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Estimates of the Relative Roles of Diapycnal, Isopycnal and Double-Diffusive Mixing in Antarctic Bottom Water in the North Atlantic

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Whitehead and Worthington (1982) have measured the fluxes of heat and salt due to the northward flow of Antarctic Bottom Water through a passageway 300 km wide between the Ceara Rise and the Mid-Atlantic Ridge at about 4°N. Downstream of this "sill" the temperature and salinity of the underflowing water increase, and Whitehead and Worthington have described this change as being due to "downward" fluxes of heat and salt across isothermal surfaces. We consider the relative roles of three separate mixing processes to these "downward" fluxes of heat and salt across isotherms, and we use this information to decide between Whitehead and Worthington's two separate estimates of the volume transport of Antarctic Bottom Water: one based on current meter data, and the other based on geostrophic calculations. The slope of the θ-S locus of bottom water as it moves northward past the equator provides a valuable extra constraint on the relative importance of the three mixing processes. We conclude that the dominant mixing process is diapycnal (i.e., cross-isopycnal) turbulent eddy diffusion and that the geostrophic data set of Whitehead and Worthington is consistent with the mixing ideas presented here, whereas their current meter data set is not.

INTRODUCTION

Antarctic Bottom Water, which comes into the northwestern Atlantic basin from the south, must flow over the Ceara Abyssal Plain, which lies roughly between the equator and 4°N and is hundreds of kilometers wide. At the northern end of this plain is a gap approximately 300 km wide between the Ceara Rise on the west and the Mid-Atlantic Ridge on the east, through which Antarctic Bottom Water entering the North Atlantic must flow. *Whitehead and Worthington* [1982] located two moorings in this gap and obtained eight 360-day records, which they used to estimate the volume flux of Antarctic Bottom Water (potential temperature colder than 1.9°C) into the North Atlantic. The current meter measurements yielded an estimate which agreed closely with geostrophic calculations for water with potential temperature between 1.2°C and 1.9°C (potential temperature will always be used in this article) and disagreed strongly for water between 1.0°C and 1.2°C (Figure 1). Notably, the geostrophic calculation predicted approximately 10^6 m^3 s^{-1} of water colder than 1.2°C flowing northward, while current meter estimates were a tenth of this value. In this note we consider the implications of these two different estimates on the implied relative importance of three mixing processes; namely, diapycnal mixing, isopycnal mixing, and double-diffusive convection. Expressions are developed for the contributions of these three processes to (1) the fluxes of heat and salt across isotherms and (2) the rate of change of temperature and salinity of bottom water along its downstream path. By using both these expressions in conjunction with data from *Mantyla and Reid* [1983] and *Whitehead and Worthington* [1982] we conclude that diapycnal turbulent mixing is the dominant mixing process and that Whitehead and Worthington's geostrophic data set is consistent with the mixing constraints developed in this paper, whereas their current meter data are not.

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MIXING ACROSS ISOTHERMAL SURFACES

The temperature of water near the bottom of the water column at the sill is 1.0°C, and progressively warmer isotherms extend northward into the North Atlantic and intersect the ocean floor. Since there has been no historical evidence of migration of the isotherms upward or downward, the most reasonable first assumption is that there is a steady balance between advection into the North Atlantic from the south and mixing of properties across the isothermal surfaces. In this note we consider the contribution of three processes to this mixing across isothermal surfaces.

The first two processes are diapycnal mixing processes. The first is double-diffusive convection, which is now widely recognized as being active in regions of the ocean that have certain temperature and salinity gradients, as reviewed by *Huppert and Turner* [1981]. In our case the Antarctic Bottom Water is colder and fresher than the North Atlantic Deep Water under which it is thrusting. This is the well-known salt-fingering case for which salt is mixed more efficiently across isopycnals than heat. The second process is diapycnal turbulent mixing (across potential density surfaces) caused by turbulent processes due to shear instability or wave breaking. The third process is isopycnal mixing (along constant potential density surfaces) due to quasi-geostrophic turbulence.

Figure 2*a* is a sketch of the isothermal and isopycnal surfaces in the region of interest downstream of the current meter moorings at the sill. *Whitehead and Worthington* [1982] have deduced the "downward" fluxes of heat and salt across each isothermal surface by taking the differences between the fluxes (of heat and salt) advected in and those advected out of each control volume. These control volumes are bounded by successive isothermal surfaces and the seafloor, where the geothermal heat flux is included. This method of analysis neglects any isopycnal mixing of heat and salt into the control volume from upstream of the sill. This approximation is justified because we expect more mixing downstream of the sill and because there is little evidence of synoptic-scale mixing from the

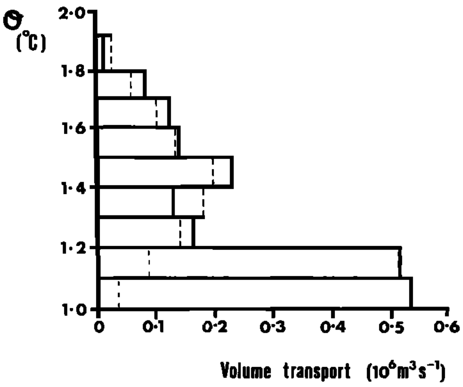

Fig. 1. Comparison of the current meter results (dashed line) and the geostrophic calculations (solid line) of *Whitehead and Worthington* [1982] for mass flux into the North Atlantic.

current-meter data. It does, however, remain an unproven assumption.

Both diapycnal mixing processes (double-diffusive convection and diapycnal turbulent mixing), by definition, transfer properties perpendicular to isopycnals, and so the contributions of these processes to the flux of heat and salt across an isothermal surface are equal to the corresponding diapycnal fluxes multiplied by cos ψ, where ψ is the angle between the isopycnal and isothermal surfaces, as shown in Figure 2*a*.

The isopycnal eddy diffusion flux of heat is given by $-K\nabla_i\theta$, where K is the isopycnal diffusivity and $\nabla_i\theta$ is the two-dimensional gradient of potential temperature in the isopycnal plane. The contribution of this isopycnal flux of heat across an isotherm is $K\nabla_i\theta \sin\psi$ in the "downwards" direction. Figure 2*b* shows two isotherms, θ_1 and θ_2, and one isopycnal. The diapycnal gradient of potential temperature is $\theta_2 - \theta_1$ divided by the distance *bc*, and we shall call this diapycnal gradient θ_z (even though the diapycnal direction is not strictly vertical). From Figure 2*b*, $\nabla_i\theta = (\theta_2 - \theta_1)/ab = (bc/ab)\theta_z = \tan\psi\ \theta_z$. The flux of heat across an isotherm due to isopycnal diffusion is then $K\theta_z \tan\psi \sin\psi$, which is approximately equal to $K\theta_z\psi^2$ because ψ is small. Note that the sign of this flux is independent of the sign of ψ. The corresponding flux of salt through an isothermal surface is $K\nabla_i S \sin\psi$, and using the isopycnal identity $\beta\nabla_i S = \alpha\nabla_i\theta$ (where α and β are the coefficients in the equation of state $\rho^{-1}(d\rho - \rho_p dp) = \beta dS - \alpha d\theta$), this flux is equal to $(\alpha/\beta)K\theta_z\psi^2$ in the direction perpendicular to an isotherm.

The total fluxes of heat and salt across isotherms due to (1) double-diffusive convection, (2) diapycnal mixing, and (3) isopycnal mixing are

$$Q^\theta = F^\theta \cos\psi + D\theta_z \cos\psi + K\theta_z\psi^2 \quad (1)$$

$$Q^S = F^S \cos\psi + DS_z \cos\psi + \frac{\alpha}{\beta} K\theta_z\psi^2 \quad (2)$$

where F^θ and F^S are the double-diffusive fluxes per unit area across isopycnals, D is the diapycnal eddy diffusivity, and θ_z, S_z are the gradients of potential temperature and salinity in the vertical (almost diapycnal) direction. Since the angle ψ is expected to be quite small, we can approximate cos ψ by unity.

A revealing comparison between the contribution of the different mixing processes can be made by examining the following number, which is the ratio of the contributions of Q^S and Q^θ to the flux of density:

$$\frac{\alpha Q^\theta}{\beta Q^S} = \frac{\alpha F^\theta + D\alpha\theta_z + K\alpha\theta_z\psi^2}{\beta F^S + D\beta S_z + K\alpha\theta_z\psi^2} = \frac{R_f\beta F^S + R_\rho D\beta S_z + K\alpha\theta_z\psi^2}{\beta F^S + D\beta S_z + K\alpha\theta_z\psi^2} \quad (3)$$

Here we have adopted the shorthand notations $R_f(\equiv \alpha F^\theta/\beta F^S)$ for the buoyancy flux ratio of salt finger, double-diffusive convection and $R_\rho(\equiv \alpha\theta_z/\beta S_z)$ for the stability ratio of the water column. From (3) we see that if double-diffusive fingering is the dominant mixing process then $\alpha Q^\theta/\beta Q^S$ will be approximately $R_f \approx 0.7 \pm 0.1$ [*Schmitt*, 1979; *McDougall and Taylor*, 1984]. If isopycnal eddy mixing is the most important mixing process then $\alpha Q^\theta/\beta Q^S$ will be close to 1.0, and if diapycnal eddy mixing is the most vigorous process, then $\alpha Q^\theta/\beta Q^S$ will be close to R_ρ, which has the value of 1.98 at this location in the ocean.

Table 1 shows the estimated values of Q^θ, Q^S, and $\alpha Q^\theta/\beta Q^S$. The values of Q^θ were obtained from *Whitehead and Worthington* [1982, Table 4, column 2]. The values of Q^S were obtained from their Table 6, column 2, except the values are multiplied by ρ to get units of grams of salt per gram of seawater. There is also a correction due to an interpolation error in the bottom salinity in their column 3, Table 5, which also makes small differences to the values of Q^S in their Table 6.

These tabled values of $\alpha Q^\theta/\beta Q^S$ are shown in Figure 3 together with the three limiting values: 0.7, 1.0, and 1.98. All the data, based both on the current meter measurements and the geostrophic calculations, lie between the allowable limits, but the implied relative importance of salt fingering and diapycnal eddy mixing is quite different for the two data sets. The current meter results can be explained by mixing that is principally isopycnal with significant salt-fingering activity below 1.2°C and some diapycnal eddy mixing above 1.2°C (perhaps one third as much as the isopycnal and double-diffusive mixing combined). In contrast the geostrophic calculations imply that diapycnal turbulent eddy mixing is approximately four times as important as both salt fingering and isopycnal eddy diffusion at producing fluxes of heat and salt across isotherms. We note that as there are three "mixing" processes and only two trace materials the relative importance of the mixing processes cannot be uniquely determined. For instance a certain percentage of fingering activity along with diapycnal mixing will act like isopycnal mixing.

In order to resolve the discrepancy between the transports

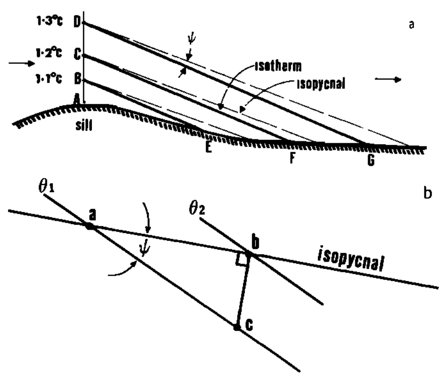

Fig. 2. (*a*) A sketch showing the isotherms and isopycnals for the flow of Antarctic Bottom Water (from left to right) downstream of the sill. The current meters were located at the sill. (*b*) Sketch of the two isotherms, θ_1 and θ_2, and an isopycnal.

TABLE 1. Table of "Downward" Fluxes Q^θ and Q^S Across Isotherms $\theta = 1.1$ to $\theta = 1.9$ Using Volume Transports Based on Both Current Meter Measurements and Geostrophy [after *Whitehead and Worthington*, 1982]

θ, Potential Temperature, °C	Q^θ, Downward Diffusion of Temperature 10^{-6}, °C cm^{-2} s^{-1}	Q^S, Downward Diffusion of Salt 10^{-10}, g cm^{-2} s^{-1}	$\alpha Q^\theta/\beta Q^S$
	Current Meter Transport		
1.9	3.45	5.93	1.34
1.8	3.43	6.07	1.30
1.7	3.53	6.23	1.30
1.6	3.29	5.76	1.31
1.5	2.85	5.18	1.26
1.4	2.78	5.25	1.22
1.3	3.14	6.16	1.17
1.2	2.62	6.02	1.00
1.1	1.45	4.10	0.81
	Geostrophic Transport Referenced to 1.9°C		
1.9	13.92	18.65	1.71
1.8	14.84	20.04	1.70
1.7	16.62	22.20	1.72
1.6	17.97	23.25	1.76
1.5	19.90	25.29	1.81
1.4	26.31	33.17	1.82
1.3	43.42	54.45	1.83
1.2	56.54	73.93	1.76
1.1	50.91	64.64	1.81

The last column is the ratio of Q^θ and Q^S expressed in density units.

calculated by the two methods we consider the evolution on the θ-S diagram of the water at the ocean floor as it moves downstream from the sill, using the theory developed below and the data of *Mantyla and Reid* [1983].

The θ-S Locus of Bottom Water as it Moves Northward

The conservation of potential temperature at a point in the ocean may be written

$$\theta_t + \mathbf{V}\cdot\nabla\theta = -F_z{}^\theta + (D\theta_z)_z + \nabla_i\cdot(K\nabla_i\theta) \qquad (4)$$

where $\mathbf{V}$ is the three-dimensional velocity vector and ∇_i is the two-dimensional gradient operator in the isopycnal plane. A similar equation applies for the conservation of salt. At the seabed, $\mathbf{V}\cdot\nabla\theta$ is simply the velocity parallel to the bottom (say, of magnitude U) multiplied by the gradient of θ along the bottom in the direction of flow (say θ_b). Since the fluxes at the seafloor are zero (apart from the geothermal heat flux), the conservation equations may be written

$$U\theta_b h = F^\theta \cos\phi + D\theta_z \cos\phi - K\nabla_i\theta \sin\phi \qquad (5)$$

$$US_b h = F^S \cos\phi + DS_z \cos\phi - K\nabla_i S \sin\phi \qquad (6)$$

Here h is the depth of the bottom flowing current, steady state is assumed, and ϕ is the angle of the isopycnals to the seafloor (see Figure 4). Using $\nabla_i\theta = \tan\psi\ \theta_z$ (as before) and approximating $\cos\phi$ by 1 and $\sin\phi$ by ϕ, we obtain the ratio of the θ

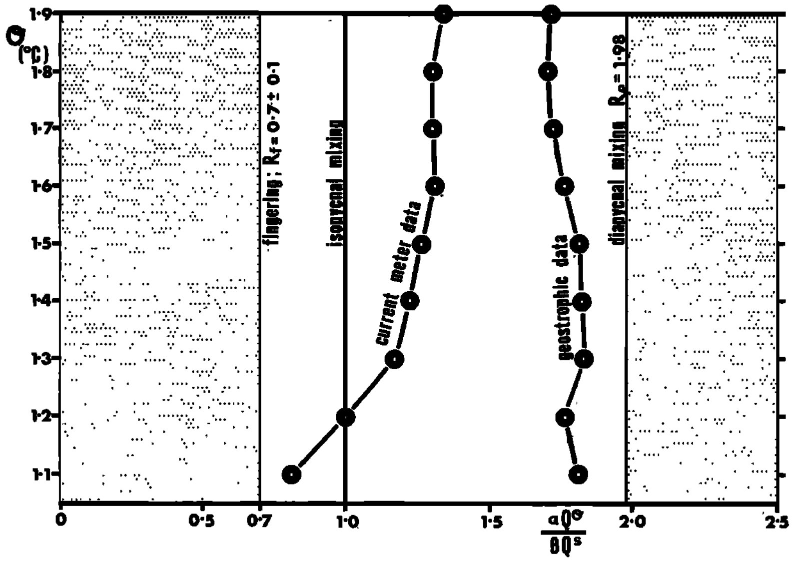

Fig. 3. Graphs of the ratio $\alpha Q^\theta/\beta Q^S$ for the two separate data sets of *Whitehead and Worthington* [1982]. Values less than $R_f \approx 0.7$ or greater than $R_\rho = 1.98$ are not physically possible, as indicated by the shading.

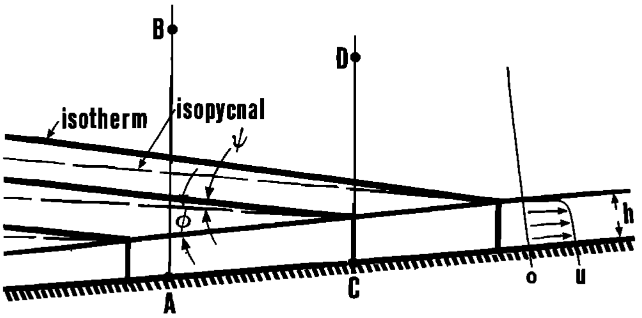

Fig. 4. Sketch of the bottom water moving to the right at speed U and in a layer of depth h. ϕ is the angle between the isopycnals and the sea floor and is positive as shown. The lines AB and CD indicate the positions of vertical CTD casts.

and S changes in the bottom water as it flows northward as

$$\frac{\alpha\theta_b}{\beta S_b} = \frac{R_f \beta F^S + R_\rho D\beta S_z - K\alpha\theta_z\psi\phi}{\beta F^S + D\beta S_z - K\alpha\theta_z\psi\phi} \tag{7}$$

The isopycnal mixing terms have opposite signs in (3) and (7) because equation (3) is concerned with the fluxes across an isothermal surface, whereas equation (7) considers the fluxes into the bottom boundary layer. The slopes of these two surfaces (the isotherms and the top of the boundary layer) measured from the isopycnals are $+\psi$ and $-\phi$, respectively.

The geometry of Figure 4 is such that the gradient of θ along the bottom, θ_b, is equal to $\theta_z \sin(\phi + \psi)$, where $\phi + \psi$ is the angle between the seafloor and the isotherms. Similarly, the gradient of density $(\beta S_b - \alpha\theta_b)$ is equal to $(\beta S_z - \alpha\theta_z) \sin \phi$. Combining these two relations, we find

$$\frac{\psi}{\phi} = \frac{\left(R_\rho - \dfrac{\alpha\theta_b}{\beta S_b}\right)}{R_\rho\left(\dfrac{\alpha\theta_b}{\beta S_b} - 1\right)} \tag{8}$$

and this geometric relationship is used in the discussion below.

Discussion

In Figure 5 we show θ-S data on the evolution of bottom water properties from *Oceanus* cruise 52, with stations from 7°42′S to 38°21′N in the western Atlantic. The slope of the locus of the bottom-most points of the θ-S curves is close to the local R_ρ of the water column. Figure 5 also shows the slopes of the three mixing processes which vectorically add together to give the slope θ_b/S_b (see equation (7)). The isopycnal flux is in the "cold-fresh" direction with $(\alpha d\theta/\beta dS) = 1$, whereas diapycnal turbulent diffusion proceeds with slope $(\alpha d\theta/\beta dS) = R_\rho = 1.98$ and double-diffusive convection has $(\alpha d\theta/\beta dS) = R_f < 1$. The data in Figure 5 have been sorted into three groups: (1) those stations south of 4°N, (2) those between 5°N and 16°N, and (3) those from 18°N to 38°N. *Mantyla and Reid* [1983] show that the properties of bottom

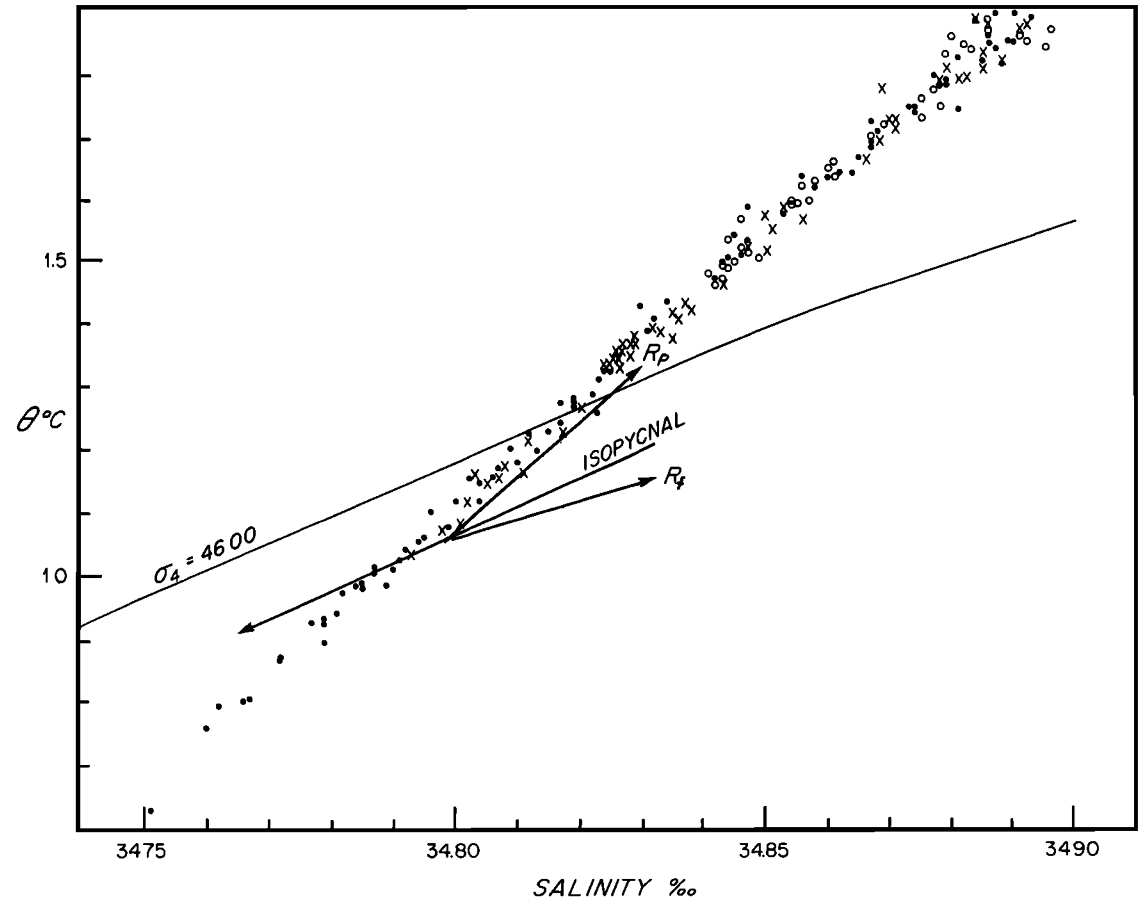

Fig. 5. Potential temperature-salinity diagram from *Oceanus* cruise 52, stations 61-99, showing the slopes near the bottom of 38 successive Nansen casts. Three mixing processes cause the lowermost θ-S point of a cast to describe a locus on this diagram. These three mixing processes are shown by the arrows on the figure and are explained in the text: (solid circles) stations 61–78 (7°42′S to 4°1.7′N); (crosses) stations 79–90 (5°6.9′N to 15°59.6′N); (open circles) stations 91–99 (17°44′N to 38°21′N).

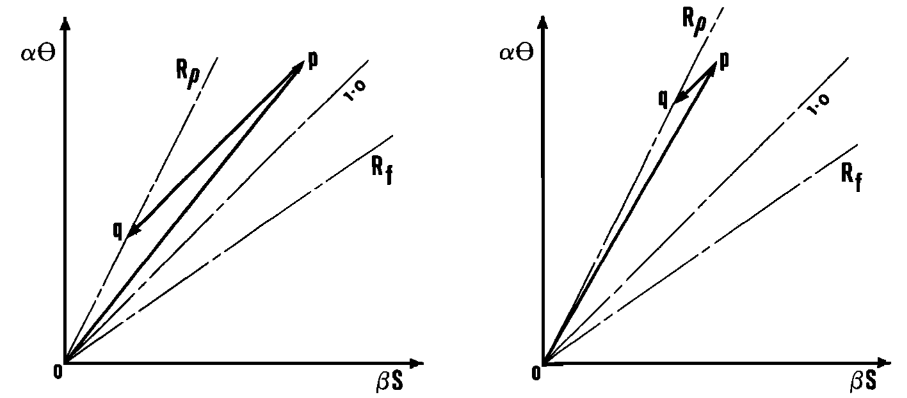

Fig. 6. (*a*) The large arrowed vector **op** from the origin represents the vector sum of diapycnal turbulent and double-diffusive mixing based on the current meter data of Whitehead and Worthington [1982]. A large "isopycnal mixing" vector **pq** is required to reach the R_ρ line. (*b*) The same as (*a*) except that the geostrophic data of Whitehead and Worthington has been used.

water change only gradually in ocean basins, and "major transitions occur at prominent sills and passages; the most notable example is evident in the equatorial Atlantic." Our data also bear this out.

Taking $(\alpha\theta_b/\beta S_b) \approx R_\rho$ (from Figure 5 and from *Mantyla and Reid* [1983, Figure 4]) and $R_\rho \approx 2$, we find from (8) that ψ is much smaller than ϕ. Because of this, it can be seen by comparing (3) and (7) that, if isopycnal mixing makes a significant contribution to (3), then it must dominate the balance of the three mixing terms in (7). Since $\alpha\theta_b/\beta S_b$ is not close to 1, we conclude that isopycnal mixing must make a negligible contribution to (3). The ratio $\alpha Q^\theta/\beta Q^S$ of the heat and salt fluxes across isotherms, which we have determined from *Whitehead and Worthington*'s [1982] two data sets, must then be regarded as the sum of two mixing processes only, namely, diapycnal turbulent diffusion and double-diffusive convection. The current meter data in Figure 3 imply that diapycnal turbulent diffusion and double-diffusive convection are of approximately equal strength. The sum of these two contributions to the vector $(\alpha\theta_b, \alpha S_b)$ is shown by the long arrowed vector **op** in Figure 6*a*. Since $\alpha\theta_b/\beta S_b$ is observed to be close to R_ρ, the remaining isopycnal mixing term $-K\alpha\theta_z\psi\phi$ must be the length shown in Figure 6*a* by vector **pq**. In this way we see that the current meter data of Whitehead and Worthington imply that each of the three mixing processes have approximately equal lengths on Figure 6*a*.

In the absence of isopycnal mixing the geostrophic data shown in Figure 3 imply that diapycnal turbulence is six times as strong as salt fingering, and the resultant of these two vectors is shown in Figure 6*b* by the vector **op**. The required amount of isopycnal mixing to bring the ratio $\alpha\theta_b/\beta S_b$ back to R_ρ is now quite small.

The data of *Mantyla and Reid* [1983, Figure 4] and our data (Figure 5) show that $\alpha\theta_b/\beta S_b$ is close to R_ρ over a large range of latitude (from 65°S to 35°N), and it is inconceivable that a delicate balance is everywhere maintained between the three mixing processes (of comparable magnitude) so that their vector sum always points in the local R_ρ direction. At the most southerly stations, for example, R_ρ is very large, and so double-diffusive convection would be absent. We conclude that the value of $\alpha Q^\theta/\beta Q^S$ (from Figure 3) for the current meter data set is incompatible with the bottom water data. The geostrophic data of Figure 3 together with our geometrical insight that $\psi \ll \phi$ imply that diapycnal mixing dominates salt fingering not only in equation (3) for $\alpha Q^\theta/\beta Q^S$, the ratio of fluxes across isotherms, but also in the $\alpha\theta_b/\beta S_b$ ratio, which is the slope of the bottom water locus on the θ-S diagram. The predominance of the diapycnal mixing process yields consistency between the two very different types of data of Mantyla and Reid on the one hand and the *Whitehead and Worthington* [1982] data on the other, but only if the geostrophically based data set of Whitehead and Worthington is used. This implies that the diapycnal diffusion coefficient downstream of the Ceara Rise is likely to be near the 3.9 $cm^2\ s^{-1}$ value of Whitehead and Worthington's geostrophic data set rather than the value of 1 $cm^2\ s^{-1}$ found from their current meter data. Such a large value of the diapycnal diffusion coefficient should not be too surprising, since the flow is in contact with the seafloor. Also, the dominance of diapycnal diffusion over isoycnal diffusion should not be alarming for the same reason.

Since the bottom water flow receives a small amount of geothermal heat from the seabed, we briefly note that consistency with the observed θ-S bottom water locus can be interpreted as requiring that salt fingers be a little more active than would otherwise have been the case. This can be seen by adding a small fourth vector to Figure 5 in the vertical direction to account for the geothermal heat flux. The requirement that $(\alpha\theta_b/\beta S_b) \approx R_\rho$ can then be satisfied by a slight increase in the salt finger flux in the direction R_f.

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