Reprinted from Nature, Vol. 336, No. 6194, pp. 59-61, 3 November 1988
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Wave transport of deep mantle material

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Fluid with a certain density and viscosity can rise by buoyant Poiseuille flow through a conduit[1] within a second fluid of greater density and viscosity. Such conduits exhibit a rich behaviour characteristic of nonlinear systems, an aspect of which is the formation of solitary waves[2,3]. Here we present theoretical and experimental studies of these systems. Both approaches reveal that solitary waves trap material in a cell with closed streamlines and that the central streamline velocity is faster than the wave speed. Hence, parcels of deep material are transported directly upward over large distances. This is in contrast to the usual situation in which wave propagation through a medium causes only small displacement of fluid particles. Material in these parcels will be far less contaminated by diffusion from the surroundings than would be material in ordinary pipe flow. In addition, solitary waves are more efficient than buoyant spheres at conveying material upward. We suggest that such waves might exist in the Earth's mantle, conveying uncontaminated deep mantle material to the surface of the Earth.

There has been considerable recent interest in buoyant conduits, for several reasons. First, geological and geophysical counterparts may exist in the Earth: applications have been made to magma chambers[4], mantle convection[5,6] and hotspots[7,8]. Seismic evidence shows that the core-mantle boundary and the associated D″ layer (which can be interpreted as a thermal boundary layer) is rough. Numerical simulations[6] of mantle convection in a fluid with a strongly temperature-dependent viscosity show conduits in which unstable solitary waves are generated. Second, the system is a simple analogue of compaction-driven flow in a porous viscous matrix[2], which bears upon such diverse issues as the formation of melt under mid-ocean spreading centres, the formation of melt under hotspot islands such as Hawaii or Iceland, and the nature of mud volcanoes and diapirs. Scott[10] noted that solitary waves in the compaction problem do not convey material over large distances because the waveform ascends faster than the background flow. Thus our new findings distinguish the conduit problem from its compaction analogue.

The clearest demonstration of closed streamlines are theoretical. In the limit of small amplitude, characterized by $\varepsilon \equiv A_m/A_0 - 1 \ll 1$, the wave speed is $c = (g'\ A_0/4\pi\nu)[1+\varepsilon/3]$, whereas the maximum (centre-line) vertical fluid velocity in the conduit is $u_m = (g'\ A_0/4\pi\nu)[1+\varepsilon]$, so $u_m > c$. Here A_m is the maximum area or amplitude of the wave, A_0 is the area of the undisturbed conduit, $g' = g\Delta\rho/\rho$ is gravity multiplied by normalized density difference between exterior and conduit fluids and ν is the viscosity of the conduit fluid. In a frame co-moving with the wave, fluid moves forward at the exact centre of the wave and backward in the undisturbed pipe (both upstream and downstream), and stagnation points are found both in front of and behind the centre of the wave. Fluid is trapped in closed streamlines between these two points and conveyed with the wave. The above argument is valid for small ε, but this result is found at arbitrary wave amplitude. Figure 1 shows theoretical streamlines for $\varepsilon = 9$ (K.R.H. and J.A.W., preprint).

We have performed simple laboratory experiments which clearly demonstrate fluid trapping. The apparatus is the same as described previously[9] and essentially consists of a glass vessel containing corn syrup with a tubular source of syrup-water mixture on the bottom. Figure 2 contains a time series of photographs that follow a single solitary wave. The conduit and much of the wave are made up of clear fluid which is difficult to distinguish from the external fluid. Some dyed fluid was injected into the wave, and circulates within the wave by ascending along the centre-line and descending close to the periphery. On reaching the bottom of the wave, it repeats the circuit. These photographs indicate that the fluid is trapped within the waves and that the circulation pattern is qualitatively similar to the theory (Fig. 1). Clear observations of trapped fluid have also been made in waves of very small amplitude and in rank ordered packets of waves of extremely large amplitude[9].

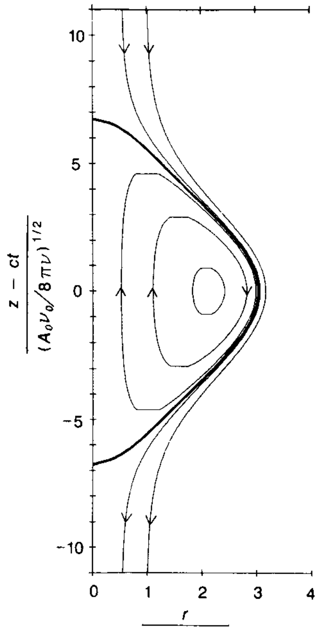

Fig. 1 Streamlines of a solitary wave calculated in the frame moving with the solitary wave speed c for the case $\varepsilon = 9$. Radius r is normalized by the radius of the conduit and the conduit axial dimension z is normalized by a natural length scale of the problem[3]. Note the large region of closed streamlines.

If conduits extend upward from some great depth in the Earth's mantle, they might contain such waves in response to changes in volume flow from transient boundary conditions. They will then convey isolated packets of deep mantle material upward in their centres and deposit the material in the upper mantle, possibly under hotspots. The following calculations indicate that the dilution of deep mantle material from sideways diffusion in waves would be quite small relative to the corresponding dilution in ordinary pipe flow.

We consider first the dilution in pipe flow. Taylor[11] showed that matter dispersed in a solvent in pipe flow possesses a virtual coefficient of diffusivity $D_\nu = a^2u_0^2/192D$ in the direction of pipe flow, where a is pipe radius, u_0 is the maximum velocity at the axis and D is the molecular diffusivity of the material. This relation is valid when $4L/a \gg u_0a/D \gg 6.9$ (ref. 12), with L the conduit length. If we use for typical values[13] a mantle conduit radius $a = 5$ km, conduit length $L = 3{,}000$ km (depth of mantle), molecular diffusivity $D = 10^{-8}\ \mathrm{m^2\ s^{-1}}$ and a velocity $u_0 = 10^{-9}\ \mathrm{m\ s^{-1}}$ (equivalent to a rate of $3\ \mathrm{cm\ yr^{-1}}$, which is the spreading rate of moderately fast tectonic plates and gives a rise of 3,000 km in 100 Myr) then the inequalities are satisfied. Consider a pulse of new material injected in a conduit undergoing waveless pipe flow. The concentration anomaly of the new material can be represented as a gaussian function which diffuses vertically with width $\sim\sqrt{D_\nu t}$. The pulse amplitude decreases as the inverse of this width. If the initial width is equal to the conduit radius a and rises through the mantle from the core, it will have been diluted with surrounding conduit material by an amount

$$\frac{a}{\sqrt{D_\nu L/u_0}} = \sqrt{\frac{192D}{Lu_0}} = 2.5 \times 10^{-2}$$

which might make it undetectable in upper mantle material.

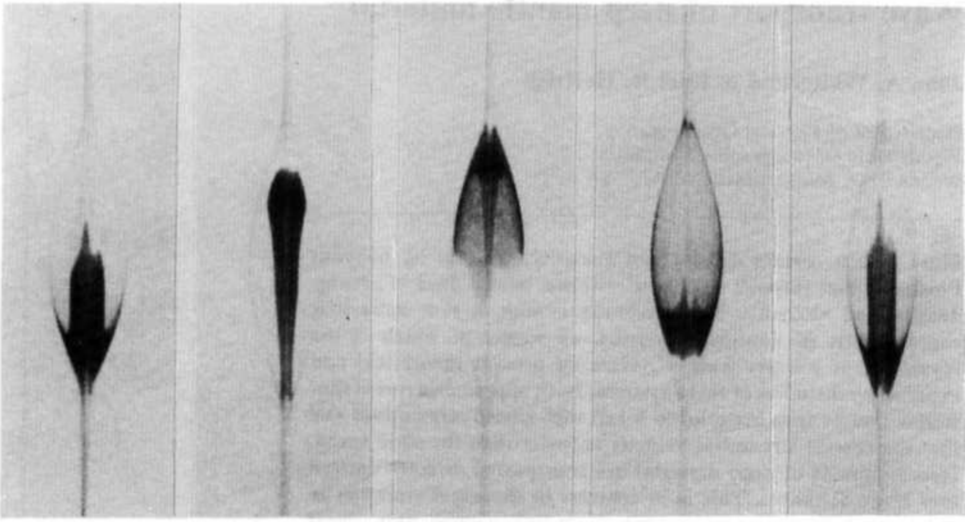

Fig. 2 Sequential photographs (with equal time intervals) of a solitary wave with a small amount of dye. This reveals the circulation in closed streamlines. The conduit and most of the wave contains clear fluid which is scarcely distinguishable.

Using a smaller value of D leads to even more dilution, although one of the inequalities begins to be violated. We infer, therefore, that in pipe flow, materials would get strongly dispersed as they rose through a conduit in the mantle.

One reason for the extensive mixing of material in pipe flow is that the surface area between the material and surrounding fluid increases with time. In contrast, material confined within a wave with closed streamlines will retain a constant surface area. The law governing the evolution of concentration C in a well mixed volume V with surface area S as it rises through a conduit of thickness d is

$$V\frac{\mathrm{d}C}{\mathrm{d}t}=\frac{-SD}{d}C$$

with solution concentration $C=C_0\,\mathrm{e}^{-SDt/Vd}$. Using the values $t=L/u_0=100$ Myr and thickness of both the wave and the boundary layer $V/S=d=5$ km, along with the previous value of D, the concentration will decrease by $\sim\mathrm{e}^{-1.2}=0.3$. With such a dilution, a substance originating from the core–mantle interface or at some other deep mantle source might be chemically or isotopically unaltered by the time it reaches the surface of the Earth. We note that hotspot basalt has different trace-element and isotope signatures from mid-ocean ridge basalt[14], indicating a lack of dilution of the hotspot material between its source and the Earth's surface.

Waves in conduits are also a preferable mode of vertical mantle transport because they transport material more efficiently than spherical cavities. We can estimate the size of a wave required for it to rise through the mantle in a reasonable geological timescale. Olson and Christensen[3] show that the speed of large solitary waves is (in dimensional form)

$$c=\left(\frac{g'a^2}{4\nu}\right)\left(\ln\left(\frac{A_{\mathrm{m}}}{A_0}\right)-\frac{1}{2}\right)$$

To obtain for c the geophysically relevant value of $10^{-9}\ \mathrm{m\,s^{-1}}$, with the parameter $g'=0.01\ \mathrm{m\,s^{-2}}$, $\nu=10^{14}\ \mathrm{m^2\,s^{-1}}$ (assuming conduit viscosity is 10^{-4} of the commonly accepted mantle estimate $\nu_0=10^{18}\ \mathrm{m^2\,s^{-1}}$ (ref. 13)) and $a=5.0$ km, we get $A_{\mathrm{m}}/A_0=8.17$, which implies that the wave radius $a_{\mathrm{m}}=14.3$ km. The volume of material in a wave is then

$$\Delta V=\frac{\pi a^3 A_{\mathrm{m}}}{A_0}\sqrt{\left(\frac{\pi\nu_0}{2\nu}\right)\left(\ln\left(\frac{A_{\mathrm{m}}}{A_0}\right)-\frac{1}{2}\right)}=3.9\times10^5\ \mathrm{km^3}$$

The volume flux of the base conduit is

$$Q_{\mathrm{c}}=\frac{A_0^2 g'}{8\pi\nu}=7.7\times10^5\ \mathrm{m^3\,yr^{-1}}$$

Dividing ΔV by Q_{c}, we see that one solitary wave of this type every 500 million years would double the net volume flux from the conduit. Thus, wave transport is a very efficient means of increasing net conduit volume flux. In contrast, a sphere must be significantly larger to rise through the mantle in geological time. The sphere velocity v_{s} is[1]

$$v_{\mathrm{s}}=\frac{g' r_{\mathrm{s}}^2}{3\nu_0}$$

To give the same velocity as before, we require a radius $r_{\mathrm{s}}=550$ km, and therefore a much greater volume of $7.0\times10^8\ \mathrm{km^3}$. Spheres are inefficient for other reasons as well. Griffiths[15] has shown that spheres smaller than 200 km are too slow ever to reach the surface.

Thus we envisage a process of mantle flow in which solitary waves trap parcels of material and easily transport them vertically with little dilution. Note that mantle (especially lower mantle) characteristics are poorly constrained, with uncertainties much greater than factors of ten, so the estimates of diffusion velocities are only suggestive. We have also ignored thermal and multicompositional effects, although thermal effects are modelled well by laboratory compositional experiments. But the description presented here avoids problems associated with very wide mantle plumes of uniform viscosity or with very large spheres in the mantle.

Received 14 July; accepted 12 September 1988.

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Printed in Great Britain by Turnergraphic Limited, Basingstoke, Hampshire