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SELECTIVE WITHDRAWAL OF ROTATING STRATIFIED FLUID

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ABSTRACT

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A simple theory and some experimental observations are presented of the transient withdrawal of rotating, stratified fluid in a field of gravity. The problem is confined to axisymmetric geometry and negligible viscosity. It is predicted that the withdrawal initially proceeds like non-rotating selective withdrawal, but at a time equal to $3\sqrt{3}/2f$ there is a transition to a rotation-dominated selective withdrawal process which requires that fluid come from distances above and/or below the inlet given by the time-dependent formula $(fQt/2\pi r_0 N)^{1/2}$. Experimental observations are given which are in approximate agreement with the predictions.

1. INTRODUCTION

Selective withdrawal, the flow of stratified fluid into a sink, has received a considerable amount of study over the last two decades, but little attention has been paid to such problems when the fluid is rotating. In many problems in meteorology, oceanography, planetary physics and astrophysics, and doubtless engineering, it would be useful to have a technique available for the estimation of uplift and/or downlift of surfaces of constant density into a sink when the entire system is rotating.

Here we will develop a rather simple method for estimating the maximum height above or below a sink from which stratified fluid can descend or rise at it is sucked in. It will be assumed that the fluid is linearly stratified and motionless at time $t = 0$. Gravity is downward and the entire system is rotating with angular velocity $\Omega = f/2$. After $t = 0$ the sink, which will be uniformly distributed around a circle of radius r_0, will be turned on with total mass flux Q. The sink may be along a wall which faces inward, toward the central axis of the sink; or it may be along a wall which faces outward, away from the central axis of the sink. Fluid may rise from below the sink upward, or descend from above the sink downward. For clarity we will discuss only the upper half-plane, so fluid only comes down to the sink. We will then test the estimate against some measurements in a laboratory experiment.

This small initial effort is most analogous to the early works by Yih on selective withdrawal and the experiments by Debbler which are described by Yih (1980) and Turner (1973). Recent works which concern viscous and transient effects and detailed velocity profiles are far beyond the scope of the present work.

The basis for the theory lies in the analysis by Whitehead and Porter (1977) (WP), who predicted the upstream height at some radius of origin for the axisymmetric withdrawal of a layer of homogeneous fluid in a rotating frame which lies under a stagnant, deep second fluid. An explicit formula was obtained which related mass flux Q with the upstream height h_1 at the radius of origin r_1, the radius of the sink r_0, the density difference between the fluid being removed and the stagnant layer overhead $\Delta\rho$, the Coriolis parameter (two times angular rate of rotation) f, and the force of gravity g. This formula is

$$Q = 2\pi r_0 \left(\frac{2}{3}\right)^{3/2} (g')^{1/2} \left[h_1 - \frac{f^2}{8g'}\left(r_0 - \frac{r_1^2}{r_0}\right)^2\right]^{3/2} \tag{1}$$

(WP, eq. 14 with $g' << g$ and $v_1 = 0$), where $g' = g\Delta\rho/\rho$. If one could develop à method to predict the appropriate r_1 and g' for a continuously stratified fluid, (1) might be useful for predicting h_1, which one would normally consider to be a height of selective withdrawal. Here it will be assumed that Q is constant for $t > 0$ and that r_1, h_1 and g' vary with time. Before proceeding to develop the appropriate r_1 and g' and discussing what h_1 is equivalent to in the stratified fluid, it is useful to review the derivation of (1). The steady Navier—Stokes equations in a cylindrical coordinate system with friction ignored, for fluid of density ρ which lies under a fluid of density $\rho - \Delta\rho$, are

$$\frac{\partial}{\partial r}(ruh) = 0$$

$$-fv + u\frac{\partial u}{\partial r} - \frac{v^2}{r} = -g'\frac{\partial}{\partial r}\left(h - \frac{f^2 r^2}{8g}\right)$$

$$u\left(\frac{\partial v}{\partial r} + \frac{v}{r} + f\right) = 0$$

Since $u \neq 0$, velocity in an angular direction can be determined by integrating the term in parentheses in the third equation to give

$$v = \frac{fr_1^2}{2r_0} - \frac{fr_0}{2} \tag{2}$$

where it is assumed that v at r_1 is zero. Solution (2) can be used in combination with the other two equations to derive

$$g'h^3 - g'Hh^2 + \frac{Q^2}{8\pi^2 r^2} = 0 \tag{3}$$

(eq. 8 in WP with $g' << g$ and $v = 0$ at r_1), Here, H is a spatially varying Bernoulli head

$$H = h_1 - \frac{f^2}{8g'r^2}(r^2 - r_1^2)^2 \tag{4}$$

Equation 3 varies from its counterparts in non-rotating hydraulic problems principally in that the "head" H varies with position. Henceforth this will be called a "virtual head". Such an effect was pointed out by Sambuco and Whitehead (1976). Although eq. 3 contains all the dynamics necessary to predict h as a function of r and Q, the equations allow many values to exist, depending upon the value of h at r_1. There is, however, a miminum value of h at r_1 which allows the fluid to flow to r_0, below which fluid runs out of virtual head before it reaches r_0, and this (unique) value can be determined in many ways. One of the most convenient is to say that

$$\frac{\partial r}{\partial h} = 0 \text{ at } r = r_0 \tag{5a}$$

although this strictly violates one of the assumptions used in deriving the starting equation (but only in a small region close to r_0). An equivalent one is that the Froude number be equal to one at r_0, i.e.

$$\frac{Q}{2\pi r_0 h_0} = (g'h_0)^{1/2} \tag{5b}$$

Either (5a) or (5b) will lead to (1) when applied to (3).

What does this two-layer problem have in common with selective withdrawal of a stratified fluid? It appears that it has the two most important elements:

(1) $F = 1$ at the inlet (density surfaces are sucked up and down to the inlet in response to Bernoulli's law).

(2) An advected inertia is balanced by pressure in the fluid by virtue of a law like Bernoulli's law (at least for inertial selective withdrawal, which is being considered here).

It thus seemed reasonable to hope that (1), with suitable interpretations of g' and r_1, could lead to a useful method for estimating h_1 (suitably interpreted) as a function of flux Q and rotation rate. We will proceed under that assumption and test the result against experiment. It will be shown in Appendix 1 that the solutions so generated do not generate large temporal acceleration forces, and in Appendix 2 large viscous forces for water.

There are, of course, a number of pitfalls in applying the above equations to a laboratory experiment with continuous stratification. Obviously there is really no unique value of r_1 and h_1 for the continuously stratified case. We interpret r_1 as being that radius which the fluid at the sink at time t had at the beginning of the experiment. In reality, fluids may come from different radii, so this may be a very poor assumption. It does, however, lead to a simple relation between h_1 and r_1, since fluid which originated at r_1 will reach the inlet when a volume of size $\pi h_1(r_1^2 - r_0^2)$ has been removed, so that

$$Qt = \pi(r_1^2 - r_0^2)h_1$$

Although this is a Lagrangian argument to a problem which has been posed with Eulerian dynamics, we believe this may be quite correct dynamically for the following reason. The principal function of r_1 in (1) is to create velocity in the angular direction at the inlet as given by (2) which modifies the virtual head (4). Since the angular velocity depends on conservation of angular momentum, (2) is also true in a Lagrangian sense on any ring of fluid.

To solve for $\Delta\rho$ in (1) the relation

$$\Delta\rho = -\frac{1}{2}\frac{\partial\rho}{\partial z}h_1$$

will be used. Such a relation is so widely used to model a stratified fluid that little need be said. With these assumptions, the parameters r_1 and $\Delta\rho$ can be eliminated in (1) and one predicts

$$Q = 4\pi r_0 \left(\frac{1}{3}\right)^{3/2} N h_1^2 \left[1 - \left(\frac{fQt}{2\pi r_0 N h_1^2}\right)^2\right]^{3/2} \tag{6}$$

where $N \equiv [(g/\rho)\partial\rho/\partial z]^{1/2}$ is the Brunt—Väisälä frequency.

Before discussion of the features of the solution and comparison of the prediction with experimental observations, a few remarks will be made about the dynamics which have been included in the problem. When fluid is withdrawn from a reservoir containing stratified fluid, surfaces of constant density are observed to be uplifted from below the level of the sink and depressed from above the level of the sink due to a lowering of pressure at the sink. Here we have simply arrived at a form of Bernouilli's law that is used to estimate a height change that surfaces of constant density must undergo to generate that pressure. There is a balance between velocity head at r_0 and hydrostatic head. The concept of critical control is only necessary in order to give an estimate of what that maximum possible withdrawal will be. The problem of solving for r_1 is important only because the swirl velocity v is very strongly dependent on r_1.

As time progresses from zero, prediction (6) is as follows. Initially the term within the square brackets will be close to one so

$$h_1 \approx 27^{1/4}\left(\frac{Q}{4\pi r_0 N}\right)^{1/2}, \qquad t < 27^{1/2}/2f \tag{7}$$

Naturally, this relation holds for a fluid without rotation as well. For instance, it differs from the non-rotating Froude number criterion (which is a prediction for a steady problem) $h_1 = (Q/2r_0N)^{1/2}$ (Yih, 1980; Brooks and Koh, 1969, eqs. 11b, c) by less than 10%. After

$$t = 27^{1/2}/2f, \tag{8}$$

the term in the square brackets in eq. 6 would become negative unless height becomes close to the value

$$h_1 \approx \left(\frac{fQt}{2\pi r_0 N}\right)^{1/2}, \qquad t > 27^{1/2}/2f \tag{9}$$

In this limit the swirl velocity v becomes greater than the radial velocity, and the geostrophic and cyclostrophic balance at the sink radius determines the height of selective withdrawal.

The fact that this formulation agrees with that of Brooks and Koh (1969) in the absence of rotation lends some validity to the many assumptions which have been used. As a further test, the predictions were tested against experiment.

2. AN EXPERIMENTAL TEST

The objective of the experiments was to obtain quantitative data on the selective withdrawal height above and below axisymmetric inlets. Two experimental containers were used. The first was a rectangular plexiglass container 35 cm wide, 35 cm broad and 30 cm deep, mounted on a 1 m diameter turnable. On the bottom, a copper manifold was embedded in 6 cm of crushed rock. The manifold was connected, by flexible tubing and a swivel connector, to two tanks which were set up to supply water whose salinity increased uniformly in time by the Oster method, as sketched in Fig. 1. The second tank was also square with an internal cylindrical false wall 1 m in diameter. All else, except the volume of the filling tanks, was the same. In each experiment the container was mounted on a carefully levelled 1 m turntable which was set to uniform rotation and slowly filled from below via the manifold so that after 2 h it was filled with uniformly stratified water with a density gradient $(1/\rho_0)\partial\rho/\partial z$ of 0.0021 ± 0.0002 cm^{-1}. The water exhibited a drift with respect to rigid rotation of less than 0.01 rad s^{-1}. Salinity was measured with an optical refractometer at every 2 cm of depth.

In the first runs a pipe with an outside diameter of 0.95 cm and inside diameter of 0.70 cm was suspended vertically halfway down into the water. The small container was used since the gyre formed around the withdrawal pipe at the center of the tank. The withdrawal pipe was filled with fresh water and was attached to a siphon hose and a pipette. By adjusting the height of the spout of the pipette with respect to the free surface of the water in the test tank, a steady volume flux of 0.2 cm^3 s^{-1} was obtained.

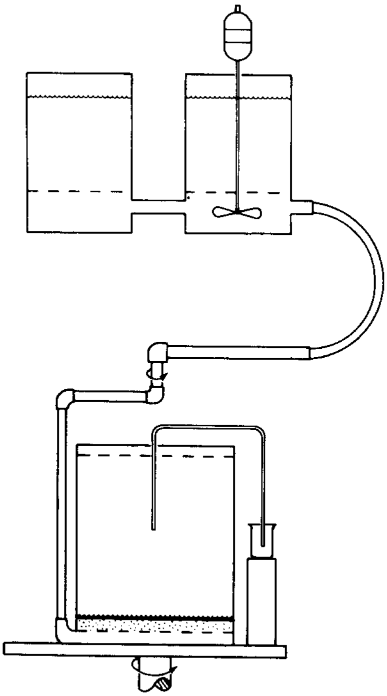

Fig. 1. Sketch of the laboratory apparatus and the method of generating a stratified fluid by the Oster method. The broken lines correspond to the water surfaces after the tanks have been filled.

After the sink was turned on, pellets of potassium permanganate were dropped into the tank along a straight line stretching from the withdrawal tube to one side of the tank. The pellets rapidly sank and left a vertical dyed column. After these had been somewhat distorted by lateral flows, photographs were taken of them against a white background with a 35 mm camera equipped with a 135 mm lens. The purpose of the telephoto type of lens was to minimize apparent optical distortion to the light rays which reached the camera by observing an image formed by rays which left the tank at an angle which was almost normal to the wall of the tank. Periodically, new pellets were introduced and pictures were subsequently taken. The distorted columns give a clear measure of the velocity flow structure.

Figure 2 shows a typical photograph of vertical columns which have been distorted by the flow. Of course, the flow field is really smooth rather than discrete, and our intention is to find the vertical extent for most of the withdrawal, not a height where there is a clear jump in water properties. We presume that the height of large flow corresponds to the height of selective withdrawal. Therefore, the heights which will be reported here were heights

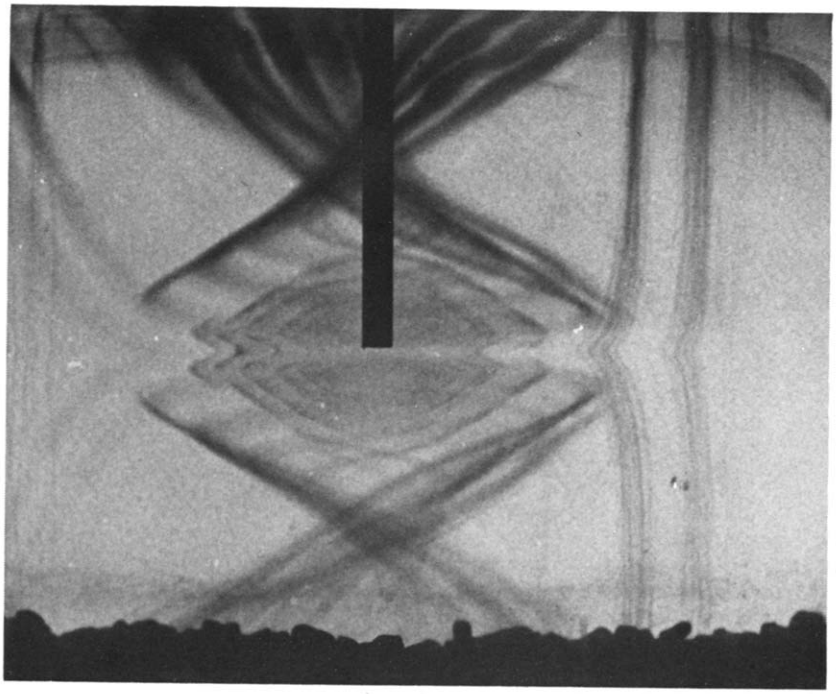

Fig. 2. Photograph of dye columns in an experiment which has been running for some time. The old columns have been swept around by rather deep, sluggish gyres which lie above and below the dynamic withdrawal region. New columns, on the right, display a distortion which marks the region of true selective withdrawal, which is characterized by radial as well as angular flow.

of 90 to 95% of the displacement. Figure 2 and virtually all the other photographs give some evidence to support this conclusion. The vertical lines on the left and right show evidence of the region of velocity in the angular direction, while the older, remnant dye lines display a clearer layer near the level of the inlet presumably because clear water has come in from further out.

Clearly, the flow varies with distance away from the sink and there is no single depth of selective withdrawal. Fortunately, with a small exit pipe most of the vertical excursions of the water took place near the sink, where a pronounced vortex developed. There was little change in depth away from the sink. Thus we report here those measurements taken over a variety of radii, with no bias except that the radius was always a few centimeters, but not large enough to show no shear. We also report only measurements where r_1 is predicted to be less than the size of the tank.

As a note of caution to those who might try such a measurement technique, the old pellet columns were slowly swept by large, sluggish gyres (possibly Taylor columns) above and below the region where the currents were intense, and one could easily be deceived into thinking that the region including

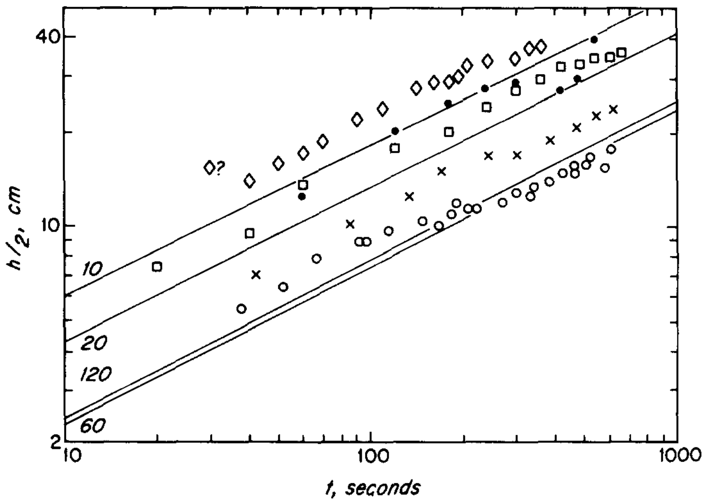

Fig. 3. Observations of height of withdrawal as a function of time. As explained in the text, the height presented is half the entire vertical height observed in the experiments because of symmetry above and below the level of the sink. For r_0 = 0.475 cm and Q = 0.2 cm^3 s^{-1}, symbols denote rotation period as follows: Diamonds: 10 s. Squares and filled circles: 20 s, Crosses: 60 s. Lines labelled 10, 20 and 60 denote theoretical predictions for those periods. For r_0 = 50 cm, Q = 50 cm^3 s^{-1} and a period of 120 s, the data are circles, and the theoretical prediction is labelled 120. All theory arises from eq. 9.

the gyres was the region of selective withdrawal dynamics. Figure 2 shows a photograph of such old columns and some newly created columns on the right. The region of rapid flow is clearly much more restricted (and energetic) than the large dome-like region.

One run was conducted at a rotation period of 10 s, two at 20, one at 60, and one at 200 in the first apparatus. Figure 3 shows the results of the 10, 20, and 60 s runs. Because of symmetry above and below the level of the sink, $h/2$ denotes half the entire vertical extent of the selective withdrawal region. We presume that the net sinking of the water from above is not yet great enough to create a strong asymmetry about the level of the sink. Data from the theoretical prediction, in the rotating limit, eq. 9, are also shown in Fig. 3 as lines for the three different rotation periods. For this prediction the volume flow rate of 0.2 cm s^{-1} was divided by two because of symmetry above and below the mid-plane of the sink. The data lie parallel to the curves but are systematically higher by tens of per cent.

A second set of experiments was conducted in the large tank with a 1 m cylindrical false wall and a 1 m diameter sink along the outer wall. The sink consisted of a 1 m diameter loop of 1 cm diameter copper pipe with holes every 2 cm. The loop was suspended halfway down into the tank. Three runs were conducted at a rotation period of 120 s, and at a pumping rate of 50 cm^3 s^{-1}. The same eq. 9 applies and data for all runs are also plotted in

Fig. 3, with a corresponding line from eq. 9. Again, because of symmetry, half the height of selective withdrawal is given, and half the volumetric pumping rate is used in the calculation. In this case the shear zone between selective withdrawal was easier to identify in real time than in the photographs and measurements are believed to be good to ±15%.

A third experiment was done in the large tank. The sink was the small central pipe which had been previously used in the small tank. The objective was to see whether the transition from non-rotating to rotating flow could be observed. In order to have sufficient time to observe the distortion of the vertical dye columns it was necessary to make rotation as slow as possible; in this case a period of 355 s was set for the turntable. It was also believed necessary to have sufficient flux out of the sink to overshadow the effect of viscosity; the volume flux of 2.54 cm^3 s^{-1} was believed to be sufficient for this.

The dye columns were photographed every 20 s. Results are shown in Fig. 4. Fresh pellets were re-introduced after every second or third photograph; a datum based on a new column is denoted by an arrow. The data lie parallel but above the two predicted asymptotes from eqs. 7 and 9. The qualitative agreement is quite good; however it is not entirely clear whether there are enough data to show clear agreement with the non-rotating case. Measurements of the photographs with an optical micrometer with errors smaller

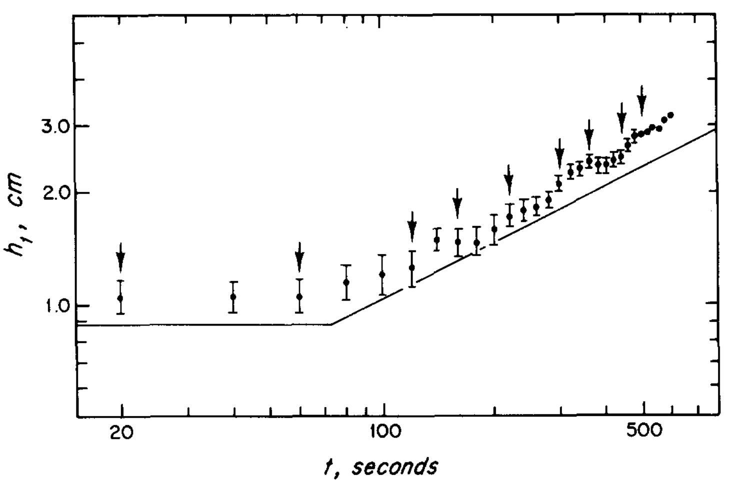

Fig. 4. Observations of the height of withdrawal as a function of time for r_0 = 0.475 cm, Q = 2.54 cm^3 s^{-1}, and a period of rotation of 355 s. The non-rotating prediction of height, eq. 7, and the rotating prediction of height, eq. 9, are shown as straight lines. Because of symmetry above and below the midplane of the tank the mass flux Q must be divided by two and the observed height h_1 is half the observed vertical extent of withdrawal. Arrows denote where data were based upon a newly introduced column of potassium permanganate.

than the equivalent of ±0.1 cm were made twice with extra caution to ensure that the data were as accurate as possible.

Thus it appears that there is some experimental evidence that the rather simple formula given by eq. 6 would give a useful estimate of selective withdrawal into an axisymmetric sink. Nowhere has the assumption been made that r_1 is bigger or smaller than r_0, so fluid flowing outward toward a cylindrical sink has the same features as fluid flowing inward. Also, in the limit of r_0 being much bigger than $r_1 - r_0$, the problem approximates flow into a line sink in a Cartesian coordinate system.

A result which is simpler than anticipated is that frame rotation always becomes important at a time equal to $3\sqrt{3}/2f$ which, in a geophysical context, would be equal to $0.21/\sin\theta$ (time units are days) where θ is latitude. This is considerably earlier than one normally expects to see the effects of rotation being set up and raises the possibility of whether considerable swirl from the earth's rotation may be generated near axisymmetric or line sinks in nature such as cumulus clouds (turbulent plumes act as sinks to adjacent fluid), deep ocean convection plumes, the proposed ocean thermal energy conversion (OTEC) power plants and sea breezes.

Let us apply these estimates to two typical thermal difference power plants assuming they were sitting in a motionless fluid, or freely drifting with the current so there was little differential motion between the inlet and the neighboring current. For an estimate of the evolution of selective withdrawal height with time, Table I has been constructed using $Q = 250$ and $2500\ \mathrm{m^3\ s^{-1}}$ (Designs of 100 MW have been proposed with $Q = 506$ and $303\ \mathrm{m^3\ s^{-1}}$ and 400 MW with $Q = 2420\ \mathrm{m^3\ s^{-1}}$ for the hot water inlet (TRW System Group, 1975).) Height and r_1 were calculated from eq. 7 or 9,

TABLE I

Some numbers for the selective withdrawal solutions as applied to small and large OTEC power plants

Time after start (h)	Slow or fast	$Q = 250\ \mathrm{m^3\ s^{-1}}$ (100 MW)				$Q = 2500\ \mathrm{m^3\ s^{-1}}$ (400 MW)			
		h_1 (m)	r_1 (m)	ΔT (°C)	U ($\mathrm{m\ s^{-1}}$)	h_1 (m)	r_1 (m)	ΔT (°C)	U ($\mathrm{m\ s^{-1}}$)
6	S	58	167	1.1	0.012	183	296	3.3	0.021
8	F	61	193	1.1	0.010	192	343	3.3	0.018
12	F	76	213	1.4	0.008	240	378	4.4	0.014
16	F	87	229	1.7	0.006	275	407	5.3	0.011
20	F	97	242	1.9	0.005	306	430	6.1	0.009
24	F	107	253	2.2	0.005	338	450	6.9	0.009

and the conservation of mass respectively. Temperature degradation was calculated assuming that water for the warm water inlet was sucked up into the inlet from an average depth of $h_1/2$ below the top surface, through the use of an OTEC design bathythermal profile which has been used as a "typical" ocean temperature profile for engineering purposes (TRW System Group, 1975, pp. 3—6). Lastly, the ocean current necessary to stop growth at the listed values of h_1 and r_1 by replacing the mass flux in an area of $2h_1r_1$ is given.

The table shows that there may be some degradation of the inlet temperature due to selective withdrawal. A glance at typical current meter records, such as are shown in the MODE atlas (Lee and Wunsch, 1977) will show that ocean currents are often in excess of the velocities given in Table I as being necessary to supply the water to the inlet. We presume that when this happens the growth of the withdrawal region as predicted by the theory will stop. However, there are occasional time intervals of many days when currents were smaller than 1 or 2 cm s^{-1}, during which the theory would predict that considerable rotational effects would develop.

Probably the predictions for withdrawal height are biased toward numbers that are larger than a real OTEC powerplant would experience. Such oceanic processes as internal waves, vertical shear, and friction in the surface mixed layer will all counteract the mechanism presented here. However, such processes, although they might occur rarely, may degrade the output of the plant or create unique torques on the plant which are best anticipated and accounted for.

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APPENDIX 1. LIMITS OF VALIDITY OF THE STEADY APPROXIMATION

For the spatial acceleration terms to be smaller than other terms, the inequality

$$u/l > 1/t$$

must hold, where l is some horizontal length scale; l will be set equal to $r_1 - r_0$, and time t will be determined. Rearranging,

$$|ut| > r_1 - r_0 \tag{A1}$$

r_1 is eliminated by the use of

$$r_1^2 - r_0^2 = Qt/\pi h_1 \tag{A2}$$

Equation A2 can be substituted into (A1) which can be rearranged to the form

$$u^2 t + 2ur_0 > Q/\pi \mathrm{h}_1 \tag{A3}$$

To calculate u, note that the largest u occurs at r_0, and is

$$u = Q/2\pi r_0 h(r_0)$$

Using the definition of H, $\Delta\rho = h_1(\partial\rho/\partial z)$

$$u = \frac{3Q}{4\pi r_0[h_1 - f^2(r_0^2 - r_1^2)^2/4N^2 h_1 r_0^2]} \tag{A4}$$

Since the non-rotating balance is established first, let eq. 7 be used first to eliminate h_1

$$h_1 = (Q/4\pi r_0 N)^{1/2} 27^{1/4} \tag{A5}$$

Combining (A3) and (A4) with $f = 0$ and (A5), one obtains

$$9(QN)^{1/2} t/8(27)^{1/4}\pi^{1/2} r_0^{3/2} + 3/2 > 1$$

The inequality is always satisfied but becomes large when

$$t > 4(\pi r_0^3)^{1/2}/3^{1/4}(QN)^{1/2} \tag{A6}$$

Depending upon the particular physical situation, this may be a long or short time compared to the time $\sqrt{27}/2f$ when there is a transition to a rotating limit. For rapid rotation the velocity in an angular direction is important, so

$$vt > r_1 - r_0$$

From (2)

$$\left(\frac{fr_1^2}{2r_0} - \frac{fr_0}{2}\right) t > r_1 - r_0$$

Rearranging

$$\frac{ft}{2}\left(\frac{r_1 + r_0}{r_0}\right) > 0 \tag{A7}$$

Since the rotating limit does not become valid until $ft > \sqrt{27}/2$, the quasi-steady limit always appears to be valid for the rotating problem.

APPENDIX 2. ESTIMATE OF THE EFFECT OF VISCOSITY

If t is the time since the beginning of withdrawal, for viscosity to be important we want

$$t_{\text{withdrawal}} < t_{\text{viscous}} = h^2/\nu \tag{A8}$$

The viscous time scale is h^2/ν, where h is the depth of a fluid and ν is the kinematic viscosity. Using eq. 7 to eliminate h

$$t < \sqrt{27}\ Q/4\pi r_0 N\nu \tag{A9}$$

For the first experiments reported here, Q was approximately 0.2 $cm^3\ s^{-1}$, r_0 was 0.35 cm, N was 1.45 s^{-1}, and ν was 0.01.

Equation A9 predicts that viscosity becomes important in 16 s. Since the balance will exhibit a transition to a rotating balance after two-tenths of a revolution time, there will be a transition to rotating flow before the viscous effect becomes established for any experiments with rotation periods of less than 80 s. If the period is more than that, a viscous balance will occur before this transition takes place, with unpredicted results.

After the rotation balance is established, eq. 9 must be used to solve for h. This leads to

$$1 < fQ/2\pi r_0 cN\nu \tag{A10}$$

if viscous drag is negligible. Using the same numbers as before for Q, r_0, N and ν, the inequality is satisfied for rotation periods of less than 79 s; hence in this case any experiment with a period under 80 s will attain a rotating limit and not subsequently develop dominant viscous effects.

REFERENCES

Brooks, N.H. and Koh, R.C.Y., 1969. Selective withdrawal from density-stratified reservoirs. Proc. Am. Soc. Civ. Eng., HY4: 1369—1400.

Lee, V. and Wunsch, C. (Editors), 1977. Atlas of the Mid-Ocean Dynamics Experiment. POLYMODE Office, Massachusetts Institute of Technology, Cambridge, MA, 274 pp. (unpublished report).

Sambuco, E. and Whitehead, J.A., 1976. Hydraulic control by a wide weir in a rotating fluid. J. Fluid Mech., 73: 521—528.

TRW System Group, 1975. Ocean Thermal Energy Conversion Research on an Engineering Evaluation and Test Program: Executive Summary, Vol. 1, TRW System Group, 1 Space Park, Redondo Beach, CA, 201 pp.

Turner, J.S., 1973. Buoyancy Effects in Fluids. Cambridge University Press, New York, NY, 367 pp.

Whitehead, J.A. and Porter, D.L., 1977. Axisymmetric critical withdrawal of a rotating fluid. Dyn. Atmos. Oceans, 2: 1—18.

Yih, C.S., 1980. Stratified Flows. Academic Press, New York, NY, 418 pp.