

A MATLAB Toolbox for Estimating the Second Moments of Earthquake Ruptures

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Abstract

I present a set of routines in MATLAB for estimating the second degree moments of an earthquake's rupture from far field body waves. The second moments describe the length, width, duration and directivity of a rupture. The second moments approach is particularly useful when a seismic dataset is dense enough to resolve the primary finite source properties but the geodetic data needed for a well resolved finite fault inversion are not available. In particular for M3-6 earthquakes, this approach can be a useful way to estimate rupture area without the assumptions of typical corner-frequency approaches. The provided software utilizes Empirical Green's Function deconvolution to isolate the Apparent Source Time Function (ASTF) for each station and phase. The spatial variations in the duration of the ASTF are quantified and inverted for the second moments. The inverse problem is solved using Matlab's convex optimization routines for systems of linear matrix inequalities. An error analysis using the jackknife and bootstrap methods is included. An example M_w 4.7 earthquake from the San Jacinto Fault is used to demonstrate the method.

1 Introduction

The second moments of the slip-rate distribution for any earthquake describe the spatial and temporal extent of the rupture as well as its propagation, all of which contribute

24 to the apparent duration of the earthquake observed in any particular far-field phase.
 25 For an earthquake with a constant moment tensor such that the spatial variations in
 26 moment-rate are described by:

$$\underline{\underline{\dot{M}}}(\underline{x}, t) = \underline{\underline{\hat{M}}}\dot{f}(\underline{x}, t) \quad (1)$$

27 The second moments are defined as:

$$\begin{aligned} \hat{\boldsymbol{\mu}}^{(2,0)} &= \int \int \dot{f}(\mathbf{r}, t)(\mathbf{r} - \mathbf{r}_0)(\mathbf{r} - \mathbf{r}_0)dV dt \\ \hat{\boldsymbol{\mu}}^{(0,2)} &= \int \int \dot{f}(\mathbf{r}, t)(t - t_0)(t - t_0)dV dt \\ \hat{\boldsymbol{\mu}}^{(1,1)} &= \int \int \dot{f}(\mathbf{r}, t)(\mathbf{r} - \mathbf{r}_0)(t - t_0)dV dt \end{aligned} \quad (2)$$

28 where $\dot{f}(\mathbf{r}, t)$ is a scalar function that describes the spatial and temporal distribution of
 29 moment release along the fault (McGuire et al., 2001), \mathbf{r}_0 and t_0 denote the centroid
 30 location and time (i.e. the first moments) respectively. The hat denotes that these
 31 are central moments taken about the centroid. The integrals are taken over the entire
 32 source volume and earthquake duration (Backus, 1977a,b; McGuire et al., 2001).

33 When $\dot{f}(\mathbf{r}, t)$ is integrated over the volume of the source, it is known as the moment-
 34 rate or source time function (STF) $\dot{M}(t)$. The second spatial moment $\hat{\boldsymbol{\mu}}^{(2,0)}$, is related
 35 to the spatial extent of the rupture area, the second temporal moment $\hat{\boldsymbol{\mu}}^{(0,2)}$ is related to
 36 the duration of rupture, and the mixed moment $\hat{\boldsymbol{\mu}}^{(1,1)}$ is related to rupture propagation.

37 There is considerable background literature on second moments. In general they are
 38 a way to capture the overall kinematic properties of a rupture that are well constrained
 39 by the far-field waveforms. For a more detailed theoretical background and examples
 40 in various settings, see: Backus and Mulcahy (1976a,b); Backus (1977a,b); Bukchin
 41 (1995); Clévéde et al. (2004); Das and Kostrov (1997); Doornbos (1982a,b); Gusev and
 42 Pavlov (1988); McGuire et al. (2001); Silver (1983). This toolbox mostly follows the
 43 measurement scheme developed in McGuire (2004) and the inversion scheme developed
 44 initially in McGuire et al. (2001). The measurement scheme and partial derivatives are
 45 specific to far-field body waves with some sort of time domain Green's function available

for propagation corrections (empirical or theoretical). For surface-wave based schemes the inversion algorithms still apply but a different measurement and partial derivative calculation scheme is required depending on the approach (see *McGuire et al. (2001)*, *Clévéde et al. (2004)*, *Chen et al. (2005)*, and *Llenos and McGuire (2007)*).

The characteristic rupture duration τ_c , rupture length L_c , and average propagation velocity of the instantaneous spatial centroid \mathbf{v}_0 are defined following *Backus and Mulcahy (1976a)*, *Backus (1977a)*, *Silver and Jordan (1983)* *Silver (1983)* and *McGuire et al. (2001)*:

$$\begin{aligned}
 x_c(\hat{\mathbf{n}}) &= 2\sqrt{\hat{\mathbf{n}}^T \hat{\boldsymbol{\mu}}^{(2,0)} \hat{\mathbf{n}}} \\
 \tau_c &= 2\sqrt{\hat{\boldsymbol{\mu}}^{(0,2)}} \\
 v_c &= L_c/\tau_c \\
 \mathbf{v}_0 &= \hat{\boldsymbol{\mu}}^{(1,1)}/\hat{\boldsymbol{\mu}}^{(0,2)}
 \end{aligned}
 \tag{3}$$

where x_c is the spatial extent of the rupture in the direction $\hat{\mathbf{n}}$ and L_c is the maximum value of x_c (i.e. corresponding to the largest eigenvalue of $\hat{\boldsymbol{\mu}}^{(2,0)}$). W_c corresponds to the second largest eigenvalue, e.g. the rupture width. The second moments can either be calculated in three spatial dimensions or along a 2D fault-plane if the mechanism is known. To resolve the fault plane ambiguity, we typically invert for the second moments assuming each 2D nodal plane and choose the one with the higher variance reduction as the true nodal plane.

In general the characteristic dimensions give an idea of the region that contributed substantially to moment-release and the relative importance of directivity in the rupture. Figure 1 shows an example of the characteristic dimensions calculated for a theoretical crack model with unilateral propagation from *Kaneko and Shearer (2015)*. The ellipse defined by the second spatial moment captures the orientation and extent of the region with the large slip, which is smaller than the total dimension of the rupture (Figure 1). Similarly the second temporal moment captures the time period in which most of the moment was released, not the total duration (Figure 1). This definition of earthquake duration is theoretically closely related to the corner-frequency obtained from simple

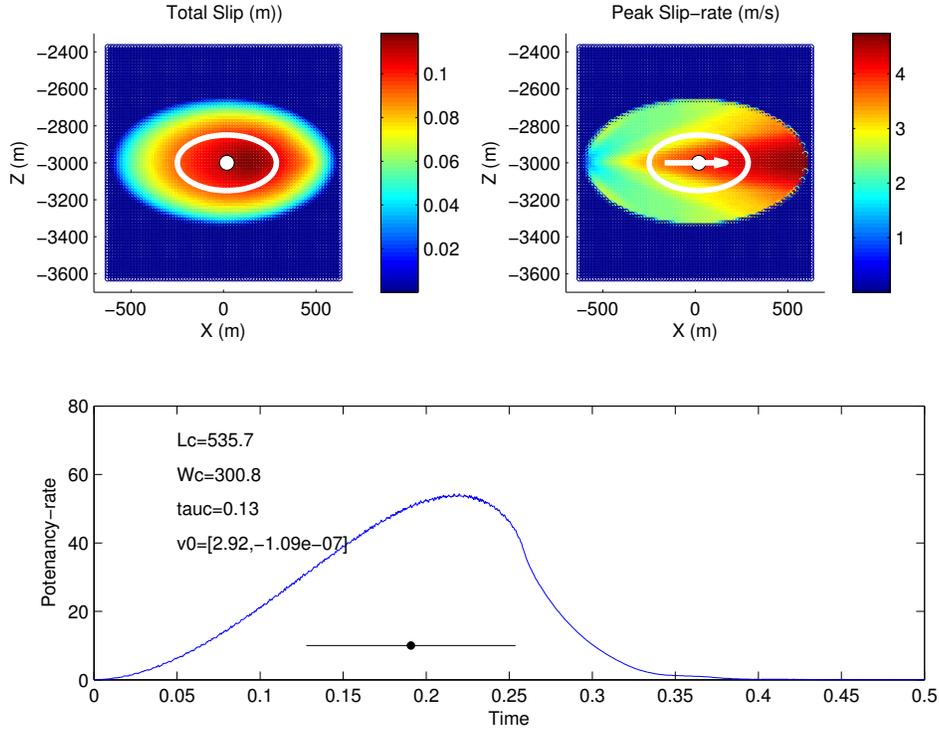


Figure 1: An Elliptical source from *Kaneko and Shearer* (2015), (their Figure 10). Top Left) Slip distribution. The white ellipse has major axis of length L_c and minor axis W_c . Top Right) The spatial distribution of the peak slip-rate. The rupture propagated unilaterally from left to right and in this model the slip-rate is highest at the righthand edge. The vector shows the mixed moment $\hat{\mu}^{(1,1)}$ scaled to the distance L_0 . Bottom, The potency-rate for the modeled rupture (blue line). The black circle gives the centroid time, and the horizontal line through it has a length equal to τ_c to illustrate the difference between τ_c and the total duration. The values for L_c and W_c are given in meters, τ_c in seconds, and v_0 in km/s. v_0 has components in the along-strike direction and the downdip direction.

70 spectral fitting (*Silver*, 1983), but in practice the second moments may result in better
 71 estimates of rupture area (*Chen and McGuire*, 2016).

72 2 Software Requirements

73 The toolbox contains three main directories. The **src** directory contains the MATLAB
 74 functions for making measurements and inversion as well as a fortran program for cal-
 75 culating takeoff angles. The **SJFex** directory contains an example M_w 4.7 earthquake
 76 from the San Jacinto Fault in southern California. The third directory contains a
 77 pdf of this manual. To use this toolbox you need only the codes in the **src** directory

78 and MATLAB. The inversion algorithm uses some routines from MATLAB's Robust
79 Control toolbox.

80 **3 Data Preparation**

81 The input to the EGF algorithm is simply pairs of velocity seismograms for each
82 station/component that you would like to try a deconvolution on. The code is currently
83 set up to allow interactive waveform windowing, this could be automated if you prefer.
84 The SJFexamp directory contains an example of the data preparation using event
85 directories of miniseed files and the Antelope software package for event information.
86 The data loading is done in the script runSJFsetup.m which could be easily modified
87 to match other databasing approaches.

88 The example event is a M_w 4.7 earthquake on March 11th 2013 that was recorded by
89 both the ANZA network and a dense temporary deployment of instruments including
90 numerous strong motion sensors (*Kurzon et al.*, 2014). The example event is run with
91 the runSJFex.m script. The flag LOADDATAFILE can be set to 1 to use a pre-loaded
92 .mat file with all of the necessary arrays, or it can be set to 0 to load the miniseed
93 files and alter filters, etc. If loading directly from miniseed, the key choices to make
94 are the 'dofilt' flag, and $fmin$ and $fmax$ which collectively set the frequency bandpass
95 for filtering the data. The code is setup to integrate all seismograms with component
96 names like 'HNZ' from acceleration to velocity. The key choice is specifying the list of
97 stations and components to be read in/tried for deconvolutions. This is done in the
98 stasm and compm variables. This could be automated to look for all pairs in the event
99 and egf directories.

100 **4 Measurements**

101 Our approach requires using the far-field body wave data to estimate the moment-rate
102 function from the P and/or S wave at a set of azimuthally distributed stations. This

103 estimate will be either a stretched or compressed version of the true moment rate func-
 104 tion due to the finite source properties of the rupture and hence is termed an Apparent
 105 Source Time Function (ASTF). Because many moderate earthquakes require working
 106 at relatively high-frequencies (>1 Hz) to retrieve the ASTF, we typically use Empirical
 107 Green's Functions (EGFs) to remove propagation effects. Many EGF deconvolution
 108 algorithms, such as the water-level technique, produce ASTF estimates that have low-
 109 amplitude ringing for an extended period of time following the main pulse of moment
 110 release that make it difficult to determine the end of the rupture. One technique
 111 that provides an objective determination of the duration of the ASTF is the projected
 112 Landweber deconvolution (PLD) algorithm of *Bertero et al. (1997)* and *Lanza et al.*
 113 *(1999)*. This algorithm performs the deconvolution with moment release restricted to
 114 a series of increasing-length time intervals and analyzes the misfit as a function of the
 115 interval length (see Fig. 2). The interval length where this trade-off curve flattens
 116 out is chosen as the interval during which moment release is allowed (*Lanza et al.,*
 117 *1999*). This technique produces ASTFs that satisfy a positivity constraint, provide a
 118 good fit to the observed seismograms, and are very consistent between nearby stations
 119 (*McGuire, 2004*).

120 The matlab files for making the measurements are:

- 121 • *makemeasurements.m* This contains the GUI for looping through velMS and
 122 velEGF arrays and making the measurements of $\mu^{(0,2)}(\underline{s})$.
- 123 • *pld.m* This file contains the implementation of the projected Landweber deconvol-
 124 ution algorithm. The key control parameter for the deconvolution is the number
 125 of iterations. Effectively, the higher this number (variable *niter*), the better the
 126 ASTF will do at fitting the highest frequency energy in the mainshock waveform.
 127 It is best to experiment with a range of values for your dataset, starting rela-
 128 tively low (10, 50) and increasing. Typically a value of 100 is a good compromise
 129 between seismogram fit and computation time.
- 130 • *findt2.m* calculates $\mu^{(0,2)}(\underline{s})$ for the ASTF that results from the deconvolution.

131 This gives you the option to pick the starting and ending time points of the ASTF
132 that you want to include in the calculation. Assuming this is working well, this
133 choice could be omitted (set pickt2=0) and the calculation will be done on the
134 entire range of the time function.

135 The flag DOMEAS lets you choose between either just loading in an example set of
136 measurements or going through the process of making the measurements yourself.

137 In making the measurements, the routine is set to store the current set of measure-
138 ments after each station in the file measurements.mat. This is intended to allow you
139 to stop part way through a dataset and restart at a later time or simply go back and
140 revise a few stations that appear to need it after seeing the whole dataset. The first
141 window that appears asks you if you want to load an existing set of measurements from
142 this file or start from scratch. The *makemeasurements.m* function will loop through
143 all of the channels in the dataset and ask you to make a series of choices for each:

- 144 • The first menu ask you if you want to start a new set of measurements or load the
145 previous set from MEASUREMENTS.mat. The script will save the measurements
146 you've made after each station. This provides an easy way to stop in the middle
147 and restart later. The MEASUREMENTS.mat file is overwritten after every
148 station is processed, so if you wish to save it, you need to copy it to some other
149 filename. If this is your first attempt with this event, choose 'Start New' to
150 initialize the variables.
- 151 • The first plot shows the MS and EGF waveforms and ask you if this channel is
152 worthy of doing a deconvolution. If you say no, DONE(i) remains =0 and this
153 channel will not be used. If you choose yes, you will move onto windowing the
154 MS and EGF waveforms.
- 155 • If you choose to try deconvolving this channel, it will ask you to manually pick
156 a time range to zoom in on the mainshock waveform. Just click roughly before
157 the beginning of the P or S phase that you want to work on. It will then replot
158 this time range of the MS and ask you to pick the time range to be fit by the

159 deconvolution. It is best to pick a window surrounding the 'main arrival'. The
160 inversion assumes that the ASTF reflects energy that left the source at roughly the
161 slowness vector of the first arrival. So it is most accurate to not try and fit the later
162 parts of the coda which likely reflect different slownesses. In practice, we usually
163 pick the start of the interval as being 20-50 samples before the first arrival (e.g.
164 so there is a relatively flat, near-zero, part before the arrival). Picking the end of
165 the fitting interval can be tricky. Ideally, the waveform would've returned to near-
166 zero amplitude relatively quickly after the main arrival. For a M4-5 earthquake
167 we typically fit 3-5 seconds of P or S-wave. If there is a clear second arriving
168 phase (Pn, etc) it is best to end the interval before that arrival. For stations
169 with a strong coda, it is often very difficult to pick the end of the interval. These
170 stations will often not work very well.

- 171 • Once the MS fitting interval has been picked, it will replot this interval and ask
172 you to pick the onset time of the P or S wave.
- 173 • It will then plot the EGF waveform and ask you for a interval to zoom into. This
174 is slightly different. You should zoom fairly tightly on the first arrival. It will
175 then replot this window and ask you to pick the first arrival time, which defines
176 the start of the Green's function.
- 177 • Once the waveforms have been picked, it will perform the PLD for a series of
178 allowed durations of the ASTF and calculate the misfit for each deconvolution.
179 It will plot this tradeoff curve (misfit vs ASTF interval). If the EGF was a good
180 choice, then the tradeoff curve will have a region where it drops steadily to a low
181 value ($\leq .3$) and then flattens out. In a ideal case there is a clear break in slope
182 of the tradeoff curve that identifies the correct value for the total duration at
183 that station. It will ask you to pick this inflection point. See Figure 2 for some
184 examples.
- 185 • if you have set the flag pickT2=1 it will plot the ASTF for your choice and ask you
186 to pick the start and end points for the integration that calculates $\mu^{(0,2)}(\underline{s})$. This

187 is sometimes desirable for earthquakes that do not end abruptly in case you want
188 to revise your choice without re-doing the deconvolution. If you had pickT2=0,
189 then it does this calculation automatically on the whole ASTF (usually fine for
190 good datasets).

- 191 • It will now plot the ASTF and the fit to the data and ask you to choose among
192 4 options for saving the result; Finished P-wave, Finished S-wave, re-do, or "No,
193 done" on this channel and move on to the next. The first two set the DONE
194 variable to 1 for this channel and record whether it was a P or S wave. re-do lets
195 you go through this channel again in case you think you got the windowing wrong.
196 The "No,done" option will set the *DONE* variable to zero and this channel will
197 not be used in the inversion.
- 198 • the script will loop through this process for the rest of the stations

199 Once this loop has been completed for all stations and the *DONE* variable has been
200 set to 1 or 0 for each the measurement process is complete and saved.

201 5 Inversion Scheme for Second moments

202 The inversion algorithm is essentially identical to that in *McGuire* (2004) and *McGuire*
203 *et al.* (2001) but has been implemented using the Robust Control toolbox in Matlab.
204 In general the inverse problem for the second moments can be posed as a simple linear
205 inversion based on the equation for the observations

$$\mu^{(0,2)}(\underline{\mathbf{s}}) = \hat{\mu}^{(0,2)} - 2\underline{\mathbf{s}} \cdot \hat{\underline{\mu}}^{(1,1)} + \underline{\mathbf{s}} \cdot \hat{\underline{\underline{\mu}}}^{(2,0)} \cdot \underline{\mathbf{s}} \quad (4)$$

206 where $\underline{\mathbf{s}}$ is the slowness vector associated with a particular measurement. See *Silver*
207 (1983) and *McGuire* (2004) for details. However, in practice if the station distribution
208 is suboptimal, this could result in unphysical estimates for the second moments (*Das*
209 *and Kostrov*, 1997). We always enforce the constraint that the source region have
210 non-negative volume (*McGuire et al.*, 2001) which is accomplished by enforcing the

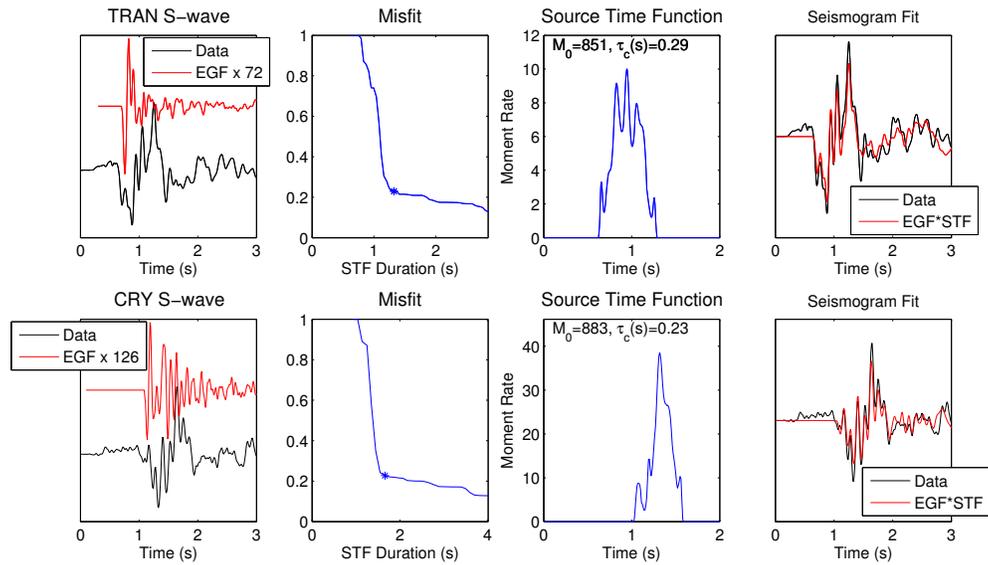


Figure 2: Examples of the EGF deconvolution measurements from the M5.1 March 2013 earthquake on the San Jacinto Fault for the S-waves recorded at stations TRAN (top row) and CRY (bottom row). The left panels show the raw velocity seismograms for the M5.1 and an EGF event. The next panel shows the tradeoff curve for waveform misfit versus source time function duration with the asterisks denoting our pick for the duration of the moment-rate function as seen by that S-wave. The third panels show the resulting moment-rate function (in units of the EGF event’s moment). The fourth panels show the fit to the mainshock seismograms. Station TRAN sees a slightly longer ASTF (characteristic duration of 0.29 s) than CRY (0.23 seconds).

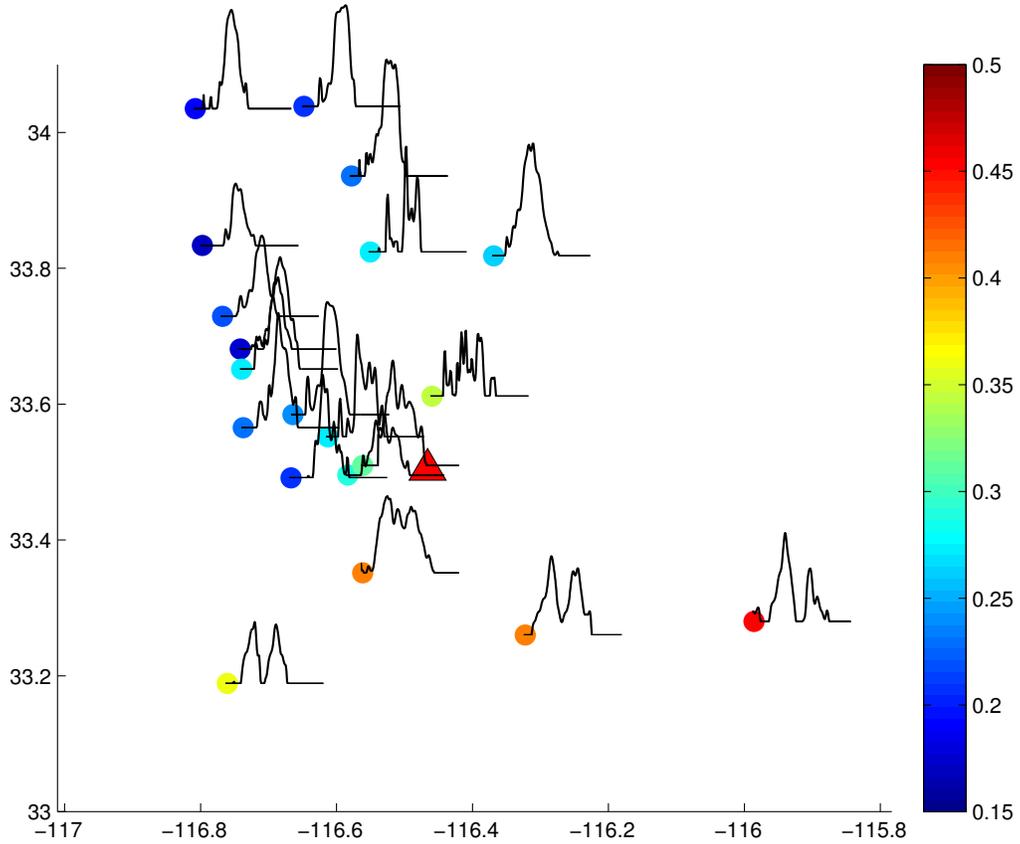


Figure 3: ASTFs resulting from EGF deconvolutions at stations in the SJFZ array as a result of the March 2013 M5.1 earthquake (red triangle). Each ASTF is plotted at the location of the station denoted by the circle. The colorscale of the circles denotes the characteristic duration of that moment-rate function, $\tau_c(s)$, in seconds. The earthquake lasted about 0.3 seconds but appears longer to the SW and shorter to the NE.

211 matrix inequality:

$$\begin{bmatrix} \hat{\mu}^{(2,0)} & \hat{\mu}^{(1,1)\text{T}} \\ \hat{\mu}^{(1,1)} & \hat{\mu}^{(0,2)} \end{bmatrix} \geq 0, \quad (5)$$

212 where ≥ 0 indicates that the matrix is required to be positive semi-definite.

213 To set up an inverse problem based on equation 4, we simply need the measurements
 214 of $\mu^{(0,2)}(\mathbf{s})$ from the previous section and the corresponding slowness vectors for each
 215 phase. We include a fortran program for calculating the takeoff angles that is run exter-
 216 nal to matlab called *topp*. Most of this code is taken directly from the hypoDD software
 217 package of *Waldhauser and Ellsworth (2000)*. See <http://earthquake.usgs.gov/research/software/>.
 218 *topp* is called from within the subroutine for calculating the partial derivatives.

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220 `[G]=getpartials_2d_generic(mlats,mlons,melevs,late,lone,depe,Vp,Vs,topl,phas,strike,dip);`

221

222 where *mlats*, *mlons*, and *melevs* give the station coordinates, *late*, *lone*, *depe* give
 223 the earthquake coordinates, *Vp*, *Vs*, and *topl* give the layered velocity model and *topl*
 224 is the depth of the top of each layer in km. *phas* is a character array the length of
 225 the number of measurements that has either the value P or S for each measurement.
 226 This routine calculates the partial derivatives for a 2-D source along a fault specified
 227 by *strike* and *dip* in the usual convention.

If the DOINVERSION flag is set to 1, the runSJFex script will call the function
 seconds_2d_v2.m to do the actual inversion. To solve equation 2 in a least-squares
 sense subject to equation 3, we recast equation 2 as a linear matrix inequality, similar
 to equation 3, and solve the system using Matlab's Robust Control toolbox. The least-
 squares problem from equation 2 is rewritten $\|Ax - b\| \leq c$ where c is a dummy decision
 variable. The problem in LMI form is then given by:

minimize c

$$\begin{aligned}
& \text{subject to } \begin{bmatrix} c & \mathbf{Ax} - \mathbf{b} \\ (\mathbf{Ax} - \mathbf{b})^T & \mathbf{I}^N \end{bmatrix} \geq 0, \\
& \text{and } \begin{bmatrix} \hat{\mu}^{(2,0)} & \hat{\mu}^{(1,1)T} \\ \hat{\mu}^{(1,1)} & \hat{\mu}^{(0,2)} \end{bmatrix} \geq 0, \\
& \text{and } \hat{\mu}^{(0,2)} \leq \max(\mathbf{b})
\end{aligned} \tag{6}$$

229 where \mathbf{I}^N is the identity matrix with dimension equal to the number of measurements,
 230 N , and ≥ 0 indicates the matrix is positive semidefinite. The equivalence between the
 231 linear least-squares problem and the above can be seen by calculating the eigenvalues
 232 of the $N + 1$ by $N + 1$ matrix, which are non-negative when the matrix is positive semi-
 233 definite. This restatement of the problem is known as using Schur complements to
 234 represent a nonlinear constraint as linear matrix inequality (*Vandenberghe and Boyd,*
 235 *1996*). The last equation ensures that the estimate of the second temporal moment is
 236 smaller than the largest measurement of $\mu^{(0,2)}(\mathbf{s})$, which should be true for any dataset
 237 that contains stations with good azimuthal coverage.

238 MATLAB's robust control toolbox contains many routines that are particularly
 239 useful for problems with matrix inequality constraints like (5). For a detailed descrip-
 240 tion of how to use these, see the MATLAB manual pages. The script *seconds_2d.v2.m*
 241 shows how to implement the problem in (6) within MATLAB's LMI syntax. For each
 242 matrix inequality, the matrix on the lefthand side is first defined in terms of the opti-
 243 mization variables (the second moments) using the function *lmivar*. Each individual
 244 inequality in (6) is added to the LMI system using the function *lmiterm*. For instance,
 245 the lines:

```

246 [Xposvolume,NDEC,XDEC]=lmivar(type,struct);
247 lmiterm([-1 1 1 Xposvolume], 1, 1);

```

248 is an implementation of equation (5) where the variable $\mathbf{X}_{\text{posvolume}}$ represents the LHS
 249 :

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$$\mathbf{X}_{\text{posvolume}} = \begin{bmatrix} \hat{\mu}^{(2,0)} & \hat{\mu}^{(1,1)\text{T}} \\ \hat{\mu}^{(1,1)} & \hat{\mu}^{(0,2)} \end{bmatrix} \geq \mathbf{0}, \quad (7)$$

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and $\mathbf{X}_{\text{posvolume}}$ has been previously specified as a 3 by 3 symmetric matrix composed of the six independent elements of the second moments (for a 2D fault) which we seek to estimate with the MATLAB to solver. The LMI is described to MATLAB by specifying the location of each individual term within the LMI by which side of the inequality it is on, which entry within the matrix it is part of, and any constant factors that multiply it. The entries in the call to *lmiterm* are the *termid*, and two constant matrices *A* and *B* (see the *lmiterm* man pages). The LMI we are describing has only one term on the LHS and it is a matrix variable type of term. It has no pre or post multiplying matrices so *A* and *B* are set equal to 1. For the *termid* = [-1 1 1 $\mathbf{X}_{\text{posvolume}}$], the entries are:

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- first entry = -1 denotes the sign of the inequality in MATLAB's convention (-1 indicates the greater side of the inequality) and that this is the first constraint equation (they are concatenated into a block diagonal system).
- second and third entries =1 give the location of the next variable in the matrix (e.g., there is only 1,1 in this example).
- fourth entry = $\mathbf{X}_{\text{posvolume}}$ is the term to be added to the location specified in the LMI.

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Note that a sequence of LMIs such as (6) is implemented as one block diagonal LMI, so the first entry is incremented (1,2,3,...) in *seconds_2d_v2.m* to accommodate each subsequent constraint equation as a new block.

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6 Error Analysis

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We present two versions of error analysis that have been used previously in second moment studies based on the Jackknife and Bootstrap approaches. Both approaches

274 are based on resampling the data vector. Often seismic arrays involve a number of
 275 stations that are located close together while other parts of the focal sphere are poorly
 276 sampled at best. Thus, it is not straight forward to apply techniques like the Jackknife
 277 that assume each data point is independent. In our case, nearby station will likely
 278 have highly correlated measurements both in terms of the true values of $\tau_c(s)$ and
 279 the errors introduced by imperfect EGFs. To account for this when sub-sampling the
 280 dataset, we delete all of the stations in an azimuthal bin, typically with a range of
 281 20° to 40° . The Jackknife calculation produces an estimate of the covariance matrix
 282 of the six second moment quantities from which errors on the derived characteristic
 283 rupture quantities can be estimated (*McGuire et al., 2001*). The jackknife calculation
 284 with 20 degree azimuth bins results in the following 1σ error estimates for the San
 285 Jacinto fault example in Figures 2 and 3: $\tau_c = 0.30 \pm 0.015s$, $L_c = 0.33 \pm 0.06km$,
 286 $W_c = 0.19 \pm 0.12km$, $V_0 = [0.84, 0.58] \pm [0.03, 0.26]km/s$.

287 In general, the Jackknife results are sufficient to capture the uncertainty. It is some-
 288 times worthwhile to perform inversions for a number of random perturbations to the
 289 earthquake location, velocity model, or fault plane orientation if the uncertainties in
 290 those quantities are well known to evaluate their influence (*McGuire, 2004*). Another
 291 interesting approach is to apply the bootstrap technique. The boot strap approach
 292 with 1000 resamples of the data vector (with replacement) yields the PDFs for τ_c and
 293 L_c shown in Figure 4. As can be seen in Figure 4, the Jackknife error bounds cap-
 294 ture well the peaks of the bootstrap PDFs for L_c and τ_c . However, when there are
 295 only a small number of stations in a critical azimuth (e.g. the forward or backward
 296 rupture directions), then the resamples that lack stations from this direction can re-
 297 sult in values that are well outside the Jackknife bounds. In particular, for the San
 298 Jacinto earthquake, there are only three stations that capture the long durations in
 299 the backward direction. For resamples that do not involve these observations, there is
 300 much greater variability in the second moment estimates as would be expected. The
 301 95% confidence limits from the bootstrap are $[0.25, 0.36]$ and $[0.31, 1.39]$ km for τ_c and
 302 L_c respectively. The extent to which these are meaningful depends on how dense the

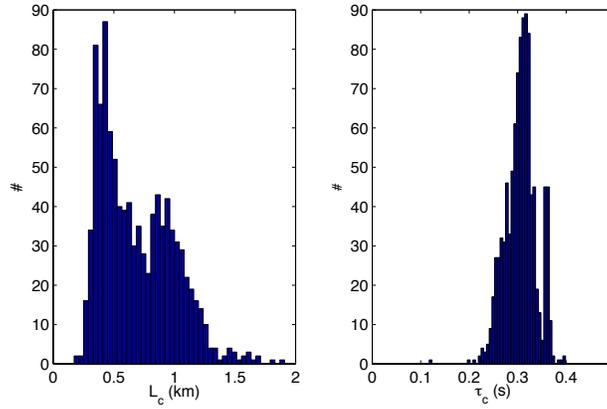


Figure 4: Examples of 1000 runs of the bootstrap calculation for the San Jacinto fault example in Figures 2 and 3 for the estimates of τ_c and L_c .

303 station distribution is.

7 Data and Resources

Seismograms used in this study were collected as part of the ANZA Network and the San Jacinto Fault Zone PASSCAL experiment. Data can be obtained from the IRIS Data Management Center at www.iris.edu (last accessed September 2016).

8 Acknowledgements

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