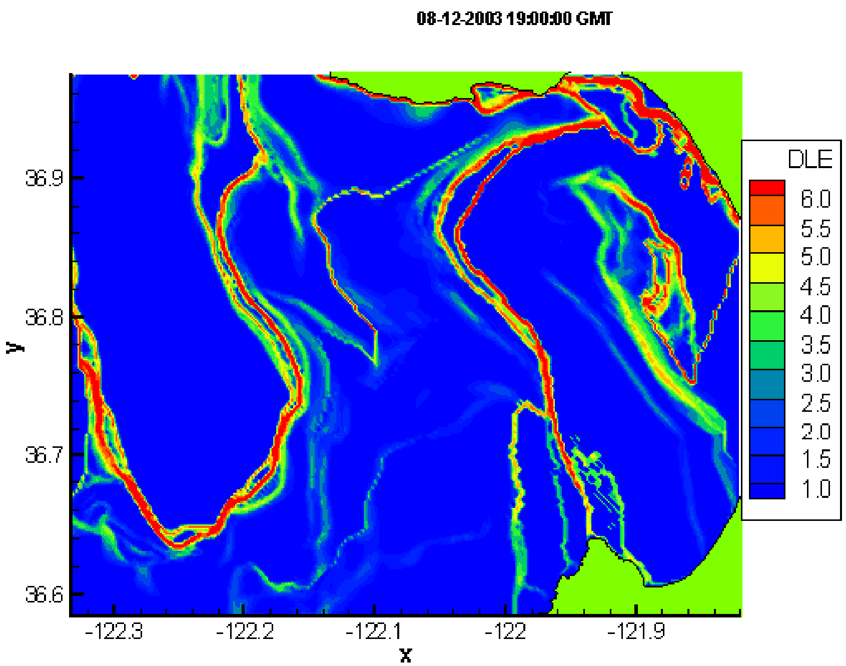
Phil Yecko (Montclair State)

Ira Schwartz (NRL)

http://drexelsaslab.appspot.com/

Coherent Structures

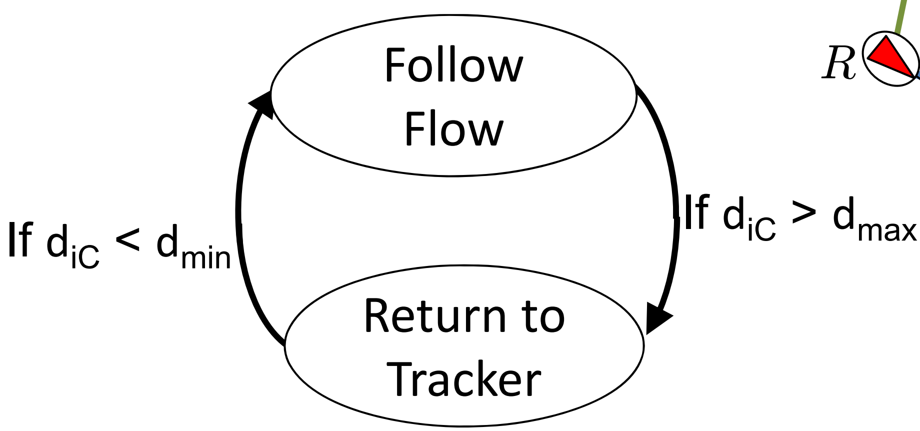


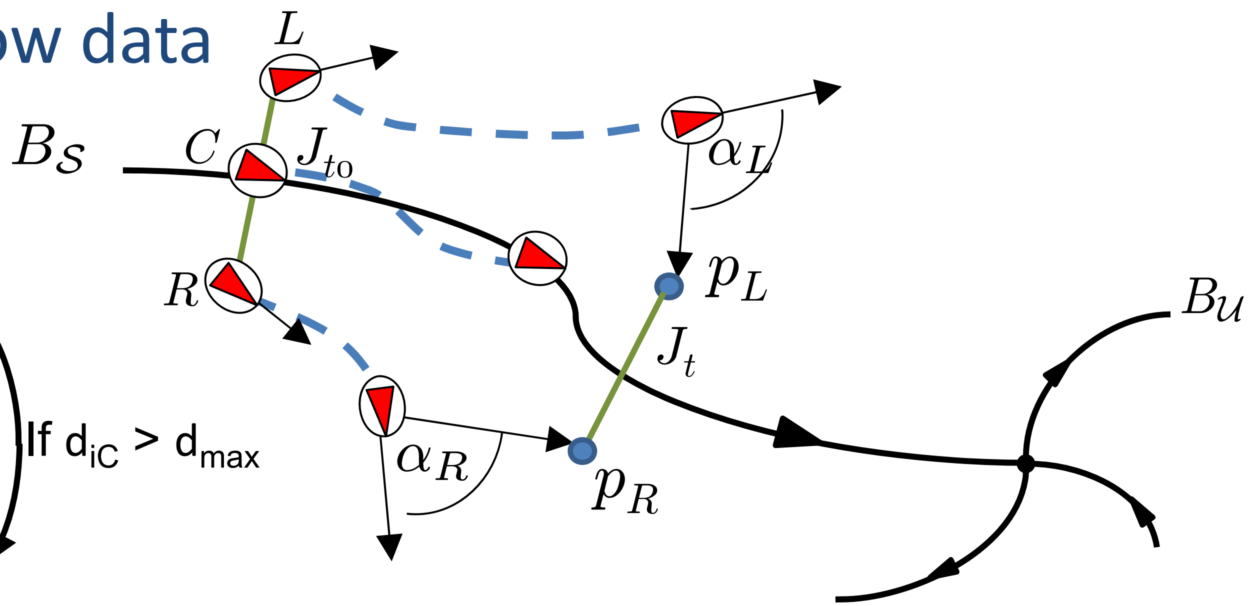
Courtesy of Paduan and Cook (NPS) and Shadden (IIT)
Source: http://mmae.iit.edu/shadden/LCS-tutorial/

- Geophysical flows exhibit *coherent structure*
- *Coherent structures* give insight into dynamics of fluids
- *Lagrangian coherent structures* give insight into transport

Straddle Formation Control

Maintains desired formation
while collecting flow data





$$V_i = \begin{cases} 0 & \text{if } d_{Min} < \|\mathbf{x}_i - \mathbf{x}_C\| < d_{Max} \\ \|(\mathbf{p}_i - \mathbf{x}_i) - \mathbf{u}_i\| & \text{otherwise} \end{cases},$$

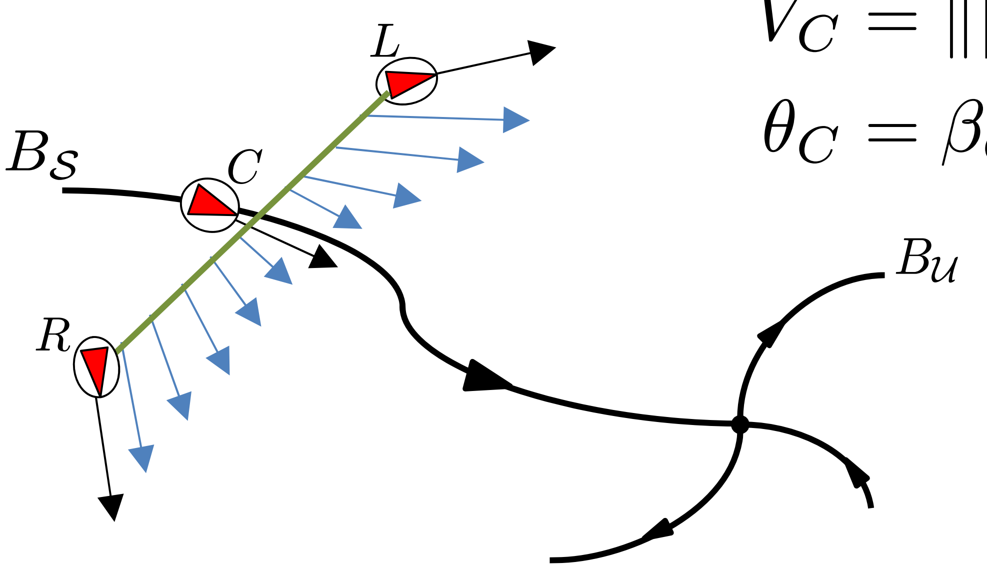
$$\theta_i = \begin{cases} 0 & \text{if } d_{Min} < \|\mathbf{x}_i - \mathbf{x}_C\| < d_{Max} \\ \alpha_i & \text{otherwise} \end{cases}.$$

Manifold Tracking Control

- Use herders position and local flow measurements to
  - » Determine vector field for points $\mathbf{q}_1, \ldots, \mathbf{q}_M$ on J_t

$$\mathbf{u}(\mathbf{q}_k) = \sum_j \sum_{i=1}^{N} \frac{w_{ij}\hat{\mathbf{u}}_i(j)}{\sum_j \sum_{i=1}^{N} w_{ij}}$$

  - » Find point on J_t closest to B_S (or B_U)



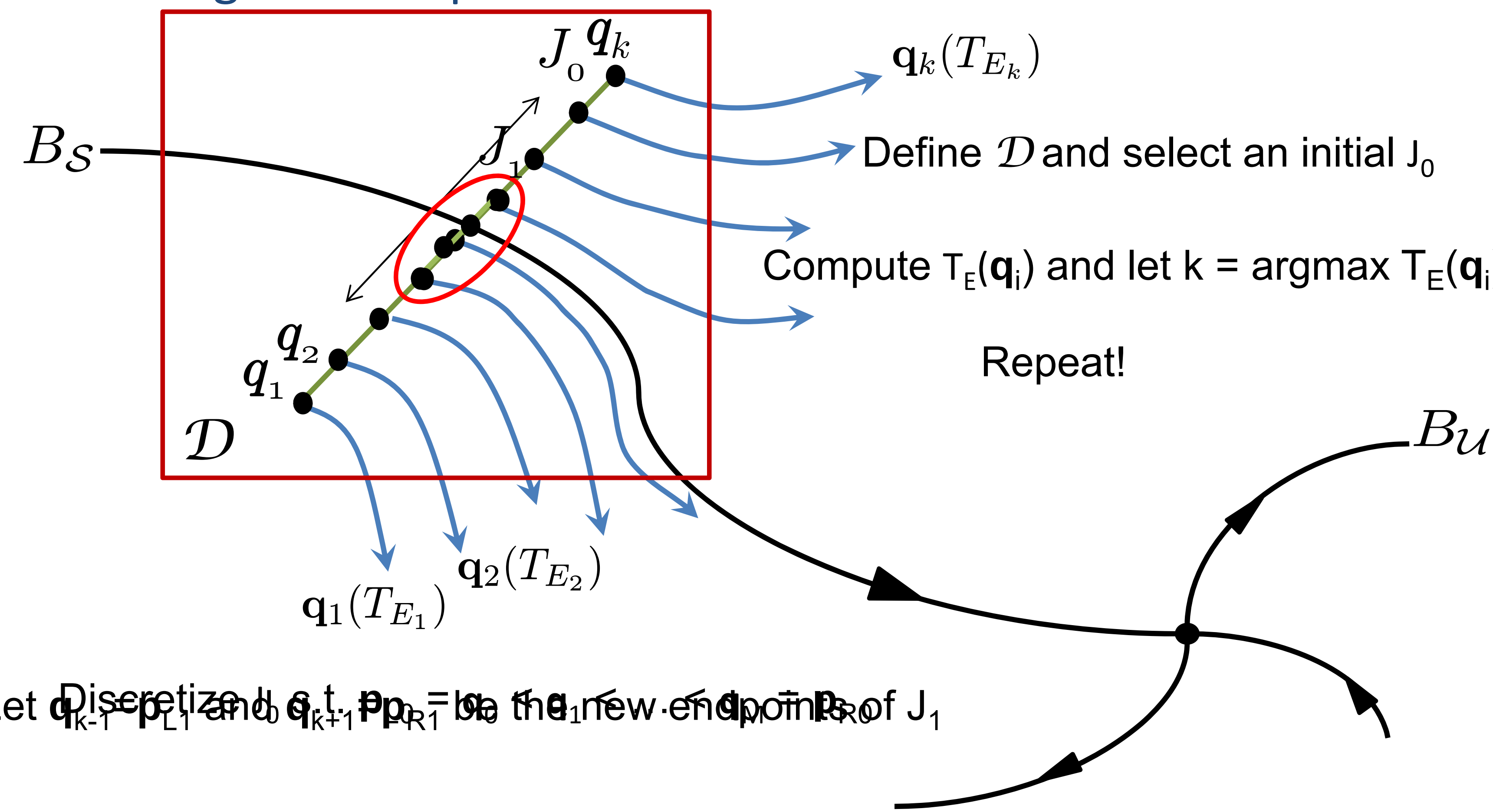
$$V_C = \|[(\mathbf{q}_B + b\hat{\mathbf{u}}_B) - \mathbf{x}_C] - \mathbf{u}_C\|$$

$$\theta_C = \beta_C$$

The PIM Triple Algorithm

Given $\dot{\mathbf{x}} = F(\mathbf{x})$

Finding a PIM Triple



Define $\mathcal{D}$ and select an initial J_0

Compute $T_E(\mathbf{q}_i)$ and let k = argmax $T_E(\mathbf{q}_i)$

Repeat!

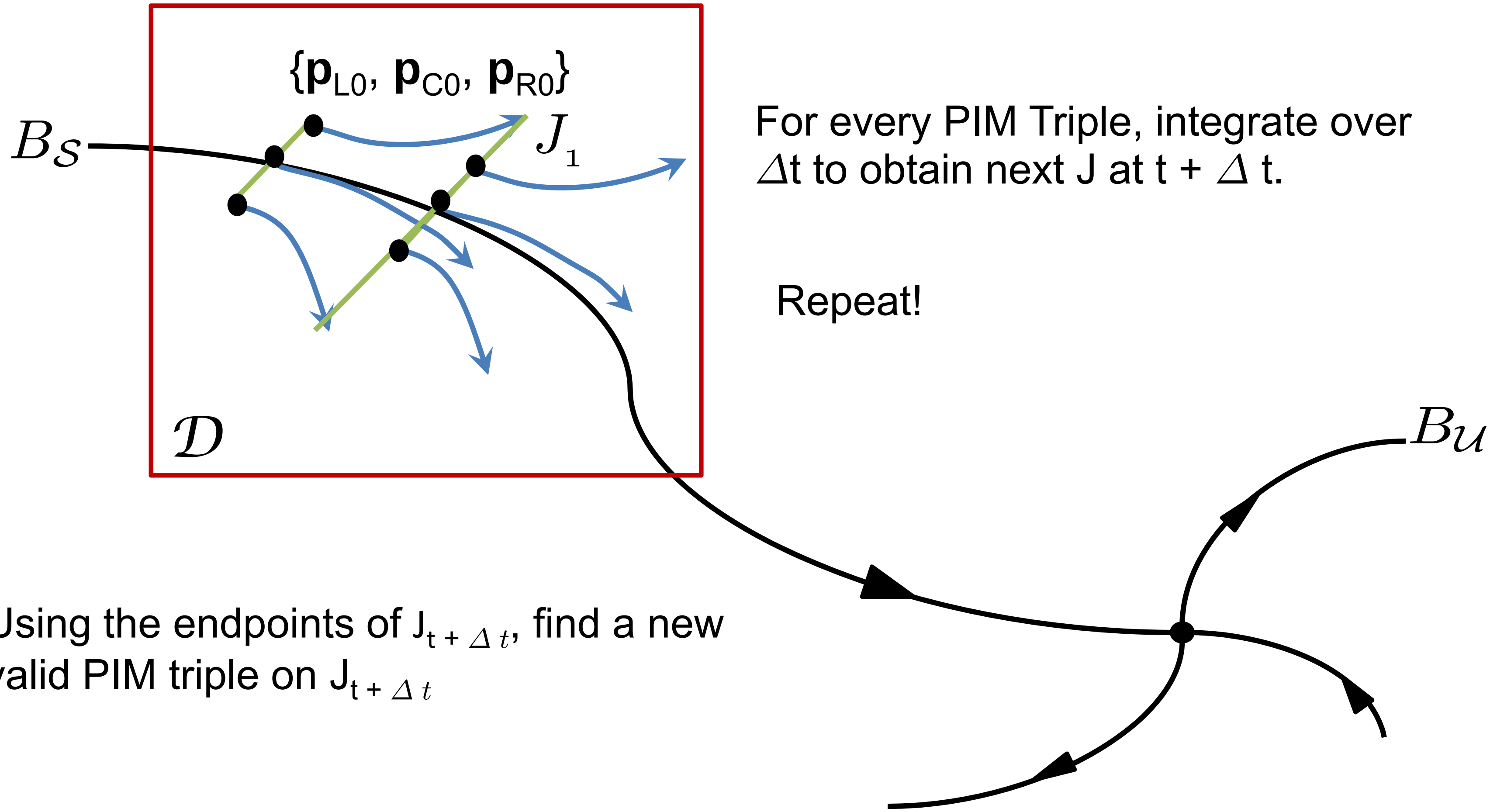
Discretize J_0 s.t. $\mathbf{p}_{L0} = \mathbf{q}_0 < \mathbf{q}_1 < \cdots < \mathbf{q}_M = \mathbf{p}_{R0}$

Let $\mathbf{q}_{k-1} = \mathbf{p}_{L1}$ and $\mathbf{q}_{k+1} = \mathbf{p}_{R1}$ be the new endpoints of J_1

The PIM Triple Algorithm I

Given $\dot{\mathbf{x}} = F(\mathbf{x})$

Mapping out the boundary



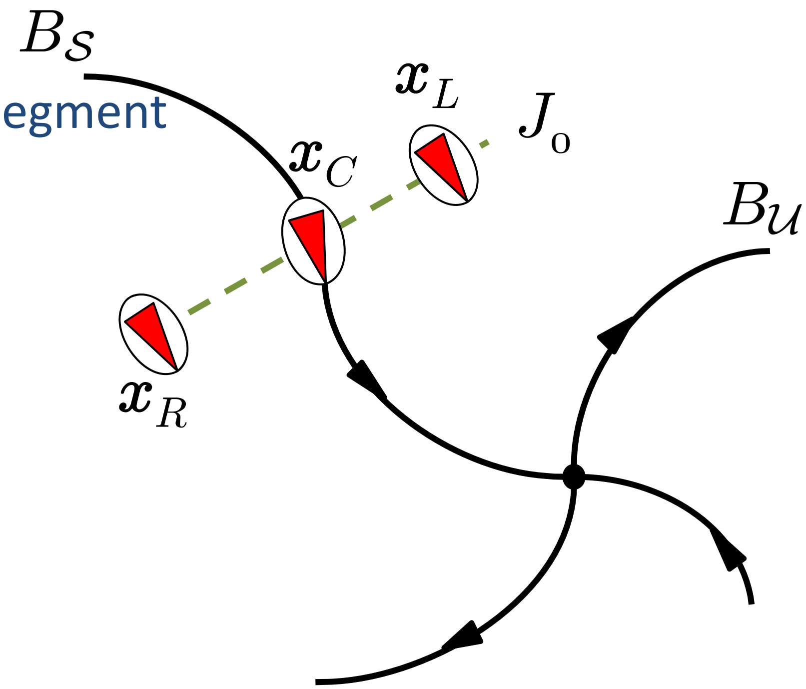
For every PIM Triple, integrate over Δt to obtain next J at t + Δ t.

Repeat!

Using the endpoints of $J_{t+\Delta t}$, find a new valid PIM triple on $J_{t+\Delta t}$

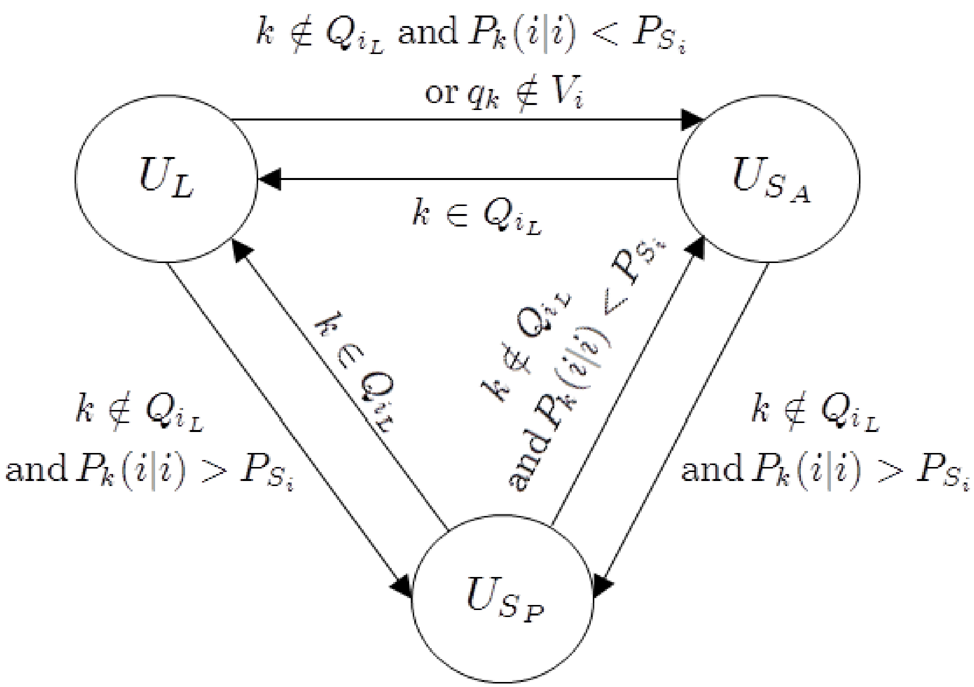
3-Robot PIM Triple Inspired Control

- Initial positions lie on J_0, a saddle straddle line segment
- Controller Objectives:
  - » End robots (Herders):
    maintain *valid* saddle straddle line segment at all times
  - » Center robot (Tracker):
    tracks the boundary B_S or B_U



Single Robot Controller

$$\mathbf{q}_k = \mathbf{u}_k + F(\mathbf{q}_k)$$



Q_{i_L} – Set assigned to LEAVE
$P_k(i|i)$ – Liklihood to stay

LEAVE

$$U_L(\mathbf{q}_k) = \omega_i \times c\frac{F(\mathbf{q}_k)}{\|F(\mathbf{q}_k)\|}$$

ACTIVE STAY

$$U_{S_A}(\mathbf{q}_k) = -\omega_i \times c\frac{F(\mathbf{q}_k)}{\|F(\mathbf{q}_k)\|}$$

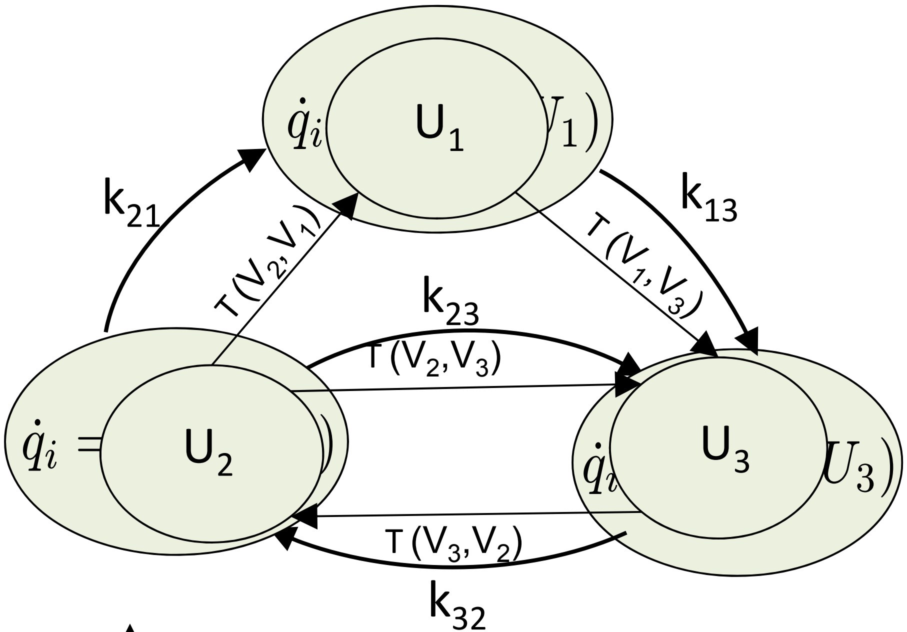
PASSIVE STAY

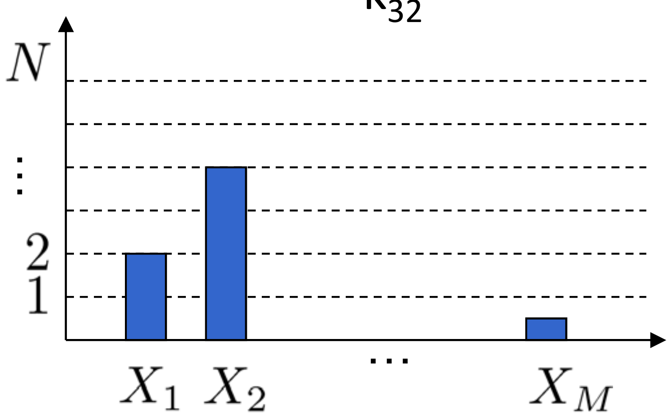
$$U_{S_P}(\mathbf{q}_k) = 0$$

From Robots to Particles

A Stochastic Hybrid System (SHS)

Single Robot Behavior





$$X_u \xrightarrow{k_{uv}} X_v$$

- Ensemble States: $X_1, \ldots, X_M$
- States defined as discrete random variables
- k_{uv} -> transition propensities

Mather & Hsieh (RSS 2011)

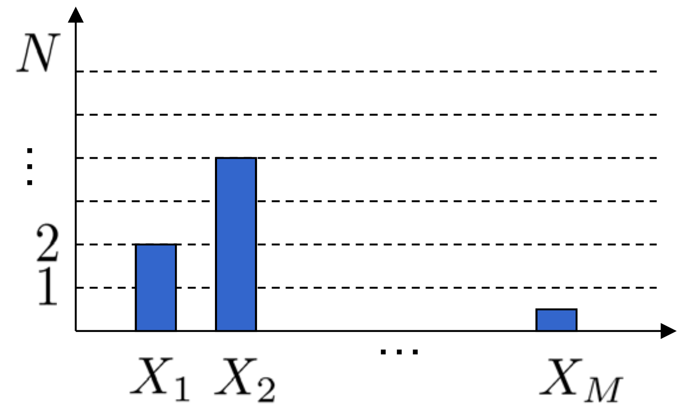
What This Gets Us

- The Extended Generator, i.e. $\frac{d}{dt}\mathrm{E}\left[\psi(X_i)\right] = E\left[L\psi(X_i)\right]$

$$L\psi(X_i) = \sum_j \left[(\psi(X_i - 1) - \psi(X_i))w_{ji} + (\psi(X_i + 1) - \psi(X_i))w_{ij}\right]$$

  - » ψ : real-valued function of X_i
  - » $w_{ij}(k_{ij}, X_i, X_j)$: frequency of change

- Examples:
  - » Let ψ $(X_i) = X_i$ to obtain $\frac{d}{dt}\mathrm{E}\left[X_i\right]$
  - » Let ψ $(X_i^2) = X_i^2$ to obtain $\frac{d}{dt}\mathrm{E}\left[X_i^2\right]$
  - » So on …



Hespanha (2008)

Two State Example

$$X_1 \underset{k_{21}}{\overset{k_{12}}{\rightleftharpoons}} X_2, \quad \text{with } \psi(X_i) = X_i, \quad w_{ij} = k_{ij} X_i$$

$$\frac{d}{dt}\begin{bmatrix} \mathrm{E}[X_1] \\ \mathrm{E}[X_2] \\ \mathrm{E}[X_1 X_1] \\ \mathrm{E}[X_2 X_2] \\ \mathrm{E}[X_1 X_2] \end{bmatrix} = \begin{bmatrix} -k_{12} & k_{21} & 0 & 0 & 0 \\ k_{12} & -k_{21} & 0 & 0 & 0 \\ k_{12} & k_{21} & -2k_{12} & 0 & 2k_{21} \\ k_{12} & k_{21} & 0 & -2k_{21} & 2k_{12} \\ -k_{12} & -k_{21} & k_{12} & k_{21} & -k_{21} - k_{12} \end{bmatrix} \begin{bmatrix} \mathrm{E}[X_1] \\ \mathrm{E}[X_2] \\ \mathrm{E}[X_1 X_1] \\ \mathrm{E}[X_2 X_2] \\ \mathrm{E}[X_1 X_2] \end{bmatrix}$$

- n^{th} moment dynamics only depends on 1^{st} - n^{th} moments
- Thus, moment equations are *closed*
- Ensemble dynamics are stable

Mather and Hsieh (RSS 2011)

Stability Analysis

Theorem: *(Mather and Hsieh, RSS2011)*

The first moment dynamics of the system $X_i \xrightarrow{k_{ij}(X_i, X_j)} X_j$ with the ensemble feedback strategy $k_{ij} = \alpha_{ij} - \beta ij \frac{X_j}{X_i}$ is stable.

Proof: $$\frac{d}{dt}\mathrm{E}[X_i] = \sum_{(i,j)\in\mathcal{E}} (\alpha_{ji} + \beta_{ij})\mathrm{E}[X_j] - \sum_{(j,i)\in\mathcal{E}} (\alpha_{ij} + \beta_{ji})\mathrm{E}[X_i]$$

$$\frac{d}{dt}\mathrm{E}[X] = (\mathbf{K}_\alpha + \mathbf{K}_\beta)\mathrm{E}[X]$$

Both K_α and K_β are Markov process matrices and negative semidefinite => stable

Monitoring of 3 Regions

No Feedback

With Feedback

Mather and Hsieh (RSS2011)

Future Work

- Tracking LCS
  - » Precision
  - » Representation
  - » Planning

- Controlling Spatial Distribution
  - » Analysis of the single robot controller
  - » Derive transition propensities from actual fluid dynamics
  - » Use ensemble models to improve single robot strategy
- Experimental Validation
  - » Develop a large scale indoor test-bed