13

Laboratory Experiments With Abrupt Thermohaline Transitions and Oscillations

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13.1. INTRODUCTION

Climate records indicate that ancient ocean temperatures occasionally change by many degrees within climatologically "fast" times (order of 50 years). Some of these are attributed to abrupt transitions of the thermohaline circulation regime [Broecker et al., 1985; Boyle, 1990; Keigwin and Jones, 1994; Keigwin et al., 1994; Bard et al., 1996; Broecker, 1997; Stocker and Wright, 1991; Stocker, 2000; Burns et al., 2003; Weart, 2003; and many others]. In addition, some numerical ocean circulation models produces abrupt transitions. The changes involve both salinity and temperature (henceforth always called thermohaline) changes [Bryan, 1986; Cessi, 1994; Rahmstorf, 1995; Manabe and Stouffer, 1995; Whitehead, 1998; Rahmstorf and Ganopolski, 1999, Weaver et al., 1999]. The dynamics of such abrupt transitions is formulated in a pioneering mathematical box model study [Stommel, 1961]. This model has two well-mixed chambers of water connected side by side with one tube at the top and a second at the bottom. Both temperature and salinity diffuse through sidewalls to the chambers at different rates. Positive temperature and salinity diffuse into one chamber, and negative values diffuse in the other. The resulting flow has a range of the governing parameters in which there are two possible states, one with temperature dominance and the other with salinity dominance, each with a different flow rate and direction. Subsequent mathematical box models (reviewed by Marotzke [1994] and Whitehead [1995]) illuminate additional aspects that help us to understand how the abrupt thermohaline transitions arise and what their context might be. For

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example, ocean estuary mathematical box models have demonstrated abrupt thermohaline transitions [Hearn and Sidhu, 1999; Bulgakov and Skiba, 2003]. In actuality, no direct observations of abrupt thermohaline transitions in estuaries exist. Finally, abrupt transitions are also mathematically predicted for wind-forced convection [Stommel and Rooth, 1968] and in basins forced by surface stress alone [Ierley and Sheremet, 1995; Jiang et al., 1995].

Mathematical box models and numerical simulations both have drawbacks. Although numerous box experiments readily produce abrupt thermohaline transitions, they are always subject to the criticism that they restrict the number of degrees of freedom that the flows can adopt. Mathematical box models cannot account for the large number of degrees of freedom that actual flows can include, making abrupt thermohaline transitions more prevalent than actually exist. Even though numerical models show that the abrupt thermohaline transitions do not vanish, a more convincing way to illustrate whether or not abrupt thermohaline transitions actually exist in a physical system is by developing controlled laboratory experiments.

Abrupt transitions in fluid mechanics are commonplace. For example, the sudden stall of an airplane wing along with all the dangers it produces to pilots and passengers, has been continually in the minds of aviators since the early part of the twentieth century. Generally, abrupt transitions occur within a finite range of the parameters that govern the flow. The flow in this range can have either one of two modes. Each mode is locally stable and the flow can be made to abruptly jump back and forth from one mode to another. Therefore, the transitions are said to have *hysteresis*, since the flow is determined by history, as well as by the governing parameters. Hysteresis is obviously a significant challenge to climate modeling if it exists (just as in airplane design). Depending on details of the model's history, any given model might have one or the other climate.

The possibility of abrupt transitions is present in many areas of geophysical fluid dynamics. Virtually every natural body of fluid has two components that affect density and thereby the buoyancy flux of convection: temperature and water vapor affect air density, temperature and salinity affect ocean water density; temperature and different elements such as silicon, magnesium and hydrogen (water) affect the mantle density; temperature and sulfur affect the Earth's core, and heat and helium affect star density.

All aspects discussed above concerning abrupt thermohaline transitions have contributed to the motivation of producing the laboratory studies reviewed in this paper. In comparison to the many numerical and box models, so far, only a few laboratory experiments have produced these abrupt thermohaline transitions. The motivation to develop experiments with abrupt thermohaline transitions was also prompted by the fact that physical observations of thermohaline transitions in the ocean are unknown in historical or modern times. Therefore, before such experiments were completed, there were no direct scientific observations of the many theoretical ideas from simulations and models concerning abrupt thermohaline transitions. Devices to investigate abrupt thermohaline transitions require very precise controls of temperature, salinity, and in some cases, pumping rates. Generally, heat loss must be minimized too. Theory provides information on the parameter ranges required for hysteresis. This important information was used to design the devices. First, the flow must be driven by two components. Buoyancy force from temperature and salinity variations suffices for this. Furthermore, the boundary conditions must allow a flux of heat and salinity into the fluid at different rates. Then, the transitions are found when the buoyancy forces generated by temperature and salinity oppose each other.

Section 13.2 of this chapter reviews experimental observations of abrupt thermohaline transitions in the laboratory. This author and colleagues have performed all experiments, and much of the material is covered in *Whitehead* [2009]. Also, there is always a possibility that instead of an abrupt transition, the system oscillates back and forth between the two flow modes. Experiments that find such temperature and salinity oscillations are described in Section 13.3. Virtually no numerical simulations or ocean models produce similar oscillations, and their mechanism needs more investigation.

13.2. FOUR LABORATORY EXPERIMENTS SHOWING ABRUPT THERMOHALINE TRANSITIONS

Experimental apparatus used to find abrupt thermohaline transitions typically has one chamber containing fluid that is either heated or cooled. The chamber is connected by a passage consisting of some sort of opening to a reservoir kept at constant temperature and salinity. All known devices with enough precision to measure the range of hysteresis are sketched in Figure 13.1. The first one, called the box experiment (Figure 13.1a), has a well-mixed chamber connected laterally to a reservoir of fresh water at ambient room temperature (20° C) by two tubes one above the other [Whitehead 1996, 1998]. The chamber is heated below with a metal plate, and salt water is pumped into sponge at the top surface. Heating causes water in the chamber to become lighter, but in contrast, the salt water influx causes it to become denser. The time constants for temperature and salinity are controlled by salt water pumping rate, chamber surface area, and depth.

The second, called the *slot experiment* (Figures 13.1b and 13.2), is like the first except that a tube replaces the sponge salinity source at the top of the chamber, and a vertical slot replaces the two tubes [Whitehead et al., 2003]. Internal mixing from sinking of the salt water under the sponge is significantly reduced compared to the box experiment. Therefore, the possibility of one or more density layers within the chamber exists.

The third one, called the *layered experiment* (Figure 13.1c and 13.3), has the chamber connected to a reservoir with three tubes at the top, middle, and bottom [Whitehead et al., 2005, Whitehead and Bradley 2006]. It is cooled from above instead of heated from below, and the reservoir contains a layer of fresh water with salt water below it, with both layers kept at 20° C by a heat transfer device labeled ILE in Figure 13.3. An early explorator upside-down version of this 3-tube experiment had a chamber heated from below with a layer of salty water on the bottom below fresh water in the reservoir. This device is the first to document reproducible thermohaline oscillations [teRaa 2001]. Earlier runs by Bulgakov and his collaborators with the slot experiment also observed oscillations, but they were not reproducible.

The fourth, called the *cavity experiment* (Figure 13.1d) is geometrically the simplest. There is simply a cavity in the floor of a reservoir of fresh water at 20° C. The cavity is heated at the bottom, and salt water is pumped into it at a steady rate. The salt water is heated in the cavity and it either spills out of the top lip of the cavity, which lies slightly above the floor of the reservoir, or it rises as a thermal to the top of the reservoir. The reservoir has a specially designed drain at both top and bottom so it can be flushed well enough to maintain fresh water at 20° C.

The experiments have progressively greater degrees of freedom. The box experiment has well mixed water inside the chamber so that no layers exist. The flows are limited to two modes in and out of the top tube, with corresponding flow out and in of the bottom tube. The slot

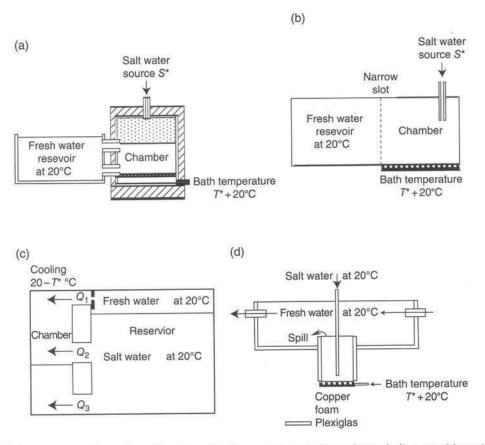
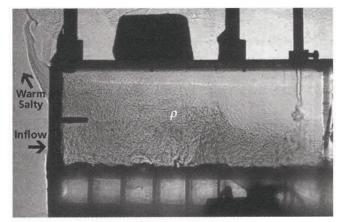


Figure 13.1. Four laboratory experimental configurations that have produced abrupt thermohaline transitions: (a) driving parameters T*, S*, and saltwater pumping rate (for a, b, and d) are indicated. (a) box experiment, (b) slot experiment, (c) layered experiment, and (d) cavity experiment. (Figure adapted from Whitehead 2009).

experiment is designed to limit the mixing so that layers in the box can exist. The layered experiment seeks to control flow in three layers, and the cavity experiment has fully three-dimensional flow.

As in theory and computer transitions, the abrupt thermohaline transitions that are produced in the experiments separate two flows that are distinctly different. This is most clearly seen in shadowgraphs of two flows on either side of an abrupt thermohaline transition in the slot experiment (Figure 13.2). Different names for the various modes are found in the literature, but for clarity in this review, we adopt a single set of names. The first such mode is flow driven primarily by temperature. The upper panel shows such a flow, and here it is called the temperature mode (called the T-mode in Whitehead et al. [2003]). This mode are found if the temperature of the bath above the reservoir temperature is greater in magnitude than a certain critical value T_T^* . Essentially, the flow is the same as the flow when salinity forcing is absent. In the top panel of Figure 13.2 this mode is shown by a shadowgraph. Water from the reservoir flows into the chamber at the bottom of the slot and hot salty water flows out at the top as indicated by the arrows. The salty water is dyed, and the shadowgraph indicates that there is a lot of small-scale turbulent mixing. The mixing is provided by convection at relatively high Rayleigh numbers. The salt water injected by the tube is mixed and diluted to such an extent that the salinity makes negligible contribution to density. The hot plume rises to the top of the chamber where it exits through the top of the slot. The lower picture shows the salinity mode (called the S-mode by Whitehead et al. [2003]). This can be found if bath temperature is below a second critical value T_S^* . Three layers exist. The salty water sinks to the floor almost as though heating were absent and it forms a hot layer of water that flows out of the chamber at the bottom of the slot. Fresh water from the reservoir flows into the chamber along the top interface of the hot salty layer and forms the middle layer. It is heated by thermal conduction from the hot salty layer and rises to the top of the chamber accompanied by convection cells. The hot, fresh water then forms the top layer what exits the chamber through the top of the slot. Hysteresis happens because the experiments show that $T_T^* < T_S^*$, consequently either flow is found for



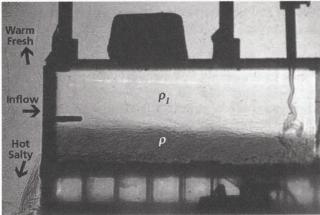


Figure 13.2. Side view of the slot experiment with the two different modes of flow at the same values of driving parameters. The top panel shows the temperature mode and the bottom panel shows the salt mode. BW rendition of a color figure [Whitehead et al., 2003].

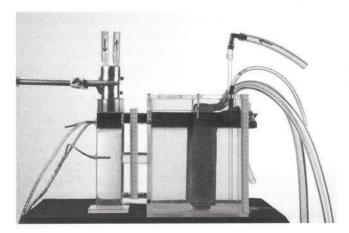


Figure 13.3. Layered experiment [Whitehead 2009].

bath temperatures between those two temperatures. The slot experiment is particularly appealing, because artificial mixing or suppression of mixing can trigger transition back and forth. If the salt mode exists, the flow can be converted to the temperature mode by inserting a small paddle into the chamber through the slot and mixing. If the temperature mode exists, the flow can be converted to the salt mode by inserting the paddle near the salinity source tube and suppressing the local turbulent mixing. Soon, a layer of salty water forms along the bottom and the salt mode forms.

The results are best summarized by plots of dimensionless temperature and salinity versus scaled bath temperature (Figure 13.4). For these runs, the value of salinity difference, and the pumping rates of salt water for the box slot, and cavity experiments are each kept at one constant value. Experimental runs with different values of pumping rate were also studied in many of the papers, but results are inferior due to poorer coverage over parameter space so they add little to the story. In all experiments, one complete run must always be conducted over a long enough time span to become convincingly steady. The necessary time is at least an hour and sometimes much longer. In regions of hysteresis, the two corresponding points with the same driving parameters are determined by how that particular run was initiated. One can start from an earlier run or start the apparatus with either fresh or salty water in the chamber. Transitions can be triggered by inserting temporary blocks in some of the tubes or by stirring. mentioned above. Naturally, in points without hysteresis. the same point is found no matter how the flow is started.

Scaling the laboratory results is quite simple. The bath temperature, the temperature in the chamber, and the salinity of water in the chamber are plotted here using the density difference between salt and fresh water: $T^* =$ $\alpha T^*/\beta S_0$, $\bar{T} = \alpha T/\beta S_0$, and $\bar{S} = S/S_0$, where α is the coefficient of thermal expansion for water at 20° C. B the density coefficient of salinity, and S_0 is salinity of the salt water pumped in at the source or, in the case of the layered experiment, the salinity of deep water. Since warm water has lower density than cold, the scaled temperature is inversely proportional to density but the scaled salinity is proportional to density. Thus, if scaled temperature and salinity are plotted together, (Panels a and c) the two opposing effects overlie each other and it is immediately observed as to whether temperature or salinity affects density more strongly. This is not true for Figure 13.4 panel b, where the scaled salinity is subtracted from unity to make the figure clear when compared to theory (shown by the curves).

Figure 13.4 shows a number of points. First density change from salinity always dominates over density change from temperature in the Salt Mode and density change from temperature always dominates over density change from salinity in the Temperature Mode. as expected. Second, the range of hysteresis is greatest for the box experiment and less so for the rest. A close relation between the experiments and box model theory similar to Stommel's are demonstrated in *Whitehead* (1996) and

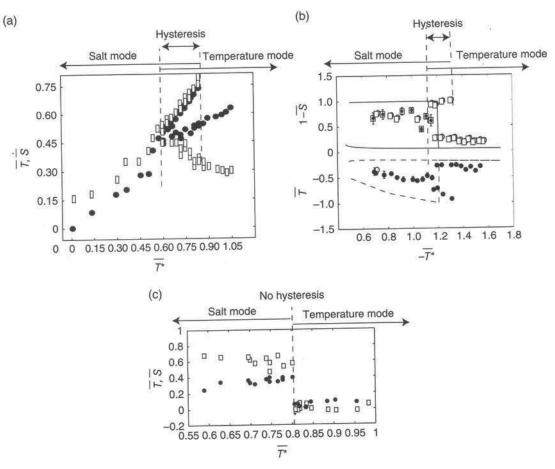


Figure 13.4. Measurements of dimensionless temperature (circles) and salinity (rectangles) versus driving temperature for (a) the box experiment, (b) the middle of the bottom layer of the layered experiment with the curves from the theory in *Whitehead*, [2000]. and (c) the middle of the cavity in the cavity experiment. The error bars in (b) are for the oscillation described in the next section and Adapted from *Whitehead* [2009].

Whitehead (1998), so it is not surprising to see sizeable hysteresis as the box model theory predicts. The theory for the layered experiment is approximately confirmed by the data in Figure 13.4b. The cavity experiment does not have a box model counterpart and hysteresis is too small to be resolved.

Figure 13.4 also indicates that density difference between the chamber and the reservoir changes sign upon an abrupt thermohaline transition from one mode to the other. This is more clearly seen with plots of dimensionless density from the box and cavity experiments (Figure 13.5). The greatest uncertainly arises because it is impossible to assign an exact value of temperature in the experiments since it varies within a chamber. Also, temperature at a point is time dependent, because active convection takes place. Only in the box experiment was the temperature in the chamber recorded over a long enough time to make good averages, and since the convection within the box is relatively even, the temperature is not

a very strong function of the location of the temperature probe. Therefore, scatter in Figures 13.4a and 13.5a is relatively small and the data exhibit straight trends. The cavity experiment temperature data are not as linear (Figure 13.4c and 13.5b), because parcels of water of differing temperature and salinity are always present inside the cavity. Although conceivably one could average over long times to smooth out the time dependence, in practice, this was not possible with our existing equipment. Also, for the cavity experiment in the temperature mode, the average temperature is a function of elevation of the probe above the bottom boundary layer. Therefore, the scatter of the temperature and density data is larger than optimal in Figures 13.4d and 13.5b.

13.3. OSCILLATIONS

Oscillations were first discovered in the slot experiment. These did not prove to be reproducible in that

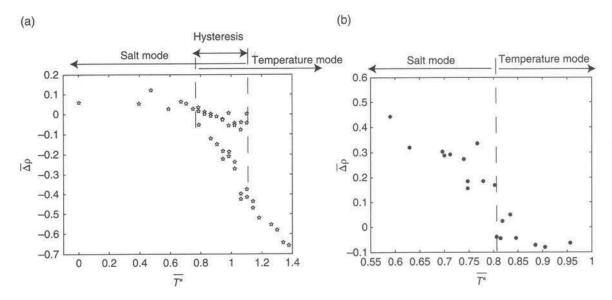


Figure 13.5. Density from (a) the box experiment and (b) the cavity experiment.

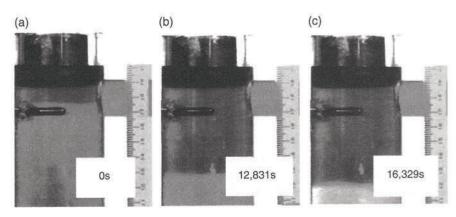


Figure 13.6. The oscillation cycle. (a) formation of a dyed freshwater layer over a very deep mixed layer of dyed salty water and clear salty water; (b) deepening of the top layer; (c) overturn and mixing of the deeper layer resulting in clear salt water lying below the top layer. Thereafter, a new fresh layer forms at the top as in (a). The previous top layer becomes the deep mixed layer, and the cycle repeats. [From *Whitehead et al.*, 2005].

experiment, so their existence is only mentioned briefly at the end of section 4.2 by Whitehead et al. [2003]. The next observations of oscillations were unexpectedly discovered in an early exploratory upside-down version of the layered experiment and were both robust and reproducible. The apparatus had a chamber heated from below with a layer of salty water flowing in along the bottom of a tank containing fresh water. The results are reported in a Geophysical Fluid Dynamics Fellow's report [teRaa, 2001]. This led to the development of the layered apparatus. The oscillations and their properties are documented more fully, and a supporting theory is developed in Whitehead et al. [2005]. This work was followed by oscillations reported by Mullarney et al. [2007]. They were unaware of the previous observations of oscillations. As

in the te Raa apparatus, a layer of salt water flows over a hot plate on the bottom of a chamber of fresh water, but the apparatus is more than ten times larger.

The oscillation process in all of these experiments is the same, and it is very simple. In the layered apparatus, a layer of dyed freshwater forms at the top (Figure 13.6a). The interface at the base of this layer begins to move downward and the layer becomes much deeper (Figure 13.6b). Below this is a deeper older layer. The color of the deeper layer begins to fade dramatically as the freshwater source to this layer has been blocked by the new top layer. After some time, the interface between the old layer and the bottom salt water develops large waves that begin to break. Mixing between this deep layer and bottom salt water increases and the color difference between the lower layer

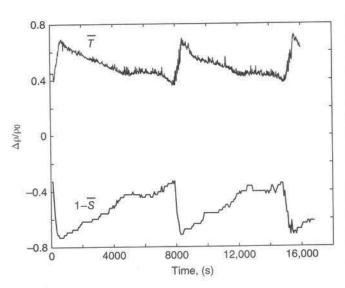


Figure 13.7. Density due to temperature and salinity versus time for a little over two experimental oscillation cycles. The top curve is from a temperature probe 2.54 cm below the top. The density (from salinity of the water) is withdrawn at a depth of 10 cm. The forcing strength is $-\alpha T^*/\beta S_0 = 0.85$.

and the salt water below it decreases. Suddenly the bottom layer's interface mixes away by direct overturning so that the top layer lies over purely salt water (Figure 13.6c). The cycle begins again as a new layer forms near the top. A time series of temperature at a depth of 2.54 cm and salinity at 10 cm depth is shown in Figure 13.7. The criterion for oscillations is $-\alpha T * /\beta S_0 < 1.2$. There is a small range where this overlaps the steady temperature mode, which is found for $1.15 < -\alpha T * /\beta S_0 < 1.35$.

The oscillation in the Mullarney et al. [2007] experiment is similar to the above oscillation although their apparatus was inverted compared to the layered experiment. It

had an electric heating pad on one end of the bottom with ambient fresh water in the tank. A layer of salt water flowed along the bottom and over the heater. The oscillation occurred because the layer of salt water simply spread over the heater and after some time interval it mixed away. This was followed by another layer spreading that was followed by mixing, repeating the cycle. The criterion they found for oscillation is $0.038 < B_S/B_T < 0.067$, which differs from the criterion for the layered apparatus. The first parameter is the buoyancy flux of heat, B_T $g\alpha F_T WL/\rho_0 c_p$. The second is the buoyancy flux of salinity, $B_S = g\beta S_0 q$. Here, g is the acceleration of gravity; heat flux per unit area is F_T ; the volume flux of salt solution is q; the box length is L; the box width is W; the reference water density is ρ_0 ; the water's specific heat is c_p ; and the other symbols are previously defined.

Neither experiment produces clear evidence of the complete criterion needed for oscillation. The balance is clearly between the formation of a layer and its destruction from mixing along the interface, separating the layer from ambient water. A simplified model of the oscillation [Whitehead et al., 2005] includes a freshwater layer that deepens with time from interfacial mixing with salt water below the layer. This is followed by complete mixing and the start of a new layer at the top. It seems to produce the effects observed in the laboratory (Figure 13.8).

13.4. SUMMARY AND DISCUSSION

Temperature-salinity abrupt transitions and oscillations can be produced in the laboratory, as demonstrated by four different experiments. It is necessary to precisely control all driving temperatures and pumping rates to hold the experiment in the range of hysteresis. The findings generally confirm the presence of abrupt thermohaline transitions that were predicted by mathematical box models

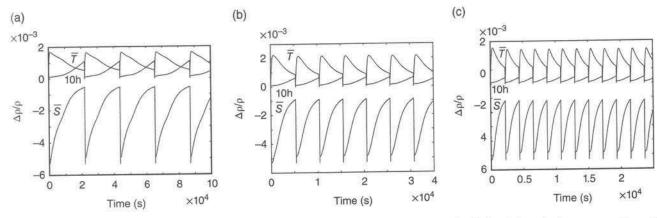


Figure 13.8. Time series from numerical calculations of density from temperature and salinity, (given in the same units as in Fig. 13.7) and layer depth h, (units multiplied by 10 and given in meters): (a) $-\alpha T^*/\beta S_0 = 0.5$ (b) $-\alpha T^*/\beta S_0 = 0.85$ (the value in Figure 13.7), (c $-\alpha T^*/\beta S_0 \le 1.15$. Adapted from Whitehead et al. [2005]).

and numerical simulations. Hysteresis appears to occupy a smaller range in experiments that allow more layers of stratified water in the interior [Whitehead, 2009]. In the cavity experiment the hysteresis range is less than the 1% range of resolution. The experiments uncovered nonlinear self-sustained oscillations that are not reported by box models and numerical simulations. These new oscillations exist in the hysteresis range of the layered experiment. The oscillations represent an excursion back and forth between layer formation and destruction by mixing so that the temperature, salinity, and flow patterns lie in two different "climates."

Double-diffusion processes include salt fingering, layer formation and flux between layers of two components. The two components affect buoyancy and have different diffusivities such as temperature and salinity [Turner, 1973, Schmitt, 1994]. The experiments described here most strongly indicate that double diffusive heat and salinity flux is clearly important during layer evolution in a number of the experiments. However, a close link between abrupt thermohaline changes and double-diffusion processes is almost completely unexplored, even though double diffusion can produce abrupt transitions (Veronis, 1965, Whitehead 2002). Experiments that are designed to clarify the role of double diffusion in conjunction with abrupt transitions or oscillations would be valuable. They would form a more complete understanding of two-component fluid mechanics.

We are not aware of any existing numerical experiments that possess no hysteresis with abrupt transitions as in the cavity experiment. Thus, this issue remains an open area of research. It is also clear that the manner in which thermohaline flows either oscillate or jump back and forth between different states as abrupt transitions is still poorly understood. All these are surprising since both nonlinear oscillations and abrupt transitions are understood mathematically.

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