Laboratory Studies of Turbulent Mixing

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Laboratory measurements are required to determine the rates of turbulent mixing and dissipation within flows of stratified fluid. Rates based on other methods (theory, numerical approaches, and direct ocean measurements) require models based on assumptions that are suggested, but not thoroughly verified in the laboratory. An exception to all this is recently provided by direct numerical simulations (DNS) mentioned near the end of this article. Experiments have been conducted with many different devices. Results span a wide range of gradient or bulk Richardson number and more limited ranges of Reynolds number and Schmidt number, which are the three most relevant dimensionless numbers. Both layered and continuously stratified flows have been investigated, and velocity profiles include smooth ones, jets, and clusters of eddies from stirrers or grids. The dependence of dissipation through almost all ranges of Richardson number is extensively documented. The buoyancy flux ratio is the turbulent kinetic energy loss from raising the potential energy of the strata to loss of kinetic energy by viscous dissipation. It rises from zero at small Richardson number to values of about 0.1 at Richardson number equal to 1, and then falls off for greater values. At Richardson number greater than 1, a wide range of power laws spanning the range from −0.5 to −1.5 are found to fit data for different experiments. Considerable layering is found at larger values, which causes flux clustering and may explain both the relatively large scatter in the data as well as the wide range of proposed power laws.

Diffusion; Eddy diffusivity; Entrainment; Mixing efficiency; Stratification; Stratified shear flow; Turbulence

Introduction

As described elsewhere in this encyclopedia, turbulence and diffusion act both vertically in mixing up the density field of the ocean and laterally in mixing adjacent density and tracer fields. Vertical mixing of density in a stratified fluid dissipates turbulent kinetic energy by raising the potential energy of the density field, and the ratio of this dissipation to viscous dissipation of turbulence is a fundamental quantity needed to understand the energy balance of the ocean. Estimates of these rates by theory are not yet readily available in a form useful for the ocean. Numerical approaches and computational fluid dynamics are under development, but they are not capable of investigating a wide range of parameter space in present form, nor can results be recovered that are verified by experimental benchmarks. Direct ocean measurements rely on theoretical assumptions about the form of turbulence, which must ultimately be verified in the laboratory. Therefore, laboratory measurements continue to be essential to the quest of determining the rates of turbulent dissipation within flows of stratified fluid.

Both the production of turbulence through instability and its dissipation are markedly different from the case of instability and dissipation within a homogeneous fluid. Ignoring internal wave radiation from turbulent regions, dissipation is partitioned between viscous dissipation and the work that increases potential energy. This is shown below by the two integrals for energy of a simple system with no internal body forces and closed bottom and top boundaries in a field of gravity. In this example, the flow must start with an initial value of kinetic energy, and the decrease in kinetic energy of a volume of incompressible fluid is equal to the rate of buoyancy work plus viscous dissipation:

$$\frac{1}{2}\frac{d\langle \tilde{v}\cdot\tilde{v}\rangle}{dt} = -\frac{g}{\rho_0}\langle \rho w\rangle - \nu\left\langle \left(\nabla\tilde{v}\right)^2\right\rangle$$

The change in potential energy above a bottom plane set to zero is the rate of buoyancy work plus buoyancy flux multiplied by elevation and it reduces to:

$$g\frac{d\langle z\rho\rangle}{dt} = -g\langle \rho w\rangle + gh\left[\rho w\right]_{z=h}$$

where angle brackets are averages over all three spatial dimensions, and the square brackets are over the two lateral dimensions. Here, $\tilde{v}$ is the three-dimensional velocity vector, ν is viscous diffusivity, the variables z, w are the vertical direction and velocity (in the direction of gravity g), and density is ρ with a constant reference density ρ_0. Clearly, if the following three conditions are met, then the only two terms that can dissipate the kinetic energy are viscous dissipation and buoyancy (heat) flux: first, the potential energy is not changing in time; second, dense (considering it to be cold) fluid enters the bottom with lighter (warm) fluid leaving the top (it would require a downward heat flow into the volume to allow this); and third, the volume is given an initial value of kinetic energy that is allowed to run down. It is the purpose of laboratory experiments to allow measurements of density and velocity fields and to obtain the partition between viscous dissipation and buoyancy flux. No instrument exists for precisely measuring every term within any of the above brackets, so simplified approaches have been necessary.

Experiments

Two types of experiments generate the turbulence, either a shear-flow instability is set up or eddies are directly generated. In addition, there are two groups of density distribution, one with sharp interfaces and the other having continuous stratification. Since both of the dissipation terms shown above are negative, there are no cases with both of the equations in their steady form. Experiments incorporate either transient setups that run down with time, or utilize flowing tanks (mostly with salt-stratified water but a few wind tunnels with thermal stratification are used too). In the latter case, the flows are transient following a fluid parcel.

The techniques to produce eddies are numerous. Figure 1 shows some of the laboratory configurations. Some generate turbulence behind grids in tunnels (Figure 1(a)) and others are driven by buoyancy (Figure 1(b)). In some flumes and wind tunnels, narrowing the sides enhances the shear in a test region. Other experiments have a moving grid or rod stirrer (Figure 1(c)), and still others have a moving lid (Figure 1(d)). Not sketched are special studies of pumped jets directed toward an interface, and experiments with double-diffusion driven flows, both being directed toward explicit mechanisms of mixing. Experiments have been motivated by numerous phenomena in addition to oceanographic ones; some examples are fire prevention, ventilation, snow avalanches, ecosystem studies, solar ponds, mixing of industrial chemicals, and turbidity currents.

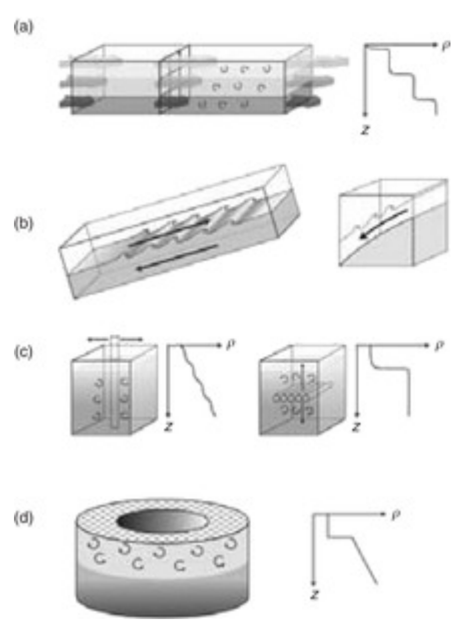

Figure 1

Configurations of various laboratory experiments to measure stratified mixing.

Flows have been produced in pumped tunnels, as sketched in Figure 1(a), with the stratification produced by either temperature (using heat-transfer devices) or salinity (using multiple sources). A variation is a closed-circuit tunnel with a special device for turbulence-free propulsion of the water. In some cases the vertical salinity distribution is initially set and the density profile runs down with time. Another variant has layers pumped in at multiple elevations with the same amount withdrawn at each corresponding level downstream and returned to cisterns. Also, there are currents driven by buoyant flow in tanks with sloping tops and bottoms, as shown in Figure 1(b), or with exchange flows in passages between reservoirs containing waters of differing salinity. Then, there are experiments in closed containers with oscillating grids or moving rods as sketched in Figure 1(c). In some cases the experiments are in annular chambers and some are rotating on a turntable (Figure 1(d)). Salt-stratified experiments are the most numerous. They possess a vertical salinity distribution that evolves with time, although there are also thermal experiments using air or water motivated by engineering applications.

The dynamics are all characterized by velocity scale of the turbulence u_* (which may be the same size as velocity difference in a sheared laminar flow whose instability generates the turbulence) and density difference $\Delta\rho$. The force of gravity makes the density difference equivalent to a buoyancy difference $g'=g\Delta\rho/\rho_0$. Geometrically, there is the separation distance d between regions of different velocity and density. Finally, there are two additional fluid properties, the viscosity ν and density diffusivity D. Velocity, reduced gravity, length, viscosity, and diffusivity can be reduced to many combinations of three dimensionless numbers. For stratified mixing, they are picked sequentially to represent the important balances in the flow in rank order. The primary dimensionless number is the bulk Richardson number $Ri = g'd/u_*^2$. It is a measure of the ratio of buoyancy to fluid inertia. The second number is the Reynolds number $Re = u_*d/\nu$, which is the ratio of inertial to viscous force. The third is the Schmidt number $Sc=\nu/D$. It is the ratio of viscosity to density diffusivity. Experiments generally have the objective to determine a buoyancy work rate (frequently called buoyancy flux) as a function of these three numbers. By the early 1990s it was clear that the buoyancy flux obeyed a range of power-law relations with *Ri*, and that these are sensitive to details of actual experiments and values of *Re* and *Sc*.

Virtually any source of turbulence can be used to mix stratified fluid, and one of the challenges of experiments is to separate the influence of the spatial distribution of the turbulent source from the actual processes within the fluid. Turbulence that is shed from a vertical rod that moves laterally and sheds a turbulent wake in fresh water above a salt layer (Figure 2(a)) produces striations that are strongest near the interface and weaker at higher and lower levels. This produces a divergence in buoyancy flux so the interface gets progressively thicker with time. However, continuous stratification (Figure 2(b)) can spontaneously break down to internal layers. Therefore, in both cases, the flux varies locally. To complicate matters, the variation of the stratification is usually about the same size as the scale of the turbulence, so in almost all experiments it is not obvious that statistical turbulence theory applies.

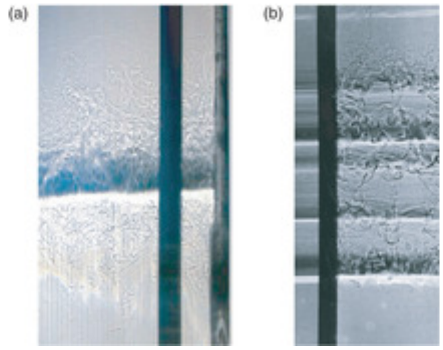

Figure 2

Shadowgraphs from a parallel beam of light falling onto a screen. They show the effect of turbulence produced by many excursions of a moving rod (a) for a layered fluid as in [Figure 1(c)] right (salt water under fresh, the rod has recently reversed direction near the tank wall); and (b) for a stratified fluid that breaks down into layers as in [Figure 1(c)] left (stratified salt water, the rod is moving toward the left into a placid, previously mixed fluid).

Probably the simplest laboratory configuration has salt water under fresh water with grid stirring confined to one layer, for example, the top layer. This easily attains high Reynolds numbers using a horizontal grid moving up and down with vigorous oscillatory motion. To determine u_* at the level of the interface, grid velocity should be multiplied by a suitable constant to account for spatial variation of the turbulence between grid and interface. As time progresses, for $Ri>1$ the turbulence in the upper layer causes the interface to remain sharp and it mixes salt water up into the top layer. This 'entrains' salt water into the top layer, which increases both the volume and salinity of the top layer and decreases the volume of the bottom one but leaves its salinity unchanged. The interface moves downward with entrainment velocity u_e, and the speed quantifies the mixing rate. As density difference between the layers decreases, Ri decreases. The entrainment velocity increases steeply with decreasing Richardson number as shown by solid circles in Figure 3. For $Ri<1$, the interface deflection is as large as d and the subsequent mixing rate is rapid and soon the two layers mix completely.

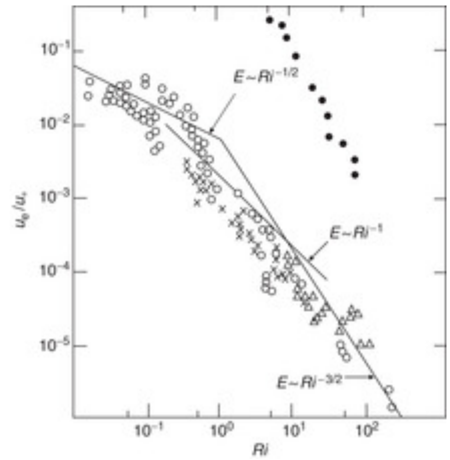

Figure 3

Entrainment velocity as a function of Richardson number for assorted experiments. Solid circles: Grid experiments where the grid velocity is to define u_*. Open circles: Buoyant outflows. Triangles: Density currents. Crosses: Counter-flows. Data and slopes taken from Fernando HJS (1991) Turbulent mixing in stratified fluids. *Annual Review of Fluid Mechanics* 23: 455–493, figure 15, with permission from author, and Turner JS (1973) Buoyant convection from isolated sources. *Buoyancy Effects in Fluids*, pp. 165–206, Figure 9.3. Cambridge, UK: Cambridge University Press, with permission from author.

Many other experiments have produced entrainment velocity measurements; a collection of some is shown in the lower cluster of Figure 3. In some of them, the mixing is supplied by shear instability driven by a rotating screen in an annulus with stratified fluid (Figure 1(d)). A mixed layer with a sharp density jump at the bottom penetrates into the stratified fluid. Others involve gravity currents (Figure 1(b)), with buoyant outflows and with counter-flows. All of these configurations successfully give useful data for large stratification (Ri<1). The scatter in the points shown in Figure 3 is typical and is due to the statistical nature of the data rather than instrumental error. In such experiments, the horizontally averaged density typically breaks up into patches and layers so that local regions have different local values of Richardson number. In spite of such scatter, the results of such experiments are overwhelmingly consistent with each other with respect to the general trends of the data shown in Figure 3. Primarily, the Richardson number is the most important variable governing mixing rate if it is of order 1 or more. The Reynolds number does exhibit some role especially if less than *c*. 500. Experiments to date range up to almost $Re<10^5$ and generally speaking the mixing is sensitive to Reynolds number for the entire range. It is thought that mixing will finally become insensitive for very large Re but data up to such a possible limit are not yet available.

Three power laws are sketched as straight lines in this figure. To the left the open circle data have a smaller slope. To the right, the data are inversely proportional to a higher power of Richardson number. The constants of proportionality are functions of the actual configuration and the manner of defining velocity and density. As an example, the relation found by Kato and Phillips for shear-driven experiments is

$$\frac{u_e}{u_*} = 2.5\ Ri^{-1}$$

This can be rearranged to $g\Delta\rho u_e d = 2.5\rho_0 u_*^3$, which states that the rate of change of potential energy is proportional to the rate of delivery of kinetic energy for mixing.

The value of the exponent for power-law dependence has been extensively studied and discussed. There are some circumstances in which there is no dependence because stratification effects are smaller than viscous, diffusive, or turbulent effects. In other cases there are over 40 proposed relations with *Ri*, but there is still no clear consensus about the range of validity for each of these in *Ri*, *Re*, and *Sc* space. This lack of agreement seems to arise for a number of reasons. First, no tight cluster about one line is found because of the scatter mentioned above. Second, it has always been found that the results tend to be specific for each experimental configuration. Third, the experiments are in water or air, so only a few values of *Sc* are investigated. One proposed relation between the three dimensionless numbers has the dimensionless entrainment at successively increasing *Ri* proportional to $Ri^{-3/2}$, $ScRi^{-1}$, and $Sc^{-1/2}Re^{-1/4}$. It shows that there is an increasingly important role for molecular and turbulence effects as stratification is increased. However, in listing 30 such relations, Fernando found that the −1.5 power law generally tended to be for higher values of *Ri* rather than lower values.

A gravity current down a slope is of particular interest to oceanography because of its relevance to deep overflows in polar regions and salt plumes in arid regions. In such studies the Froude number $Fr = u_*/\sqrt{g'd\cos\theta}(\approx Ri^{-1/2})$ (θ is angle of the slope) is often used to measure the intensity of mixing compared to stratification. Results for a wide range of gravity-driven currents are shown in Figure 4. As in Figure 3 it is clear that large stratification suppresses entrainment velocity, and that $u_e \approx u_*$ with smaller stratification. In addition, it is clear that mixing is enhanced at larger Reynolds numbers.

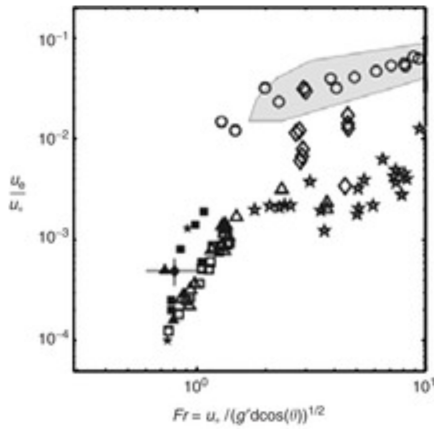

Figure 4

Entrainment velocity in laboratory gravity currents down a slope compared to estimates of ocean overflows. The solid triangle is for Lake Ogawara, the solid square is for Mediterranean outflow into the Atlantic, the small star is for the Denmark Strait overflow, and the solid diamond is for the Faroe Bank channel overflow. For rotating density currents, the open squares, open triangles, and large stars are experiments with *Re*<100 and the open diamonds are with *Re*≥100. The shaded area and open circles are found for large Reynolds number nonrotating density currents. Supplied by C. Cenedese.

Continuous Stratification

The layers of mixed fluid in mechanical stirring experiments are separated by very sharp interfaces, so for large values of *Ri* the layers remain well defined. Therefore, the results are relatively precise and easy to interpret. Of course, experiments with continuous stratification are more similar to the ocean. Continuously stratified fluid exposed to turbulence tends to break up into layers for large *Ri*. Therefore, *N* varies locally and hence the dimensionless number varies locally. As a result, the mixing becomes concentrated in local regions. In addition, as time progresses the layers evolve and slowly change flux. The dynamics that determine the size of the layers remain controversial. In some cases, the layer depth scales with the scale u/N, and in other cases the scales are linked to vortical modes shed from the stirrers, or even to the stirrer size itself. In continuously stratified experiments, the Richardson number can be defined as $Ric=(Nd/u_*)^2$ (defined either locally or over longer distances) or, if shear du/dz is imposed, as $Ris=(N/(du/dz))^2$.

The invention of the bathythermograph led to the discovery of extensive layering within the ocean. Although this layering can be explained as a consequence of localized wave breaking, its universal character suggests that there is a more fundamental cause. Laboratory experiments all exhibited the spontaneous growth of layers for $Ric \gg 1$, including early experiments with stirring near the sidewall of a stratified fluid and later ones with continuously stratified fluid stirred with a rod or towed grid. Figure 5 shows layer growth in salt-stratified water with grid-generated turbulence (with *d* set to the mesh size) for *Ric*=10.7. In some elevations the local stratification (as measured by locally defined *N*) increases, and at other elevations it decreases toward zero. This has a profound influence on propagation of both internal and acoustic waves through the water. As time progresses beyond the stage shown in the figure, the layers begin to interact with each other and some will be eliminated, so finally two layers remain with only one interface in between. Ultimately, the density difference between these two layers decreases to zero, and the fluid becomes fully mixed. Thus, the entire sequence is inherently transient in nature. It has proved to be difficult to attribute the emergence of the layers and their subsequent evolution to one particular mechanism and the transient character may be responsible for the difficulty.

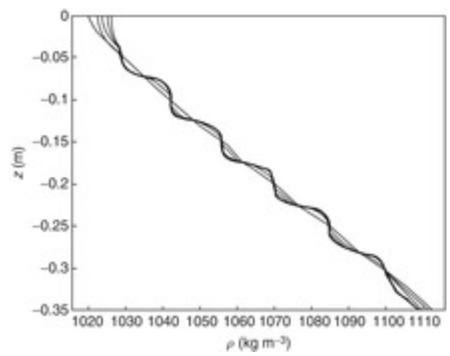

Figure 5

Density profiles at successive times showing the spontaneous emergence of layers in a continuously stratified fluid for large *Ric*. Adapted from Rehmann CR and Koseff JR (2004) Mean potential energy change in stratified grid turbulence. *Dynamics of Atmospheres and Ocean* 37: 271–294, with permission from Elsevier.

In contrast, such an experiment with values of *Ric* approximately 1 or lower (which includes experiments before the 1980's) does not produce layers (Figure 6). Instead, a mixed layer forms at the bottom and top of the fluid. Simultaneously, the interior stratification gradually decreases. The result is a fluid with values of *N* decreasing everywhere.

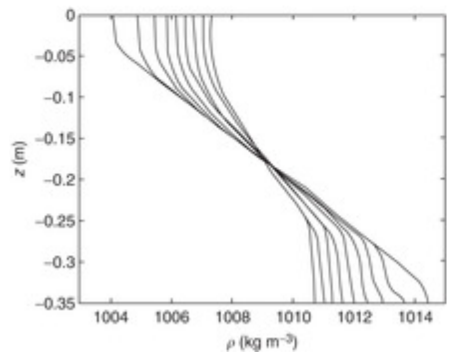

Figure 6

The evolution of a density field with stirring source throughout the entire fluid and with *Ri*≤1. Adapted from [Rehmann CR and Koseff JR (2004)] Mean potential energy change in stratified grid turbulence. *Dynamics of Atmospheres and Ocean* 37: 271–294, with permission from Elsevier.

Figure 5 and Figure 6 were produced with data from experiments with the configurations shown in Figure 1(c). Accurate resolution of the vertical density field by a conductivity microprobe (developed for stratified turbulent flume measurements) allows precise measurements of the change of potential energy with time. This change is quantified by a flux Richardson number *Rfc*, defined as rate of change in potential energy divided by power (rate of energy) exerted by the stirrer (which is estimated for the grid using known drag laws). The resulting data are shown in Figure 7. Recent DNS results approximately agree with this. Starting from *Ric*=0, experiments with increasing values have increasing values of *Rfc*, which level off at a value *Rfc*≃0.067 at *Ric*≃1.

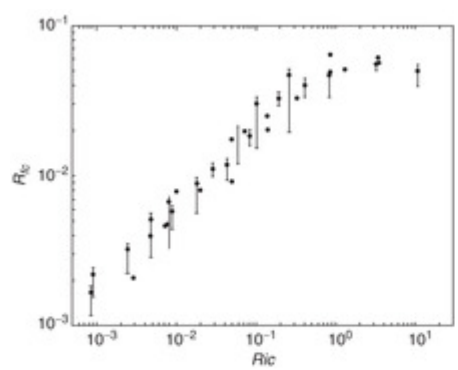

Figure 7

Mixing efficiency in grid-stirring experiments with continuous stratification and small *Ri*. Adapted from [Rehmann CR and Koseff JR (2004)] Mean potential energy change in stratified grid turbulence. *Dynamics of Atmospheres and Ocean* 37: 271–294, with permission from Elsevier.

Both layered and continuously stratified experiments have a flux Richardson number that reaches its maximum value of Rfc=0.1 at a Richardson number of order 1, with values decreasing toward 0 for larger and smaller values. The exact theoretical value of the maximum has been widely discussed, with some estimates approaching a maximum value of 0.3 and others only reaching a maximum value of 0.05 or so. Naturally, the exact definition depends on the choice of a length and velocity scale, which is not only a matter of choice, but also subjected to the details of each apparatus and analysis technique. In addition, since most experiments are either transient or possessing a variation in space, the measurement location and time can influence a value. Thus, a maximum of Rf≈0.1 with a range of roughly ±50% has remained unchanged over the last 20 years. Most of the experiments have been conducted with Reynolds numbers of a few thousand or less, and with salt rather than temperature but both the range and the number of components has increased recently. The coverage in *Re* and *Sc* space remains limited. Generally, larger Reynolds number produces smaller maximum values (Figure 8).

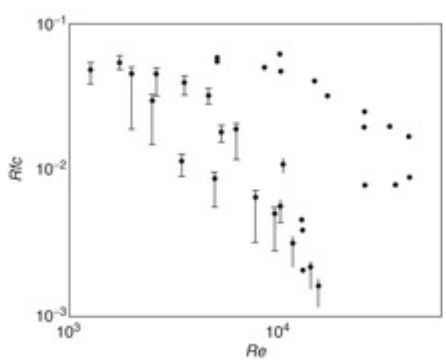

Figure 8

Mixing efficiency as a function of Reynolds number in mixing grid experiments of a uniformly stratified fluid. Stratification was from salinity in some experiments, temperature in others, and from both in one case. Adapted from [Rehmann CR and Koseff JR (2004)] Mean potential energy change in stratified grid turbulence. *Dynamics of Atmospheres and Ocean* 37: 271–294, with permission from Elsevier.

Other experiments have been conducted with salt and temperature contributing to the density. If the value of *Ric* for each component differs, the phenomenon called ‘differential mixing’ is said to exist. The experiments show a strong dependence on the intensity of the turbulence and the diffusivity ratio ranges from 0.05 to 1. Differential mixing (of components with different diffusion) is expected to be most important in oceanic regions where both temperature and salinity variations influence density. Therefore, for climate studies it is important to incorporate differential mixing accurately in numerical models since the salinity field is known to influence deep wintertime convection.

Summary

In summary, laboratory experiments to measure the rate of mixing of stratified fluids by turbulence are conducted using a wide range of devices. Both qualitative and quantitative data provide information to oceanographers and numerical modelers. To estimate the mixing rates and their consequences in many ocean regions from the top mixed layer to the abyss, information about flux Richardson number from controlled laboratory measurements continues to be vital. The buoyancy flux ratio rises from 0 at small Richardson number to values of about 0.1 at Richardson number equal to 1, and then might or might not fall off for greater values. At Richardson number greater than 1, a wide range of power laws spanning the range from −0.5 to −1.5 are found to fit data for different experiments. Effects from different diffusivities are large. The layering in this range causes flux clustering and may contribute to the relatively large scatter in the data and to the wide range of proposed power laws.

See also

Differential Diffusion. Energetics of Ocean Mixing. Estimates of Mixing. Vortical Modes.

Further reading

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