Dislocations in convection and the onset of chaos

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High Prandtl number convection possesses a square flow pattern that is steady and is apparently stable to infinitesimal disturbances. This pattern is unstable to finite-amplitude disturbances, however, because a more chaotic (in time and space) spoke pattern of convection eats its way into the squares from the lateral boundaries. Experiments are described in which the breakup of the squares is initiated by dislocating one square in the middle of the apparatus with the use of a small, heated resistor. Once a critical heating rate and time is exceeded, the dislocation initiates a spoke cell which then systematically destroys neighboring square cells, resulting in the more chaotic spoke pattern. If the critical rate is not exceeded, the cell becomes severely deformed during the heating, but will relax back to a square convection cell after heating has ceased.

I. INTRODUCTION

Much progress has recently been made toward an understanding of the onset of chaos in systems which can be described by ordinary differential equations. A common approach in linking them to the Navier–Stokes equations is to expand the velocity fields as a series of sinusoidal functions of the space variables. After substitution into the Navier–Stokes equations, it is necessary to reduce the series to a manageable number through some truncation assumption. In reducing the convection equation to three equations, which was the lowest number to make sense physically, Lorentz[1] discovered interesting transitions to and from chaotic behavior. Moreover, laboratory experiments have been conducted which exhibit many of the same phenomena such as subharmonic bifurcation and quasiperiodic motion.[2]

The above theoretical procedure produces flow patterns that are clearly unlike turbulent flows in the sense that the correlation lengths are periodic in space. In contrast, the correlation lengths in a turbulent fluid are finite. The purpose here is to relate observations of one experiment in which spatial inhomogeneities preclude and trigger spatial and subsequent temporal chaos. The lead of Donnelly *et al.*[3] is followed in suggesting that such dislocations are important in initiating turbulence and should be called turbators. This type of emergence of chaos may not be addressed by current theoretical studies.

In Rayleigh–Benard convection theoretical analyses of flows which are periodic in the two lateral directions have predicted a large number of transition states.[4] These agree well with experiments in which the long-range order has been established by special initial conditions.

In experiments with Prandtl number of order 1 the flows eventually become disordered to some extent by transitions to a variety of forms including oscillating flows. With larger Prandtl number there are fewer transitions, and a square form of convection can remain stable at Rayleigh numbers of order half a million for a long period of time.[5] Disordered cells, which are called spoke convection, were not observed to emerge from a spontaneous breakdown of the square cells, but ultimately propagated in from the wall. The present experiments have been conducted to determine the energy necessary to break down a square cell in the middle of the tank, and to illustrate that a local disruption in the squares can cause spoke convection.

II. THE EXPERIMENT

The apparatus (Fig. 1) has been used in previous studies of high Prandtl number fluid.[5] A horizontal layer of Dow Corning 200 silicon oil (physical properties: kinematic viscosity $\nu = 10$ cm^2 sec^{-1}; thermometric diffusivity $\kappa = 1.16\times 10^{-3}$ cm^2 sec^{-1}; thermal expansion coefficient $\alpha = 9.6\times 10^{-4}$ °C^{-1}; Prandtl number $\nu/\kappa = 8.6\times 10^3$) 7 ± 0.003 cm deep was bounded above and below by transparent plate glass water baths with a 92 cm $\times$ 102 cm working area. The glass surfaces were suitably flattened by matching hydrostatic pressure, and leveled to better than 25 sec of arc. Thermostatically controlled water precise to 0.05 °C flowed through each bath, and controlled the temperature above and below the layer of oil. The chamber was insulated on the sides by 5 cm thick polyvinylchloride walls.

The method of Chen and Whitehead[6] (Fig. 1) was used to induce square convection cells. This method consists of placing a grid made up of alternating blocked and clear areas over the top transparent bath (in this case the grid had a wavelength of 7.63 cm; wavenumber $2\sqrt{2}\pi d/\lambda = 8.15$). Light from a 300 W incandescent lamp is then directed down through the pattern, so the test fluid lying below is slightly radiatively heated in the desired pattern. The system is left for at least 2 h. This time is a little more than the thermal time constant $d^2/\pi^2\kappa = 1.25$ h, which is the smallest thermal time constant of the system. After the 2-h interval, the temperature difference between top and bottom baths is raised to $25.0° \pm 0.05°$ C ($R = 5.7\times 10^5$) and a convection pattern with the wavelength of the initiated pattern is seen to develop. After the pattern has developed the light is turned off, the square grid removed, and the evolution of the convection structure observed. The square pattern which was generated here is discussed in more detail by Whitehead and Parsons.[5]

The test area was bordered laterally by an artificial wall of polyvinylchloride 6.9 cm high and 2.54 cm thick. Outside

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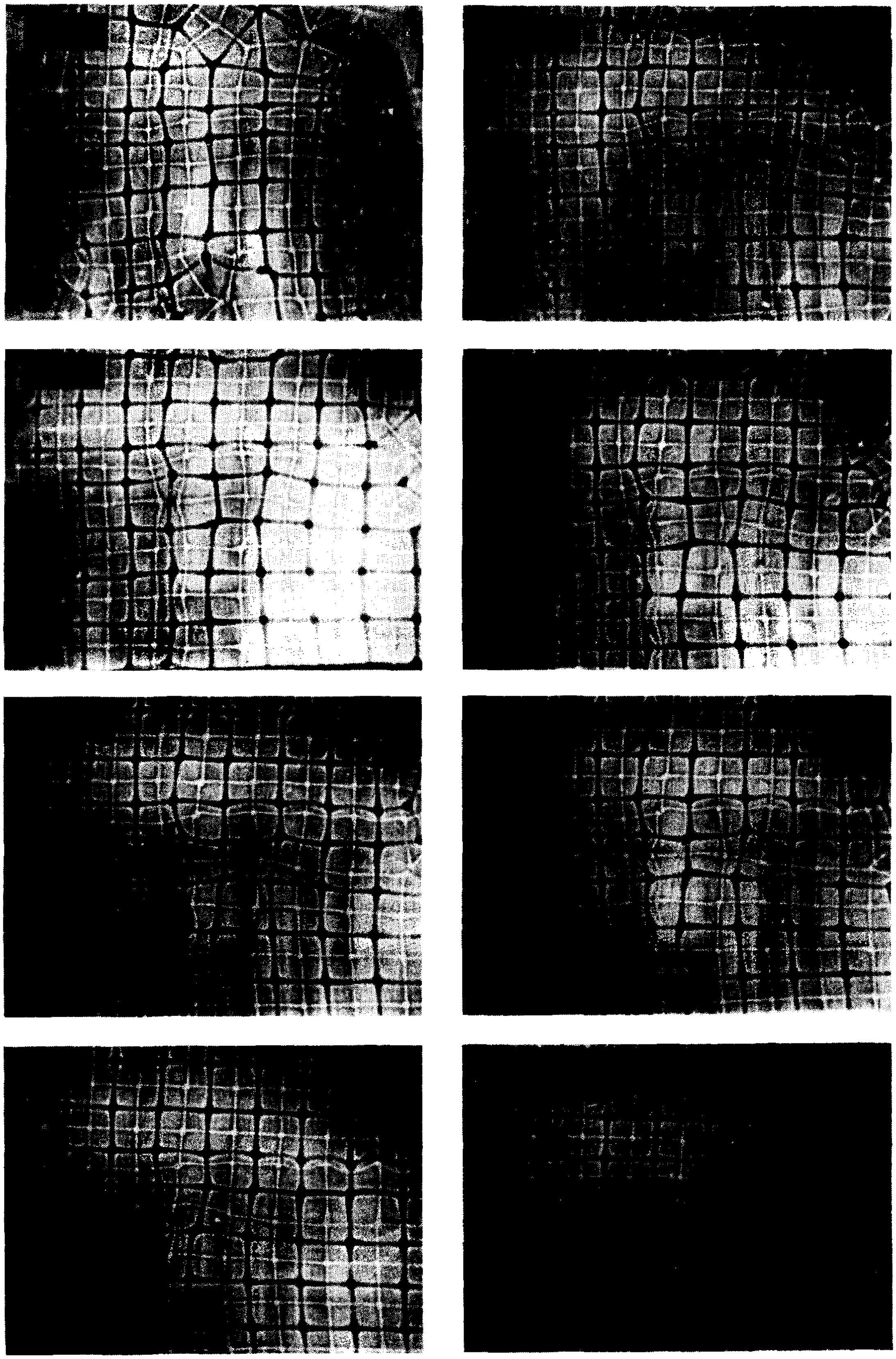

FIG. 1. Shadowgraph taken when the square cells were severely deformed by the heated thermistor (and to a lesser extent the wires which hold the thermistor), but which healed after the voltage to the thermistor was turned off. The voltage was 25 V and the distance from the heater to the original crossing of the black lines was 1.6 cm. Times after start of the thermistor heating were, respectively, 0, 87, 147, 171 (heater just off), 192, 207, 267, and 387 min. Overturn time of the convection cell is approximately 12 min.

this was a region a few centimeters wide in which silicon oil was convecting, but whose outer limit was a glass wall in contact with the room air. These boundaries always cause the square cells to break up into spoke convection. The time this takes is many overturn times (in our case, approximately 20 or more). Thus, although the square cells are never realized forever in an experiment, they can resist for a time the disrupting effect of the boundaries. It is believed that the fact that the squares will remain for a very long time (at least until the disordered cells propagate in from the wall) implies that the squares are stable to infinitesimal perturbations but unstable to finite-amplitude perturbations.

III. BREAKUP OF A SQUARE CELL AND THE INITIATION OF A DISLOCATION

The role of the boundaries in breaking up the pattern was not clear. In order to pose a more clearly quantified question, we report here measurements of the heat which was necessary to irreversibly dislocate one square cell from virtually a point source in the middle of the apparatus.

The source of heat was a small oval thermistor (1×1.4 mm) which was soldered to two fine copper wires (of diameter 0.005 cm) stretched at right angles to each other across the tank, one wire at 3 cm above the bottom, and the other at 4 cm. The wires were then connected to a dc voltage supply. The size of the wires was dictated by the desire to have them as small as possible so they would have a minor mechanical presence. However, the choice was unfortunate because the currents necessary to break up a cell turned out to be larger than expected, large enough in fact to heat the wires, which thus had a thermal presence, acting as a disturbance second in size only to the heated thermistor. The distortion of a square cell is influenced by the placement of the thermistor, the voltage, and, to a lesser extent, the position of the two fine wires—all in respect to the placement of the induced square flow field.

Experiments have not been done with heaters at different depths, but 6 runs were made with the heater located at different distances close to the black crossings.

In all cases at 20 and 25 V (three runs each), the square cell became severely deformed after approximately 100 min, but did not break up during the next 100 min. When the voltage was teminated, the cells relaxed back to the square pattern in approximately an hour and a half (Fig. 1). Turnover time of the roll was approximately 12 min.

In two runs at 30 V, with the heater located at a distance of about 1.6 cm from the intersection of the black lines (which was also the maximum distortion distance at 25 V), the square broke down to an irrepairable dislocation.

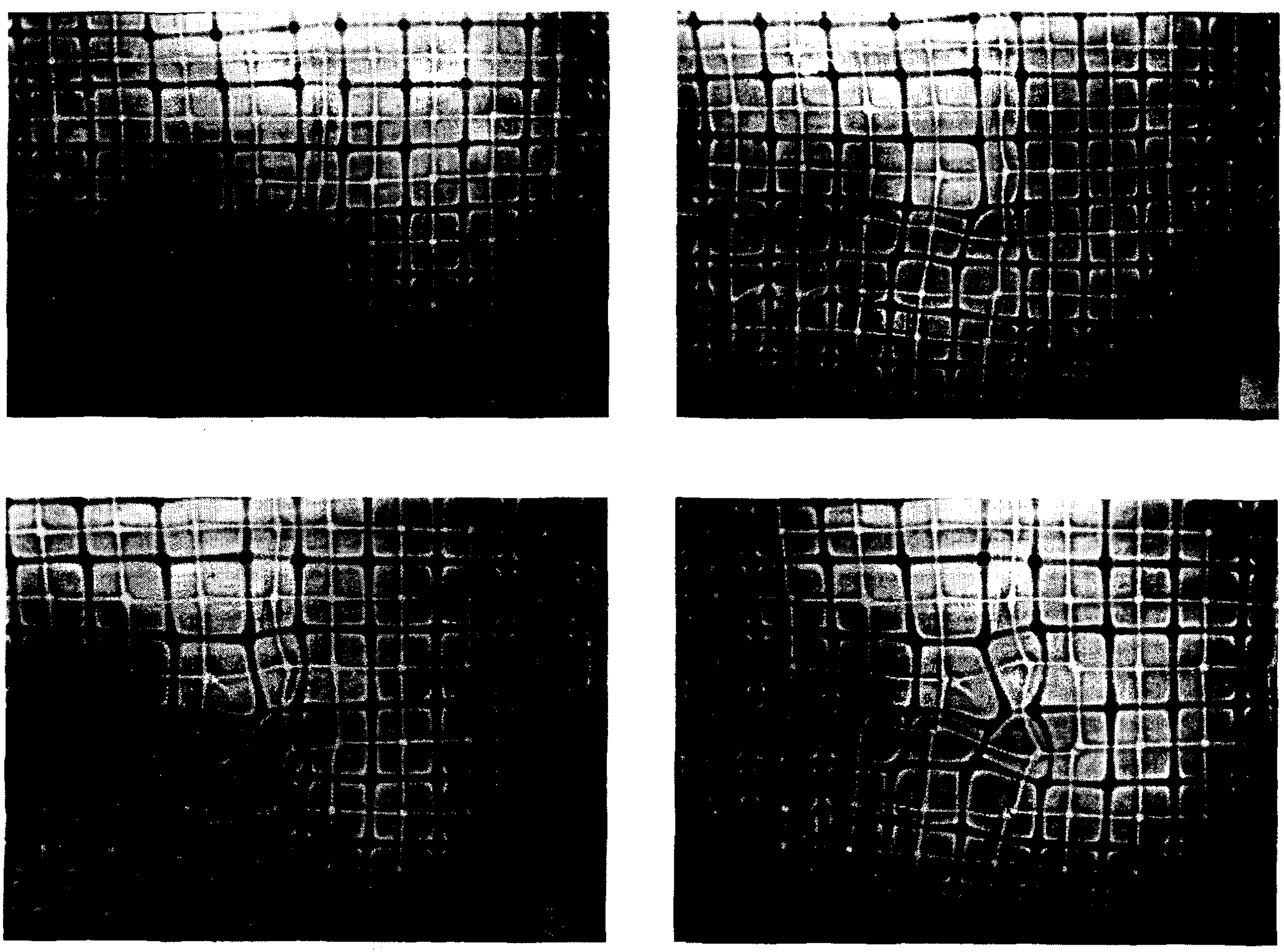

FIG. 2. Shadowgraph from one of the two runs in which the square was broken down by the heater. Times are 0, 45, 75, and 105 min after the heater was turned on, respectively. We judge the square was first irreparably broken down at the Y-shaped intersection at 105 min.

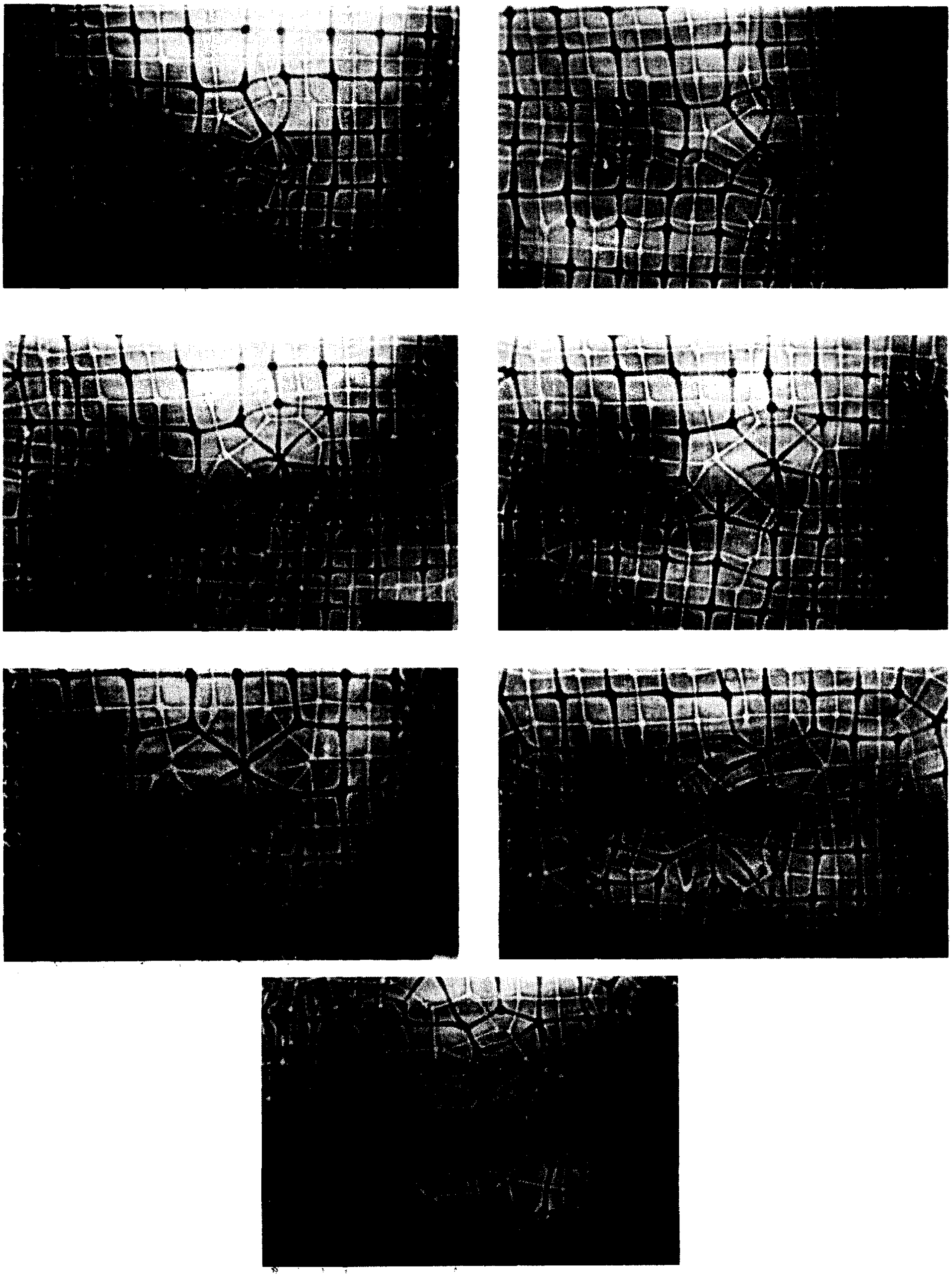

FIG. 3. The subsequent destruction of neighboring square cells by spokes in the same experiment as Fig. 2. Times are 120, 130, 165 (heater off at 164), 180 (the squares which were seriously distorted by the heating wires in the lower left are restored), 210, 240, and 360 min after start, respectively.

Figure 2 shows sequential photographs of the square as it breaks down. The qualitative feature that was most noticeable in the breakdown was that one side of a square was eliminated, so two squares became triangles. Also, the intersection of two hot thermals moved directly over the thermistor. Figure 3 shows the subsequent evolution to spoke convection. The triangles were unable to revert back to squares and soon became spoke cells. The spoke cells subsequently began to break down neighboring cells, and the new convection pattern has the time-dependent behavior characteristic of spoke convection.[5,7]

IV. PARAMETERS OF THE BREAK UP

In order to estimate the effective heater strength, it must be compared to the Rayleigh numer of the convection which was predicted at 5.7×10^5. To estimate this, account was taken of the temperature drop across the glass walls above and below the convection chambers. To obtain this correction it is necessary to estimate the heat flux through the layer. To do this, the law for Nusselt number $N = 0.19\ R^{0.282}$ was used where R is the Rayleigh number. This was the best fit curve observed by Somerscales and Gazda[8] at these Rayleigh numbers, but a somewhat lower Prandtl numbers. Since the difference between bath temperatures and the temperature jump across the layer is less than 20%, the above calculation should estimate the Rayleigh number to a few percent accuracy.

To calculate the Rayleigh number of the heater, use is made of the formula

$$R_Q = g\alpha Q d^2/\nu k \kappa,$$

where $Q = IV$ is the heat produced by the thermistor, with V as the voltage across the thermistor and I as the current. The current I was measured directly with a current meter. There was a voltage drop along the wire, so voltage across the thermistor was measured in the experimental tank after the experiments were completed, and the top header removed so there was a free surface. Based upon the manufacturer's specifications of the thermistor, the temperature was determined to be in the 200 °C to 300 °C range for these experiments.

The values of R_Q for the three voltages are estimated as 2.1×10^5, 2.7×10^5, and 2.9×10^5, respectively. Therefore, the square is destroyed when the heater Rayleigh number is approximately 2.9×10^5, which is about half the Rayleigh number of the fluid layer. This implies that the square cells are relatively robust and must be subjected to a sizable perturbation, of order Rayleigh number, to break down. Once they break down they do not heal themselves. The point heater, therefore, seems to be a reliable method of generating a quantifiable finite-amplitude perturbation.

V. SUMMARY

Obviously the dislocation triggered a phase change in the fluid flow, but an important question is whether these dislocations are unlike spatially periodic disturbances. They are, after all, possibly similar to the collective instability observed at lower Rayleigh and Prandtl numbers.[9] In other words, one may ask "is the dislocation different in principle from the finite amplitude perturbations which are used in theory (which are spatially periodic)?"[4] Rephrasing this in more mathematical terms, is a perturbation represented as a Fourier integral different in principle from a perturbation represented as a Fourier series? Can we hypothesize that all systems which are stable to finite-amplitude disturbances are also stable to dislocations? There is the complementary question that asks, do all systems which are unstable to finite-amplitude disturbances go unstable to dislocations? Furthermore, what distinguishes those dislocations which lead to chaotic behavior?

No clear answers are given yet, but the photographs give some clues. If one closely examines Fig. 2, there is little evidence of a spatially periodic disturbance which decays away from the dislocation (apart from the flows which are from the heating wires). Nor can any readjustment be seen to any squares that are not immediately next to the spokes in Fig. 3. The disturbance appears to be entirely local. Thus, it is possible that periodic arrays of cells are unstable to dislocations and thereby generate short spatial correlation lengths. At least one other system has exhibited behavioral sensitivity to dislocations,[3] i.e., increased degrees of freedom, line broadening, extra broadband noise, loss of spatial coherence, and hysteresis with dislocations. Donnelly (private communication) has suggested that such dislocations be called turbators (L. troublemakers). Manalotte-Rizzoli[10] has numerically observed that random finite-amplitude perturbations break up solitons more easily than more-coherent perturbations.

It would not be surprising to see that as Rayleigh number was increased, more voltage is required to break up the squares. There are two reasons for this. First, the squares are unstable at Rayleigh numbers below 150 000 and may be expected to be progressively more rugged at higher Rayleigh numbers. Second, preliminary experiments were conducted at lower Rayleigh numbers and these exhibited a breakdown of the squares due to the mechanical presence of the thermistor and wires alone, with no heating. This is consistent with less stable cells at lower Rayleigh numbers.

Note that the total energy flux necessary to initiate the finite-amplitude perturbation can be made arbitrarily small compared to the energy flux of the entire convecting fluid by conducting the experiment in a tank with a very large lateral extent, and dislocating only one cell. Although one must then wait a very long time for the disorder to propagate throughout the entire fluid, the "finite-amplitude" perturbation can thus be made as small as desired. However, it is not known whether all systems which are unstable to finite-amplitude disturbances will break up when subjected to a perturbation at only one point.

ACKNOWLEDGMENTS

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