

Depth-Resolved Surfzone Wave and Roller Transformation

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Key Points:

- A two-phase Reynolds-averaged Navier-Stokes model simulates time series of surface elevation and water velocity well in the surfzone
- Vertical inertia and turbulent transfer dominate the momentum balance beneath the roller, while roller forcing dominates near the surface
- The thickness of the roller and turbulent-production dominated regions scale with local significant wave heights

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Abstract The vertical structures of cross-shore wave, roller, and other momentum forcing terms are examined using a two-phase Reynolds-averaged Navier-Stokes model driven with field and laboratory measured waves and bathymetry. Modeled wave height, setup, and undertow agree well with field observations along a cross-shore transect extending from the shoreline to about 5-m water depth, and with measurements in a wave flume. Momentum flux carried by wave, roller, vertical turbulence transfer, pressure slope, and vertical inertia primarily balance near the surface, while wave, vertical turbulence transfer, vertical inertia, and setup primarily balance in the mid-to lower-water column. Turbulence transports momentum downward, while vertical inertia transfers momentum upwards. Combined wave and roller forcing agrees with a linear wave theory parametrization offshore of the sandbar and on a near-planar beach, and combined wave, roller, and vertical inertia agrees with linear theory onshore of the sandbar. Roller thickness is related to local wave height, similar to previous studies. Sub-surface offshore-directed mean currents (undertow) can be estimated from the mid-to lower-water column momentum balance. Turbulence production dominates the near-surface turbulence-energy-flux balance, and its penetration depth in the sandbar trough is correlated with local wave height.

Plain Language Summary In the surfzone water carried toward the shoreline on the surface of breaking waves is balanced by water flowing offshore below the surface, often called “undertow.” The offshore flowing water carries sediment and biota from the shoreline to the open ocean. The strength of the undertow varies with location below the surface, reflecting a balance of different forcing terms, such as waves and turbulence whose strength varies between the surface and the seafloor. Here, the vertical structure of the forcing terms through the water column is investigated with a numerical model that includes waves, currents, and the foamy air-water mixture near the surface of breaking waves (the roller). The model shows that waves and the vertical transfer of breaking-wave-induced turbulence are the dominant forces in the mid-to lower-water column, while the forces in the air-water roller dominate in a region near the surface. The roller has thickness proportional to the local wave height. The results of this study could be useful to improve engineering models of waves and currents near the shore, providing better tools for coastal planners.

1. Introduction

Breaking waves in the surfzone transfer momentum and energy to the water column, causing an increase of the mean water level (wave setup) and a sub-surface offshore-directed current (undertow) that balances the onshore mass flux from the breaking waves (Garcez Faria et al., 2000). Previous studies have shown that the cross-shore profile of wave setup can be estimated accurately with the 1-dimensional cross-shore depth-integrated momentum balance between wave, wave setup, and the roller, the foamy air-water mixed region at the surface of depth-limited breaking waves (Apotsos et al., 2007; Chen et al., 2024a; Longuet-Higgins & Stewart, 1962, 1964). However, the vertical structure of the undertow, which is critical for estimating transport of sediment, pollutants, and biota (Grant et al., 2005; Kerpen et al., 2020) is due primarily to a vertical imbalance of the momentum fluxes caused by the cross-shore wave radiation stress (the momentum flux owing to waves) gradients, wave setup gradients, and vertical turbulence momentum transfers (momentum diffused vertically via turbulence, Apotsos et al. (2007); Dyhr-Nielsen and Sørensen (1970); Garcez Faria et al. (2000); Mellor (2021)). Therefore, accurately estimating the vertical structure of the momentum balance is important to determine the vertical structure of undertow.

Numerical field-scale surfzone models parameterize wave breaking and assume that the wave forcing is depth uniform (Feddersen & Trowbridge, 2005), surface concentrated (Apotsos et al., 2007; Deigaard, 1993; Martins et al., 2022; Mellor et al., 2008; Walstra et al., 2001), consistent with linear wave theory (Mellor, 2015), or customized (Kumar et al., 2012; Uchiyama et al., 2010). However, these models often do not reproduce observed

undertow profiles well. In contrast, two-phase (air and water) Reynolds-averaged Navier-Stokes (RANS) and Large Eddy Simulation (LES) models that do not require assumptions about wave breaking or forcing predict the cross-shore wave transformation, setup, and undertow reasonably well (Chen et al., 2024a; Larsen et al., 2020; Tsai et al., 2024). However, few studies have used two-phase model output to examine the vertical structure of, or parameterizations for surfzone forcing.

Prior studies also assume that the roller forcing either acts as a surface stress (Apotsos et al., 2007; Martins et al., 2022), is distributed within a finite region near the surface (Walstra et al., 2001), or has similar vertical distribution as the breaking wave forcing (Kumar et al., 2012; Uchiyama et al., 2010). Studies that assume roller momentum is distributed within a finite region near the surface commonly assume that the “roller thickness” is proportional to the local wave height (Duncan, 1981; Svendsen, 1984; Walstra et al., 2001). The momentum balance within the roller thickness is not well understood because roller generation and dissipation usually are parametrized.

Additional momentum terms, such as vertical advection (vertical flux carried by vertical mean flow), turbulence radiation stresses, and pressure slope typically are neglected in models (Apotsos et al., 2007; Mellor, 2021). However, turbulence radiation stresses, the momentum flux owing to turbulence, may be responsible for about 20% of the transfer to setup (Chen et al., 2024a; Roelvink & Stive, 1989). In addition, the pressure slope, a vertical flux term that represents the dynamic pressure acting on a tilted internal material surface, may be responsible for redistributing wave forcing when waves propagate over a slope (Ardhuin et al., 2017). However, the impacts of these additional terms are difficult to quantify with models that make a priori assumptions about them.

Here, the vertical structure of surfzone momentum transfers is investigated using field and laboratory measurements of waves, velocities, and setup (Section 2) to drive and evaluate a two-phase Volume of Fluid (VOF) two-equation RANS model (Section 3). Momentum forcing terms calculated from model output are converted to vertically Lagrangian, horizontally Eulerian σ coordinates before wave averaging (Section 4). The vertical structure of momentum forcing, the roller thickness in the upper water column, and the undertow profile in the mid-to lower-water column are investigated over barred and unbarred bathymetries (Section 5). Differences between the model estimates and linear theory and the vertical structure of turbulence are discussed and study limitations are noted (Section 6).

2. Field and Laboratory Observations

The model is evaluated using field and laboratory observations of waves, setup, and undertow (see Appendix B for offshore conditions). Field data were collected between the shoreline and about 5-m water depth over a barred beach profile at the U.S. Army Corps of Engineers Field Research Facility in Duck, NC for 6 hr spanning a rising tide. The waves, which had near constant offshore heights of around 0.8 m and energy weighted mean period T_w around 9.7 s (Table B1), broke on the offshore slope of (low tide) and over (high tide) the sandbar (Apotsos et al. (2007); Chen et al. (2024a); Raubenheimer et al. (2001), Figure 1a). Laboratory measurements of surface fluctuations and velocities were obtained for a 500-s data run over a near-planar beach in a wave flume (Bond et al. (2023); Tsai et al. (2024), Figure 1b). The significant wave height $H_s = 0.58$ m and the energy weighted mean period $T_w = 4.0$ s (Table B1).

For the field data, sea-surface fluctuations are estimated from 3,072-s-long time series of pressure measured with near-bed or buried sensors (Figure 1). Pressure time series are demeaned and quadratically detrended to remove tides and motions with periods longer than roughly 1 hr, and sea-surface fluctuations are calculated using linear theory for wave attenuation through the water column and exponential attenuation through the sand bed (Raubenheimer et al., 1998). Wave setup is estimated from field data using 512-s time series from buried sensors assuming hydrostatic pressure (Raubenheimer et al., 2001). Undertow is estimated from 3,072-s-long 2 Hz time series of cross-shore current measured using electromagnetic current meters. Alongshore flow magnitudes were smaller than 0.2 m/s except for the 13 September 1997, 12:00 field case when alongshore flows reached 0.4 m/s in the surfzone (Table B1).

In the laboratory, sea surface elevations were sampled at 100 Hz with capacitance wave gauges and velocities were measured at 100 Hz with acoustic Doppler velocimeters (Bond et al., 2023; Tsai et al., 2024). Setup is estimated as the time-averaged super-elevation of the sea surface over the final 327.6-s time series of the 500-s run relative to the still water level. Undertow is estimated as the time average of the final 200 s of the 500-s run.

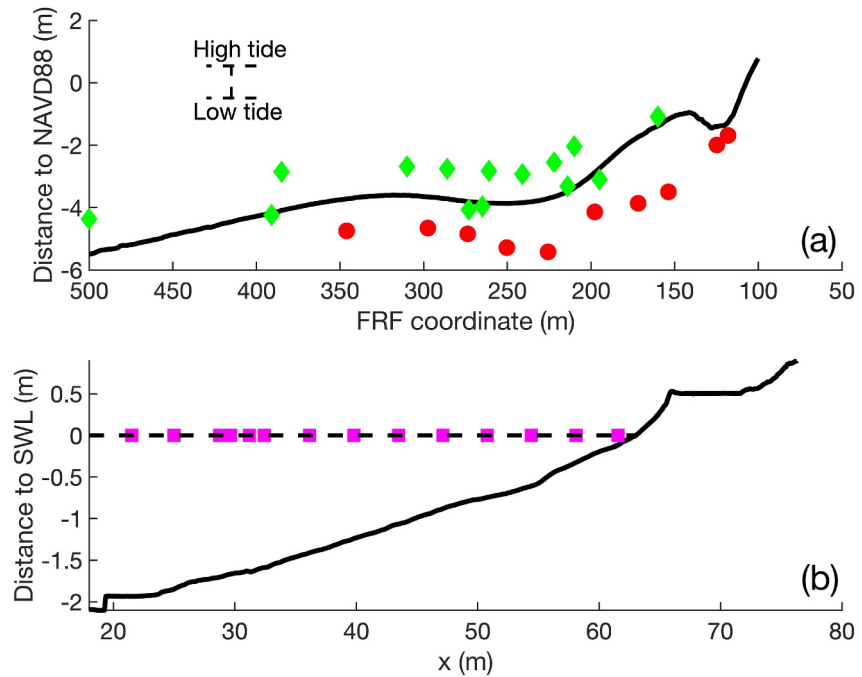


Figure 1. Cross-shore seafloor elevation (black solid curves) and sensor locations (colored symbols) for (a) the field observations (Field Research Facility coordinate system, green diamonds and red circles are near-bed and buried pressure sensors, respectively) and (b) the laboratory experiment (wave flume coordinate system, magenta squares are capacitance wave gauges, only cross-shore locations are shown). The range of (a) mean and (b) still water levels is indicated by the black dashed bars in (a) and by the black dashed line in (b).

Significant wave heights (H_s) are estimated as 4 times the standard deviation of the surface-elevation fluctuations over the frequency (f) range $0.06 < f < 0.30$ Hz for field observations and $0.04 < f < 4.00$ Hz for lab measurements. The field bathymetry was surveyed within a day and within 27 m north and south of the instrument transect, and the lab bathymetry was surveyed immediately prior to the start of the experiment (Bond et al., 2023; Chen et al., 2024a; Tsai et al., 2024).

3. Model

3.1. Model Description and Set-Up

The two-phase *OpenFOAM* RANS model (Chen et al., 2024a; Larsen & Fuhrman, 2018; OpenFOAM, 2018) uses a stabilized $k-\omega$ Shear Stress Transport (SST) turbulence closure, in which the parametrization for eddy viscosity has the form (Larsen & Fuhrman, 2018; Menter, 1994):

$$\nu_t = \frac{a_1 k}{\max(a_1 \omega, F_2 \sqrt{p_0}, a_1 \lambda_2 \frac{\beta}{\beta^* \alpha} \frac{p_0}{p_\Omega} \omega)}, \quad (1)$$

where a_1 , F_2 , λ_2 , β , β^* , and α are closure coefficients (given by Larsen and Fuhrman (2018)), k is the turbulent kinetic energy (TKE), and ω is the specific dissipation rate. The variables p_0 and p_Ω are defined as (Chen et al., 2024a; Larsen & Fuhrman, 2018):

$$\begin{aligned} p_0 &= 2S_{ij}S_{ij}, & S_{ij} &= \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \\ p_\Omega &= 2\Omega_{ij}\Omega_{ij}, & \Omega_{ij} &= \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i} \right) \end{aligned} \quad (2)$$

where S_{ij} is the mean strain rate tensor, Ω_{ij} is the mean rotational rate tensor, U_i is the i th component of the ensemble flow (mean and wave orbital flow), and x_i is the i th direction (x , y , z).

The stabilized k - ω SST closure adds the third limiter in the denominator of ν_t in Equation 1 compared with the classical k - ω SST closure (Menter, 1994; Wilcox, 2006), such that eddy viscosity in the near potential flow region (i.e., offshore of the breaking region where $p_0 \gg p_\Omega$) is kept small so that wave energy is not dissipated prematurely via turbulence offshore of the surfzone.

A sub-grid model is used for a more accurate distribution of two incompressible and immiscible fluids within a cell near the air-water interface (*isoAdvector*, Roenby et al. (2016a, 2016b)). The model is set up in two-dimensional cross-shore vertical coordinates with the positive x axis pointing onshore and the positive z axis pointing upward. For simulating field observations, a 100-m-long wave generation region and a 100-m-long wave absorption region with a flat seafloor are added offshore of FRF coordinate 450 m (Figure 1a) to provide stability and to absorb reflected waves (Chen et al., 2024a). No wave absorption or generation zone is added when simulating the laboratory experiment, and the model offshore boundary is at the wave flume coordinate $x = 0$ m to simulate the lab conditions fully. The field simulations use a base mesh of $\Delta x = 0.1666$ m and $\Delta z = 0.1250$ m that is refined to $\Delta x = 0.0833$ m and $\Delta z = 0.0625$ m near the air-water interface and to $\Delta x = 0.0417$ m and $\Delta z = 0.015$ m near the bottom. The lab simulation uses a base mesh of $\Delta x = 0.12$ m and $\Delta z = 0.04$ m that is refined to $\Delta x = 0.06$ m and $\Delta z = 0.02$ m near the air-water interface and to $\Delta x = 0.030$ m and $\Delta z = 0.003$ m near the bottom, similar to prior studies (Tsai et al., 2024). Further refining the mesh does not improve simulations of surface elevation and velocity significantly for either field or lab cases (Appendix A).

Waves are generated at the model offshore boundary by inputting sea-surface-elevation energy spectra to a wave generation and absorption toolbox (*waves2Foam*, Chen et al. (2024a); Jacobsen et al. (2012)). For field cases the input spectra are based on measurements in about 5-m water depth (with randomized phases) and for the lab case the spectrum is based on the offshore wave generator (including the phases).

The instantaneous water depth $D = h + \langle \eta \rangle$, where h is the still water depth and $\langle \eta \rangle$ is the instantaneous wave fluctuation, is estimated as the vertical integration of α_w , the volumetric ratio of water to the grid volume (Chen et al., 2024a). The sea-surface-elevation fluctuations $\langle \eta \rangle$ can be decomposed further into $\bar{\eta}$, the time-averaged wave-induced setup, and $\tilde{\eta}$, the demeaned surface fluctuations. The significant wave height H_s is estimated in the same way that H_s is estimated from the field data. The details of wave spectra and output data processing (e.g., wave height, setup, and velocity) for field (Chen et al., 2024a) and lab (Tsai et al., 2024) cases can be found in previous studies.

3.2. Model Verification

The model reproduces the observed transformation of wave heights, setup, and undertow reasonably well for both field and lab observations (Figure 2 and Appendix B). The root-mean-square error (RMSE) for simulated H_s is 0.09 m for the field and 0.05 m for the lab cases with a correlation $r^2 = 0.74$ (field) and $r^2 = 0.99$ (lab) (Figure 2a). The RMSE for the simulated setup is 0.006 m for the field and 0.002 m for the lab cases with a correlation of 0.97 (field) and 0.99 (lab) (Figure 2b). These results are more accurate than prior studies that used bulk radiation stress estimates (Table 1 of Apotsos et al. (2007)) and are similar to previous Large Eddy Simulation (LES) estimates (Tsai et al., 2024). Offshore wave heights and RMSE of wave heights and setup for each of the simulated cases are listed in Table B1. The RMSE for simulated undertow $\bar{u}$ is 0.02 m/s for combined field and lab cases with a correlation $r^2 = 0.7$ (Figure 2c). The model simulates the evolution of the wave spectra well (field case: Chen et al. (2024a), lab case not shown), including predicting the location and magnitude of the spectral peak and the growth of harmonics prior to wave breaking (Chen et al., 2024a).

The model simulations of wave height, setup, and undertow are more accurate for the lab than for the field, possibly because field conditions have large directional spreads ($\sim 25^\circ$) of wave energy around the incident wave direction (Feddersen, 2004) and alongshore variability (Feddersen & Guza, 2003) that are not accounted for in the 2D cross-shore vertical model.

Modeled instantaneous values of demeaned surface elevations $\tilde{\eta}$ and cross-shore velocities $\tilde{u}$ also agree reasonably well with measured surface fluctuations $\hat{\eta}$ (RMSE = 0.08 m and $r^2 = 0.51$) and cross-shore velocities $\hat{u}$ (RMSE = 0.30 m/s and $r^2 = 0.51$) in the lab surfzone, even though $\hat{\eta}$ and $\hat{u}$ include both wave and turbulent

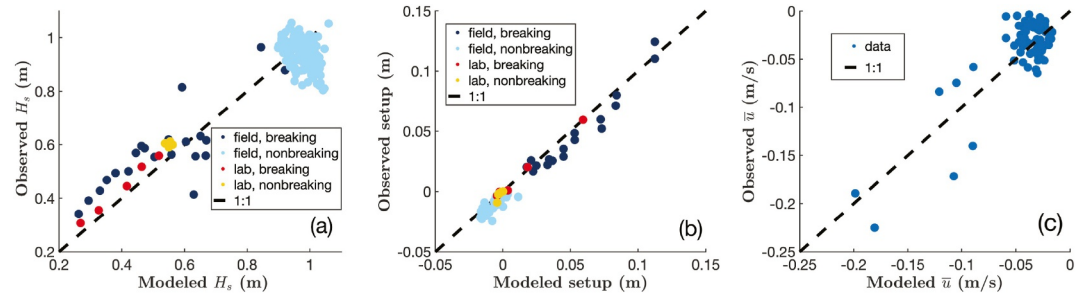


Figure 2. Observed versus modeled (a) significant wave height H_s , (b) wave setup, and (c) undertow $\bar{u}$ for field (dark blue breaking and light blue unbroken waves in (a) and (b)), lab (red breaking and yellow unbroken waves in (a) and (b)), and combined field-lab data sets (c). The dashed black lines in (a)–(c) are 1:1.

fluctuations (Figures 3a and 3b). In contrast, although binned mean values (solid black circles in Figure 3c) are correlated, modeled instantaneous vertical velocities $\bar{w}$ are not correlated well with observed $\bar{w}$ (Figure 3c), with RMSE of 0.09 m/s and $r^2 = 0.13$. The poor agreement for instantaneous values likely occurs because unbiased (zero mean) turbulence, which is not included in the modeled velocities, but which was not removed from the lab estimates, is relatively large (Figure 3c).

Modeled $\bar{\eta}$ is larger than observed $\bar{\eta}$ when modeled $\bar{\eta}$ is larger than about 0.2 m, or near the wave crest (Figure 3a). This discrepancy likely is due to sparse data around the nonlinear wave crest, and model failure to simulate the short-lived wave crest accurately.

The Index of Agreement (IA) between modeled P_i and observed O_i signals is (Tsai et al., 2024; Willmott, 1981):

$$IA = 1 - \frac{\sum_1^n (P_i - O_i)^2}{\sum_1^n (|P_i - \bar{O}| + |O_i - \bar{O}|)^2} \quad (3)$$

where i is a sample counter, n is the total number of samples, and $\bar{O}$ is the time average of the measured signal. The IA is 0.87 for both surface elevation and cross-shore velocity (not shown), similar to that using an LES model (Tsai et al., 2024). The IA is 0.47 for vertical velocity. The field simulations are driven with spectra and random phases, and thus time series comparisons are not possible.

4. Depth-Resolved Momentum Balances

For each time step, velocities (u, w), pressure (p), and turbulence variables (TKE k , specific dissipation rate ω , and eddy viscosity ν_t) measured at Eulerian coordinates are mapped into vertically Lagrangian (σ coordinates, bottom and surface following) and horizontally Eulerian (VL) coordinates (Figure C1, Mellor (2015), detailed σ

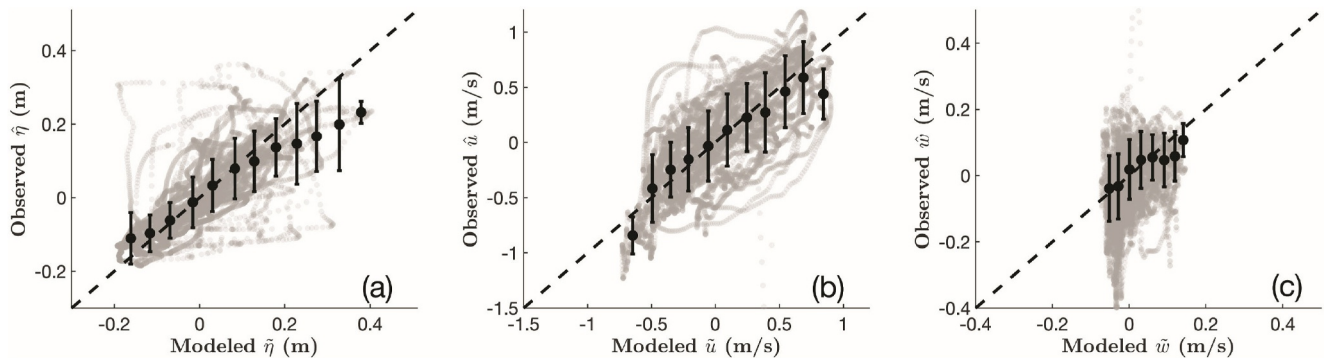


Figure 3. Laboratory observed ($\bar{\eta}$, $\bar{u}$, $\bar{w}$) versus modeled ($\bar{\eta}$, $\bar{u}$, $\bar{w}$) instantaneous (gray dots) (a) demeaned sea surface elevations at $x = 54.42$ m, (b) demeaned cross-shore velocities at $x = 54.42$ m and $z = -0.49$ m, and (c) vertical orbital velocities at $x = 54.42$ m and $z = -0.49$ m between 300 and 400 s. Solid black circles and vertical bars are means and standard deviations over bins of (a) 0.05 m, (b) 0.15 m s⁻¹, and (c) 0.03 m s⁻¹. The dashed lines are 1:1.

coordinate transformation can be found in Appendix C). Variables in each layer are then used to calculate the momentum balance.

Assuming alongshore uniformity, normally incident waves, negligible wind stress, and a gentle bottom slope, the depth-resolved time-averaged momentum balance equation is approximated as (Ardhuin et al., 2017; Chen et al., 2024a; Mellor, 2003, 2015):

$$\underbrace{\frac{\partial}{\partial x}(\bar{D}\bar{u}_e^2)}_{\text{horiz. inertia}} + \underbrace{\frac{\partial}{\partial \sigma}(\Omega\bar{u}_e)}_{\text{vert. inertia}} + \underbrace{\frac{\partial}{\partial x}(\bar{D}S_{xx,wave})}_{\text{wave}} + \underbrace{\frac{\partial}{\partial x}(\bar{D}S_{xx,roller})}_{\text{roller}} + \underbrace{\frac{\partial}{\partial x}(\bar{D}S_{xx,turbulence})}_{\text{turbulence}} + \underbrace{\bar{D}\frac{\partial}{\partial x}(g\bar{\eta})}_{\text{setup}} + \underbrace{\frac{\partial}{\partial \sigma}\tau_{turb}}_{\text{vert. turb. transfer}} - \underbrace{\frac{\partial}{\partial \sigma}\left(\frac{\partial \tilde{s}}{\partial x}(\bar{p} - g\tilde{s})\right)}_{\text{pressure slope}} = 0. \quad (4)$$

where $\bar{u}_e$ is the mean Eulerian velocity plus wave Stokes drift (Mellor, 2015), Ω is the time-averaged velocity normal to the σ layer, $\bar{D}$ is the mean water depth, g is the gravitational acceleration, $\tilde{s}(\sigma)$ is the fluctuation at a given σ layer, and $\bar{\eta}$ is setup (Mellor, 2003, 2015). The depth-dependent wave radiation stress $S_{xx,wave}$ is estimated in the water column below the level where α_w first decreases to 0.95, defined as (Chen et al., 2024a; Mellor, 2015):

$$S_{xx,wave}(\sigma) = \left(\overline{\tilde{u}(\sigma)^2} - \overline{\tilde{w}(\sigma)^2} + \frac{1}{D} \frac{\partial}{\partial \sigma} \left(\overline{\tilde{p}(\sigma)\tilde{s}} - \frac{g}{2}\tilde{s}^2 \right) \right) \Big|_{\min(\sigma(\alpha_w=0.95))} \quad (5)$$

where the overbar is time averaging over the final 2,048 data points from the model. The depth dependent roller radiation stress $S_{xx,roller}$ is defined as (Chen et al., 2024a):

$$S_{xx,roller}(\sigma) = \left(\overline{\alpha_w \tilde{u}(\sigma)^2} - \overline{\alpha_w \tilde{w}(\sigma)^2} + \frac{1}{D} \frac{\partial}{\partial \sigma} \left(\overline{\tilde{p}(\sigma)\tilde{s}} - \frac{g}{2}\tilde{s}^2 \right) \right) - S_{xx,wave} \quad (6)$$

where $\tilde{p}$ is the dynamic pressure, calculated as (Ardhuin et al., 2017):

$$\tilde{p}(\sigma) = p(\sigma) - gD(1 - \sigma) \quad (7)$$

where p is total pressure. The turbulence radiation stress $S_{xx,turbulence}$ cannot be obtained directly from the model and is approximated as:

$$S_{xx,turbulence}(\sigma) \approx \alpha_w \overline{(u'^2 - w'^2)} \approx \alpha_w c_t k(\sigma) \quad (8)$$

where u' and w' are the cross-shore and vertical turbulent velocity fluctuations, c_t is a parameter that measures turbulence anisotropy, and $k(\sigma)$ is the turbulence intensity mapped onto σ coordinates (Chen et al., 2024a, 2025; Roelvink & Stive, 1989). Previous studies have empirically estimated c_t ranging from 0.22 (analogous to wake turbulence, Townsend (1976) and Roelvink and Stive (1989)) to 1.3 (observation, Hally-Rosendahl et al. (2014); Hally-Rosendahl et al. (2015); Hally-Rosendahl and Feddersen (2016)) and have shown vertical variations (Chen et al., 2025; Yoon & Cox, 2010). Here, c_t is set to 1 because the depth-integrated turbulent radiation stress when $c_t = 1$ closes the momentum balance well (Chen et al., 2024a). The turbulent shear stress τ_{turb} is first defined in Eulerian coordinates in the two-equation RANS model, and then mapped to σ coordinates represented by the $(\cdot)_\sigma$ operator as:

$$\tau_{turb} = \left(-\nu_t \left(\frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} \right) \right)_\sigma \quad (9)$$

However, velocities are sampled only at vertical transects, and thus the horizontal gradient for velocity is approximated using time series and the wave phase speed, c_p , as:

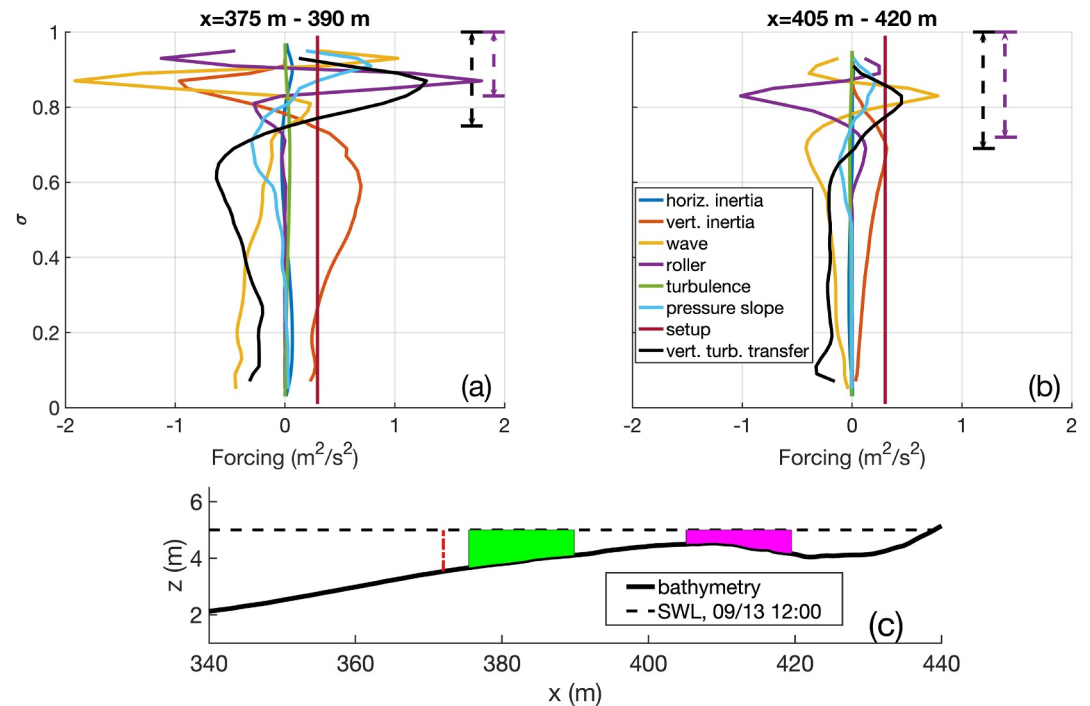


Figure 4. Momentum forcing terms (Equation 4) as a function of σ coordinate when roller momentum flux (a) increases and (b) decreases for a barred field case. Positive (negative) values indicate momentum flux gains (losses). (c) Seafloor elevation (black solid line) and still water level (black dashed line) versus horizontal coordinate for the field observation case on 13 September, 12:00 LT. The vertical dashed lines in (a) and (b) are roller thicknesses estimated using the momentum (purple) and turbulence (black) methods, discussed in Section 5.2. The green and magenta regions in (c) are the regions used in the estimates of (a) and (b), respectively. The vertical red dotted dashed line in (c) indicates the break point.

$$\frac{\partial u_i}{\partial x} \approx -\frac{1}{c_p} \frac{\partial u_i}{\partial t} \quad (10)$$

where u_i is a generic (orbital or ensemble) velocity component where $i = 1, 2, 3$ are cross-shore, alongshore, and vertical coordinates. The phase velocity c_p (and the group velocity) is approximated as $\sqrt{gD}$ assuming the shallow water limit.

5. Results

5.1. Vertical Momentum Structure

Near-surface forcing (e.g., when σ is larger than about 0.8) is different from that in the mid-to lower-water column (Figures 4 and 5). When waves start to break offshore of the sandbar and the roller is increasing (Figure 4a), the near-surface momentum balance is between wave (yellow curve) and roller (purple curve) radiation stresses, vertical advection (brown curve), the pressure force (light blue curve), and vertical turbulence transfers (black curve), while the roller and pressure forces are small in the lower-water column. Specifically, near the surface the momentum flux in breaking waves decreases (negative yellow curve), while that in wave rollers increases (positive purple curve). Vertical turbulence transfers increase near the surface (positive black curve when $\sigma > 0.8$) and become negative in the mid-to lower-water column (black curve for $\sigma < 0.8$), indicating a downward momentum transfer. Similarly, the pressure slope term transfers momentum from the near-surface to the mid-water column, but is small in the lower-water column (light blue curve). The vertical advection term is negative near the surface (brown curve) and positive below, indicating an upward momentum transfer. Setup is depth uniform (red line).

The momentum balance over and onshore of the sandbar is similar to that offshore of the sandbar, but with smaller forcing magnitude (Figure 4b). However, the signs of the wave and roller fluxes are reversed (compare yellow and

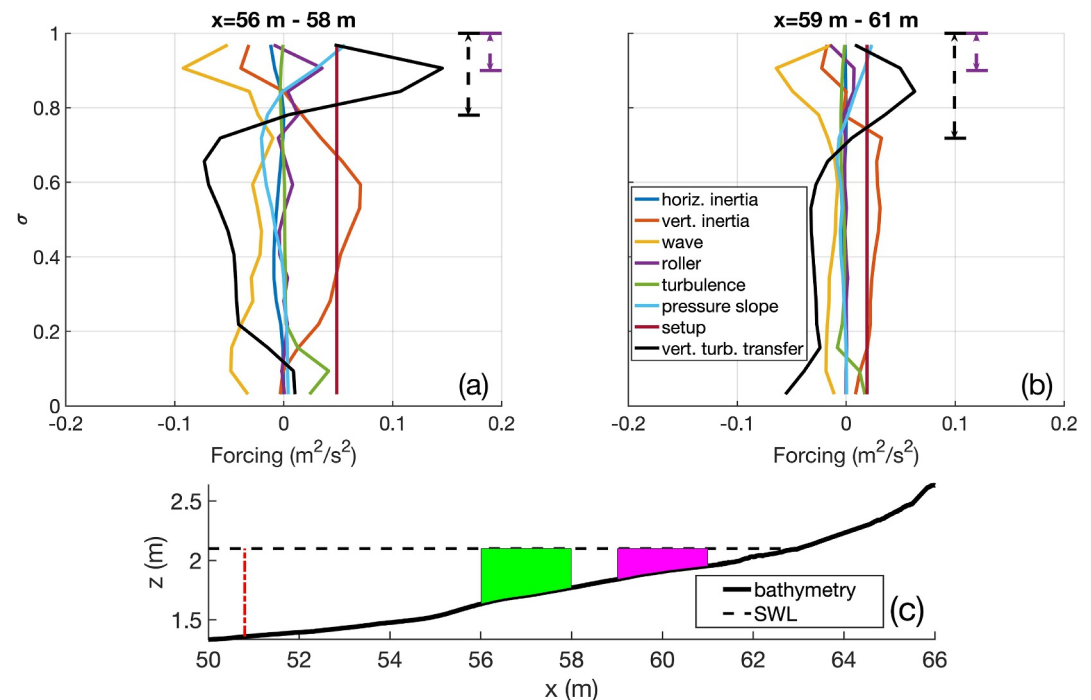


Figure 5. Momentum forcing terms (Equation 4) as a function of σ coordinate for (a) the outer and (b) inner surfzone for the laboratory case. Positive (negative) values indicate momentum flux gains (losses). (c) Seafloor elevation (black solid curve) and still water level (black dashed line) versus horizontal coordinate. The vertical dashed lines in (a) and (b) are roller thicknesses estimated using the momentum (purple) and turbulence (black) methods, discussed in Section 5.2. The green and magenta regions in (c) are the regions used in the estimates of (a) and (b), respectively. The vertical red dotted dashed line in (c) indicates the break point.

purple curves in Figure 4a with those in Figure 4b). This sign reversal likely is owing to a rapid decrease in the roller radiation stress as the waves reform on the onshore side of the sandbar. Momentum balances for cases with larger water depth are similar, albeit with smaller magnitude and shifted onshore (not shown).

The momentum balance is similar for the unbarred beach lab case and the barred field cases. However, the momentum transfers on the planar beach always are from the wave (losing momentum) to the roller (gaining momentum) as the water depth always is decreasing (Figures 5a and 5b, negative yellow curves and positive purple curves for $\sigma > \sim 0.8$). The turbulence radiation stress may gain some momentum near the bed (green curves, Figures 5a and 5b) where the bottom boundary layer dissipation may be enhanced by an overlap with the surface generated breaking wave turbulence. However, the turbulence radiation stress term remains small in most locations, and therefore using different values of the anisotropy parameter c_t as discussed above will not affect the results significantly.

5.2. Upper Water Column: Roller Thickness

A wave-averaged “roller thickness” (d_{roller}) is defined as a qualitative measure of the distance between the near surface, where the roller is important, and the mid-to lower-water column using two methods. Here, the roller thickness is defined using both estimated roller radiation stress (the momentum method, vertical dashed purple lines in Figures 4 and 5), or the vertical turbulent transfer (turbulence method, vertical dashed black lines in Figures 4 and 5). In the momentum method d_{roller} is defined as the distance between the surface and the vertical location where the roller radiation stress (cross-shore integrated purple curves in Figures 4 and 5) decreases to an ambient value equal to that resulting from numerical diffusion across one or two grid cells near the surface interface offshore of the breaking region (surface “smearing”) (Chen et al., 2024a). In the turbulence method d_{roller} is defined as the distance between the surface and the vertical location where vertical turbulence transfers (black curves in Figures 4 and 5) change from positive (gaining momentum) to negative (diffusing momentum).

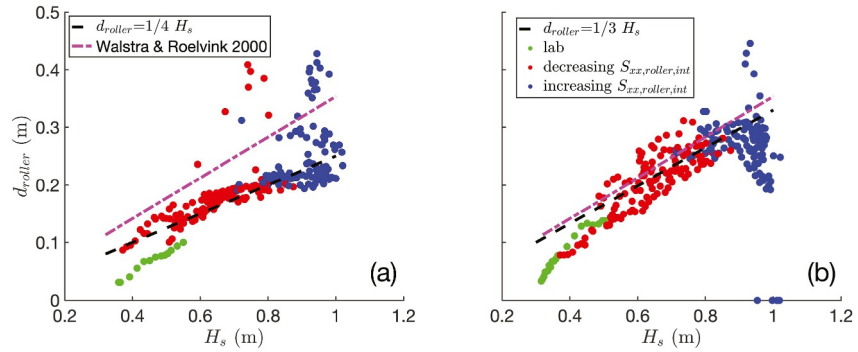


Figure 6. Roller thickness d_{roller} for the lab case (green) and for the field cases when the vertically integrated roller momentum flux increases (blue) and decreases (red) versus wave height using the (a) momentum and (b) turbulence methods.

Estimated roller thicknesses for all locations and cases are correlated with local significant wave height (Figure 6). Using the momentum method, d_{roller} is roughly 1/4 of the local H_s (Figure 6a), which is smaller than prior estimates and common (e.g., SWAN) model parameterizations (Svendsen, 1984; Walstra et al., 2001). However, the turbulence method gives $d_{roller} \approx 1/3 H_s$ (Figure 6b), consistent with prior estimates and parameterizations of roller geometry (Duncan, 1981; Svendsen, 1984; Walstra et al., 2001). The d_{roller} estimated using the turbulence method is slightly larger than that using the momentum method, possibly because additional turbulence is generated by shear below the roller-water interface (Na et al., 2016, 2020). The roller thickness is correlated better with H_s onshore of the field sandbar, where the vertically integrated roller radiation stress, $S_{xx,roller,int} = \int_{\sigma} D S_{xx,roller} d\sigma$, is decreasing (red circles in Figure 6), than offshore of the sandbar where $S_{xx,roller,int}$ is increasing (blue circles in Figure 6) for both methods (Figure 6a, momentum method $r^2 = 0.4$ and 0.1 and Figure 6b, turbulence method $r^2 = 0.7$ and 0.0, for decreasing and increasing vertically integrated roller radiation stresses, respectively). The roller thickness also is correlated with $S_{xx,roller,int}$ (momentum method $r^2 = 0.5$ and 0.1, turbulence method $r^2 = 0.8$ and 0.2 for decreasing and increasing vertically integrated roller radiation stresses, respectively, not shown), suggesting that the roller thickness may be parametrized with the integrated roller radiation stress. The roller thickness of the lab cases estimated using the momentum method is smaller than that of the field cases with similar wave height, probably because the roller radiation stress is smaller in the lab cases, while the lower threshold of roller radiation stress due to numerical smearing is similar to that in the field cases. Further refining the interfacial grids may decrease d_{roller} , but the resolution for convergence would be prohibitive to the current field-scale simulation and the correlation with H_s largely would be preserved (Appendix A).

5.3. Mid-to Lower-Water Column: Undertow

Momentum terms in the mid-to lower-water column are more depth-uniform than those in the upper water column (Figures 4 and 5). However, the vertical structure is important to the resulting undertow, and models based on parameterized wave breaking and forcing often do not reproduce observed undertow profiles well, possibly because additional momentum terms, such as vertical advection (vertical flux carried by the mean vertical flow), turbulence radiation stresses, and pressure slope are neglected in models (Apotsos et al., 2007; Mellor, 2021).

To examine the importance of different forcing terms and their vertical structure, the undertow is estimated below the wave roller by inverting a simplified version of Equation 4:

$$\underbrace{\frac{\partial}{\partial \sigma} \tau_{turb}}_{\text{vert. turb. transfer}} = - \underbrace{\frac{\partial}{\partial \sigma} (\Omega \bar{u}_e)}_{\text{vert. inertia}} - \underbrace{\frac{\partial}{\partial x} (\bar{D} S_{xx,wave})}_{\text{wave}} - \underbrace{\frac{\partial}{\partial x} (\bar{D} S_{xx,roller})}_{\text{roller}} - \underbrace{\bar{D} \frac{\partial}{\partial x} (g \bar{\eta})}_{\text{setup}} + \underbrace{\frac{\partial}{\partial \sigma} \left(\frac{\partial \bar{s}}{\partial x} (\bar{p} - g \bar{s}) \right)}_{\text{pressure slope}}, \quad (11)$$

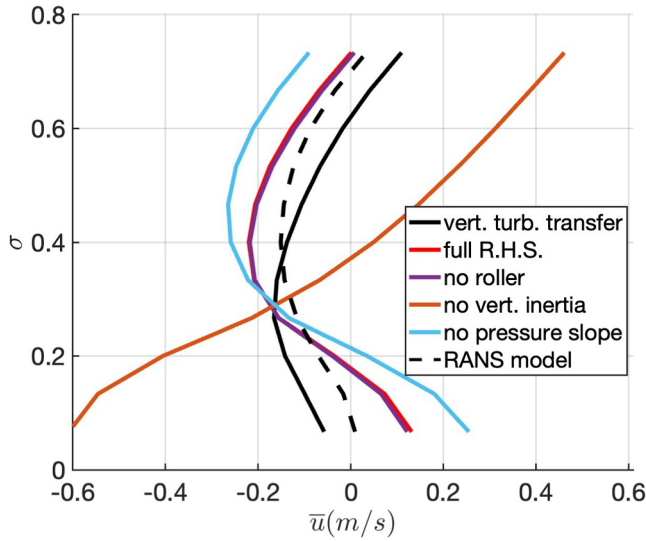


Figure 7. Vertical profile of the undertow estimated from the vertical turbulence transfers (solid black curve, Equation 14), and the inverted undertow profile from Equation 15 including all terms (red curve), without the roller term (purple curve, overlays the red curve), without the vertical inertia term (orange curve), and without the pressure slope term (blue curve) for the laboratory case (no sandbar) between $x = 56$ and $x = 58$ m. The black dashed curve is the undertow profile from the Reynolds-averaged Navier-Stokes model.

which neglects the small horizontal inertia and turbulence terms (dark blue and green curves in Figures 4 and 5). The simplified balance applies in the mid-to lower-water column, up to the zero-crossing of the vertical turbulence transfer profiles (black curves of Figures 4 and 5), which is $\sigma_t = 1 - H_s/(3\bar{D})$ according to the empirical relation between roller thickness and wave height (Figure 6b). Integrating both sides from $\sigma = 0$ to $\sigma_t = 1 - H_s/(3\bar{D})$ and assuming $\frac{\partial W}{\partial x}$ is small, Equation 11 suggests the terms on the right hand side (R.H.S.) determine the vertical structure of the undertow at a given cross-shore location as:

$$\left(\nu_t \frac{\partial U}{\partial z} \right)_\sigma \Big|_{\sigma=0}^{\sigma_t} = \int_0^{\sigma_t} R.H.S. d\sigma + b_1 \quad (12)$$

where b_1 is a constant at a given cross-shore location that is related to the top and bottom shear stresses. Numerical results show that the nonlinear interaction between eddy viscosity ν_t and the horizontal shear, $(\nu_t \partial U / \partial z)_\sigma - (1/\bar{D})(\bar{\nu}_t)_\sigma \partial \bar{u}(\sigma) / \partial \sigma$, is within about 20% of the linearized time-averaged shear stress $(1/\bar{D})(\bar{\nu}_t)_\sigma \partial \bar{u}(\sigma) / \partial \sigma$ except close to the bed, where the nonlinear shear stress can be approximately two times the linearized shear stress. Therefore, a nonlinear shear stress factor, $c_\nu(x, \sigma)$ is introduced as:

$$\left(\nu_t \frac{\partial U}{\partial z} \right)_\sigma = \frac{1}{\bar{D}} c_\nu \bar{\nu}_t(\sigma) \frac{\partial \bar{u}(\sigma)}{\partial \sigma} \quad (13)$$

where for this study, c_ν is assumed to be cross-shore uniform, $c_\nu = 2$ for the bottom 1/6 of the water column and $c_\nu = 1$ for the remainder of the water column. The shear stress estimated with the chosen $c_\nu(\sigma)$ profile using the R.H.S. of Equation 13 agrees well with τ_{turb} estimated using Equation 9, so that:

$$\frac{1}{\bar{D}} c_\nu \bar{\nu}_t \frac{\partial \bar{u}(\sigma)}{\partial \sigma} \approx \tau_{turb}. \quad (14)$$

Moreover, the undertow profile estimated from the vertical turbulence transfers by solving Equation 14 for $\bar{u}$ agrees reasonably well with the RANS-modeled undertow profile (RMSE = 0.06 m/s, compare solid black curve with black dashed curve, Figure 7).

Dividing both sides by $c_\nu \bar{\nu}_t$, integrating both sides by σ , and using Equation 12, Equation 13 becomes:

$$\begin{aligned} \bar{u}(\sigma)_{inverted} = \int_0^\sigma \left[\frac{1}{c_\nu \bar{\nu}_t} \left(\int_0^\sigma - \underbrace{\frac{\partial}{\partial \sigma} (\Omega \bar{u}_e)}_{\text{vert. inertia}} - \underbrace{\frac{\partial}{\partial x} (\bar{D} S_{xx, wave})}_{\text{wave}} - \underbrace{\frac{\partial}{\partial x} (\bar{D} S_{xx, roller})}_{\text{roller}} \right. \right. \\ \left. \left. - \underbrace{\bar{D} \frac{\partial}{\partial x} (g \bar{\eta})}_{\text{setup}} + \underbrace{\frac{\partial}{\partial \sigma} \left(\frac{\partial \bar{s}}{\partial x} (\bar{p} - g \bar{s}) \right)}_{\text{pressure slope}} d\sigma + b_1 \right) d\sigma + b_2, \end{aligned} \quad (15)$$

where b_2 is a constant that is related to the total undertow-driven offshore-directed mass flux. The constant b_2 can be calculated by setting b_1 so the shear stress below the roller equals the numerical output, and by assuming the total offshore mass flux below the roller balances the total onshore wave (M_w) and roller (M_r) mass flux, as:

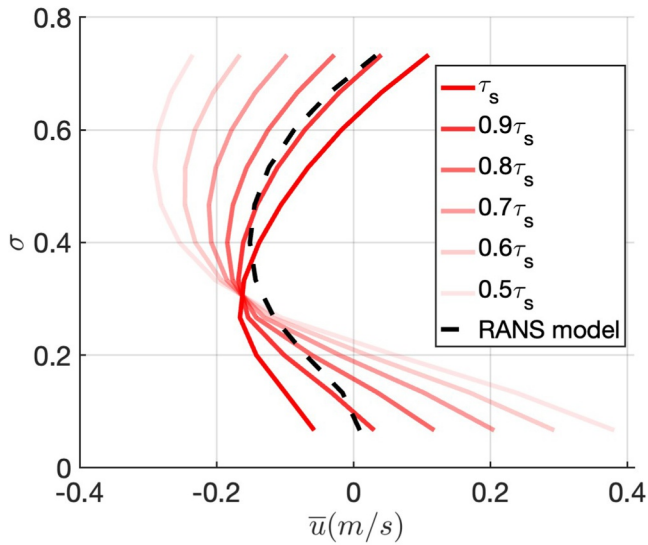


Figure 8. Vertical profiles of the inverted undertow (Equation 15) with different shear stresses below the roller. Lighter colors indicate smaller top shear stresses (given in the legend).

$$\bar{D} \int_0^{\sigma_1} \bar{u}_{inverted} d\sigma = M_w + M_r \quad (16)$$

where $M_w = \frac{1}{16} g H_s^2 / c_p$ and $M_r = \frac{1}{c_p} \int_0^{\bar{D}} \bar{D} S_{xx,roller} d\sigma$ (Apotsos et al., 2007).

When using all terms in Equation 15, the inverted undertow profile agrees reasonably well with the RANS-modeled undertow profile (compare solid red with black dashed curve, Figure 7). Uncertainty in estimating $c_v(\sigma)$ and the remaining residual momentum flux terms contributes to the discrepancies between the inverted and the modeled undertow profiles. The roller term is negligible in the momentum balance in the mid-to lower-water column (Figure 7 red and purple curves overlay). Excluding the pressure slope term increases the vertical gradient of the undertow, indicating that a stronger vertical turbulent transfer is needed to compensate the pressure slope term (Figure 7, blue curve). Excluding vertical inertia (Figure 7, orange curve) changes the undertow profile significantly because it is the dominant process balancing the downward turbulent transfers in the mid-to lower-water column.

The inverted undertow profile is sensitive to the choice of shear stress below the roller, $\tau_s = \left(\nu_t \frac{\partial U}{\partial z} \right)_\sigma$ ($\sigma = 1 - H_s / (3\bar{D})$), which is related to the choice of

b_1 . Given the same vertical profile of momentum fluxes, a reduced shear stress below the roller results in stronger offshore currents in the mid-to upper-water column and stronger onshore currents in the lower water column, therefore increasing the vertical gradient of undertow (Figure 8). Previous studies used the cross-shore gradient of wave and roller radiation stresses as the top shear stress by assuming that the wave and radiation stresses act only at the surface (Apotsos et al., 2007), neglecting the depth-dependent forcing. Accurate parametrization of surface-applied wave and roller radiation stresses and their contributions to the top shear stress of the undertow profile remain to be investigated.

6. Discussion

6.1. Deviation From Linear Theory

Assuming linear wave theory, wave radiation stress can be approximated as (Mellor, 2015):

$$S_{xx,linear}(\sigma) = k_w E (F_{cc} F_{cs} - F_{ss} F_{sc}) + \frac{E}{2\bar{D}} \frac{\partial}{\partial \sigma} (2F_{cc} F_{ss} - F_{ss}^2) \quad (17)$$

where k_w is the wavenumber, $E = g \bar{\eta}^2$ is the wave energy and

$$\begin{aligned} F_{cc} &= \frac{\cosh(k_w \bar{D}\sigma)}{\cosh(k_w \bar{D})} & F_{cs} &= \frac{\cosh(k_w \bar{D}\sigma)}{\sinh(k_w \bar{D})} \\ F_{sc} &= \frac{\sinh(k_w \bar{D}\sigma)}{\cosh(k_w \bar{D})} & F_{ss} &= \frac{\sinh(k_w \bar{D}\sigma)}{\sinh(k_w \bar{D})}. \end{aligned} \quad (18)$$

Modeled estimates of wave radiation stress are tested using 3 cases of progressive waves over a flat bottom (see Appendix C). Here, the nonlinear wave is treated as the sum of linear waves at different frequencies (with multiple spectral peaks) with wavenumbers based on linear theory. The model estimates of wave radiation stress agree well with linear wave theory (Figure 9). The model also estimates the Stokes drift well for these cases (not shown). Surface intensification of the Stokes drift during breaking in the surfzone (Deike et al., 2017; Lenain et al., 2019) is resolved when transforming the cross-shore velocity to σ coordinates.

Linear theory estimates of wave forcing (Equations 17 and 18) do not explicitly separate the roller contributions (Mellor, 2015). Thus, for comparison, wave and roller forcing estimated directly from the modeled velocity and

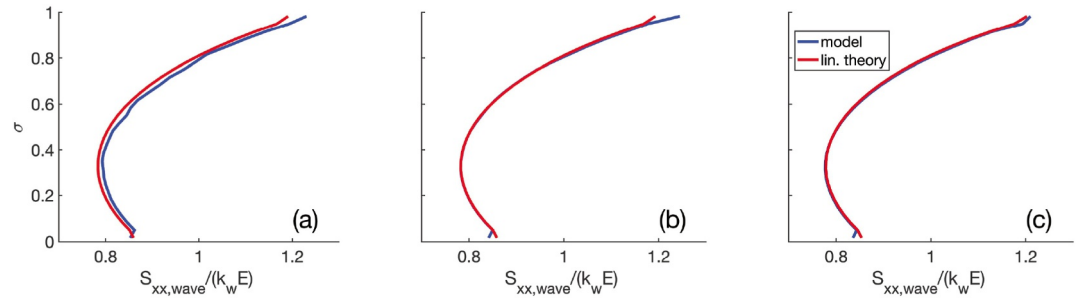


Figure 9. Wave radiation stress normalized by wavenumber and wave energy estimated from the model (blue curves) and from linear theory (red curves) as a function of σ coordinate for (a) a linear, (b) second-order Stokes, and (c) fifth-order Stokes wave.

pressure time series (Equations 5 and 6) are combined. Wave and roller radiation stresses estimated using Equations 5 and 6 (Figure 10b), which include wave nonlinearity, are surface and bottom intensified compared with the linear theory estimates (Figure 10a), which show weakly bottom intensified vertical structure. The surface intensification likely is contributed by the near-surface roller radiation stress, while the bottom intensification likely is caused by the nonlinear waves.

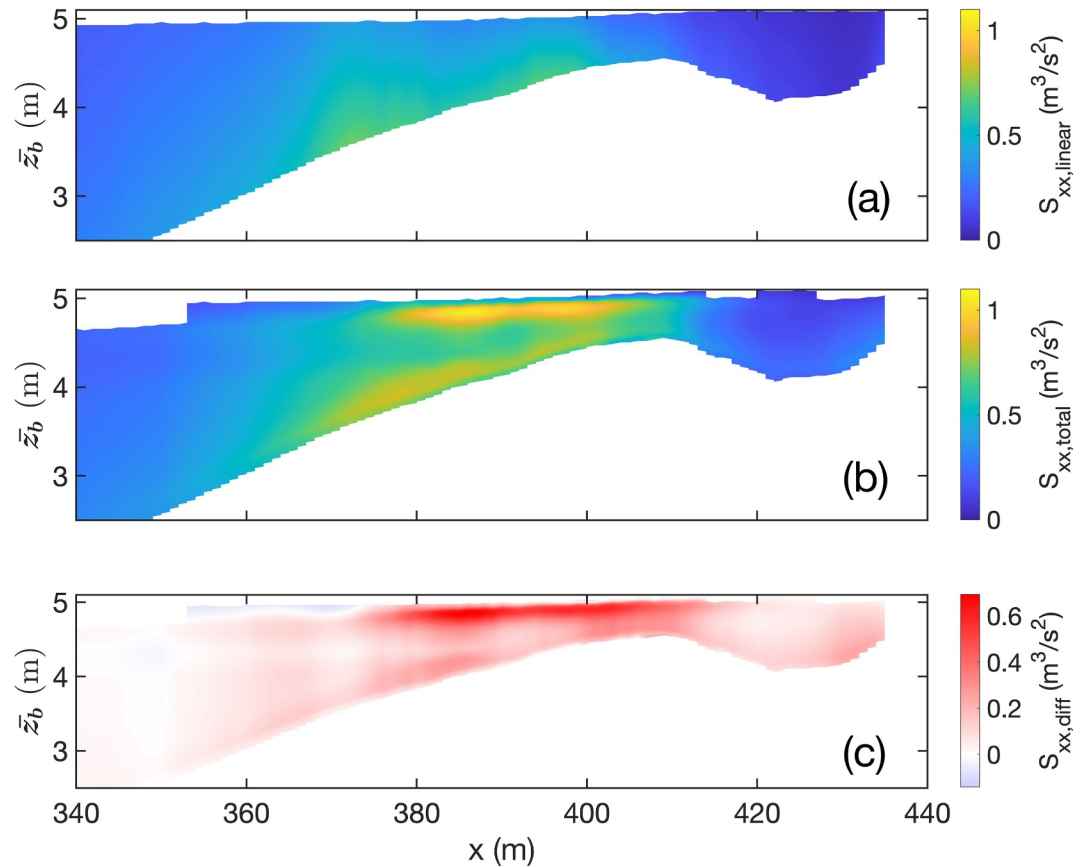


Figure 10. Wave radiation stress as a function of wave-averaged elevation $\bar{z}_b$ (σ coordinate $\times \bar{D} + h_b$) (a) estimated using linear theory $S_{xx, linear}$, (b) calculated as the sum of wave and roller radiation stress from the Reynolds-averaged Navier-Stokes model (Equations 5 and 6) $S_{xx, total} = S_{xx, wave} + S_{xx, roller}$, and (c) the difference between linear theory estimates and model estimates $S_{xx, diff} = S_{xx, total} - S_{xx, linear}$ versus cross-shore coordinate for the field observation case on 13 September 12:00 LT. Modeled radiation stress is estimated every 1 m for $x \in [360, 420]$, every 2.5 m for $x \in [350, 360]$ and $x \in [420, 430]$, every 5 m for $x \in [430, 435]$, and every 10 m otherwise.

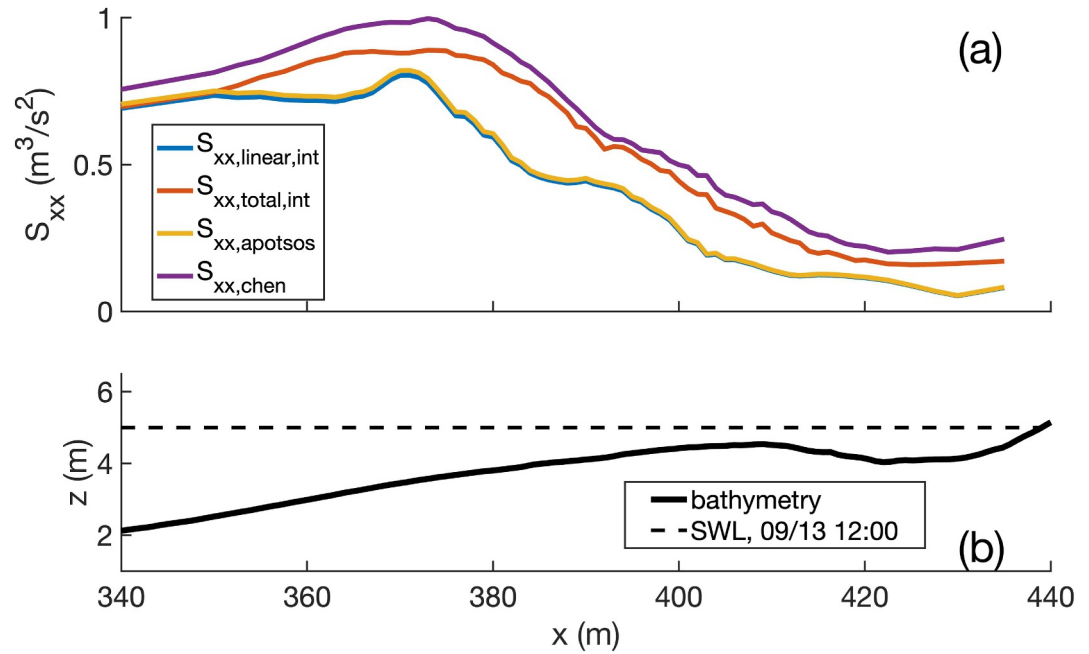


Figure 11. (a) Wave radiation stress estimated using integrated linear theory (Equation 17, blue curve), integrated definition (Equations 5 and 6, orange curve), and bulk estimates of linear theory (Equation 20, yellow curve) and definition (Equation 19, purple curve) for the field observation case on 13 September 12:00 LT, and (b) profile elevation z versus model cross-shore coordinate x .

Vertically integrated wave radiation stresses can be obtained from vertically integrating the full term contained in brackets in Equation 6, ($S_{xx, \text{total}, \text{int}}$), Equation 17, ($S_{xx, \text{linear}, \text{int}}$), or can be estimated using velocity and surface elevation time series (Chen et al., 2024a)

$$S_{xx, \text{chen}} = \int_{-h}^{\eta} (\rho \tilde{u}^2) dz - \int_{-h}^{\eta} (\rho \tilde{w}^2) dz + \frac{1}{2} \rho g \bar{\eta}^2. \quad (19)$$

where ρ is density, or using linear theory and time-averaged (bulk) properties, as (Apotsos et al., 2007):

$$S_{xx, \text{apotsos}} = \frac{1}{16} g H_s^2 \left(2 \frac{c_p}{c_g} - \frac{1}{2} \right) \quad (20)$$

where c_g is the wave group velocity. The resulting bulk estimates (Equations 19 and 20) are similar to the integrated estimates (Figure 11, compare yellow with blue curve and orange with purple curve) as expected. The nonlinear estimates using the definition of radiation stresses (Figure 11 orange and purple curves) are larger than the linear theory estimates (Figure 11 blue and yellow curves), similar to prior studies (Chen et al., 2024a; Torres-Freyermuth et al., 2007). In addition, the nonlinear estimates have maximum radiation stresses farther onshore than the linear estimates, indicating that including wave nonlinearity delays momentum transfers from breaking waves to the water column, consistent with prior studies (Chen et al., 2024a; Torres-Freyermuth et al., 2007).

Offshore of the sandbar where $S_{xx, \text{roller}, \text{int}}$ increases, the linear theory wave forcing term, estimated as the horizontal gradient of Equation 17 (Figure 12a, orange curve), is qualitatively consistent with the combined wave and roller forcing estimated directly from the model in the lower 60% of the water column (third and fourth terms in the L.H.S. of Equation 4, Figure 12a, blue curve), whereas the combined wave and roller forcing for the top 40% of the water column is in general more positive than the linear theory wave forcing because of the increasing surface intensification of wave and roller radiation stress offshore of the sandbar (Figure 10b). Over or onshore of the sandbar, wave forcing estimated using linear theory (Figure 12b orange curve) is similar to the combined wave and roller forcing and vertical inertia (second, third, and fourth terms in the L.H.S. of Equation 4, Figure 12b, blue

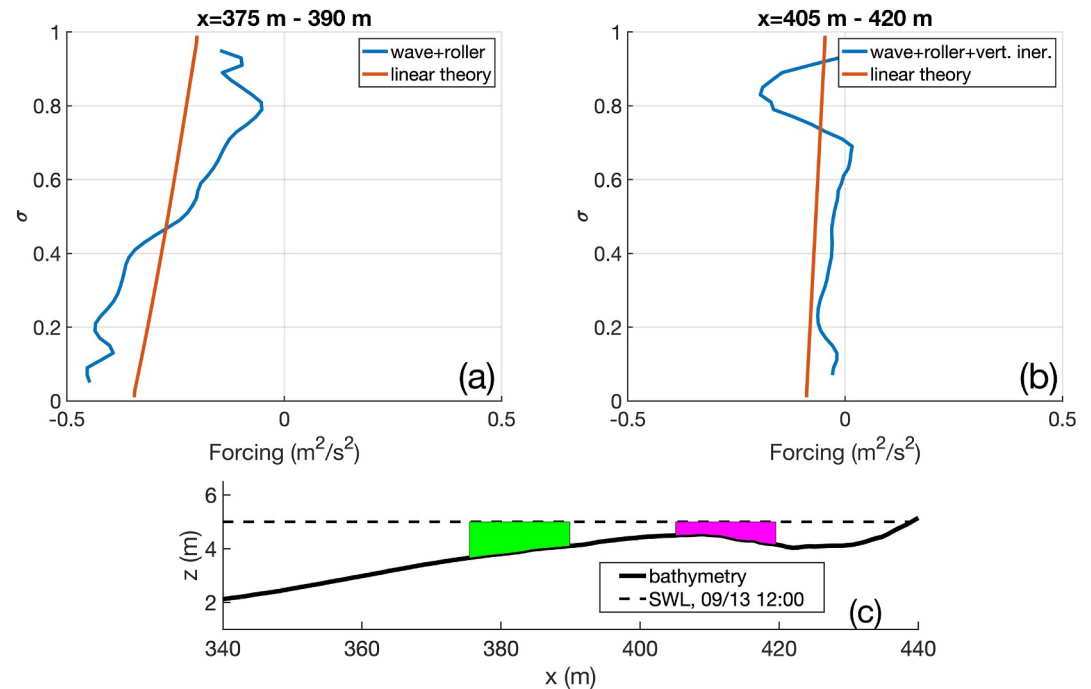


Figure 12. Wave forcing as a function of σ coordinate using linear theory (horizontal gradient of Equation 17, orange curves) and model estimates (blue curves) based on (a) wave and roller forcing when integrated $S_{xx,roller}$ increases, and (b) wave, roller, and vertical inertia when integrated $S_{xx,roller}$ decreases for the field observation case on 13 September 12:00 LT. Positive (negative) momentum fluxes indicate gains (losses). (c) Seafloor elevation (black solid curve) and still water level (black dashed line) versus horizontal coordinate. The green and magenta regions in (c) are the regions used in the estimates of (a) and (b), respectively.

curve), especially in the bottom 60% of the water column, whereas the combined wave and roller forcing from the model for the top 40% of the water column is more negative than the linear theory wave forcing because the surface intensification of radiation stress vanishes in the sandbar trough (Figure 10b).

Deviations of wave radiation stress and forcing from linear wave theory for the unbarred beach lab case are similar to the field results offshore of the sandbar, with surface intensified radiation stress when using methods that include wave nonlinearity (not shown). The similarities in forcing in the outer and inner surfzone of the unbarred beach case with those offshore of the sandbar of the field cases suggest differences over and onshore of the sandbar in the field primarily are owing to the presence of the bar-trough system (not shown).

6.2. Wave-Averaged Turbulence Estimates

Below the roller, a vertical balance of turbulent kinetic energy (TKE) between turbulence production, dissipation, and diffusion often is assumed, while horizontal turbulence advection usually is ignored (Feddersen & Trowbridge, 2005; Moghimi et al., 2016). Therefore, a 1D vertical turbulence equation that uses wave properties only at a single cross-shore location as input is needed to calculate vertical turbulence momentum mixing (Feddersen, 2012a, 2012b; Feddersen & Trowbridge, 2005; Martins et al., 2022). However, the horizontal advection of turbulence may be significant under breaking waves, especially near the surface (Van der Zanden et al., 2019). Yet, direct comparisons of laboratory and modeled TKE balances often are difficult because different methods are used for wave averaging (Van der Zanden et al., 2019).

The full evolution equation of turbulence kinetic energy (TKE) k can be described by (Larsen & Fuhrman, 2018):

$$\underbrace{P_k}_{\text{prod.}} - \underbrace{\frac{Dk}{Dt}}_{\text{advection}} - \underbrace{P_b}_{\text{buoy.}} - \underbrace{\beta^* \omega k}_{\text{diss.}} + \underbrace{\frac{\partial}{\partial x_j} \left[\left(\nu + \sigma^* \frac{k}{\omega} \right) \frac{\partial k}{\partial x_j} \right]}_{\text{diffusion}} = 0 \quad (21)$$

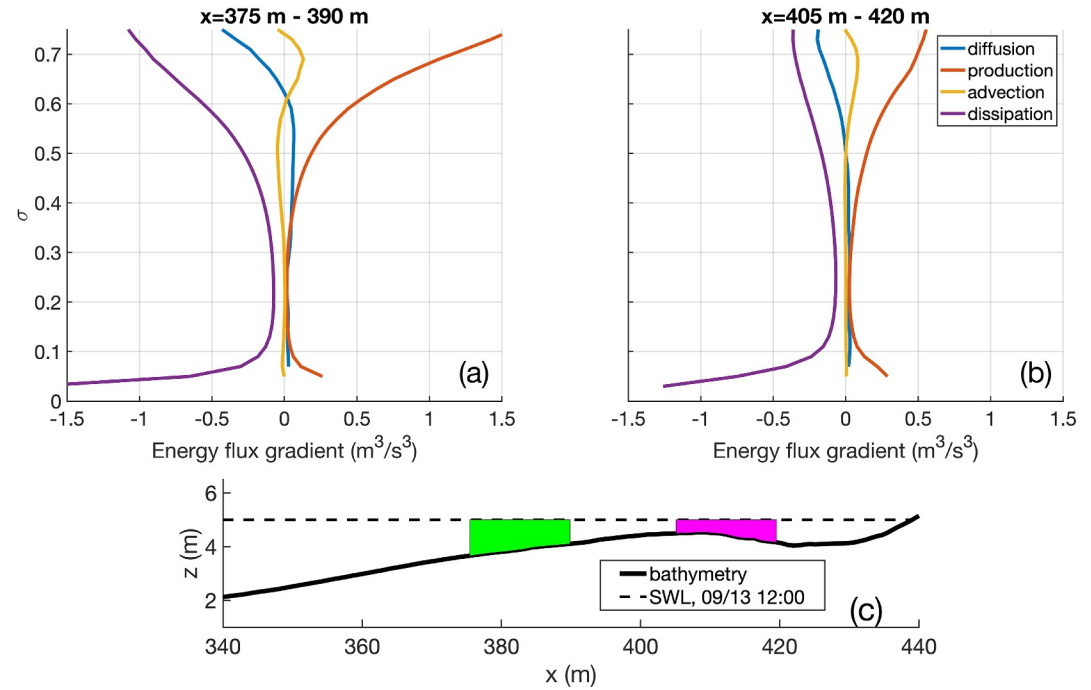


Figure 13. Turbulence energy flux gradient terms as a function of σ coordinate when the roller momentum flux (a) increases and (b) decreases for the field observation case on 13 September 12:00 LT. Positive (negative) values indicate momentum flux gains (losses). (c) Seafloor elevation (black solid curve) and still water level (black dashed line) versus horizontal coordinate. The green and magenta regions in (c) are the regions used in the estimates of (a) and (b), respectively.

where D/Dt is the material derivative, ν is the kinematic viscosity of water, σ^* and β^* are closure coefficients (Larsen & Fuhrman, 2018). Turbulence production P_k and buoyancy production P_b are given by (Chen et al., 2024a; Larsen & Fuhrman, 2018):

$$P_k = \nu_i p_0, \quad P_b = \alpha_b^* N^2, \quad N^2 = \frac{g_i}{\rho} \frac{\partial \rho}{\partial x_i}. \quad (22)$$

where N is the Brunt–Väisälä frequency, α_b^* is a coefficient, g_i is the i th component of the gravitational acceleration, and ρ is density. Here, the air–water mixed region (roller) is excluded, and the buoyancy term P_b is neglected because the water is unstratified (Mellor, 2015, 2021).

The depth-resolved wave-averaged turbulence production below the air–water mixed region of the roller is concentrated near the top of the water column (Figure 13, orange curve), whereas vertical diffusion of turbulence distributes energy flux through the water column (Figure 13, blue curve is negative near the surface and positive below). This pattern is similar offshore (Figure 13a) and onshore (Figure 13b) of the sandbar. However, turbulence production can dominate the entire water column over the sandbar where the water is the shallowest (not shown). Advection of turbulence is surface concentrated and directed offshore near the surface and onshore in the lower water column (Figure 13, yellow curves are positive near the surface and negative below), similar to previous studies (Van der Zanden et al., 2019), but it contributes the least to the TKE balance. Dissipation is concentrated near the surface due to breaking wave generated turbulence and near the bottom due to bottom boundary layer turbulence (Figure 13, purple curves). Dissipation is smaller than production near the surface because some turbulence is diffused downward, and roughly equals the sum of diffusion and production for the bottom $\sim 40\%$ of the water column, except for the lowest 10%.

The penetration depth of the production dominated region (where the orange curve is larger than the blue curve, Figure 13) spans a greater fraction of the local water depth as wave heights decrease owing to wave breaking offshore of the sandbar, and is roughly constant onshore of the sandbar (not shown). Onshore of the sandbar (and offshore of the secondary breaking on the foreshore slope, $405 \leq x \leq 435$ m), the mean penetration depth is about

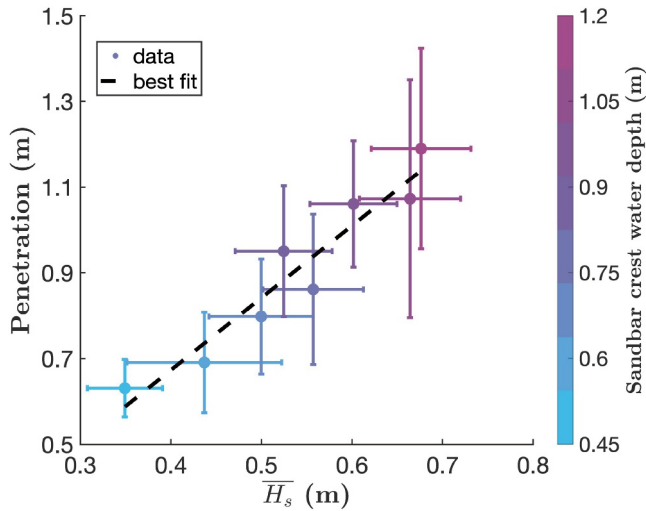


Figure 14. Spatially averaged (solid circles) and standard deviations (vertical bars) of shear-production-dominated penetration depth versus spatially averaged (solid circles) and standard deviation (horizontal bars) of the significant wave height across the sandbar trough ($405 \leq x \leq 435$ m) in field cases, colored by water depth over the sandbar crest, which limits the wave energy in the trough.

1.6 times the mean wave height averaged over the same region (Figure 14, $r^2 = 0.9$), suggesting that larger waves force deeper shear production. Some previous models of breaking wave turbulence suggest diffusion dominates near the surface and shear production dominates in the mid-water column (Moghimi et al., 2016), whereas others (Van Der A et al., 2017) suggest that shear production dominates near the surface and is in near balance with dissipation as shown here. Further modeling and observational studies on the structure of surfzone-generated turbulence are needed to resolve these discrepancies.

6.3. Limitations

The $k-\omega$ SST turbulent closure used here has been demonstrated to be one of the best two-equation RANS closures for simulating wave transformation, setup, and undertow in the surfzone (Brown et al., 2016; Chen et al., 2024a). However, most two-equation turbulent closures rely on the Boussinesq approximation, which relates Reynolds stresses τ_{ij} to the mean current strain as (Boussinesq, 1877):

$$\begin{aligned} -\tau_{ij} &= \overline{u'_i u'_j} = 2\nu_t S_{ij} - \frac{2}{3}k\delta_{ij} \\ k &= \frac{1}{2}\overline{u'_i u'_i}. \end{aligned} \quad (23)$$

where u'_i is the i th component of the turbulent velocity fluctuations and δ_{ij} is the Kronecker delta. This approximation assumes maximum efficiency of turbulent energy transfer across spatial scales (Ballouz & Ouellette, 2018; Chen et al., 2025). However, recent studies indicate that cross-scale turbulence energy transfer efficiency can be low (e.g., $\approx 25\%$ for the inertial subrange in a simulation of isotropic turbulence, Ballouz and Ouellette (2018); Pope (2000)). Therefore, spatial turbulence energy advection may be larger than estimated here, resulting in greater mixing and a more depth-uniform undertow profile. Two-equation turbulent-closure RANS models tend to predict bottom intensified undertow profiles, consistent with under-estimating spatial turbulence advection, whereas Reynolds Stress (RSM) or LES models, both of which do not relate turbulent Reynolds stresses with ensemble flow strain, simulate undertow profiles more accurately (Larsen et al., 2020; Li et al., 2022; Tsai et al., 2024). Further studies are needed of the Reynolds stress and ensemble flow strain rate tensors, and their alignment and effect on the production of TKE, turbulence dissipation, and undertow structure.

Bubble entrainment and bursting, which may generate turbulence in the roller or near the interface and may be responsible for 1/4 to 1/2 of the total wave energy dissipation in the surfzone (Derakhti & Kirby, 2014; Lamarre & Melville, 1992; Na et al., 2016, 2020), are not resolved fully by the RANS model. Two-phase Direct Numerical Simulations (Deike et al., 2016) may be useful to investigate the amount and distribution of energy dissipation and momentum transfer due to bubbles.

Wave breaking is a 3D phenomenon. The 2D simulations conducted here may confine excessive wave and turbulence energy in the cross-shore and vertical directions (Larsen et al., 2020), whereas 3D phase-averaged simulations reproduce wave transformation and setup more accurately (Martins et al., 2022). Moreover, the superior performance of LES may result from improved resolution of turbulence or from 3D simulations (Tsai et al., 2024). Further comparisons of 2D and 3D models with field and laboratory observations are needed to determine the relative importance of turbulence and alongshore variability.

7. Conclusions

A two-phase RANS model driven with measured random waves over barred and unbarred bathymetries in the field and laboratory is used to investigate the vertical structure of the cross-shore momentum balance in the surfzone. Modeled wave heights, setup, and undertow agree reasonably well with field and lab measurements.

The surface-following (σ coordinates) wave-averaged near-surface momentum flux includes wave and roller radiation stresses, setup, vertical turbulence transfers (which transports momentum downward), pressure slope (which transports momentum downward), and vertical inertia (which transports momentum upward). Waves transfer momentum flux to the roller seaward of the sandbar. However, the roller radiation stress decreases as waves propagate into the deeper water in the trough onshore of the bar crest and the wave radiation stress increases, suggesting that waves can regain momentum from the decaying roller. The roller thickness, where the roller dominates the momentum balance and turbulence generation, is proportional to the local significant wave height, with the highest correlation over and onshore of the sandbar where all waves are broken and the roller radiation stress starts to decrease. The undertow inverted from a simplified momentum balance in mid-to lower-water column agrees well with the directly estimated undertow, with the vertical inertia being the most dominant term.

Model estimates of wave radiation stress that include wave nonlinearity are surface intensified relative to estimates based on linear wave theory. The vertically integrated wave radiation stresses agree well with the respective (nonlinear or linear) bulk estimates, suggesting that the differences between the depth-integrated wave radiation stresses are largely due to surface intensification in the depth-resolved radiation stress. In the mid-to lower-water column offshore of the sandbar and on the near-planar laboratory beach the combined wave and roller forcing from the model is similar to linear theory estimates. Onshore of the sandbar, the sum of modeled vertical inertia and the radiation stresses has similar magnitude and vertical structure as linear theory.

Turbulence production roughly is balanced by turbulence dissipation near the water surface (below the roller). Downward turbulence diffusion is more depth uniform and more important in the mid-to lower-water column. The ratio of the thickness of the production-dominated turbulence to the local water depth remains roughly constant in the deeper water of the trough onshore of the sandbar. The mean penetration depth of turbulent production in the sandbar trough is correlated with the mean wave height, which is controlled at least partly by the depth of the sandbar crest.

Appendix A: Model Convergence Tests

Model convergence was evaluated by conducting a higher-resolution simulation for the 13 September, 15:02 LT field case. The simulation used a base mesh of $\Delta x = \Delta z = 0.1$ m that is refined to $\Delta x = \Delta z = 0.05$ m near the air-water interface and to $\Delta x = 0.025$ m and $\Delta z = 0.0125$ m near the bottom. The wave phases are kept identical to those in the coarse-resolution simulation to allow direct comparison of the time series.

The modeled fluctuations of surface elevation, horizontal and vertical velocities, and pressure at both breaking and non-breaking locations agree well between the two resolutions (Figure A1). The agreement is slightly worse at the breaking location than at the non-breaking location (Figures A1a–A1d, blue dots have slightly more scatter than red dots), but r^2 is above 0.98 for all variables except the vertical velocity in the surfzone, where $r^2 = 0.91$.

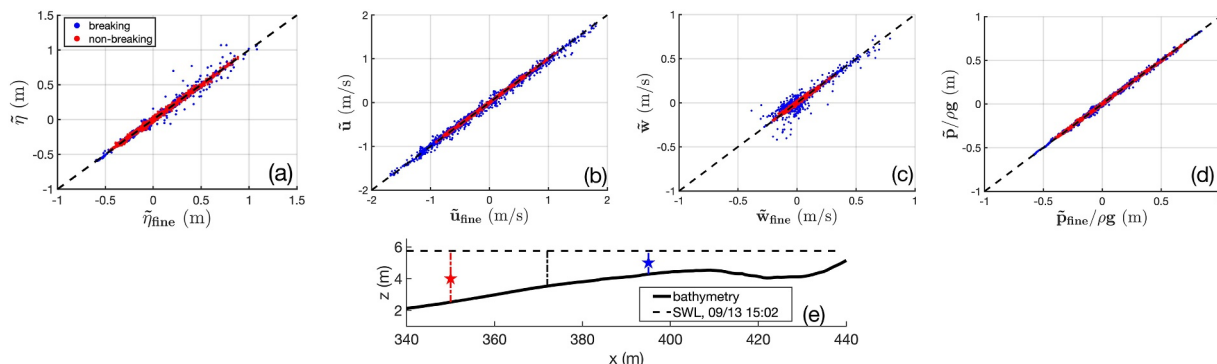


Figure A1. Fluctuations of (a) surface elevation, (b) horizontal velocity, (c) vertical velocity, and (d) pressure divided by ρg from the model used in this study versus those from the finer resolution model for both breaking (blue dots) and non-breaking waves (red dots). The locations of the dots in (a)–(d) are shown in (e), with the breaking values obtained along the vertical dashed blue line for surface elevation and at the blue star for velocities and pressure and non-breaking values obtained along the vertical dashed red line for surface elevation and the red star for velocities and pressure. The dashed lines in (a)–(d) are linear least squares fits. The dashed horizontal line in (e) is the still water surface.

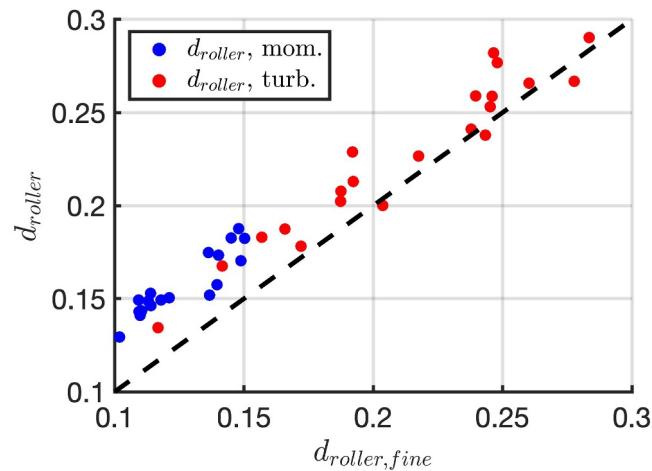


Figure A2. Roller thicknesses estimated using the momentum method (blue dots) and turbulence method (red dots) from the model used in this study versus those in the finer resolution model.

Since terms in the depth-resolved momentum balance (Equation 4) are estimated based on surface elevation, velocity, and pressure time series, good agreement of these time series indicates that the momentum terms have converged sufficiently at the current resolution.

Roller thicknesses estimated in the finer-resolution model are in general smaller than those estimated from the model used in this study using both methods, although the correlation is high (Figure A2, momentum method, $r^2 = 0.83$; turbulence method, $r^2 = 0.92$). The best-fit slope of d_{roller} is 0.91 for the momentum method and 0.88 for the turbulence method, indicating that having a finer-resolution model may decrease the roller thickness by a near-constant value. This phenomenon suggests that the model with finer resolution may be needed to resolve the wave roller fully. However, since the wave roller is a mixture of water and air bubbles, the required grid scale may be comparable to the scale of bubbles, which is prohibitive in a field-scale simulation such as performed here (Deike et al., 2016).

Appendix B: Simulated Cases

Simulated field and lab cases are summarized in Table B1.

Table B1

Simulated Data Runs, Offshore Significant Wave Height (H_s), Energy Weighted Mean Wave Period (T_w), and the RMSE Between Modeled and Observed H_s and Setup

Time (LT) or trial no.	Offshore H_s (m)	T_w (s)	RMSE of H_s , RMSE of setup (m)
12:00	0.86	9.62	0.09, 0.009
13:00	0.83	9.73	0.06, 0.007
13:32	0.86	9.76	0.06, 0.008
14:02	0.84	9.78	0.06, 0.005
14:32	0.82	9.81	0.08, 0.006
15:02	0.80	9.84	0.11, 0.005
15:42	0.79	9.88	0.06, 0.002
16:50	0.81	9.87	0.12, 0.004
Trial 5	0.58	4.00	0.05, 0.002

Note. The first 8 cases are from 13 September 1997, and Trial 5 is from the laboratory (Tsai et al., 2024). Spectral wave components for $0.06 < f < 0.3$ Hz are used to calculate H_s and T_w for field cases, while the full spectrum is used for the lab case.

Appendix C: σ Coordinate Transformation

With non-breaking waves, the vertical Lagrangian σ coordinate is related to the vertical Eulerian z coordinate as (Figure C1, Mellor (2015)):

$$z = \sigma(\bar{\eta} + h) + \tilde{s}(\sigma) + h_b \quad (C1)$$

where $z \in [h_b, h_b + h + \bar{\eta} + \tilde{\eta}]$ is the vertical Eulerian coordinate, $h_b(x)$ is the local bottom elevation, $h(x)$ is the local still water depth, $\bar{\eta}$ is setup, $\tilde{\eta}$ is surface fluctuation, $\sigma \in [0, 1]$ is the sigma coordinate representing the distance to the bottom divided by the total instantaneous water depth, and $\tilde{s}(\sigma)$ is the fluctuation at a given σ layer where $\tilde{s}(0) = 0$ and $\tilde{s}(1) = \tilde{\eta}$.

The fluctuations at each layer ($\tilde{s}(\sigma)$) have been estimated using linear wave theory (Mellor, 2015):

$$\tilde{s}(\sigma) = a \frac{\sinh(k_w \bar{D} \sigma)}{\sinh(k_w \bar{D})} \cos(\phi) \quad (C2)$$

where a is surface wave amplitude, k_w is the wavenumber, $\bar{D} = h + \bar{\eta}$ is total water depth, and ϕ is the wave phase as a function of time. However, this relationship has not been tested in nonlinear waves. Moreover, the fluctuations also can be determined numerically in the model using particle tracking independent of wave linearity. To verify the performance of Equation C2, cases are conducted with a flat bottom, 2-m mean water depth, and a propagating wave with a period of 3 s and height of 0.05 (linear wave), 0.15 (Stokes second-order wave), and 0.30 m (Stokes fifth-order wave) with 30 passive particles seeded 4 times (120 particles in total for each case) at a vertical transect far away from the model inlet and outlet (offshore and onshore) boundaries. The fluctuation is recorded as the vertical location of the particles and the wave amplitude at a given layer, s_w . Modeled layer fluctuations compare well with the amplitude derived from linear theory (Equation C2) for both linear and nonlinear waves (Figure C2). Moreover, the waves in the test cases approach the shallow water limit ($k_w h \approx \frac{1}{6}$), similar to surfzone waves, and the layer fluctuation is approximately correlated linearly with σ as (Figure C2):

$$\tilde{s}(\sigma) \approx a \sigma \cos(\phi). \quad (C3)$$

These results suggest the layer fluctuation does not need to be tracked via particles in the model and can be scaled with the instantaneous σ coordinate. Thus, Equation C3 has been employed in the coordinate transformation in this study.

In the VOF-RANS model used here, the instantaneous surface is estimated via the vertical integration of the instantaneous profile of α_w , as discussed in Section 3.1. However, to include the surfzone breaker in the σ coordinate transformation, the instantaneous surface has to be above the breaker. Thus, the coordinate transformation equation (Equation C1) is revised to:

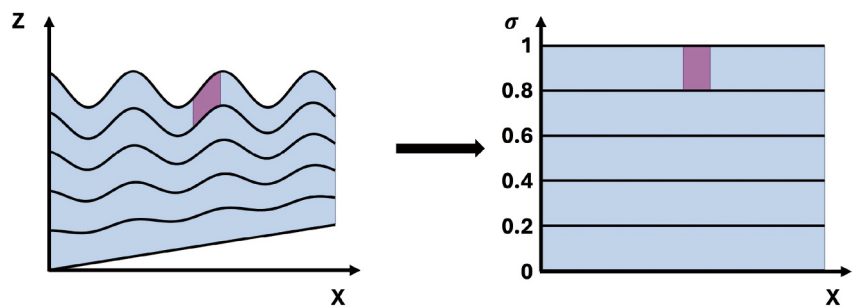


Figure C1. Sketch of the σ coordinate transformation for 5 σ layers, from Eulerian coordinates (z and x , left) to vertically Lagrangian and horizontally Eulerian (VL) coordinates (σ and x , right), with a sampled region (purple) transforming from the Eulerian coordinates to the VL coordinates.

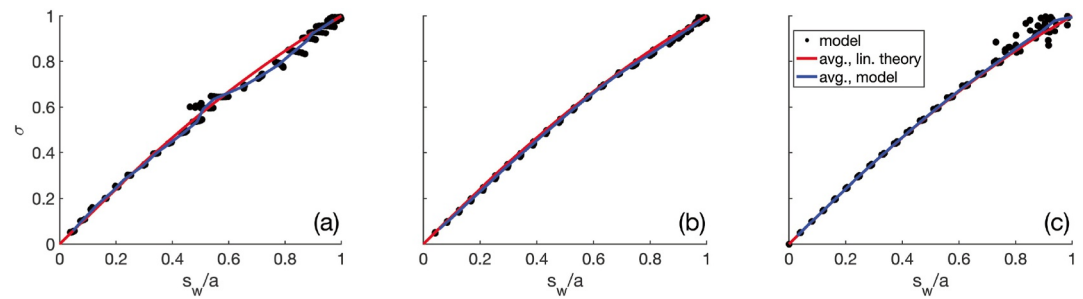


Figure C2. Fluctuation amplitude s_w nondimensionalized by the surface wave amplitude a as a function of σ coordinate for the Reynolds-averaged Navier-Stokes model data (black circles, blue curves for layer average) and linear theory (red curves) for (a) linear, (b) second-order Stokes, and (c) fifth-order Stokes waves.

$$z = \sigma(\bar{\eta}_b + h) + \tilde{s}_b(\sigma) + h_b \quad (\text{C4})$$

where $\tilde{s}_b(0) = 0$ and $\tilde{s}_b(1) = \tilde{\eta}_b$, with $\tilde{\eta}_b$ and $\bar{\eta}_b$ estimated by finding the top most level where $\alpha_w = 0.05$. However, both $\tilde{\eta}_b$ and $\bar{\eta}_b$ are used only in the coordinate transformation. Interface location estimation via vertical integration is used for estimating depth D , wave setup, and in the analysis.

Conflict of Interest

The authors declare no conflicts of interest relevant to this study.

Availability Statement

The data and code used to produce the figures are available online (Chen et al., 2026). The 1997 (SandyDuck) field data are available online (Elgar & Raubenheimer, 2024). The laboratory experiment data are available on the U.S. National Science Foundation Natural Hazards Engineering Research Infrastructure data portal (Bond & Wengrove, 2023; Pontiki & Puleo, 2022). Field model configuration files are available online (Chen et al., 2024b). Simulations are performed on OpenFOAM v1812 (OpenFOAM, 2018). Field bathymetry profiles and offshore wave data were obtained from the USACE Field Research Facility THREDDs server (USACE, 2026). NOAA water levels can be found at the station's website (NOAA, 2026). Data are processed and analyzed using MATLAB R2024a (MathWorks, 2024).

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