

A Laboratory Model of Vertical Ocean Circulation Driven by Mixing

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ABSTRACT

A model of deep ocean circulation driven by turbulent mixing is produced in a long, rectangular laboratory tank. The salinity difference is substituted for the thermal difference between tropical and polar regions. Freshwater gently flows in at the top of one end, dense water enters at the same rate at the top of the other end, and an overflow in the middle removes the same amount of surface water as is pumped in. Mixing is provided by a rod extending from top to bottom of the tank and traveling back and forth at constant speed with Reynolds numbers >500 . A stratified upper layer ("thermocline") deepens from the mixing and spreads across the entire tank. Simultaneously, a turbulent plume ("deep ocean overflow") from a dense-water source descends through the layer and supplies bottom water, which spreads over the entire tank floor and rises into the upper layer to arrest the upper-layer deepening. Data are taken over a wide range of parameters and compared to scaling theory, energetic considerations, and simple models of turbulently mixed fluid. There is approximate agreement with a simple theory for Reynolds number >1000 in experiments with a tank depth less than the thermocline depth. A simple argument shows that mixing and plume potential energy flux rates are equal in magnitude, and it is suggested that the same is approximately true for the ocean.

1. Introduction

Ocean circulation is a result of boundary conditions imposed by the atmosphere (airspeed, temperature, and humidity), by sunlight, and by tidal forces from the moon and sun. Many aspects of the circulation are well documented and understood. The greatest velocities, exceeding tens of meters per second from surface waves, are concentrated in an upper region (<100 m depth). Typical velocities of the wind-forced currents range up to 1 m s^{-1} ; the currents are concentrated in the top few hundred meters, although their roots penetrate to more than 2000 m deep and even to the bottom in some cases. Such wind-driven flows are both extensively measured and the subject of much theoretical study. Many numerical models of ocean circulation are refined enough to give a good first approximation

of the horizontal circulation when compared with data. These flows also produce time-dependent internal flows in the form of eddies and a number of topographic and internal waves. The vertical circulation is less well understood. To some extent, the sinking and spreading of waters such as the Antarctic Intermediate Water, North Atlantic Deep Water, or Antarctic Bottom Water are a form of convection driven by the buoyancy flux of heat and freshwater at and near the sea surface. However, the horizontal circulation certainly plays a part in determining the locations most suited to the subduction from convection in winter.

The simplest conceptual model of deep ocean overturning ignores wind driving completely and is instead driven by differential heating (Stommel 1962). A buoyancy flux is imposed along a level surface. Under the warmed surface (which is a model of the tropics), there is a wide region of warmed water, corresponding to the heated water in the upper ocean. Under the coldest part of the upper surface (a model of the polar region), a relatively small sinking flow forms and cold fluid descends from the surface to the abyss. Cold water

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spreads along the bottom and rises uniformly under the heated region. Rossby (1965) studied such a flow in a laboratory model by cementing a metal bar along the bottom of a Plexiglas tank that had thick, thermally insulated walls. A temperature gradient imposed on the bar produced circulation in the tank, thereby producing an inverted model of a differentially heated ocean. The observed circulation had narrow upwelling and broad downwelling regions (verifying Stommel's picture), with thermal conduction producing a cold layer that is the inverse counterpart of the ocean thermocline. The basic scaling relations in that experiment express a balance between viscous drag and buoyancy and between heat conduction and convection for nonrotating fluid with viscous (Stokes) flow. The boundary layer thickness scale is $\delta_v = (\rho C_p \kappa^2 \nu / g \alpha H)^{-1/6} L^{5/3}$, where ρ_0 is density (Boussinesq approximation), C_p is specific heat, κ is thermal diffusivity, ν is viscous diffusivity (also called kinematic viscosity), g is gravity, H is heat flux into the top boundary, and L is lateral length of the tank. This formula is in the form given by Mullarney et al. (2004), who repeated and extended Rossby's pioneering experiments. Molecular values of the diffusivities are used in the laboratory, but these are intended to represent turbulent diffusivities in the ocean. Hughes and Griffiths (2006) find in their experiments that the descending current provides extra mixing, which can help to bring observation and estimates of mixing within the ocean closer to agreement.

The rotating extension of these experiments was initiated by Speer and Whitehead (1988) and studied more extensively by Park (1996; see also Park and Whitehead 1999). Because in that case the dynamics of a rotating fluid apply, it was suggested that the balance is between buoyancy and Coriolis force rather than viscosity, and the thickness scale is $\delta_f = (\rho_0 C_p L_x L^3 \kappa^2 f / g \alpha H)^{-1/4}$, where L_x is width of the tank in the direction lateral to the temperature gradient. Molecular thermal diffusivity is involved in both the rotating and nonrotating experiments as well as in the theory described here. The molecular diffusion of heat is presumed to act as a laboratory substitute for the ocean's mixing.

Hughes and Griffiths (2006) repeated the nonrotating experiments of Rossby with a larger tank and showed that descending currents are localized even with a large cooling area, as argued by Stommel (1962). Their results agree with a transient experiment by Pierce and Rhines (1996, 1997). Hughes and Griffiths also showed that the descending current provides extra mixing, and they indicate that the general size and shape of the ocean density distribution are in accord with known ideas and facts as long as the mixing from the descending current is included. Their results indi-

cate that mixing of the sinking plume would be turbulent and that the inclusion of mixing from sinking regions in the ocean can help to bring observation and estimates of mixing within the ocean closer to agreement. Such deep sinking regions in the ocean are called many names, including stream tubes, chimneys, and cataracts. The large amount of mixing there is well known and extensively documented.

For the ocean itself, Munk (1966) quantified the balance between upwelling of cold deep water and downward turbulent mixing of warm, light surface water in the ocean, estimating an upwelling speed of about 10^{-7} m s^{-1} and a vertical eddy diffusivity (from unknown sources of turbulence) of $10^{-4} \text{ m}^2 \text{ s}^{-1}$. Fallor (1966) investigated the sources of energy for the oceanic circulation and concluded that only wind mixing and tidal energy dissipation are big enough to be important energy sources. Wright (1972) showed that the potential energy released by the sinking of cold-water masses (the northern plume in Fig. 1) is about the same magnitude as wind stress into the North Atlantic. Many other estimates were made about the magnitude of mixing and stress in those times, but it took measurements of mixing in the ocean to clarify the picture.

Both direct and indirect methods have been used to obtain more precise values of mixing. First, the direct measurement of microstructure pioneered by Cox, Osborn, and Gregg (see review in Gregg 1987) measured an average vertical diffusivity of about $10^{-5} \text{ m}^2 \text{ s}^{-1}$ in the upper ocean ($<2000 \text{ m}$ depth), which is smaller than the above numbers. The values agree with measurements of ocean tracer spreading in the upper ocean (e.g., Ledwell et al. 1993, 1998; Matear and Wong, 1997), in the Santa Cruz Basin (Ledwell and Bratkovich 1995), and in the Santa Monica Basin (Ledwell et al. 1986; Ledwell and Watson 1991; Ledwell and Hickey 1995). The numbers were incompatible with Munk's rough budget of the ocean, and the discrepancy initiated a persistent idea that there was "missing mixing." However, the above measurements were taken only in the upper 2000 m of the ocean, and the many recent microstructure measurements taken at a greater depth do not support the missing mixing idea. Microstructure measurements (Polzin et al. 1997) find a wide range of values extending up to the enormous turbulent diffusivity value of $1.5 \times 10^2 \text{ m}^2 \text{ s}^{-1}$ (although only in small regions). In addition, ocean tracers that spread over both rough topography (Ledwell et al. 2000) and the continental shelf (Sundermeyer and Ledwell 2001) also reveal turbulent diffusivity up to $3 \times 10^4 \text{ m}^2 \text{ s}^{-1}$. Therefore, it has become clear that values in the upper ocean are lower than Munk's estimates, but other regions in

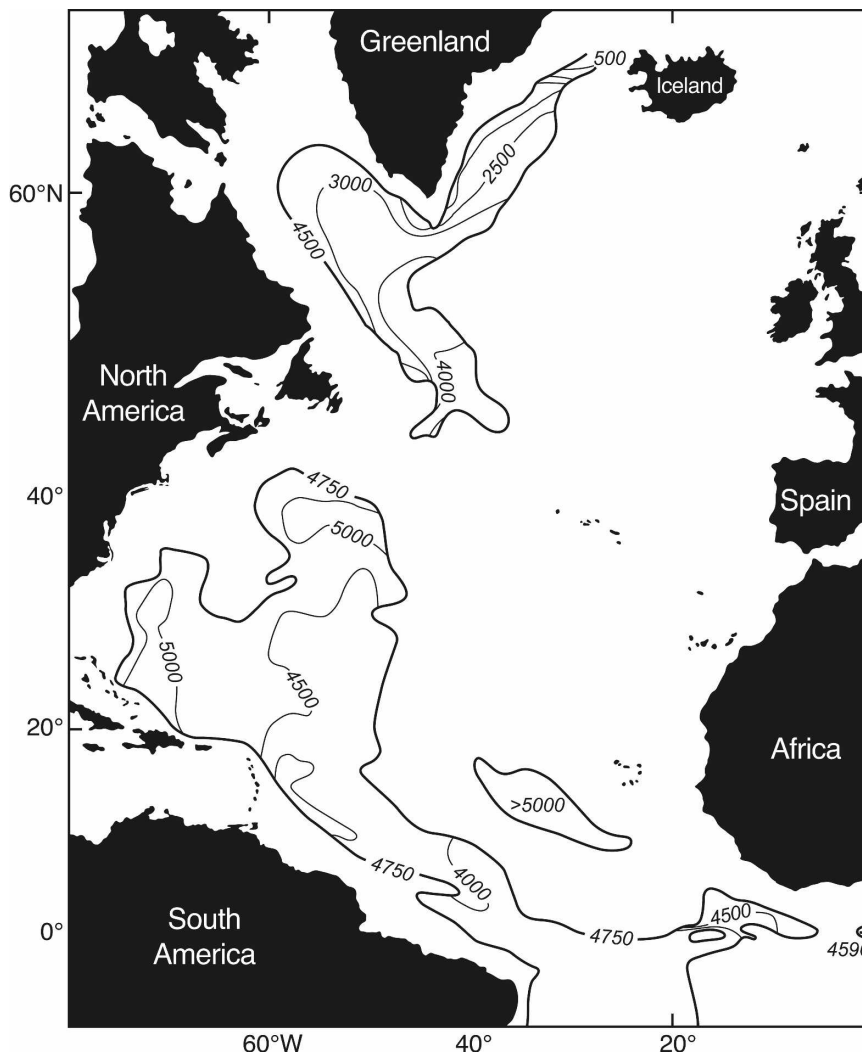


FIG. 1. The depth of the 1.8°C potential temperature surface in the western North Atlantic showing the spreading of two plumes from dense-water sources (from Worthington and Wright 1970). The upper, northern plume descends from a few hundred to over four thousand meters. The lower, southern plume enters over a gap and plain with a depth of about 4500 m, and it descends to more than 5000 m. The terminus of the southern plume is in the western North Atlantic, whose water must rise through the isotherm and join the water going south from the northern plume. All the water flows into the South Atlantic along the western boundary of the Atlantic as a component of the abyssal circulation.

the deep ocean have values of comparable or greater size.

Second, values of mixing have been estimated indirectly using heat or tracer budgets. Worthington and Wright (1970) showed that deep tongues of the coldest water extend into the deep North Atlantic from the north and south (see Fig. 1), and Worthington pointed out that they could be used to provide an estimate of mixing if the flux into the tongue at a specific upstream location was measured. Assuming that tongues have a constant volume, the downward mixing by turbulence

must equal upward motion of the cold water through the isotherm. Using volume flux estimates through passages, the value of vertical eddy diffusivity found in the deep western North Atlantic for the tongue entering from the south, as shown in Fig. 1, is $(1.1\text{--}4.6) \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ (Whitehead and Worthington 1982); for another such tongue in the Brazil Basin of the western South Atlantic, the eddy diffusivity is estimated to be $(3\text{--}4) \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ (Hogg et al. 1982). These are slightly greater than Munk's value but are confined to water deeper than 4000 m in the western Atlantic. However,

the plumes cannot supply all the mixing because they are the *result* of ocean mixing: the deep water into which they flow is warmer than the source waters (Whitehead 1989a,b).

Subsequently, estimates of the mixing rates near bottom topography arrive at smaller values of diffusivity (Rudnick et al. 2003), and the diffusion of salinity is not necessarily the same as for heat (McDougall and Whitehead 1984), but others find much larger values of up to $10^{-1} \text{ m}^2 \text{ s}^{-1}$ (Ferron et al. 1998). There are also large-scale estimates of diffusivity from basinwide volume flux calculations (Macdonald 1998; Ganachaud and Wunsch 2000; Wunsch and Ferrari 2004). These often recover the magnitude of the initial Munk value or even a greater value. Wunsch and Ferrari (2004) discuss processes that might accomplish the mixing needed to produce such a magnitude. Although the ocean is poorly sampled so far, coverage is steadily increasing, for example, through the use of velocity and strain profiles to estimate dissipation (Kunze et al. 2006).

At the same time, the understanding of mixing and its contribution to deep ocean circulation has developed. To our knowledge, Huang (1999) was the first to emphasize that the large-scale oceanic turbulence that mixes the deeper parts of the ocean serves as *the energy source* for the vertical circulation. This is in accord with estimates of the magnitude of the energy required for mixing (Munk and Wunsch 1998; Ganachaud and Wunsch 2000; Wunsch and Ferrari 2004).

The buoyancy flux into the ocean (flux of heat and freshwater) takes place mostly along the surface, with only a tiny heat flux component along the bottom from within the Earth. As discussed by Faller (1966), Huang (1999), and Vallis (2006), the sources and sinks of buoyancy flux within an ideal fluid at the same pressure (neglecting diffusion) exert a force that expands the fluid by warming, which is exactly balanced by the work absorbed by contracting the fluid by cooling (pointed out by Sandström 1916). Therefore, heating and cooling that are located strictly at the surface of a body of water cancel each other and provide no energy to generate currents (because positive energy is needed to make kinetic energy from a current that is ultimately dissipated by friction). Naturally, there are some difficulties with applying the idealized argument directly at the ocean surface. First, heat is driven to deeper elevations by molecular diffusion, turbulent mixing, and the penetration of solar heating. In addition, the surface buoyancy flux of heating and cooling is conveyed upward and downward through the agency of the wind-driven flow. These all provide vertical velocity in the top regions of the ocean, which allows heat gain at higher

pressures than heat loss, but this is limited to the upper 1500 m in most cases, and perhaps to 3000 m for some water connected to the Southern Ocean. For greater depths, only deep turbulent mixing can do the work to convey heat to great depths, where the pressures are high enough to be most effective in producing deep circulation. Therefore, as pointed out by Huang, the deeper component of the abyssal circulation is a result rather than a cause of this deep ocean mixing.

We report here the results of a laboratory study in which mixing is imposed in a tank of water with salt- and freshwater flux imposed along the top surface. Therefore, instead of density being changed at the boundary by heat flux and transmitted by advection and thermal diffusion, the density is injected differentially at the top by the flux of saltwater and transmitted in the interior by advection and turbulent mixing. It is a steady version of a circulation studied numerically by Pierce and Rhines (1997). In particular, we seek to answer the following question: Is a steady state achieved with a balance between energy loss because of plume descent and energy gain because of turbulent mixing (Pierce and Rhines's observed cyclic behavior)? If so, what are the quantitative contributions of mixing rate and surface buoyancy flux to density distribution in the experiment?

The results (section 3) show a density distribution and circulation very similar to those in a heated experiment. The flux of the freshwater by the mixing rod and the role of the turbulent plume for supplying bottom water are quantified with two simple theories in section 4, which are then compared to experimental data. Discussion of the results (section 5) indicates that energy flux continues to be a useful tool to constrain the ocean circulation.

2. The experimental apparatus

This work adds the contribution of mechanical stirring to experiments with convection driven along a horizontal surface. The experimental tank (Fig. 2) is made of transparent polycarbonate, with inside dimensions of 121.5 cm long, 20 cm high, and 5 cm wide. In contrast to flow driven by heat flux, the buoyancy force is from salinity variation as a proxy of temperature variation. Buoyancy flux at the top surface is in the form of salt- and freshwater that are gently pumped in through sponges at the two top ends of the tank. Water is withdrawn at approximately the same elevation as the sponges through a spillway in the middle. This buoyancy flux seems at first glance to mimic a model of solar heating minus surface cooling of the ocean. It is like a mixed ocean-atmosphere boundary condition in

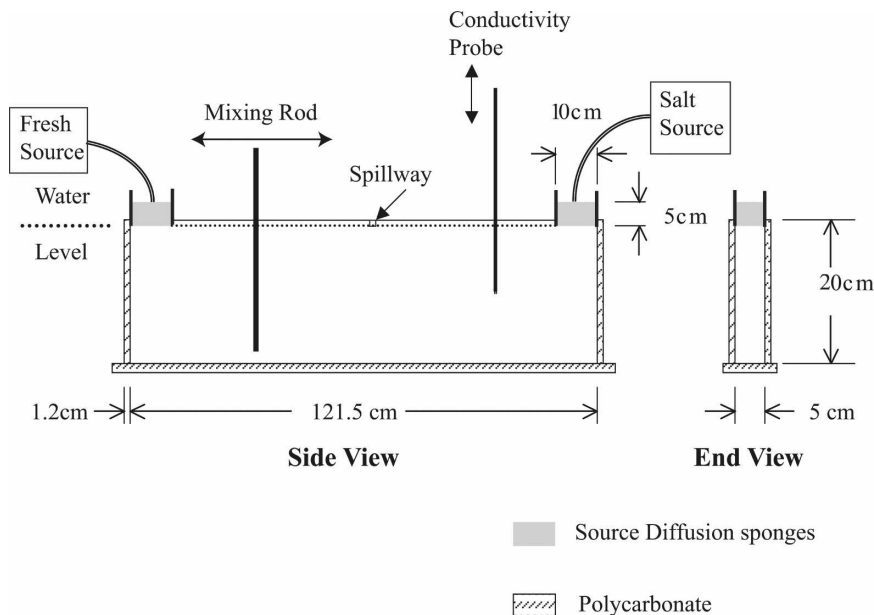


FIG. 2. Side and end views of the apparatus.

which the atmosphere adopts a mean temperature such that solar heating is removed and the ocean achieves the desired temperature distribution along its upper surface. However, in practice, the outflow condition in the experiment determines the surface density and the salty source determines the surface density at the salt-water end, so it is closer to an imposed temperature condition than a flux condition.

Turbulent mixing is produced by a vertical rod extending throughout the tank that moves laterally. It is the mixing in the wake of this rod, rather than molecular diffusivity, that mixes freshwater down into the tank to establish buoyancy-driven motion. The vertical rod traverses back and forth in a region extending 20–40 cm in length between the freshwater source and the spillway. The rod speed U and the rod diameter d are such that Reynolds numbers range between 500 and 3000 (using water viscosity of $10^{-6} \text{ m}^2 \text{ s}^{-1}$). In addition, the pumping rate Q of each source (both sources had the same rate) was set to three different values of strength in different experiments. Finally, the density difference between the salt- and the freshwater $\Delta\rho$ was also systematically varied in the experiments. The other relevant variables (the three tank dimensions, the size of the spillway, the molecular diffusivity of salinity, and viscosity) were kept constant.

The dense saltwater entering the tank possesses a buoyancy of magnitude $g'_0 = g\beta S_0$ above that of freshwater, where β is the density coefficient of salt and S_0 is the salinity of the water entering the tank at the salt source. Because freshwater is pumped in at the other

end, the volume flux of water leaving through the spillway is $2Q$. Therefore, conservation of salt dictates that the average buoyancy leaving through the spillway is $g'_0/2$. The experiment is equivalent to a tank cooled and heated from above in which the temperature at the top near the hot end has a buoyancy deficit equal to $-g'_0/2$ and the cooling produces convection with a downward buoyancy flux of $B_0 = Qg'_0$. The three variables U , d , and B_0 combine to form a dimensionless number that we will call a flux number:

$$F = B_0/U^3 d. \quad (2.1)$$

A second dimensionless number is found by combining U , d , and g'_0 , and calling the result a mixing Richardson number:

$$\text{Ri}_m = \frac{g'_0 d}{U^2}. \quad (2.2)$$

These two can be replaced by a third dimensionless number that is a combination of these two.

Because the distribution of density within the ocean is the most obvious consequence of stirring and the resulting overturning circulation, the primary measurement reported here is density obtained with a salinity microprobe located midway between the saltwater source and the spillway. A reading was taken every 0.0005 min. The microprobe was driven vertically with a stepper motor at a speed slow enough for almost perfect vertical resolution ($\leq 1 \text{ mm}$). Probe calibrations were made using samples measured by a precision den-

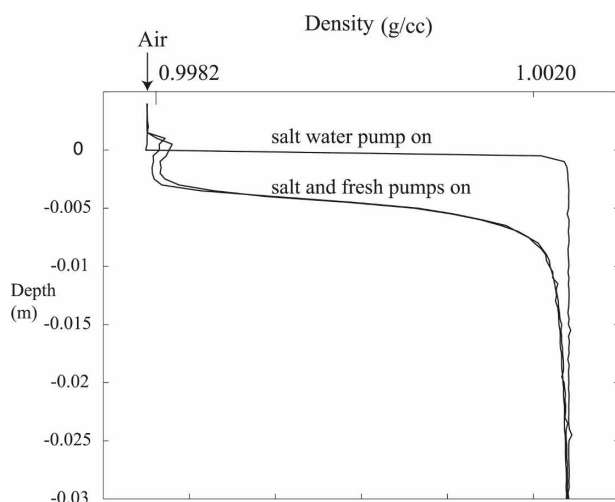


FIG. 3. Three density profiles before the mixing rod is started. The value for air marked by an arrow is not the density of air but the value corresponding to zero voltage of the probe.

sitometer. They were taken before and after each run, thus density is accurate to five decimal places.

3. Results

a. Visual observations and qualitative results

The overall features of the density distribution and velocity field are very simple to view and understand by starting the saltwater pump first, turning on the freshwater pump, and then starting the mixing. In each case, a density profile was taken periodically until a steady profile was achieved. If only the saltwater pump is turned on, the result is obvious; the tank is filled with saltwater and the surface rises to an elevation such that the flux rate of the water spilling out equals the pumping rate. This density profile is shown in the top curve in Fig. 3. Note that where the probe entered the water, the jump in voltage occurs almost completely between two adjacent readings, which illustrates the rapid response time of the probe. Below the interface, the saltwater is almost completely mixed and homogeneous. When the freshwater pump is also turned on, the water surface rises such that twice the flux leaves the spillway. Profiles were taken after the freshwater source was turned on and sufficient time elapsed for the freshwater layer to adopt a steady depth. Because the rod is not moving yet, the freshwater remains within a thin layer at the top of the tank. The two density profiles shown in Fig. 3 were taken 5 min apart. Repeat density profiles in this and other such experiments are so reproducible that profiles literally lie on top of each other. The ≈ 5 -mm-deep freshwater layer lying above a deep saltwater

layer corresponds to the Sandström case in which the density sources and sinks are at the same level. Circulation in this model is limited to lateral flow in the freshwater layer toward the spillway and a very slow flow of deep water circulating through the entire deep part of the basin and then up to the spillway. At the base of the top layer in Fig. 3 is a gradual transition region, which is 0.5 cm in depth, to the deep salty water that fills the rest of the tank. The region is much too thick to be produced by molecular diffusion, and the thickness is thought to be related to mixing near the saltwater source, where shimmering was readily seen in the freshwater layer below the sponge.

After the mixing rod is started, turbulence behind the traveling rod mixes fresher surface water with deep saltier (denser) water (Fig. 4a; henceforth, we will refer to lighter or denser water rather than fresher or saltier water). In addition, the deep dense water is mixed upward (Fig. 4b). The layer of mixed water spreads laterally over the entire upper part of the tank. A turbulent descending plume under the dense-water source penetrates through this layer. This plume is shown at the far right-hand side in Fig. 5. The turbulent plume produces two results. First, because the sinking plume entrains and mixes, the bottom water is always lighter than the dense water at the source. Second, the entrainment accompanying this mixing draws in fresher water from the top layer and causes lateral motion toward the sinking plume. We readily saw the light water from the mixer flowing toward the turbulent plume to supply entrainment in the plume. Then, to conserve mass, the bottom dense water flows away from the dense water end (left-hand side). The objective of the experiment is to quantify the vertical density distribution.

b. Quantitative results

After achieving a steady state for numerous experiments (Table 1), the density profiles were analyzed in a variety of ways to find any systematic structure they might have. The most satisfactory results were found by normalizing the density by the total density difference between top and bottom for each run. All of the normalized profiles exhibited two layers (Fig. 6), each with a relatively constant value of stratification. Therefore, we could define values of Brunt-Väisälä frequency as

$$N_i = \sqrt{g d \rho / (\rho_0 dz)} \quad (3.1)$$

for top ($i = 1$) and bottom ($i = 2$), respectively. The stratification allows a Richardson number to be determined for each layer:

$$Ri_i = \left(\frac{N_i d}{U} \right)^2. \quad (3.2)$$

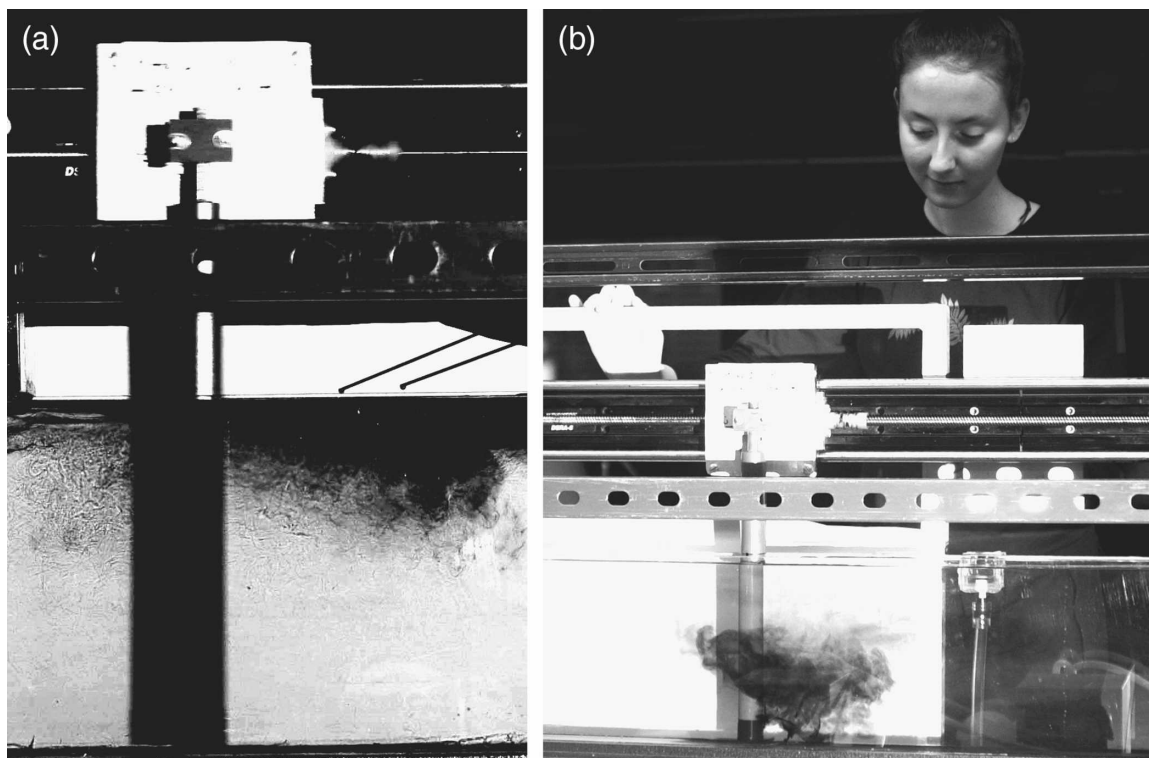


FIG. 4. (a) Shadowgraph of dye mixing down with the freshwater pumped in at the top. (b) Mixing up of the bottom dense water near the mixing rod.

This definition for the Richardson number uses the rod diameter d as a length scale because eddies that are shed from the rod will have this length scale; thus, it can be considered a Richardson number for the turbulence.

Based on the two-layer concept, we can also pick out a depth of the top layer δ for each profile. Depth location (selected by eye) is shown by the circles, triangles, and squares in Fig. 6.

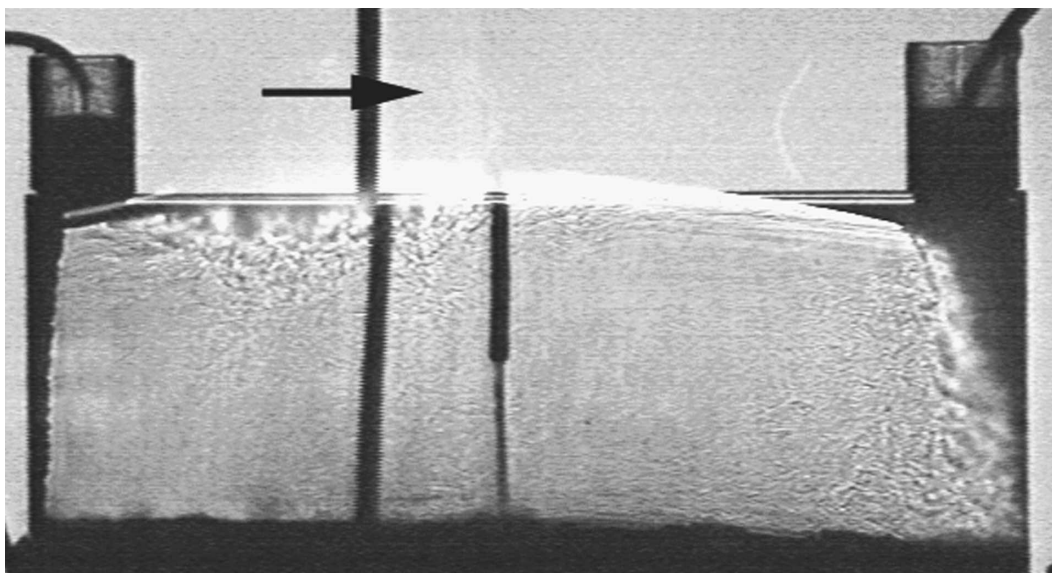


FIG. 5. Enhanced-contrast shadowgraph of the experiment. To the left is mixing behind the moving rod (that travels toward the right) and to the right is the sinking turbulent plume. The tubing that drains the spillway is in the middle. This apparatus is for demonstration and is smaller than the main experimental apparatus.

TABLE 1. The parameters of the experiments.

$g'(\text{cm s}^{-2})$	U (cm s^{-1})	Q ($\text{cm}^3 \text{s}^{-1}$)	d (cm)	D (cm)	Duration (h)
22.3	7.00	1.38	1.27	19	9
			1.80	19	10
			2.54	19	24
		3.00	1.27	19	6
			1.80	19	5
			2.54	19	5
		4.85	1.27	19	10
			1.80	19	5
			2.54	19	6
	6.00	1.38	1.27	19	12
			1.80	19	12
			2.54	19	12
		3.00	1.27	19	12
			1.80	19	5
			2.54	19	5
		4.85	1.27	19	12
			1.80	19	6
			2.54	19	5
	5.00	1.38	1.27	19	12
			1.80	19	10
			2.54	19	13
		3.00	1.27	19	12
			1.80	19	10
			2.54	19	6
		4.85	1.27	19	8
			1.80	19	9
			2.54	19	7
	3.14	1.38	2.54	19	11
		3.00	1.27	19	42
12.1	7.00	3.00	1.27	19	6
			2.54	19	5
	6.00		2.54	19	6
4.0	5.00		2.54	19	5
	7.00	3.00	2.54	19	5
	6.00		2.54	19	5
	5.00		2.54	19	5
			1.27	19	7
22.3 (outflow at the surface)	7.00	1.09	2.54	19	20
	7.00	3.00	1.27	21	12
			1.80	21	7
			2.54	21	5
	6.00		1.27	21	12
			1.80	21	5
			2.54	21	5
	5.00		1.27	21	11
			1.80	21	12
2.54			21	6	
22.3 (depth of water is 39 cm)	7.00	3.00	1.27	39	14
			1.80	39	8
	6.00		1.80	39	12
			2.54	39	7
	5.00		1.27	39	16
			2.54	39	13

The data reveal surprisingly systematic results, even though a variety of driving parameters was used. First, the data confirm that F is an important dimensionless driving parameter. Figure 7 shows the measurements of

Ri_1 that are taken from values of g' and δ from all the profiles shown in Fig. 6. The data (circles) display a linear log-log trend plotted against the flux number F and show a good correlation. The slope is close to the 5/3 slope shown by the line. For comparison, the plus symbols show Ri_1 plotted against $F/Ri_m = (Q/Ud^2)$, and there is no indication of a linear trend and a very low value of correlation of about 0.06. Therefore, the data indicate that F is the main driving parameter.

4. Theoretical interpretation

Simple models of the experiment

A streamfunction was created from the density distributions of each profile so that the results can be compared with those presented by Rossby (1965), whose theory for the thermally driven flow assumed viscous (Stokes) flow driven by differential heating along a horizontal boundary. To start with the simplest possible case, we use the equations of viscous flow in a Hele-Shaw cell. This is found using the assumption that the primary drag on the fluid is from the sidewalls; therefore, we set the velocity proportional to the pressure gradient:

$$u(z) = c \frac{\partial p}{\partial z}. \quad (4.1)$$

Second, we assume that the density in the mixing area is constant as a result of the stirring of the rod. The velocity profile of the lower layer of the circulation is

$$u(z) = c[p(z) - p_0(z)] = c \int_{\delta}^z g(\rho - \rho_0) dz, \quad (4.2)$$

and the streamfunction Ψ is given by

$$\Psi = \int_{\delta}^D u dz = cg \left[\int_{\delta}^D \int_{\delta}^z \rho(z') dz' - \frac{1}{2} \rho_0 (D - \delta)^2 \right]. \quad (4.3)$$

Figure 8 exhibits the dependence of the maximum value of the streamfunction on the Rayleigh number (defined by the density difference in the upper layer $g'_1 = \delta N_1^2$). The absolute value of $\psi_{\max}$ signifies much greater flow rates than those observed because molecular values of viscosity and salt diffusion are used, so we only consider the trend of the dataset. The 1/5 power law is the same as that derived theoretically by Rossby.

Another simple model also includes drag, but it is embedded in plume theory and therefore not readily apparent. It starts with a balance in the thermocline between mixing, which drives light water downward

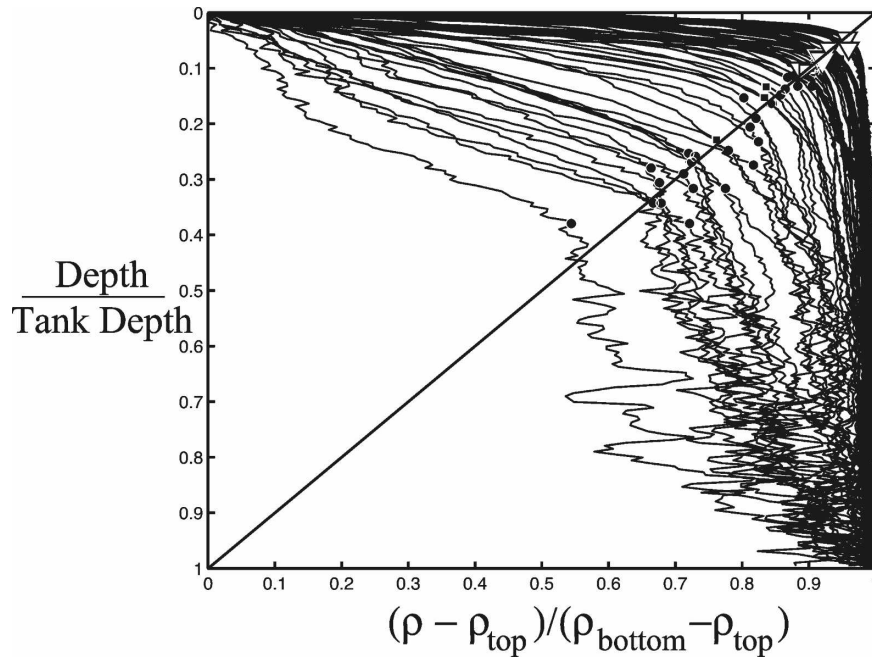


FIG. 6. Profiles of normalized density for all the runs. The depths of the upper layers are shown by dots and triangles.

and an upward flow of dense water from the bottom to the surface. As in the experiment described in section 2, light water is pumped in at the surface at one end of the tank and dense water with excess gravity g'_0 is pumped in at the other end, both with a volume flux rate Q . Water leaves through a spillway at the top in the middle with a volume flux rate $2Q$ and a reduced gravity $g'_0/2$.

The Reynolds number of the mixing rod is large enough so that the mixing is turbulent, and the turbulence mixes light water downward and dense water upward, resulting in a mixture of dense and light water. We take this mixture to be a layer of uniformly mixed water of depth δ above a layer of uniform-density deep water. The origin to measure this depth is the floor of the spillway rather than the free-water surface, which varies with Q . The dense water descends as a turbulent plume through this layer and mixes, emerging through

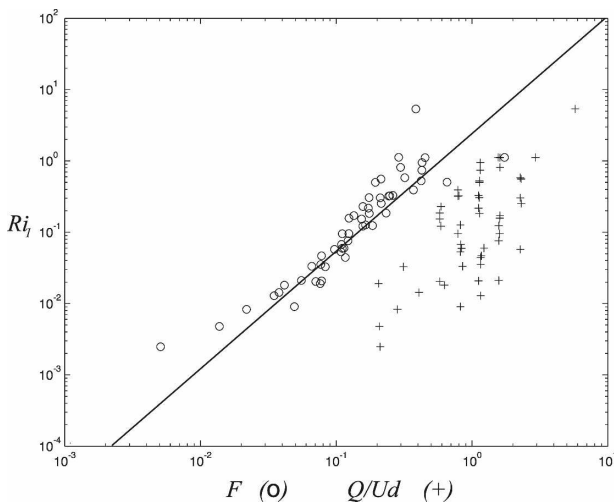


FIG. 7. Top-layer Richardson number vs the two dimensionless numbers for the experiment: flux number F (o) and Q/Ud^2 (+) [this is Eq. (2.1) divided by (2.2)]. The regression value r^2 of the log comparison is 0.92 for the first and 0.06 for the second.

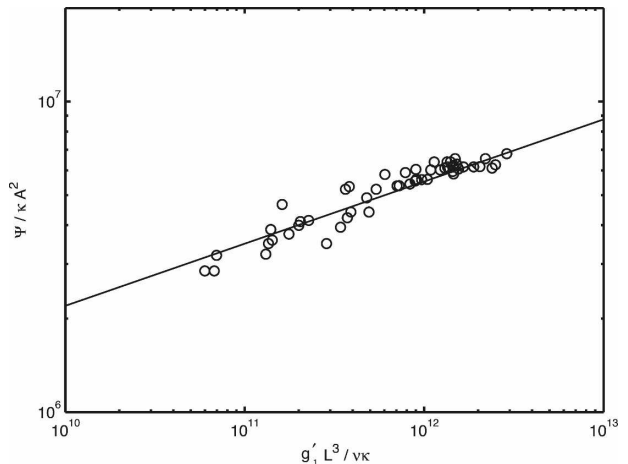


FIG. 8. Relation between the streamfunction and Rayleigh number. The regression value r^2 of the log comparison is 0.96.

the bottom of the layer as a fluid of reduced gravity g' . It then spreads out under the entire layer of light water and uniformly upwells under it to produce an advective-mixing balance. We desire to predict values of δ and g' as functions of the experimental settings. As in the experiment, there are four forcing variables: volume flux rate, reduced gravity of the dense water, velocity of the rod, and rod diameter (Q , g'_0 , U , d). We assume for this simplest model that surface area of the chamber, the length of the excursion of the rod, and the depth of the tank are not important.

For the mixing, we take the dense flux governed by a simple advection-diffusion balance:

$$w \frac{dS}{dz} = \kappa \frac{d^2 S}{dz^2}, \quad (4.4)$$

with constant vertical flow. Three simplifications are made: first, that the continuous profile is replaced by a layer of light water above heavy water; second, that there is a vertical velocity w from below into this layer; and third, that diffusivity acts downward to keep this layer at a constant depth. Replacing the differential equation with a simple scaling equation gives

$$\frac{wS}{\delta} = \frac{\kappa S}{\delta^2}, \quad (4.5)$$

then

$$\delta = \frac{\kappa}{w}. \quad (4.6)$$

Next, let

$$w = \frac{Q + \frac{1}{2} Q_e}{A}, \quad (4.7)$$

where Q_e is the flux rate of water from the layer entrained into the plume and A is a cross-sectional area of the mixing. The area of mixing is taken to be the area where the upwelling actually occurs in the layer, which is the area behind the traveling rod in the layer

$$A = d\delta. \quad (4.8)$$

This might seem to be a peculiar definition because the area in the wake of the rod is not even normal to the upwelling direction. It is best to imagine that the area gets tilted by 90° behind the traversing rod and that there is upwelling everywhere in that area in the upper layer. Alternatively, one can think of the water in this area as being mixed behind the rod and then moving up.

The next component in the simple model is that the water in the plume descends through the layer of light-

ened water and supplies deep water to the bottom. The equation for conservation of density in the plume leads to a bulk formula, which is found by writing the salt conservation first, and then multiplying by $g\beta$:

$$g'_0 Q + \frac{1}{2} \left(\frac{1}{2} g'_0 + g' \right) Q_e = g' (Q + Q_e), \quad (4.9)$$

which is easily rearranged to

$$Q_e = \frac{2(g'_0 - g')}{g' - \frac{1}{2} g'_0} Q. \quad (4.10)$$

Equations (4.6)–(4.8) are combined to give

$$\frac{\kappa d}{Q + \frac{1}{2} Q_e} = 1, \quad (4.11)$$

and using (4.10), the bottom buoyancy is

$$g' = \frac{1}{2} g'_0 \left(\frac{Q}{\kappa d} + 1 \right). \quad (4.12)$$

This can be rewritten in dimensionless form dependent on F by adopting $\kappa = Ud$ [this is the only combination of U and d with units of diffusivity ($\text{m}^2 \text{s}^{-1}$)] and multiplying by dU^{-2} , namely,

$$\frac{\left(g' - \frac{1}{2} g'_0 \right) d}{U^2} = \frac{1}{2} F. \quad (4.13)$$

Two comparisons with data are made to see whether this has any correspondence with the data. In the first (Fig. 9), the densities at the bottom on the upper layer are compared directly with (4.12) using a linear plot. Better agreement was found when we set $\kappa = 0.7Ud$, which was used in the figure. The data with $\text{Re} \geq 1000$ track the sloping line that corresponds to (4.12) better than the data with $\text{Re} < 1000$. There is significant scatter, which is appropriate for such a crude first approximation. This theory has no stratified lower layer, but because Fig. 6 shows that in the experiment the stratification of the upper and lower layers is proportional, the stratification of the lower layer also has a similar graph.

To make the second comparison, we calculate the density difference between the top layer and the deep fluid, and then divide it by the layer depth to create a relation for stratification:

$$N_1^2 = \frac{\left(g' - \frac{1}{2} g'_0 \right)}{\delta}; \quad (4.14)$$

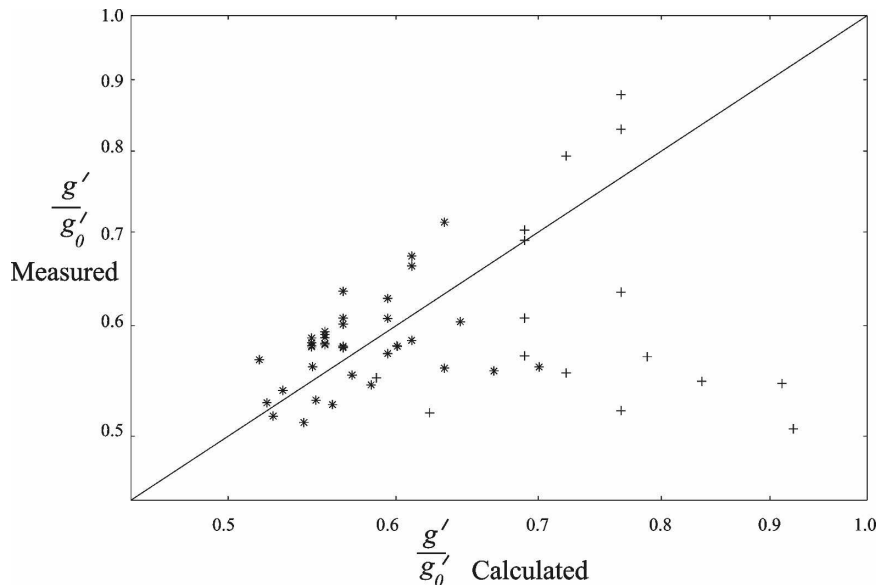


FIG. 9. Comparisons between Eq. (10b) and measurements of reduced density at the bottom of the upper layer. The data shown by an asterisk have $Re \geq 1000$ and a regression value r^2 of 0.40. The crosses have smaller values of Re and the regression value is close to zero.

using (4.13), it becomes

$$\frac{\delta dN_1^2}{U^2} = \frac{1}{2} F. \quad (4.15)$$

This is compared to measurements of N_1^2 and δ in Fig. 10 with a log-log plot. Over a broad range of F from a bit over 0.005 to about 2, data fit the linear relation given by (4.15) very well, with an almost perfect least squares fit to the slope of 0.999. The fit is far from perfect, however, because the scatter is large enough to make the correlation coefficient only about 0.8, and the constant in front of F is closer to 2.7 than to 1/2. Certainly, influences other than those expressed by (4.4)–(4.15) could produce considerable scatter, and certainly the theory does not use well-established constants of proportionality, so the results are probably as good as can be expected.

Next, the depth of the upper layer is compared with theory. To determine this depth, turbulent plume theory is used. The plume is descending through a layer whose average buoyancy is $\frac{1}{2}(\frac{1}{2}g'_0 + g')$, and it has a “reduced” buoyancy flux with respect to the buoyancy of the layer of magnitude $B_r = 2[g'_0 - \frac{1}{2}(\frac{1}{2}g'_0 + g')]Q$. Using Turner (1973; his Eq. 6.1.6) for the change of buoyancy with depth of a plume (using the value $\alpha = 0.1$), the buoyancy at the bottom of this sinking plume is

$$g' = 9^{1/3} \frac{50}{6} B_r^{2/3} \delta^{-5/3}. \quad (4.16)$$

This contains a factor of 2, because in the experiment only half the plume can entrain ambient water owing to the presence of the wall. Using Eq. (4.12), this results in

$$\begin{aligned} \delta &= \left(\frac{72Q^2}{g'_0} \right)^{1/5} \left[\left(1 - \frac{1}{2} \frac{Q}{\kappa d} \right) \right]^{2/5} \left[\frac{50}{6} \left(1 + \frac{Q}{\kappa d} \right)^{-1} \right]^{3/5} \\ &= \delta_0 \left(1 - \frac{1}{2} \frac{Q}{\kappa d} \right)^{2/5} \left(1 + \frac{Q}{\kappa d} \right)^{-3/5}, \end{aligned} \quad (4.17)$$

where $\delta_0 = 8.39[(Q^2/g'_0)]^{(1/5)}$.

The scale of this depth can be understood using an energy balance in which the potential energy increase is

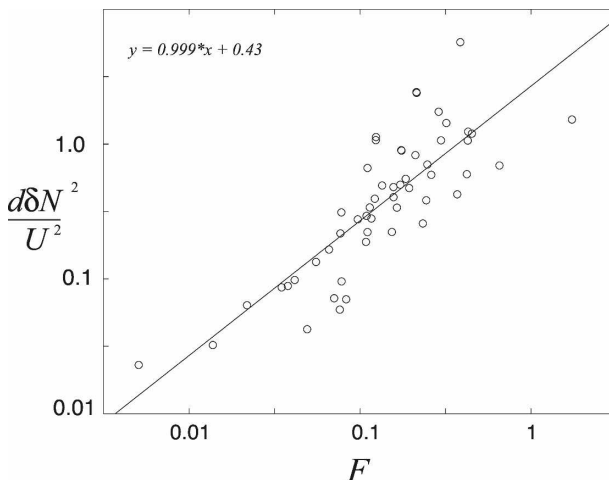


FIG. 10. Test of Eq. (4.15) in log space. The least squares fit line has a slope of 0.999 with a regression value r^2 of 0.82.

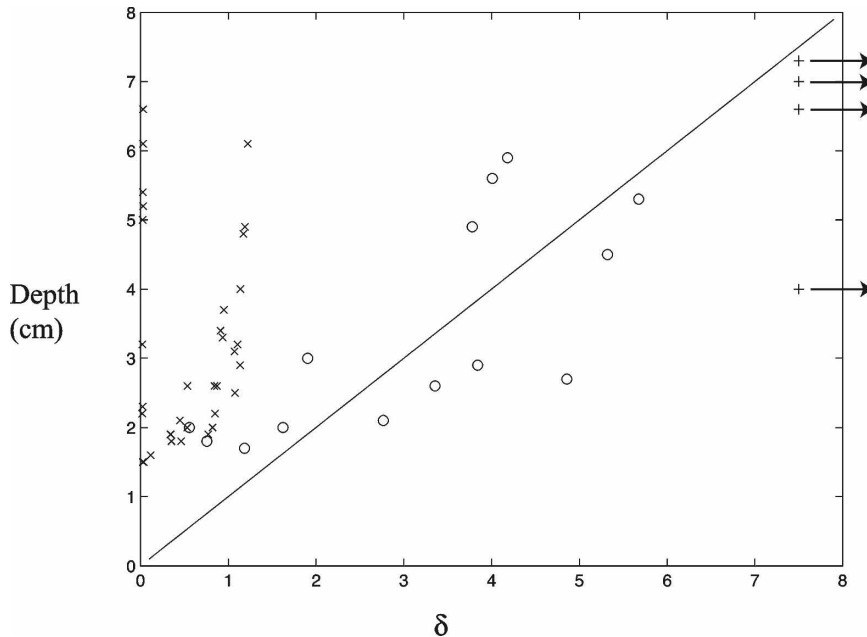


FIG. 11. Prediction of depth of the layer from Eq. (4.17) compared with measured depth. Here, x represents $(\delta_0/d) < 0.5$, o represents $0.5 < (\delta_0/d) < 3.6$, and $+$ represents $(\delta_0/d) > 3.6$. The middle data have a regression value of $r^2 = 0.74$. The others are close to zero and are therefore uncorrelated.

produced by turbulent stirring of a stratified fluid. The rate of energy from the kinetic energy of stirring E_k is proportional to the volume per unit time intersected by the moving rod times the kinetic energy imparted to the fluid so that $E_k \sim U^3$. A small portion of E_k raises potential energy by working against buoyancy, and the rest is dissipated by viscous energy dissipation. We adopt a simple layer concept in which mixing results in the freshwater being pumped down at a rate Q to depth δ in a field of buoyancy proportional to g'_0 so that the potential energy increase is $E_p \sim g'_0 Q \delta$. Setting $E_p \sim E_k$ gives $\delta \sim (U^3/Qg'_0)$. The mixing in the wake of the traveling rod is governed by overturning of the stratified fluid. Therefore, one might expect that it is regulated by a “governing” Richardson number, $Ri_g = (g'_0 \delta / U^2)$. If we eliminate U , this gives the depth scale of $\delta \sim (8Q^2 Ri_g^3 / g'_0)^{1/5}$; if Ri_g is of some fixed value, then $\delta \sim \delta_0$.

The results of the experiments are tested against Eq. (4.17) in Fig. 11. We found that all the data for $0.5 < (\delta_0/d) < 3.6$ had rough agreement with such a model and that values above and below this range had poor agreement between Eq. (4.17) and the measurements. The values above this range predicted a layer depth δ much deeper than the tank, as indicated by the arrows. The values below this range had unreasonably small measured values of δ . Many of these experiments had the smaller Reynolds numbers $Re < 1000$, and thus mixing was probably not very effective. However, there

may be some other unknown criterion violated by the experiments below the range as well.

5. Discussion

The intent is to reproduce the essential features of Rossby’s experiment and Munk’s picture of abyssal circulation using turbulence to mix a salt-stratified fluid. The experiment accomplished this successfully. The water developed a clearly defined upper-stratified layer above a second deep layer with another value of stratification. A simple theory shows a vertical circulation balance between downward buoyancy flux from mixing opposed by the upwelling of water that is emplaced along the bottom by a turbulent sinking plume. Turbulence is found both in the region that is artificially mixed by a rod (corresponding to the tropics and mid-latitude) and in the sinking plume region (corresponding to the polar regions and the deep ocean cataracts).

The qualitative nature of the flow pattern is in agreement with the crude picture of a region with broad upwelling and narrow sinking, although admittedly the source sizes were not systematically varied. Visual observations clearly confirmed that the broadness of the upwelling regions is produced by lateral spreading of lighter fluid in a stratified environment, both in the source region, where freshwater spreads away, and in any mixed location, where a lens of mixed water seeks

its neutral level. The sinking region is confined not only because the dense water sinks straight down but also because there is a flow toward the turbulent plume at midlevel to supply water for entrainment. The upwelling is found in conjunction with mixing from the rod.

The assumption that the area of upwelling is given by the area behind the moving rod [Eq. (4.8)] is not directly tested and must be regarded as still tentative. In reality, the active turbulence is found over a wider area than the rod diameter and the turbulence extends from top to bottom of the rod rather than the layer. Both would support the use of an area proportional but larger than the area used here. Also, the turbulence exists behind the moving rod over a distance that depends on how quickly the turbulence collapses. This distance has not been taken into account.

The second assumption that $\kappa \sim Ud$ is also tentative, because mixing should be dependent on stratification. The greatest dependence on stratification is found for a large local Richardson number (Fernando 1991), which in this model is $g'_0 d/U^2 \gg 1$. However, these experiments have a local Richardson number that is not very large. Of the 53 runs, 22 have $g'_0 d/U^2 < 1$, another 25 have $1 < g'_0 d/U^2 < 2$, and only 6 have $g'_0 d/U^2 > 2$; thus perhaps the local Richardson number is small enough for many of these experiments to be in the range where $\kappa \sim Ud$. For small stratification, there is experimental evidence that dependence on stratification is weak (Fernando 1991), because the flux Richardson number is proportional to Ri. We stress again that $\kappa \sim Ud$ has not been directly tested here. A third assumption that the buoyancy flux of the plume is constant is valid for $\delta/D \ll 1$.

The experiment suggests a simple balance of potential energy generation rates. The evolution of the potential energy above a plane $z = 0$ is described by the equation

$$\frac{\partial \langle gz\rho \rangle}{\partial t} = -gz \frac{\partial \langle w\rho \rangle}{\partial z} + gDz \frac{\partial^2 \langle \rho \rangle}{\partial z^2}, \quad (5.1)$$

where the brackets denote horizontal averages and D is the molecular diffusivity of salt. Zero flow and zero diffusion boundary conditions have been applied at the sides of the container. The Boussinesq approximation has been used in the continuity equation. For the experiment, the diffusion term is very small and can be set to zero; then, if one integrates by parts,

$$\frac{\partial \langle gz\rho \rangle}{\partial t} = g \langle w\rho \rangle - g \frac{\partial \langle zw\rho \rangle}{\partial z}. \quad (5.2)$$

The evolution of the density field through one increment of time is sketched in Fig. 12. It is assumed that

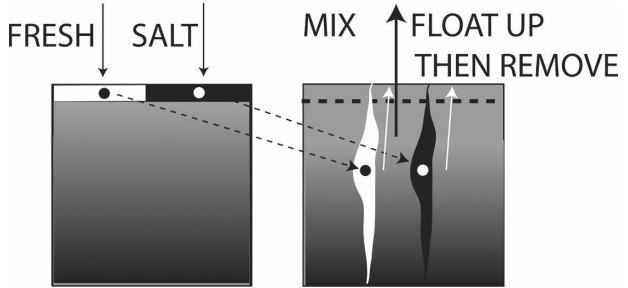


FIG. 12. Sketch of the potential energy changes of fresh- and saltwater.

the system has come to a steady equilibrium so that the graduated gray fluid, which represents the stratified fluid, is steady. The time increment illustrated in the figure is meant to be small compared to the time it takes for the pumps to renew fluid in the tank, and the balance we describe below is valid for arbitrarily small time increments. First, in the left-hand side, the fresh-water (white) and saltwater (black) are added at the top. The center of gravity of each box of injected fluid on the left is just below the top surface. Then, both the fresh- and saltwater are carried down as shown in the right-hand side, where they are mixed. The distribution of white and black with depth is arbitrary, but both are the same. The filled circles and the dashed arrows show the change in the center of gravity of fresh- and saltwater between the left-hand and right-hand sides. The potential energy increase from the downward movement of the fresh (white) fluid and the corresponding upward movement of the ambient stratified fluid represent the increase from downward flux of freshwater by turbulent mixing in the experiment. The potential energy decrease from the sinking of the salty (dark) stratified fluid and the upward displacement of lighter stratified fluid represent the energy release of the plume. In both cases, the ambient stratified water is pushed sideways and slightly displaced upward to satisfy conservation of volume. Then, we mix the bodies of salt- and freshwater, and because the mixed fluid has the same density as the fluid at the top of the stratified layer, it buoyantly floats to the top (white arrows) and the stratified water returns to its initial shape. Therefore, the density distribution of the stratified water plays no part in the final energy budget. Finally, the mixed water is removed by the spillway, as shown by the dashed line and the arrow. The cycle starts again as the salt- and freshwater fill the void left by the removed mixed water.

In summary, the potential energy change for the white fluid moved downward is positive, and the change of the black fluid moved downward is negative; they have the same magnitude of change compared to

the rebounding mixed fluid, so the change in potential energy of the stratified fluid is the same for both. The net potential energy change is zero because the steady-state density field is fixed and unchanged, thus the sum of the two is zero. This suggests that the potential energy increase from mixing (of heat) down within the model ocean equals the potential energy release of the cold dense polar plumes.

A similar argument holds for the thermal convection experiments of Rossby (1965), Mullarney et al. (2004), and Hughes and Griffiths (2006), and for the experiments with rotation by Park and Whitehead (1999). The rate of potential energy production by vertical conduction (assuming top differential heating) equals the rate of decrease of potential energy by convection in the sinking region. For an experiment with differential heating along the top surface, the decrease rate for potential energy from sinking at the dense plume is

$$\dot{E}_p = g\rho_0 Q \delta \alpha \Delta_\delta T, \quad (5.3)$$

where the dot denotes a time derivative, the plume volume flux rate at the top of the plume is Q , the temperature difference between the plume source temperature and the bottom temperature is $\Delta_\delta T$, and the total thermal boundary layer depth is δ . The terms comprising a buoyancy flux,

$$\dot{B} = g\rho_0 Q \alpha \Delta_\delta T, \quad (5.4)$$

are frequently grouped together. Because this flux is conserved as the plume fluid descends, the rate of decrease of potential energy is

$$\dot{E}_p = \dot{B} \delta, \quad (5.5)$$

and energy released equals the buoyancy flux times the boundary layer depth.

Does the energy balance implied by Fig. 12 hold true in the ocean? Is the increase of potential energy from turbulent mixing throughout the ocean in fact equal to the release of potential energy in ocean sinking regions? It would seem so, at least to the extent that the net potential energy of the oceans is not changing with time. Because we do not have direct measurements of the total potential energy of the ocean for various instants in time, there is no way to know if the potential energy remains constant. Thus, we postulate that in the ocean, if potential energy is constant, the increase in potential energy from all of the internal mixing equals the decrease in potential energy from surface negative buoyancy flux.

It is beyond the scope of this study to make a precise value of the ocean's potential energy as a function of time. However, the upper ocean is sampled sufficiently

to give a simple picture of the possible magnitude of potential energy change rate. Recent estimates indicate a net increase of temperature [therefore, an increase in heat content Lyman et al. (2006)]. The thermal expansion moves the center of gravity of the water mass away from the earth's center of mass. In a box of uniform temperature with the dimensions of the ocean, a temperature change $\Delta_i T$ over some period of time t would result in a change of potential energy of magnitude:

$$\Delta_i E_p = \frac{1}{2} g \rho_0 A D^2 \left[1 - \frac{1}{(1 - \alpha \Delta_i T)^2} \right] \cong g \rho_0 A D^2 \alpha \Delta_i T \quad (5.6)$$

(because $\alpha \Delta_i T \ll 1$), where D is the box depth and $A = 3.21 \times 10^{12} \text{ m}^2$ is the area (Sverdrup et al. 1942, their Table 4). The thermal energy change in the box is $\Delta_i E_T = \rho_0 C_p A D \Delta_i T$ and the ratio is $E_r = \Delta_i E_p / \Delta_i E_T = g D \alpha / C_p$. Lyman et al. (2006) report a worldwide warming rate of the upper 700 m of the ocean of 0.33 W m^{-2} for 13 yr, which translates to a potential energy change rate of the upper ocean $\dot{E}_{pu} = 0.33 E_r = 5.8 \times 10^{-5} \text{ W m}^{-2}$ (using $g = 10 \text{ m s}^{-2}$, $\rho_0 = 1027 \text{ kg m}^{-3}$, $D = 700 \text{ m}$, $\alpha = 10^{-4} \text{ }^\circ\text{C}^{-1}$, and $C_p = 4000 \text{ J kg}^{-1} \text{ K}^{-1}$). In the spirit of Wright (1972), let us compare this to a typical sinking plume in the ocean, which would increase potential energy at the rate $\dot{E}_p = g \rho_0 Q D \alpha \Delta T = 1.6 \times 10^{11}$ (using a plume volume flux of $Q = 2 \times 10^6 \text{ m}^3 \text{ s}^{-1}$, a temperature difference of $\Delta T = 20^\circ\text{C}$, and a mean ocean depth of 4000 m). In estimating the loss of global potential energy (GPE) from cooling, Huang and Wang (2003) made a rough estimate based on global estimates of buoyancy flux and found that it is about $2.4 \times 10^{11} \text{ W}$, which is on the same order as estimated here. However, when the diurnal cycle is considered, they found that the loss of the GPE may increase greatly to a value about 10 times larger. The plume fluid can be thought to be spread out under the entire ocean area of $3.14 \times 10^{12} \text{ m}^2$, resulting in $5.1 \times 10^{-2} \text{ W m}^{-2}$, which is a thousand times greater than $\dot{E}_{pu}$. This comparison indicates that a balance between sinking potential energy release and turbulent potential energy generation is approximately steady and that the fluxes are much greater than any observed evolution of the potential energy stored in the ocean heat field.

Naturally, this is a crude estimate. Accurate worldwide budgets for potential energy generation would be wonderful to have, but the mixing rates required for such a budget are quite complex to measure directly because of numerous challenges, including the need to measure dissipation virtually everywhere, the difficulty of measuring at sinking locations in winter, the lack of information about all sinking regions, the need to in-

clude the salt budgets, and complications involved with the complete equation of state. In addition, upward heat flux through the ocean floor is an additional factor that is not discussed here.

6. Conclusions

This study of turbulent mixing in a model ocean environment has yielded the following major conclusions:

- A steady flow is achieved.
- The sinking plume is an inevitable result of mixing a fluid that is subjected to different surface buoyancy fluxes.
- The laboratory plume potential energy flux equals the negative of the mixing potential energy flux.
- Therefore, the intensity of sinking regions in the ocean (chimneys, plumes, and streamtubes) is linked to the global intensity of ocean mixing.

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