MIT 12.521 "Computational Geophysical Modeling"

Instructors: Mark Behn, Jian Lin, and Olivier Marchal

Feb 1: Class schedule

Feb 3: Course Intro (Jian/Olivier/Mark, half hour) + BEM 1 - Intro (Jian, 1 hour)

Feb 8: FDM 1 Intro (Olivier)

Feb 10: FEM 1 Intro (Mark)

Feb 15: FDM 2 (Olivier)

Feb 17: FDM 3 (Olivier)

Feb 22: President's Day (No Class/Monday Classes Held)

Feb 24: FDM 4 (Olivier, Jian at Sea)

Mar 1: FDM 5 (Olivier, Jian at Sea)

Mar 3: FDM 6 (Olivier, Jian at Sea)

Mar 8: FEM 2 (Mark, Jian at Sea)

Mar 10: FEM 3 (Mark, Jian at Sea)

Mar 15: FEM 4 (Mark, Jian at Sea)

Mar 17: FDM 7 (Olivier, Mark Away)

Mar 22: Spring Break (No Class)

Mar 24: Spring Break (No Class)

Mar 29: FDM 8 (Olivier, Mark Away)

Mar 31: FEM 5 (Mark)

Apr 5: FEM 6 (Mark)

Apr 7: FEM 7 (Mark, Olivier Away)

Apr 12: FEM 8 (Mark, Olivier Away)

Apr 14: BEM 2 (Jian)

Apr 19: Patriots Day (No Class/Monday Class Held)

Apr 21: BEM 3 (Jian)

Apr 26: BEM 4 (Jian)

Apr 28: BEM 5 & 6 (Jian)

May 3: BEM 7 (Jian)

May 5: BEM 8 (Jian)

May 10: Student Presentations

May 12: Student Presentations

Source Codes

1) Finite-difference methods
2) Finite-element methods
3) Boundary-element methods: (a) Software package Coulomb 3.2; (b) Software package 3d-def

*Please e-mail for above jlin@whoi.edu

Homework Assignments

1) Finite-difference methods
2) Finite-element methods
3) Boundary-element methods

February 1, 2011

Course 12.521 “Computational Geophysical Modeling”
(formerly “Computational Geodynamics Modeling”)

Instructors: Mark Behn, Jian Lin, and Olivier Marchal

Course Description

This course is designed for you to learn the key concepts of computational geophysical modeling, as well as to acquire practical problem-solving skills that we hope will serve you well on your research endeavour.

The course will introduce three most commonly used approaches of boundary-element (BE), finite-difference (FD), and finite-element (FE) methods, and their hybrids. More importantly, we will illustrate how to use these methods to study the dynamics of geophysical fluids such as the mantle and the ocean. Problems of interest are, for example, the mass and heat transfer in mantle convection, the thermal structure of the lithosphere, the crustal deformation of earthquakes and magmatic dikes, and the wind- and buoyancy-driven circulations in the ocean.

There are three computer-based homework assignments and one semester-long modeling project of your own choice. You will present your project to the class at the end of the semester. There are no examinations.

Grading

Students are required to complete three computer-based homework assignments, as well as a semester-long course project of student’s interest. There will be no written examinations.

Main Textbooks

Bathe, K.-J., Finite Element Procedures, Prentice-Hall, 1996. ISBN 0-13-301458-4.

Crouch, S.L. and Starfield, A.M., Boundary Element Methods in Solid Mechanics: With Applications in Rock Mechanics and Geological Engineering, George Allen and Unwin, London, 1983. ISBN 0685-460-150 (hardcopy) ISBN 004-445-9130 (regular copy).

Ferziger, J. H., and Peric, M., Computational Methods for Fluid Dynamics, Springer, Berlin, 356 p., 1996

Durran, D.R., Numerical Methods for Wave Equations in Geophysical Fluid Dynamics, Vol. 32 of Texts in Applied Mathematics, Springer, 465 pp., New York, 1999.

Class Schedule (2011)

Feb. 1, Organization/course overview (Jian/Olivier/Mark)
1) Boundary-element (BE) methods (Jian)
2) Finite-difference (FD) methods (Olivier)
3) Finite-element (FE) methods (Mark)

Introduction to FD methods (Olivier)
1) Equations of geophysical fluid dynamics
2) Mathematical properties of partial differential equations
3) FD methods: Basic idea, (dis)advantages

Feb. 3 Introduction to BE methods (Jian)
1) Philosophy of computational geophysical modeling:
   Physical problems, computational approaches, & solutions
2) Chose an approach: BE vs. FE vs. FD methods
3) Geophysical applications
4) Computer source codes for the class

Feb. 8, Introduction to FE methods (Mark)
1) Comparison to FD & BE methods
2) Discretization, interpolation functions
3) Imposition of boundary conditions
4) Inversion
5) Simple 1-D example

Feb. 10 & 15, Components & properties of FD methods (Olivier)
1) Components: Mathematical model, grid, FD approximations, ...
2) Properties: Consistency, stability, convergence, ...

Feb. 17, FD approximations: Space-differencing (Olivier)
1) First, second, & mixed derivatives
2) Other terms: Coriolis acceleration, sources & sinks, ...
3) Implementation of boundary conditions
4) Exercises

Feb. 22, President's Day (No Class, Monday classes held)

Feb. 24, FD approximations: Time-differencing (Olivier)
1) Methods for initial-value problems
2) Implementation of initial conditions
3) Application to the generic transport equation
4) Exercises

March 1, Introduction to the FD homework (Olivier)
1) Wind-driven circulation: Model of Munk
2) Buoyancy-driven circulation: Model of Beardsley and Festa
3) Handout homework assignment #2

March 3, Essentials of FE techniques (Mark)
1) Derivation of weak form of equations
2) Discretization methods

March 8, Application of FE methods to elastic models (Mark)
1) Basic equations for linear elasticity
2) Special cases: Plane strain, plane stress, axisymmetric, antishear
3) Shells, plates, & membranes
4) Examples

March 10, Application of FE methods to tectonics (Mark)
1) Elastic-plastic deformation
2) Ductile creep
3) Visco-elasticity
4) Discrete implementation of faults (slippery nodes, split nodes, etc.)
5) Finite deformation

March 15, Discretization errors (Olivier)
1) Modeling, distretization, & convergence errors
2) Impact on stability & convergence
3) Oscillation equation
4) Differential-difference & modified equations
5) Numerical dissipation & dispersion
6) Exercises

March 17, Solution of systems of algebraic equations I (Olivier)
1) Direct methods
3) Coupled equations
4) Non-linear equations
5) Exercises

March 22 & 24, Spring Vacation, no classes

March 29, Solution of systems of algebraic equations II (Mark)
1) Iterative methods
2) Multigrid algorithm

March 31, FLAC: Hydrid FE-FD methods (Mark)
1) Overview of technique
2) Implementation of dike intrusion
3) Example calculations for mid-ocean ridges
4) Handout homework assignment #3

April 5, Application of multigrid to corner flow at ridges/subduction zones (Mark)
1) Analytical solution
2) Multigrid solution for non-newtonian flow
3) Melt migration

April 7, Parallel computing (Mark)
1) Overview
2) Architecture
3) Distributed memory
4) Parallel programming

April 12, Essentials of BE methods (Jian)
1) Boundary value problems
2) Interior/exterior problems
3) Kelvin's problem: Constant traction over a line segment
4) Crouch's solution: Constant displacement-discontinuity over a line segment

April 14, Essentials of BE methods (Jian)
1) Fictitious stress method & Influence coefficients
2) Displacement-discontinuity method & influence coefficients
3) Special conditions: Symmetry, elastic half space, and piecewise homogenous bodies
4) Design architecture of BE programs

Apr. 19, Patriots Day (No Class, Monday classes held)

April 21, Application of BE methods to earthquakes (Jian)
1) Program "Coulomb 3.1"
2) Earthquake stress interactions
3) Handout homework assignment #1

April 26, Application of BE methods to diking (Jian)
1) Program "3D-def"
2) Dikes and other examples

April 28, Other BE applications (Jian)

May 3, Application of FD methods to mantle convection (Jian)
1) Styles of mantle convection
2) Basic equations & approximations
3) Reference state & non-dimensional variables
4) Boussinesq fluid and Raleigh-Benard convection

May 5, Other computational methods (Jian)

May 10 & 12, Student Presentations

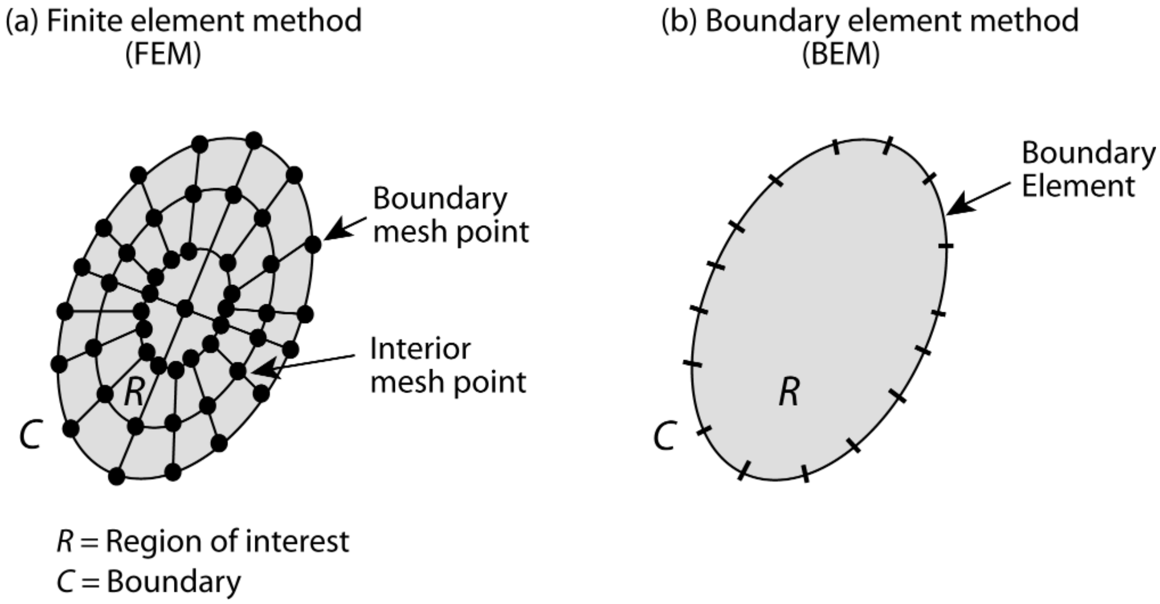
Jian Lin
February 3, 2011

12.521 Computational Geodynamics Modeling

Overview of the boundary-element method

1 Finite element versus boundary element methods

Crouch & Starfield (1983) provided an excellent overview of the essence of the boundary element method. Numerical methods for solving boundary value problems can be divided into two classes: (a) those that require approximations to be made throughout the region R, such as finite element and finite difference methods; and (b) those that require approximations to be made only on the boundary C, such as the boundary element methods.



In boundary element methods, only the boundary C is divided into elements. The numerical solution builds on the analytical solutions that have already been obtained for simple singular problems. This solution satisfies the specified boundary conditions at each element on C. Because each of the singular solutions satisfies the governing partial differential equations in R, there is <u>no need to divide R into a network of elements</u>.

If we divide C into N elements, then we seek N singularity solutions which, when superimposed, will give the required conditions at the midpoint of each element. The system of equations to be solved (total number N) is much smaller than the system needed to solve the same problem using finite element method, although the equations are no longer sparse. Once these equations are solved, the solution at any point in R can be constructed by Green's functions. Because it exploits an analytical solution that holds true throughout R, a boundary element method is potentially <u>more accurate</u> than the finite element method.

Jian Lin

February 3, 2011

2 Boundary element method source codes

The following two software packages are based on the boundary element method (BEM) and can be used to investigate 3-D stress, strain, and displacement fields of geological problems such as earthquakes, faulting, dikes, underground caves, etc. Both codes are available for academic use without fees.

2.1 Program 1: "Coulomb 3.2"

This code was developed by Shinji Toda, Ross Stein, and Jian Lin, as well as earlier by Geoffrey King, based on the program structure of Crouch & Starfield (1982) and 3D Green's functions of Okada (1992).

Jian Lin will provide the students of this class Coulomb 3.2 version, which can be run on Matlab using Mac, PC, and other computers. You can download a complete package of software, manual, and detailed examples from the following USGS web page:

http://earthquake.usgs.gov/research/modeling/coulomb/overview.php

You will need this software to complete your homework assignment #1.

Merits:

1) This is a very user friendly program and will require only minimal introduction on how to use it. It is probably one of the most commonly used boundary element program by earthquake research communities in the US, Europe, and Japan.

2) All results can be seen graphically on the computer screen as well as being saved as text output files.

Shortcomings:

You can only specify slip (not stress) boundary conditions.

References:

King, G.C.P., R.S. Stein and J. Lin, Static stress changes and the triggering of earthquakes, *Bull. Seismo. Soc. Amer., 84,* 935-953, 1994.

Toda, S. R. S. Stein, P. A. Reasenberg and J. H. Dieterich, Stress transferred by the Mw=6.9 Kobe, Japan, shock: Effect on aftershocks and future earthquake probabilities, *J. Geophys. Res., 103*, 24,543-24,565, 1998.

Lin, J. and R.S. Stein, Stress triggering in thrust and subduction earthquakes, and stress interaction between the southern San Andreas and nearby thrust and strike-slip faults, *J. Geophys. Res., 109*, B02303, doi:10.1029/2003JB002607, 2004.

2.2 Program 2: "3d-def"

The software can be downloaded from the following web page. This page also provides a detailed online user's manual.

http://www.ceri.memphis.edu/3ddef/guide.html

Merits:

This is a fully 3-D boundary element program that allows you to specify displacement or stress boundary conditions. Thus it can be applied to a wider range of geological problems than Coulomb.

Shortcomings:

1) You need to invest much more time to learn how to use this program. If a geological problem can be solved by Coulomb, you will find that it is much easier to use Coulomb unless you are quite familiar with "3d-def" already.

2) This code package does not have nearly as nice graphic output options as Coulomb.

References:

Bodin, P. and J. Gomberg, Triggered seismicity and deformation between the Landers, California, and Little Skull Mountain, Nevada, earthquake , *Bull. Seismo. Soc. Amer., 84,* 835-843, 1994.

Gomberg, J. and M. Ellis, Topography and tectonics of the central New Madrid seismic zone: Results of numerical experiments using a three-dimensional boundary-element program, *J. Geophys. Res., 99,* 20,299-20,310, 1994.

Kilb, D., Ellis, M., Gomberg, J., and Davis, S., On the origin of aftershocks following the 1989 Loma Prieta, California, earthquake , *Geophys. J. Intl., 128,* 557-570, 1997.

Behn, M.D., J. Lin, and M.T. Zuber, Evidence for weak oceanic transform faults, *Geophys. Res. Lett., 29(24),* 2207, doi:10.1029/2002GL015612, 2002.