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Observations of Convection at Rayleigh Numbers up to 760,000 in a Fluid with Large Prandtl Number†

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Observations are reported on the stability and structure of Rayleigh-Bénard convection in a fluid with a Prandtl number of 8,600, and Rayleigh numbers between 50,000 and 760,000. Under carefully initiated and controlled conditions, stationary bimodal convection was observed, even at the highest Rayleigh number. Unstable bimodal patterns broke down in a way that tends to reduce the wavelength of the cells. Oscillating spoke-shaped convection was observed in convection started from random initial conditions. The gradual increase in the occurrence of oscillations and the dependence on the initial conditions probably accounts for previous disagreements about the onset of time dependence. A square convection planform was found which was stable at the highest Rayleigh numbers but unstable for lower Rayleigh numbers. The observations demonstrate the existence of different possible solutions at a given Rayleigh number.

1. INTRODUCTION

During the past 20 years, it has become clear that Bénard convection exhibits a series of discrete transitions to a more complex state as the Rayleigh number is increased. The first manifestation of such a process to be observed experimentally was a discrete change in slope of the heat transfer curve for a fluid layer in which convection was known to already be occurring [Schmidt and Saunders (1938)]. Subsequent studies concerned the position of the first and additional

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transition points [Malkus (1954); Willis and Deardorff (1967); Krishnamurti (1970a, b)], and the structural changes in the flow at various transition points. Clear transitions are reported for fluids with Prandtl numbers greater than 6. In moderate (>6) to large (>100) Prandtl number fluids, two-dimensional rolls, stable at low Rayleigh number, give way to a three-dimensional pattern [Krishnamurti (1970a)], which consists of two sets of rolls at right angles [Busse and Whitehead (1971)] and has been called bimodal flow. A second transition occurs when the convective motions become time dependent [Krishnamurti (1970b)]. There is good agreement as to the Rayleigh number above which motion is time dependent between various experiments at low Prandtl numbers, but poorer agreement between experiments at Prandtl numbers greater than roughly twenty [Krishnamurti (1970b); Willis and Deardorff (1970); Busse and Whitehead (1974)]. More specifically, Krishnamurti reports the observation of periodic time dependent flow above a Rayleigh number of $52{,}900 \pm 5{,}290$ at Prandtl numbers of 100, 200, 860, and 8,500. Willis and Deardorff (1970) report seeing no oscillations until a Rayleigh number of roughly 100,000 is exceeded in a fluid of Prandtl number 57. In contrast, Busse and Whitehead (1974) controlled the initial conditions starting with a uniform bimodal planform. It was observed that the transition to oscillatory behavior varied with Prandtl number; the transition occurred at Rayleigh numbers of approximately 4×10^4, 7×10^4, 1.3×10^5, and 3.3×10^5 for Prandtl numbers of 16, 46, 63, and 126 respectively. It was noted that the transition to oscillations was severely affected by the inhomogeneity of the convection structure in flows which evolve from uncontrolled initial conditions.

The purpose of this paper is to report upon further investigations into the interdependence between structure of the convection and the transition to time dependent flow that was observed by Busse and Whitehead (*loc cit*). Because previous experiments disagree mainly for larger Prandtl numbers, a series of experiments are reported here for fluid with a Prandtl number of 8,600 using apparatus capable of attaining Rayleigh numbers up to approximately 7.6×10^5; the apparatus will be described in section 2. It was observed that convection can exist in two distinct states—one stationary and one oscillating—at the same Rayleigh number. In section 3 data will be shown which demonstrate that a stationary flow could be obtained up to the maximum Rayleigh number observed, which consisted of flawless bimodal convection lying within a certain area of wavenumber space. They also demonstrate that if the parameter of the bimodal structure lies outside of this space, the structure breaks down to a stationary bimodal flow with wavenumbers within the stable space.

It was further observed that this convection pattern evolved into a more random convection pattern if a large dislocation in the perfectly aligned convection pattern was present. Such a dislocation, once emplaced, jogged the neighboring patterns severely and the random pattern composed of structures similar to the spoke-shaped pattern described by Busse and Whitehead (1974) spread throughout the fluid. The spoke pattern often oscillated. If the experiment was started from random conditions, as the experiments of the other investigators are, it was observed that the statistical nature of the background noise generated natural flow regions where neighboring patches of bimodal flow fitted together poorly. These regions began to show a tendency to break down into the spoke pattern at Rayleigh numbers comparable to those at which Krishnamurti (1970b) and Willis and Deardorff (1970) reported the emergence of oscillations. The percentage of fluid which possesses a bimodal flow pattern, the percentage of fluid which has a spoke pattern, and the percentage of fluid which oscillates, is estimated from shadowgraphs. The estimates exhibit a relatively smooth transition as Rayleigh number is progressively increased.

The paper ends with a brief discussion of the implications of these results upon our understanding of turbulence.

2. APPARATUS AND PROCEDURE

A sketch of the apparatus is given in Figure 1. It is based on the apparatus used by Busse and Whitehead (1971), and has been fully described by Richter and Parsons (1975). Hence only a brief description will be given here.

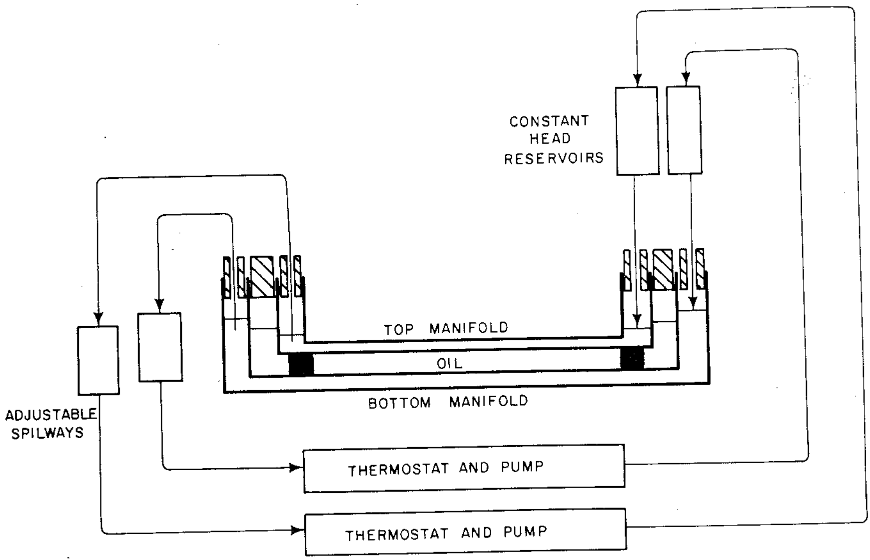

FIGURE 1 Sketch of the experimental apparatus.

A horizontal layer of Dow Corning 200 silicon oil was bounded above and below by transparent, $\frac{1}{4}$-inch plate glass water manifolds with a 1 meter by 1 meter square working area. Thermostatically controlled water flowed through each manifold such that the drop from inlet to outlet was less than 0.2°C. The oil has the following properties: kinematic viscosity, $\nu = 10\,\text{cm}^2\,\text{sec}^{-1}$; thermometric diffusivity, $\kappa = 1.16 \times 10^{-3}\,\text{cm}^2\,\text{sec}^{-1}$; thermal expansion coefficient, $\alpha = 9.6 \times 10^{-4}\,°\text{C}^{-1}$; Prandtl number, $\nu/\kappa = 8.6 \times 10^3$.

The Rayleigh number, R, is defined by

$$R = \frac{g\alpha \Delta T d^3}{\kappa \nu},$$

where g is the gravitational acceleration, ΔT is a calculated temperature difference between the boundaries, and d is the depth of the fluid. In most of the experiments d was 7 cm, enabling a Rayleigh number of approximately 8.10^5 to be reached. A few preliminary experiments were attempted with $d = 12$ cm and R up to 2.10^6.

To obtain the temperature difference across the fluid a correction for the temperature drop across the glass boundaries was subtracted from the temperature difference between water in the two manifolds. For this calculation, it was assumed that the Nusselt number of the convecting oil obeyed the law $Nu \simeq 0.19 R^{0.282}$, which was the best fit curve observed by Sommerscales and Gazda (1969). The largest correction was about 20 % of the temperature difference between the baths.

A shadowgraph technique was used to visualize convection in the oil layer. It consisted of light from a 5 mm aperture which passed through a series of mirrors, upward through the layer of oil, into another mirror, and onto a frosted screen. The convection bends light of the beam such that cold regions cause the upward passing light to converge like a convex lens, and hot regions cause the light to diverge. The frosted screen was placed so as to intercept the beam at the distance which gave the most satisfying shadowgraphs, and the shadowgraph was photographed with a 35 mm still and a 16 mm movie camera.

For some of the experiments the method of Chen and Whitehead (1968) was used to induce convection of a prescribed pattern, as described elsewhere [Busse and Whitehead 1971, 1974; Whitehead and Chan 1976]. This method consists of placing a grid made up of alternating blocked and clear areas over the top transparent channel, shining a 300 watt incandescent lamp down through the pattern, so the test fluid lying below was slightly radiatively heated in the desired pattern, and leaving the system for at least one hour. This time is a little less than the thermal time constant $d^2/\pi^2\kappa = 1.25$ hrs, which is the smallest conductive thermal time constant of the system. After this interval, the temperature of the top bath is decreased and the temperature of the bottom bath is increased at the equal rates of 2°C/min until the desired Rayleigh number was reached. It was important to change the bath at the same rates in order to exclude "asymmetric or hexagonal modes" of flow. Although the preheating is only on the order of 0.001°C, the growing convection will adopt the pattern if it is close to the region of stable finite-amplitude patterns.

Various precautions were taken to eliminate lateral inhomogeneities. The area for which observations on convection were made was bounded on the sides by walls of 2″-thick polyvinylchloride, whose thermal conductivity is close to that of the silicon oil. This provided a working area 80 cm by 80 cm. Outside of these walls was another region of convection approximately 20 cm in width so that the temperature gradients on both sides of the walls were similar. The apparatus was levelled to one part in 10^7 with a Starrett precision level, and the top channel was spaced above the bottom channel by the walls, which were accurate to $\pm 0.002''$. Hydrostatic pressure of water in the two channels was adjusted to counteract the hydrostatic pressure of the test fluid and the weight of the glass sheets, so that the glass plates would not bulge upward or downward. In all cases temperature was recorded periodically with thermometers at the inlet and outlet which were accurate to better than 0.1°C. The temperature drop from inlet to outlet was less than 0.1°C in all cases, and the flow directions in the bottom and top manifolds were opposite each other, so that vertical temperature difference was constant.

Two types of experiments were conducted. In the first, a convection pattern was induced as described above, the system was brought to the appropriate Rayleigh number, and the pattern was recorded by photographing the shadowgraph with either a motor-driven 35 mm camera or a 16 mm movie camera for an interval of many hours. In the second type of experiment, the convection was allowed to develop without inducing. After the fluid had been convecting for a period of about 8 hours, a movie camera was then run at 1/300th normal speed to make time lapse movies for at least another eight hours. These films were used to determine the transition to time dependence, the structure of the convection, and the frequency of the time dependent motion. A movie showing the results is available from either of the authors.

A probe device consisting of two thermistors 7 mm apart, one over the other, was constructed to determine the temperature field within the layer, and especially to get a measure of the heat flux. The thermistors were isotemp thermistors from Fenwall, Inc., calibrated to 0.001°C, and their resistance was measured by means of a DC bridge circuit, containing resistors whose accuracy was equivalent to an accuracy of 0.01°C. In all such measurements, the probe was left in place for sufficient time to allow transient effects to die down before data were recorded.

3. OBSERVATIONS

Artificially initiated flow

First of all, it is useful to discuss the two different ways that such convection patterns have been observed to evolve in time in past experiments. One way is by breaking down in a spatially uniform manner. A sequence of photographs showing such a breakdown is shown in the left-hand sequence of Figure 5. We presume that when such a breakdown occurs, either the pattern is not a solution to the Navier–Stokes equations, or the pattern is unstable to infinitesimal perturbations, and we will use the word *stable* if it does not break down in the above manner, and *unstable* if it does break down. The second way that such convection patterns evolve in time occurs because the sides of the tank generally trigger dislocations in the induced flow pattern, and dislocations

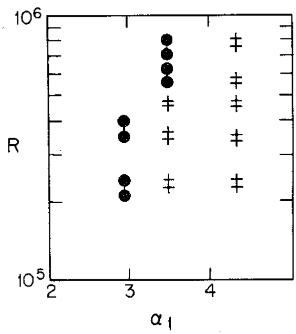

FIGURE 2 Stability range of bimodal flows with two wavelengths unequal. The cross wavenumber was 5.8. Dumbbell-unstable, square-stable.

are always observed to work their way into the pattern and break it down. This took some ten hours or longer in our experiment, which corresponds to approximately one hundred cell rotations, based on overturn times measured with neutrally buoyant particles. This places an upper limit on the time that we could observe the stability of uniform cells. Therefore, our use of the words stable and unstable can properly be interpreted as being subject or not subject to significant uniform changes which are observed within ten hours or so.

It was relatively difficult to induce bimodal flow with two different wavelengths because each wavelength had to be induced separately as described in Whitehead and Chan (1976). When attempts were made to induce two wavelengths simultaneously, one mode would grow faster than the other and one or the other mode would predominate causing the field to break down into a random pattern. The stability range is shown in Figure 2 as a function of the wavenumber of the long wavelength α_1 and Rayleigh number R for the case $\alpha_2 = 2\pi d/\lambda_2 = 5.8$ with λ_2 the wavelength of the shorter wavelength and d the depth of the fluid. The double cross denotes stable bimodal flows—stable in the sense that the bimodal flow had not broken down after 5 hours. The double circles connected by a line denotes that the flows were observed to break down to a new pattern which we call the dumbbell pattern, shown in Figure 3, in which sheets of ascending and descending fluid develop lumps which initiate new sheets of fluid at right angles to the original sheet. The dumbbells do not oscillate although they were reported by Busse and Whitehead (1974) to oscillate in a fluid with the lower Prandtl number of 126. In the present

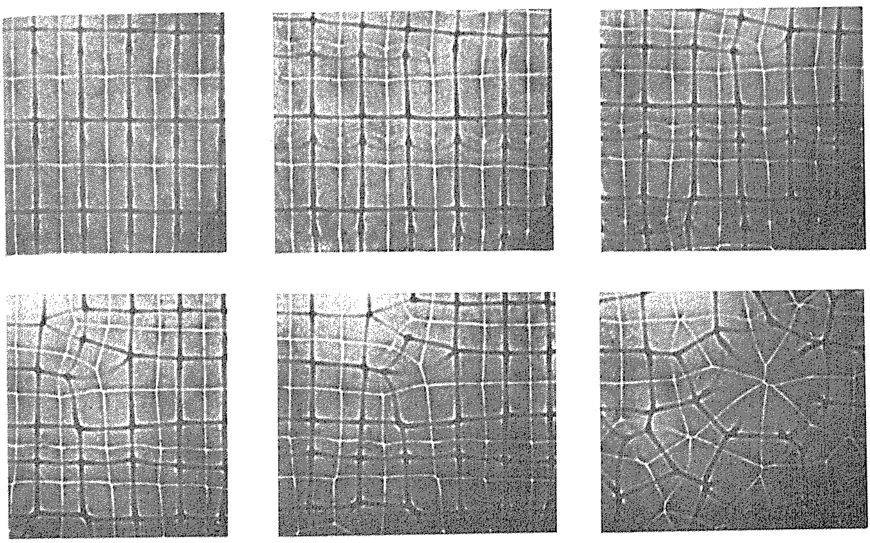

FIGURE 3 Photograph of the metamorphosis of bimodal flow by a dumbbell breakdown. $R = 372{,}000$, $\alpha_1 = 2.88$, $\alpha_2 = 5.8$. Times left to right are (first row) 0, 0.5, 1.3 and (second row) 2, 2.6, and 7.6 hours, respectively.

experiments the dumbbells seem to be a mechanism to make cells with a smaller wavelength.

It was far easier to induce bimodal flow with both wavelengths the same, especially above a Rayleigh number of approximately 300,000. This was done by using as a grid a sheet of paper with square holes cut out in a checkerboard pattern. Figure 4 shows the region of square convection studied. Various sized square arrays were stable up to the highest Rayleigh numbers observed (760,000), as denoted by a square in the figure, although the squares were observed to break down below a certain Rayleigh number, denoted by a diamond in the above figure. Photographs of the breakdown of this flow are shown in the sequence of close up pictures in Figure 5, which shows a rapidly developing instability at a Rayleigh number of 168,000 and stable flow at a Rayleigh number of 370,000. The unstable square cells broke down in a manner which in some respects resembles the zig-zag instability of Bénard rolls observed by Busse and Whitehead (1971). In the present instability, adjacent sheets of ascending and descending fluid begin alternately to zig-zag.

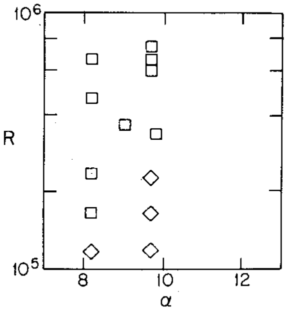

FIGURE 4 Map of the stability of square convection. The wavenumber α is defined as $2\sqrt{2}\pi d/\lambda$. Diamond-unstable, square-stable.

Random flow

The time lapse movies, filmed so that one second of movie corresponds to 300 seconds of experiment time, were started after the apparatus had been held at the desired temperature for at least the previous eight hours. It was desired to determine whether such flow resembled the artificially initiated flow. Figure 6 shows photographs of typical convective patterns at various Rayleigh numbers. It was found that the flow bore a close resemblance to bimodal flow with two different wavenumbers at Rayleigh numbers below approximately 150,000, but that above this value regions of spoke-shaped convection would become more prevalent [see Busse and Whitehead (1974) for a description of the collective instability which leads to spoke-shaped convection]. In addition, the first time dependent oscillations were observed in spokes at 150,000 and became more prevalent as R was increased. These oscillations always occurred in the spoke-shaped regions, but such regions did not always have oscillations. In addition, the convection pattern was observed to be perpetually shifting around and changing when spoke patterns existed in any number. This metamorphosis was considerably slower than the oscillation periods and also was slower than the cell overturn times which will be shown in Figure 9. This

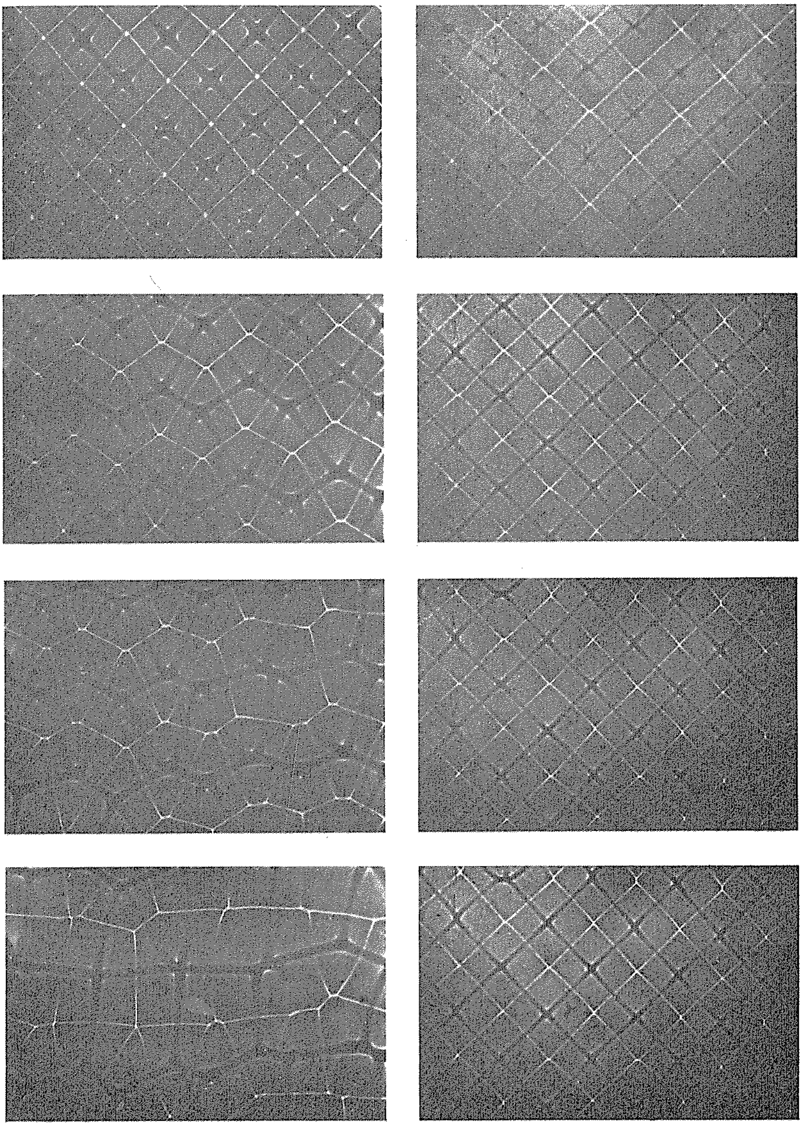

FIGURE 5 Squares with wavenumber $\alpha_1 = \alpha_2 = 6.9$ exhibiting instability on the left at a Rayleigh number of 168,000, and stability on the right at a Rayleigh number of 370,000. Time sequence is downward at 3 hour intervals.

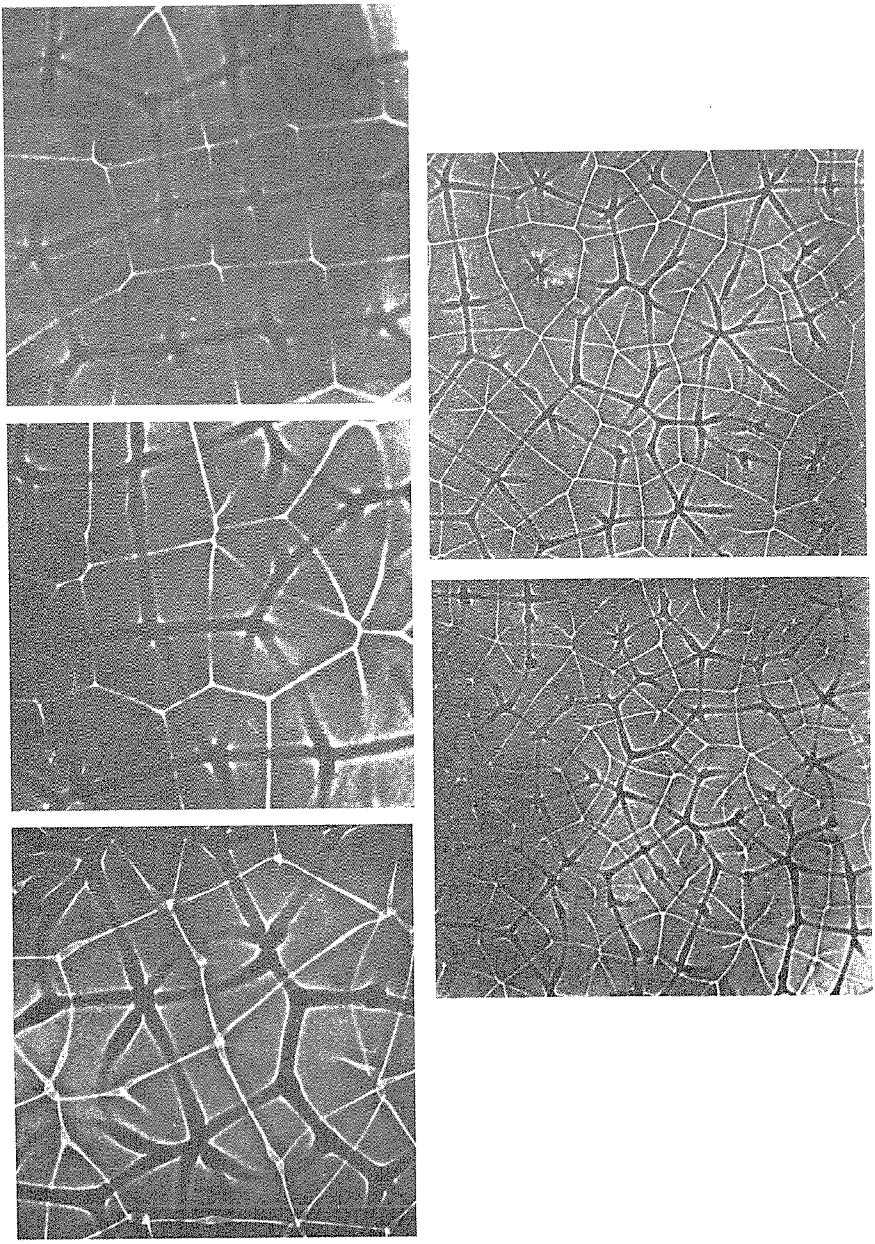

FIGURE 6 Photographs of convection after having evolved randomly for 24 hours. Top to bottom; Left $R = 120{,}000$, $170{,}000$, and $240{,}000$; Right $R = 680{,}000$ and $780{,}000$. The first three photographs encompassed an area of $40 \times 40\,\mathrm{cm}^2$, the last two encompassed an area of $60 \times 60\,\mathrm{cm}^2$.

seems to correspond to the slow drift in temperature observed by Krishnamurti (1970b, Figure 6), and was always observed to be a feature of the spoke-shaped convection for experiments up to 24 hours in duration.

Figure 7 shows the estimated percentage of area which contained spoke-shaped convection. These estimates were obtained by inspection of the time-lapse movies taken over the space of eight hours, and exhibit scatter due to the limited data base. The fifty percent crossover Rayleigh number for the emergence of spoke flows is shown in Figure 8, in comparison with the fifty percent crossover point observed by Whitehead and Chan (1976) in fluids with a Prandtl number of 16 and 126. It is evident that the dependence of the

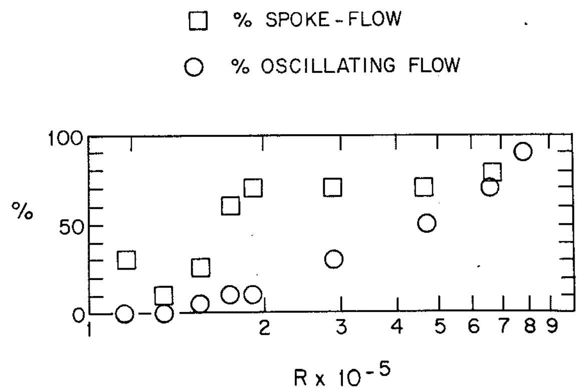

FIGURE 7 Percentage of area observed to possess spoke-like flow (denoted by squares) and time dependent flow (denoted by circles).

transition to oscillatory flow on Prandtl Number is getting weak, but there still may be some dependence. The diamond in Figure 8 denotes the Rayleigh number above which more than fifty percent of the convection was observed to be oscillating.

Figure 9 shows estimates of the turnover time, and oscillation frequencies deduced by observation of the movies with a stop watch. The times were scaled with respect to the thermal diffusion time d^2/κ. Observations of the turnover time varied by a factor of approximately 4 at the same Rayleigh number and this was clearly attributable to the variation of velocities in different cell sizes and structures. It is clear that the lowest frequency of the oscillations is the turnover frequency, in agreement with observations by Krishnamurti (1970b), but it is not possible to observe the existence of discrete higher harmonic structures due apparently to the interrelation between convection oscillations and the fluid structure.

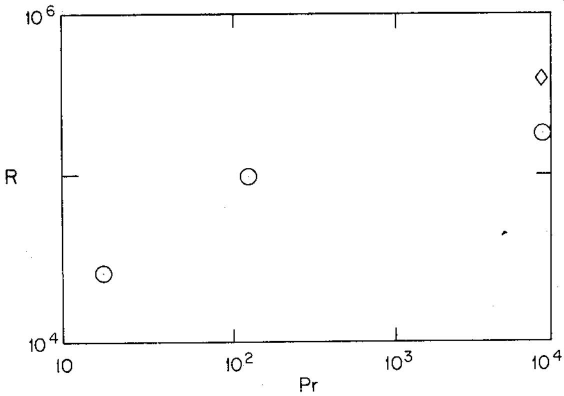

FIGURE 8 Fifty percent crossover point of the emergence of spoke-like flow as a function of Prandtl number. Data at Prandtl numbers 16 and 126 are taken from Whitehead and Chan (1975). The diamond at Prandtl number 8600 denotes the 50% crossover point for oscillation.

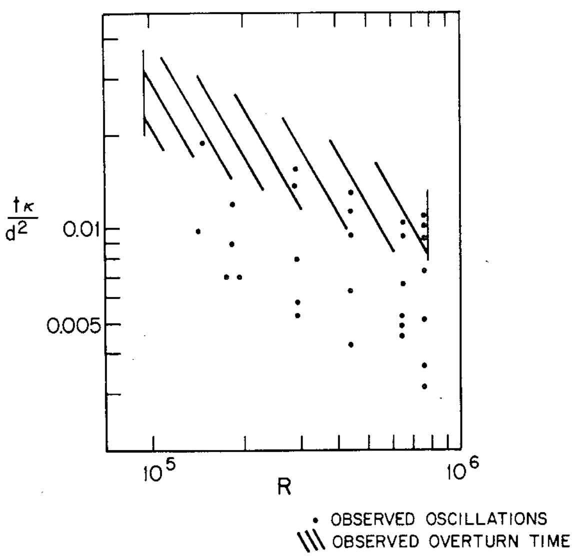

FIGURE 9 Overturn time of cells (cross hatched area) and observed oscillation frequencies of randomly oriented convection measured on films. The spread occurs because various cell geometries have differing overturn speeds and oscillation periods.

Measurements were made of temperature within the fluid by emplacing the temperature probe in the fluid, waiting for five minutes, and then taking a reading. The shadowgraph enabled us to observe that moving the probe through the fluid distorted the structure slightly, but that the convective thermals had a remarkable ability to quickly rebound back to their original position, so that within two minutes the convection had resumed its undisturbed state. By the end of five minutes, all transient effects had gone from the temperature readings.

It was possible to determine the vertical structure of the features which are depth-averaged in the shadowgraphs. Many obvious aspects were confirmed by the probe, such as that the white lines denote cold detached boundary layers which are descending from the top surface, and vice versa for the black

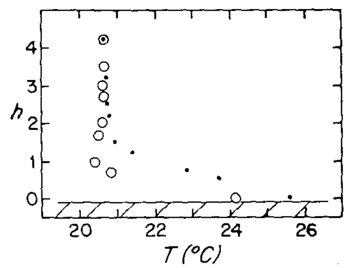

FIGURE 10 Temperature field from the bottom to 4 cm above the bottom at a point where a white line on the shadowgraph crossed a black line (dots), and above a point midway between lines (circles) where there are presumably no rising thermals. Errors of the temperature reading are less than 0.001°C. and errors of the depth reading are less than 1 mm.

lines. Other aspects were clarified, although they are more difficult to picture and describe because of the inherent three-dimensionality of the flows. For instance, there actually are rising and sinking sheets of fluid at the point where black and white lines cross in square convection, as shown in Figure 5. The probe revealed that the white line is generated by a sheet of cold fluid sinking from the top while the black line is generated by a sheet of warm fluid rising from below at right angles to the cold sheet. It was apparent that the two sheets split apart to avoid each other, so that a stagnation point apparently exists midway between the top and bottom surfaces. The temperature observations from the bottom to four centimeters above the bottom over a point where white and black lines cross are shown as dots in Figure 10, along with observations (circles) over another point midway between white and black lines where there appears to be little descending or ascending motion. Both boundary layers merge with a region of uniform temperature, but there is an excess thickness of the boundary layer above the crossing lines (dots). There is a substantial isothermal core in these cells away from the boundary layers in both cases. In spoke structures, it was clear that the spokes were sheets of fluid which protruded out of the temperature boundary layer significantly. Dye injection and observation of trace particles in movies revealed that fluid within the spokes moves as much laterally toward the central spout area, as it moves vertically. Much of the interior temperature variation occurred in the central spout.

Finally, it was possible using this two thermistor probe to assess the degree to which the finite conductivity of the glass boundaries results in departures from a constant temperature boundary condition. The greatest temperature gradients were consistently observed at stagnation points near the boundary—for instance, at the point below a descending cold sheet or spout where the fluid stagnates as it encounters the bottom boundary. The smallest were observed at separation points, for instance at the point where an ascending warm sheet or spout rises from the bottom boundary.

We ask, if the true boundary condition is represented in dimensionless form as

$$\frac{\partial T}{\partial z} + \gamma T = \text{constant}, \tag{1}$$

at every point of the glass-fluid interface, how big is γ?
Obviously if γ is very much greater than one, the boundary condition is well approximated as a constant temperature, while if γ is very much less than one, a constant heat flux condition is a better approximation. To determine γ, use was made of the approximation

$$\frac{\partial T}{\partial z} = \frac{d(T_T - T_B)}{\Delta T h},$$

so that (1) is approximated as

$$\frac{d(T_T - T_B)}{\Delta T h} + \frac{\gamma T_B}{\Delta T} = \text{constant}, \tag{2}$$

where d is depth of the fluid, ΔT is temperature difference between the top and bottom baths, h is distance between the two temperature probes (7 millimeters). Values of T_T and T_B were determined by taking values of the top and bottom thermistors in the probe when the bottom thermistor was lying on the bottom glass. Many readings were taken, and pairs were combined to solve for γ with the use of (2). γ was found to vary from 5 to 20. The variation appears to be principally due to the fact that T_T was not really taken in the conduction boundary layer. Since the effects of poor boundary conductivity predominate when this number is less than one, the large size of this number indicates that the boundary conditions can be well approximated as a constant temperature boundary condition.

Another indication that the finite conductivity of the glass does not significantly affect the oscillations is that the time constant of the glass is approximately $d^2/\kappa = 80$ seconds (using $\kappa = 5 \times 10^{-3}$ cm^2/sec.) while the oscillation times are from 14 minutes to 2 minutes.

4. DISCUSSION

The observations reported here support the observations of Busse and Whitehead (1974) that the transition to oscillations is strongly dependent upon the structure of the flow. Indeed, it appears safe to say that there are at least two distinct states which the convection can take, which are equally valid solutions of the governing equations. One solution is space-periodic, stationary, and stable to infinitesimal perturbations. The second solution is observed to be nonstationary in two different ways. First, it sometimes exhibits clearly periodic oscillations. Second the cell-structure constantly changes its disposition, although it is characterized by spoke-shaped cells. It has not been possible to induce space-periodic, non-migrating spoke convection which oscillates as was done by Busse and Whitehead (1974) by a collective instability. It appears that the two states can exist simultaneously in a statistical sense over some range of Rayleigh numbers. In such a case, it was observed that the regions with a bimodal structure never oscillated even though there were oscillations in adjacent regions of spoke convection. The question of the exact nature of the process which generates the oscillation continues to be clouded by our observations of a weak Prandtl number dependence. Using the scaled, overturning time $t\kappa/d^2$ as given in Figure 9 to be 0.01 or larger, a Reynolds number can be approximated as $d^2/t\nu$, which reduces to $100/8600 = 0.012$, a small Reynolds number indeed. It therefore seems unlikely that the nonlinear operators of the Navier-Stokes equation play a major role in the above processes.

The results presented here have two particular implications upon the work as reported by Krishnamurti (1970b and 1973). First, in a span of 16 hours no oscillations were seen in movies at Rayleigh number of 98,000 and 120,000, and only one oscillation was seen at 158,000. Krishnamurti reported seeing an oscillation at approximately 60,000 in an experiment with smaller aspect ratio, but run for a much longer time. It is not clear whether the difference between Krishnamurti's and our bottom boundary conditions or side boundary conditions is responsible for this quantitative disagreement. In view of the random nature of the oscillations as Rayleigh number is increased, it appears that the exact transition number is difficult to determine, and is not too meaningful physically, since only a tiny percentage of fluid oscillates when this number is exceeded. Secondly, even if the transition does occur at $R = 150{,}000$ the low Reynolds number suggests that a thermal type of instability is causing the oscillations, as suggested in the 1973 paper.

An interesting feature which this experiment shows is that one dislocation in an otherwise flawless and stable field of motion can trigger new flows which otherwise wouldn't exist, although perhaps they can be generated by a finite amplitude perturbation. At lower R flaws do not do this (Whitehead, 1976). In the random flow, these dislocations exist in many places and seem to allow the bimodal and spoke flows to coexist. It can be conjectured that randomness in turbulence plays a role similar to that which the dislocation plays, and enables different flow structures to coexist, and in fact sustain each other. These structures may mathematically be expressed as noncontiguous manifolds of solutions. We speculate here that perhaps the flaws provide a mechanism for coupling between the different solutions.

Acknowledgements

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