Deep-Sea Research, Vol. 35, No. 6, pp. 1069–1077, 1988.
Printed in Great Britain.

0198–0149/88 $3.00 + 0.00

NOTE

A gyre in a non-uniformly heated rotating fluid

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(*Received* 28 *May* 1987; *in revised form* 20 *November* 1987; *accepted* 8 *December* 1987)

Abstract—Convection in a rotating square basin is studied in a laboratory experiment in which the convection is driven by non-uniform heating from below. This experimental configuration is thought to be analogous to large-scale convection in the ocean, driven by non-uniform heating at the surface. As in the non-rotating case, the upper region shows relatively small temperature variation. With rotation, this region has cyclonic circulation. Fluid moves from the upper region into the thermal boundary layer near the bottom, and then moves toward and enters the rising region at the warm end of the basin.

INTRODUCTION

THE distribution of heat flux at the surface of the ocean is not uniform. This condition forces convective currents, whose behavior is not well understood even in a qualitative sense. Examples of circulation resulting from this forcing include the spreading of deep water formed in restricted areas such as the northern North Atlantic, in the Gulf of Lyons in the Mediterranean Sea, and in the Weddell Sea near Antarctica.

Laboratory experiments have played a significant role in helping to understand these geophysical systems. One of the issues that has been studied in the past, prompted by STOMMEL (1950, 1962), is the smallness of the rising region in a rectangular container of water differentially heated from below, analogous with the small sinking regions of the ocean. ROSSBY (1965) conducted experiments in a non-rotating container of rectangular cross-section which was heated from below and insulated on the top and sides. The asymmetry between the size of the rising and sinking regions was proposed to be proportional to the one-fifth power of the Rayleigh number [Ra = $(g\alpha\Delta TH^3/\nu\kappa)$, where g is gravity, α is the thermal expansion coefficient, ΔT is the temperature difference along the lower boundary, H is the vertical length scale, ν is the viscosity, and κ is the thermal diffusivity] based on the relative efficiency of convection vs conduction. BEARDSLEY and FESTA (1972) extended these studies by making a non-rotating numerical model in which this power law was roughly verified for a variety of boundary conditions and in a range of Ra $< 10^6$. The need for numerical methods or laboratory investigations at greater Ra was discussed.

HIGNETT *et al.* (1981) have presented experimental results for convection driven by non-uniform heating from below in a rotating annulus. When rotation is small, so that

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the flow is axisymmetric, flow in the direction of the temperature gradient occurs in boundary layers. In the interior a "zonal" thermal wind occurs which is at right angles to the imposed temperature gradient. This wind transfers no heat. Convection in a rotating basin with sidewalls is qualitatively different from the annulus because the sidewalls support a geostrophic flow in the direction of the externally imposed temperature gradient. This flow directly transports heat (HIDE and MASON, 1975).

The primary purpose of this report is to describe the overall flow pattern in a preliminary experiment which combines the features of the Rossby experiment with those of Hignett *et al.* We include rotation, rectangular sidewall geometry and non-uniform horizontal heating. This has not been described previously except in the experiments of SUGIMOTO and WHITEHEAD (1983) which, in addition, possessed complicated bathymetry. Without rotation, the features reported by Rossby were recovered, namely an asymmetry in rising and sinking regions, along with an upper region with relatively small temperature variation. With rotation, the asymmetry between rising and sinking areas was retained. A cyclonic gyre was observed in the upper region, presumably generated by vortex stretching. In addition, the convection region adopted helical circulation.

EXPERIMENTAL METHOD

The container was a square basin with a flat copper bottom, 90 × 90 × 12 cm (Fig. 1). The bottom was heated at one end and cooled at the opposite end by water flowing in pipes soldered to the copper. The high thermal conductivity of the copper helped to maintain a uniform temperature gradient. The sidewalls were 0.65 cm thick Plexiglas 12 cm tall. Three of the walls (excluding the wall in the convection region) were insulated with 1 in. styrofoam on the outside. A Plexiglas lid usually was placed on the top, although this was temporarily removed for photography. It had holes drilled in it for the insertion of a temperature probe. The basin was gridded with thin wires, three in each direction, for thymol blue (MERZKIRCH, 1974) flow visualization.

On either side of the table were cold and warm water reservoirs, with submersible pumps supplying the pipes on the bottom of the basin. The warm water was maintained thermostatically to a temperature of 32°C. The cold reservoir was maintained near 5°C

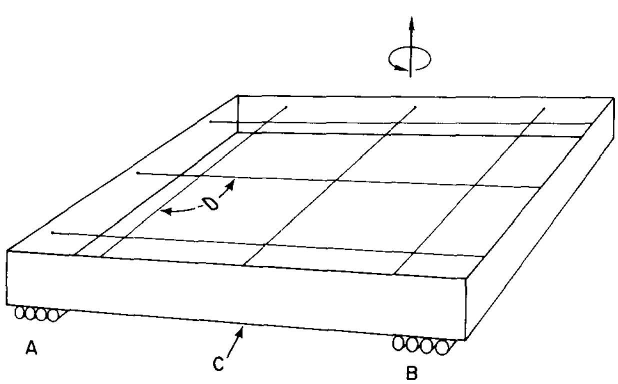

Fig. 1. Layout of the apparatus. (A) Warm water pipes. (B) Cool water pipes. (C) Copper bottom. (D) Thymol blue flow visualization wires.

with ice. Some loss of heat by the system occurred when the lid was removed, which created some patchy, small-scale convection visible in the dye.

The procedure for running an experiment was as follows: (1) The basin was spun (counter-clockwise) at a predetermined rotation rate with no differential heating. (2) Power was supplied to the pumps and the heater, and ice was put into the cold bucket. (3) The electric grid was pulsed at desired intervals, dye was injected, or temperature records were taken. During each run, a video camera or film camera recorded the evolution of the flow.

A laboratory experiment with non-uniform heating at the upper surface, rather than the bottom, is difficult to conduct. Our experiment is analogous to an oceanic situation which is differentially heated from above. To transform the experiment to the ocean, the sense of hot and cold and the sign of gravity must be reversed. With rotation, this means that the direction of gravity is reversed compared to the rotation vector. Thus, if this experiment is viewed upside down, the gyre would circulate in a clockwise direction. It would, of course, still be cyclonic, but analogous to one in the southern hemisphere.

RESULTS

Three regimes of flow were identified; two steady regimes for fast and slow rotation, and an unsteady regime for intermediate rotation. These appear to be in qualitative agreement with the early results of FULTZ (1961) for his annulus experiments. For no rotation, there was a purely "meridional" (along temperature gradient) motion. There was a rising at the hot end in a relatively narrow region, 1–2 cm, and a broad outflow at the top and inflow at the bottom to this narrow updraft. Over the heated end there was cellular convection of width about 1 cm. Figure 2a shows a vertical temperature section from a run with the parameters $T_{hot} = 30°C$, $T_{cold} = 5°C$, $Ra = 6 \times 10^8$. For comparison, a figure from the numerical calculation of Beardsley and Festa and a section from Rossby's experiment are shown. The two most striking features are a nearly isothermal upper fluid and the well-known marked asymmetry in temperature distribution.

For slow rotation (period typically 200 s), the flow in the thermal boundary layer near the bottom was from the cold end toward the hot end and was not visibly different from the flow near the bottom without rotation. However, the convection region was now about 10 cm wide and adopted a more complicated three-dimensional structure (Fig. 3). The circulation in the convection region appeared to possess a helical spiral produced by a miniature convective cell superimposed on a translation to the right (looking from the hot end to the cool end). In the upper layer, the warm fluid moved away from the hot end and moved to the right. It then circulated about the upper portion of the fluid in a counter-clockwise (cyclonic) gyre.

Dyed water was injected at intervals along the entire hot end in the run (Fig. 4a). Some of the dye recirculated near the convection region (Fig. 3), while the other fluid moved to the right, away from the convection region (Fig. 4b), where it formed a concentrated tongue of dyed water that circulated cyclonically. A small anticyclonic gyre was visible in all runs at the heated end in the right corne? (looking from hot to cold). The top dye moved cyclonically, as did most of the dye below it. However, there was a slight anticyclonic flow near the botton (Fig.4c,d). The gyre was not jetlike as Fig. 4a,b would imply, but rather possessed an inner core of almost rigid body rotation and an outer edge of greater shear.

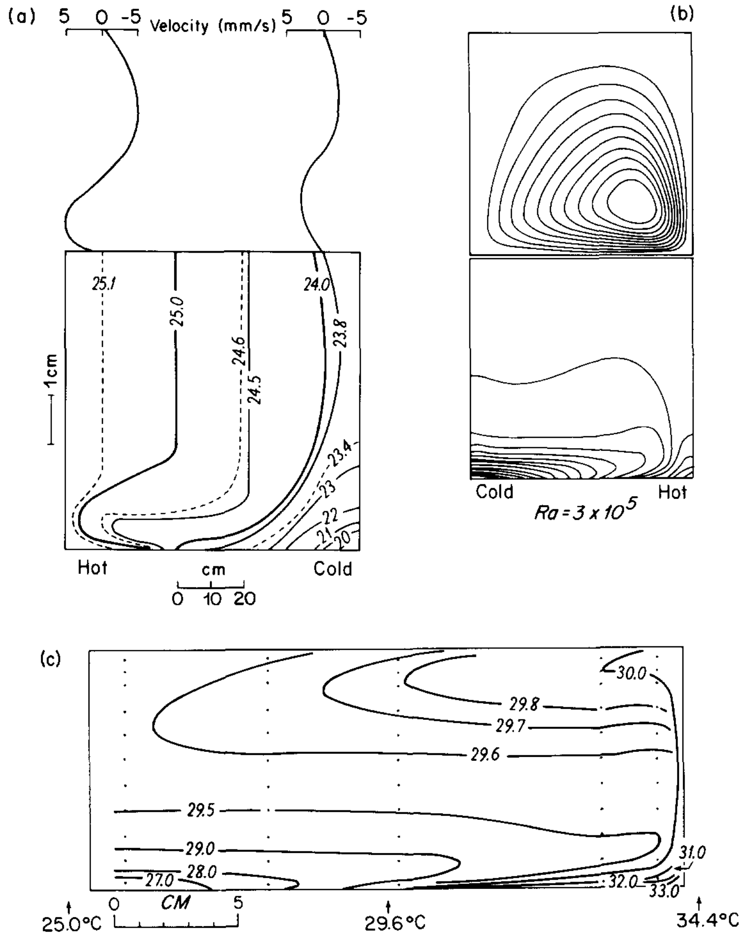

Fig. 2. (a) Vertical temperature section with two velocity profiles (shown above the basin) for the experiment with no rotation. (b) Numerical computations of streamfunction (above, interval = 1) and isotherms (below, interval = 0.05) from BEARDSLEY and FESTA (1972), Fig. 1. (c) A temperature section from ROSSBY's (1965) experiment, Fig. 5b. The Rayleigh number for our experiment is about 1×10^8; for Beardsley and Festa, 3×10^5 and for Rossby, 1.4×10^8. Note that the earlier papers had hot on the right, whereas, we have hot on the left, and that isotherms are not always drawn at equal intervals.

Two vertical temperature sections were made in the slow rotation case (Fig. 5). Individually, they looked like the zero rotation section (Fig. 2) except that the top fluid was more uniform in temperature (probably owing to a lid) and there was an inclination of the isotherms downward from the left section to the right. A geostrophic flow estimate gives a velocity difference due to thermal wind of about 1 cm s^{-1}, with upper water moving away from the heated end and lower water moving toward the heated end.

DISCUSSION

The relation of these experiments to the regimes of the annulus experiments is shown in Fig. 6. Previous discussion and photographs were for the steady case with G^* of order

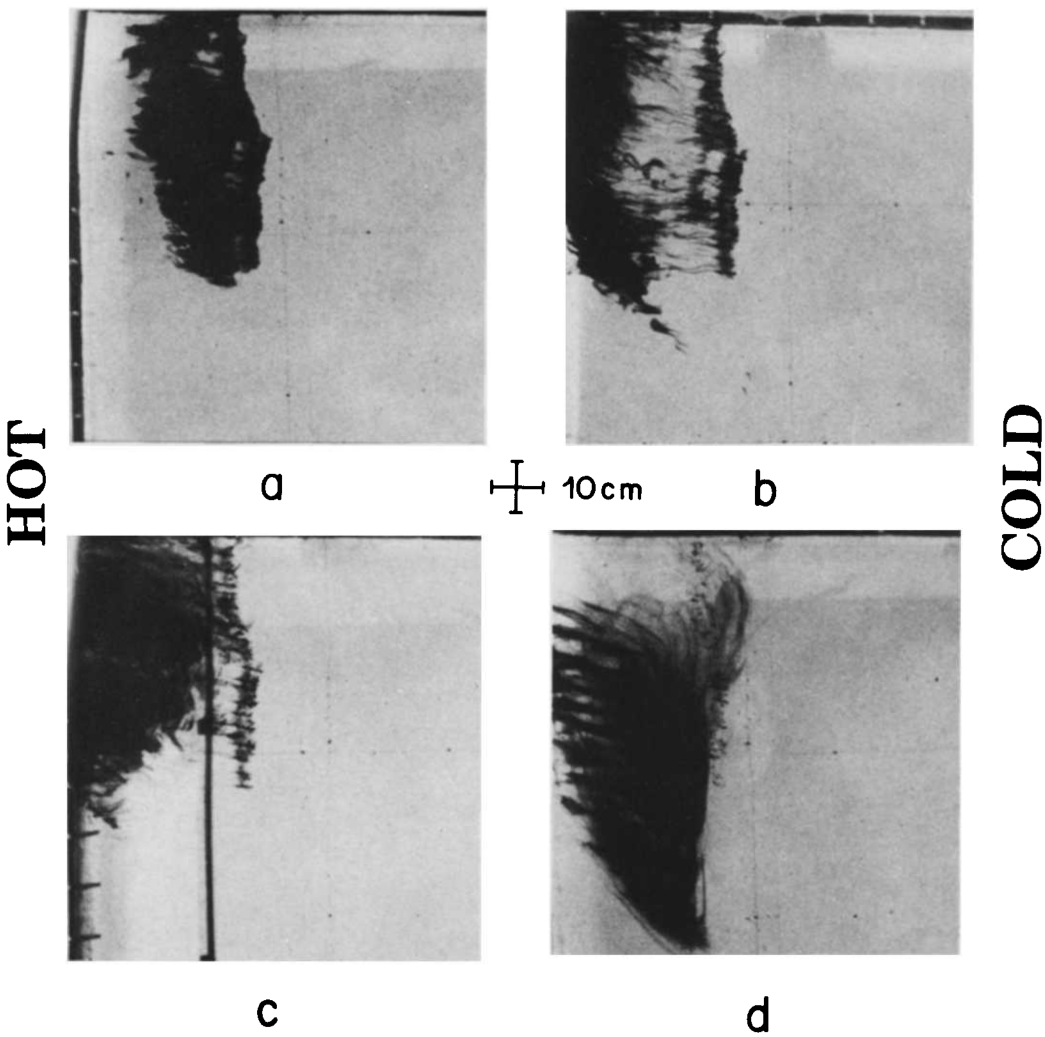

Fig. 3. Close-up of dye injected in the heated rising region of the rotating experiment, viewed from above. Immediately above the hot bottom, convection rolls are aligned in the direction of flow. (a) Immediately after injection. (b) at 56 s, the dye approaches the hot end and rises, (c) at 96 s it is swept inward and sinks, moving to the right (the vertical stripe is the shadow of the top edge of the side wall), (d) at 200 s, the dye moves to the bottom where convection rolls form again.

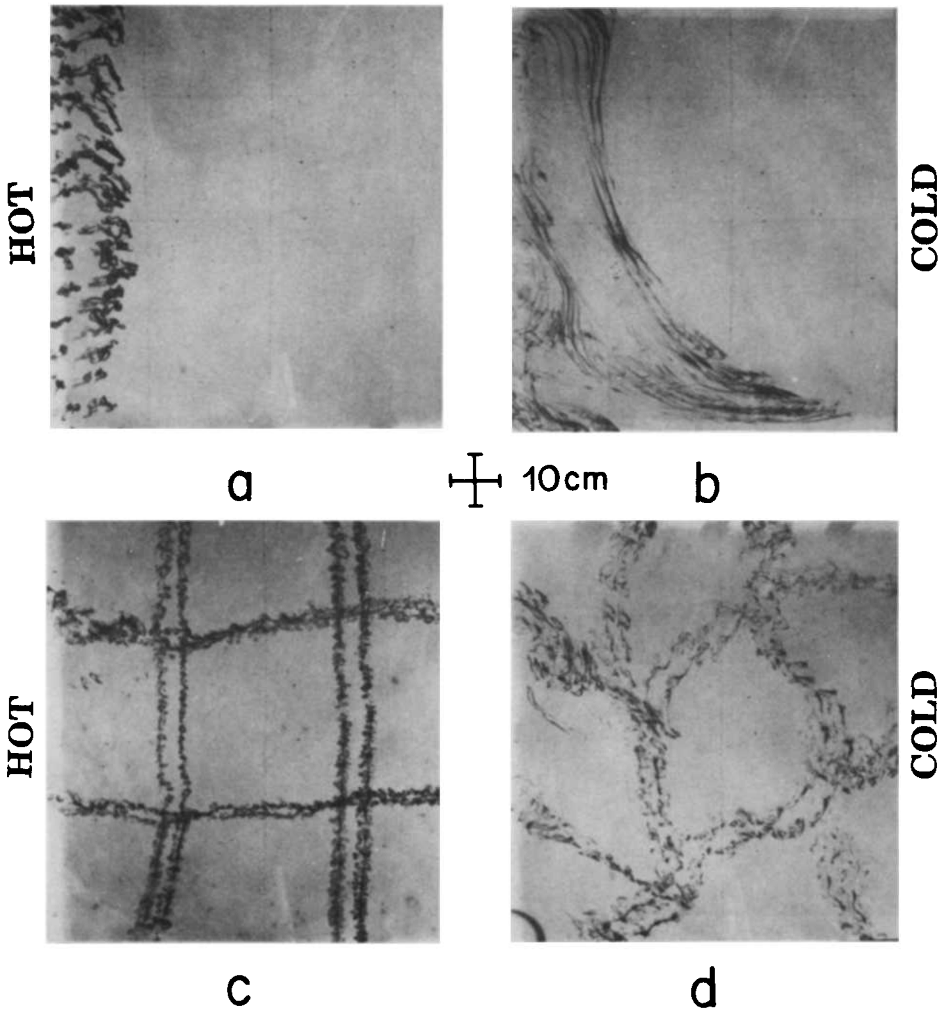

Fig. 4. (a) Dye injected at the hot end and (b) its shape 113 s later. This reveals the upper limb of the cylonic gyre. The dye is in the outer edge of the gyre and the velocity is not as jetlike as the dye streak would imply. In this and following figures, the dye patch casts a shadow on the bottom of the tank owing to an oblique light source. (c) Four horizontal lines in the upper fluid (and their shadows). (d) Their shape 45 s later.

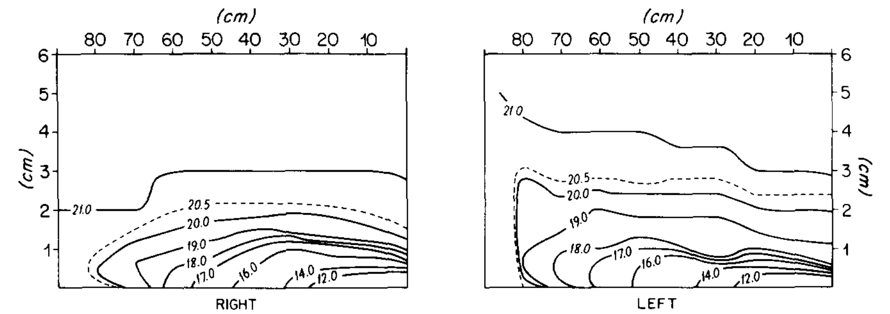

Fig. 5. Vertical sections of temperature from the hot to the cold end, located 10 cm from the left and right walls (looking from the hot end to the cold end).

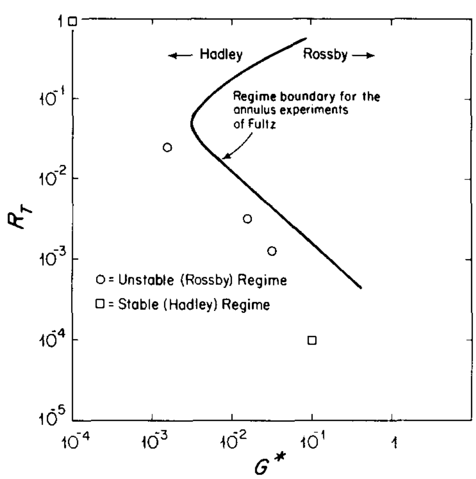

Fig. 6. Comparison of annulus and present experiments. The curve is reproduced from HOLTON (1979). The ordinate is the imposed thermal Rossby number which can be written as the square of the ratio of the deformation radius $(g\alpha\Delta TH)^{\frac{1}{2}}/f$ to the size of the basin (L, f is twice the angular velocity). The abscissa $G^* = Lf^2/g$ is a measure of the rotation rate scaled by gravity and the basin length. The symbols denote an unsteady motion, or a steady motion.

10^{-4}. At a greater rate of rotation (periods < 200 s, $G^* > 10^{-4}$) the flow was unsteady and eddies appeared which gradually filled the basin. For fast rotation (periods < 10 s, $G^* > 10^{-1}$), we observed that the flow was again in the stable regime. There is very crude qualitative agreement with the notion of a stability boundary, although its precise definition will require more experiments.

Rotation makes the vertical component of vorticity, $\zeta = v_x - u_y$, dynamically important because variations in vertical velocity (w) can now force variations in ζ through the linear vorticity balance $\zeta_t = fw_z$. Integrating throughout the upper isothermal layer of the fluid and assuming that the horizontal velocity is independent of z, that $w = 0$ at the surface and the $w = -w_b$ at the bottom of the upper layer gives

$$\zeta_t = \frac{fw_b}{H}.$$

Thus the movement of fluid into a thermal boundary layer near the bottom stretches the upper layer, spinning up cyclonic vorticity. If w_b is uniform, the motion is a rigid body rotation away from the boundaries. After a spin-up time of about 10 rotation periods (from observations), ∂_t is replaced by r with $r^{-1} \sim 10T$, where T is the period. Then the steady balance is just

$$\zeta = \frac{fw_b}{rH}.$$

For $f/r = 10$, $H = 10$ cm, $w_b = 0.01$ cm s^{-1}, $\zeta = 10^{-2}$ s^{-1}, which gives a velocity scale of about 0.5 cm s^{-1} near the sides of the basin, in accord with observations (Fig. 4). The vertical velocity is forced by convection and thus depends on Ra. The relation between the average vorticity and Ra will be explored in follow-up experiments.

Typical values of relevant physical parameters for the laboratory experiment and the ocean are given in Table 1. In the laboratory experiment, the Rossby number is greater than the ocean value, but is still small so that rotation is important. The thermal Rossby number in the experiment does span oceanic values. For small values of the thermal Rossby number, the deformation radius is small compared to the basin size, and the flow becomes unstable (Fig. 6). Numerous eddies of size approximately a Rossby radius of deformation were observed in this case.

The applicability of the experiment to the *global* ocean circulation is questionable, because of the absence of variation in rotation (planetary β). Planetary β is thought to be important for the structure of the thermocline, but on the other hand the convection we study here is absent in thermocline theory. This experiment thus compliments such studies. Our results may apply to deep convection in confined oceanic basins in which β exerts only a minor influence such as the western Mediterranean, or in the vicinity of deep convection regions.

Table 1. A comparison of relevant physical parameters between laboratory and oceanic values

Number	Lab. value	Ocean value
Rossby	10^{-1}	10^{-3}
Rayleigh	10^{8}	10^{15}
Thermal Rossby	10^{-4} to 1	10^{-4} to 10^{-2}

The Rossby number is U/fL, where the horizontal velocity $U = 1$ cm s^{-1} for the lab. and 10 cm s^{-1} for the ocean, $f_l = 10^{-1}$ s^{-1}, $f_o = 10^{-4}$ s^{-1} is the Coriolis parameter (subscripts l and o denote laboratory and ocean, respectively) and $L_l = 100$ cm, $L_o = 5 \times 10^7$ cm is the basin length scale. The Rayleigh number is $g\alpha\Delta TH^3/\nu\kappa$, $g = 10^3$ cm s^{-2} is acceleration due to gravity, $\alpha = 10^{-4}$ °C^{-1} is thermal expansion, $\Delta T_l = \Delta T_o = 10$°C is the horizontal temperature difference, $\nu\kappa = 10^{-5}$ s^{-2} is the product of viscosity and diffusivity and is unknown for the ocean (turbulent values of 1 cm^2 s^{-1} are used here), $H_l = 10$ cm, $H_o = 10^5$ cm is the depth scale. The thermal Rossby number is R^2/L^2, where $R_l = 30$–100 cm, $R_o = 5 \times 10^6$ cm is the deformation radius.

In spite of the cautionary remarks above, it is possible that the most important features observed here are generally characteristic of convection in the ocean even with the influence of β present. These central features are: fluid in the frictional boundary layer moving toward the convection region and directly feeding into the convection cells, convection cells occurring in patches, and the cells feeding fluid into a cyclonic (positive vorticity) region. Evidence for each of these features exists in the laboratory experiment.

In conclusion, the generation of a gyre by vortex stretching in a system driven by non-uniform heating seems like a simple and useful analogue of convective circulation in an ocean basin. It may have some bearing on water mass formation and circulation in confined regions.

Acknowledgements—Support for the laboratory apparatus and photography was from National Science Foundation Grant no. OCE 84-16100. KGS was supported by R. Weller (ONR N00014-84-C-0134) and B. Warren (NSF OCE 82-13967). Woods Hole Oceanographic Institution Contribution no. 6481.

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