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A Theoretical Model of the Flow in the Mouth of Spencer Gulf, South Australia[a]

John A. T. Bye[1] and John A. Whitehead Jr[2]

[1]*Flinders Institute for Atmospheric and Marine Sciences, Flinders University of South Australia,*
[2]*Woods Hole Oceanographic Institution, Woods Hole, Mass. 02543, U.S.A.*

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The salinity distribution in Spencer Gulf, South Australia, indicates an inflow of low salinity water on the West side of the Gulf, modification of the water mass at the northern end and an outflow of water of increased salinity on the East side. This system appears to be an excellent example of a theoretical model recently proposed by Whitehead, Leetmaa & Knox (1974) for flushing controlled by a buoyancy–inertial current.

Introduction

Recently an analysis was made of the thermohaline structure of the water masses of Spencer Gulf, South Australia (Bullock, 1975). It was observed that the density distribution was largely determined by the variation of salinity over the Gulf. Very high salinities (>42‰) exist at the northern end (Figure 1) and across the mouth (Figure 3) there is an increase of salinity from West to East, with a frontal zone in the center.

It was conjectured that this pattern implied an inflow of water from the Southern Ocean on the West of the Gulf, with a modification of salinity by evaporation at the northern end and an outflow on the eastern side. To test this hypothesis, a numerical model was constructed of the circulation associated with the observed density distribution. The model confirmed the circulation pattern and moreover gave an exchange rate which was consistent with the evaporation rates observed in the Gulf (Figure 2). The model also tested the idea that wind had a significant effect on this circulation, with negative results.

The cause of the salinity contrasts in Spencer Gulf was, however, not studied and it was assumed to be intimately related to the topography of the Gulf. It may be that there is a more fundamental reason. This is investigated in the following theoretical model.

The theoretical model

Whitehead, Leetmaa & Knox (1974) have recently analyzed a situation in which two basins containing waters of different densities are connected by a long, narrow strait. A flow and counterflow is predicted, driven by the density inbalance. The interface separating the two fluids in the strait is predicted to be tilted a specified amount due to the effect of rotation of the reference frame. Here we will attempt to apply the predictions of this analysis to the flow at the mouth of Spencer Gulf. To accomplish this we will imagine that one basin is infinitely

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large and contains water with a uniform salinity of 35·9‰ and a σ of 26·3 ($\sigma_t = (\rho - 1) \times 10^3$, where ρ is density in kg/l). We will imagine that the other basin represents Spencer Gulf itself, which will be assumed to have a surface of $1{\cdot}2 \times 10^{10}$ m² and have a specified evaporation rate. We will not be concerned with the mixing process in each basin but will assume that both basins are mixed enough to be homogeneous and yet are not in a state of turbulence sufficient to violate the hydrostatic assumption necessary for the theory of flow through the mouth.

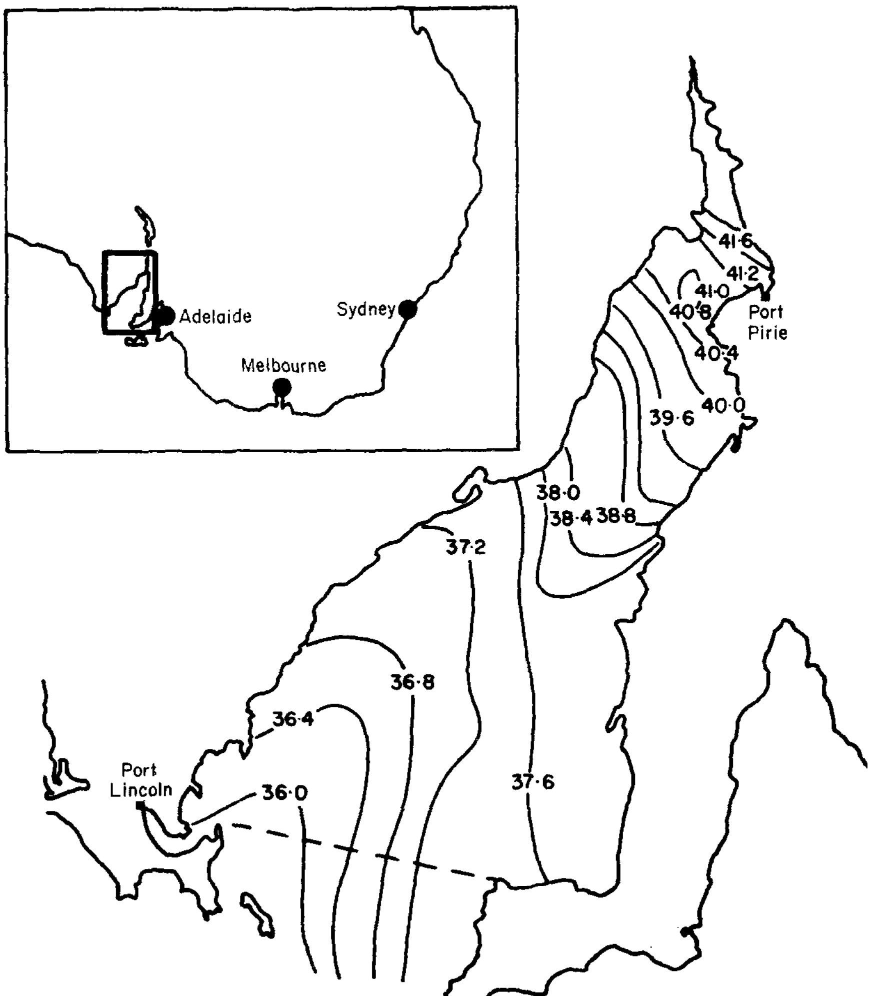

Figure 1. Depth-averaged salinity in units of parts per thousand within Spencer Gulf (from Bullock, 1975).

The mass–flux balances at the connecting channel are that the mass of water flowing in equals the mass of water flowing out plus the mass evaporated in the Spencer Gulf side. Since salt is presumed to not be appreciably produced in the Gulf, the salt flux in equals the salt flux out. The equations stating these two balances are:

$$Q_i = Q_o + Q_e, \tag{1}$$

$$S_i Q_i = S_o Q_o, \tag{2}$$

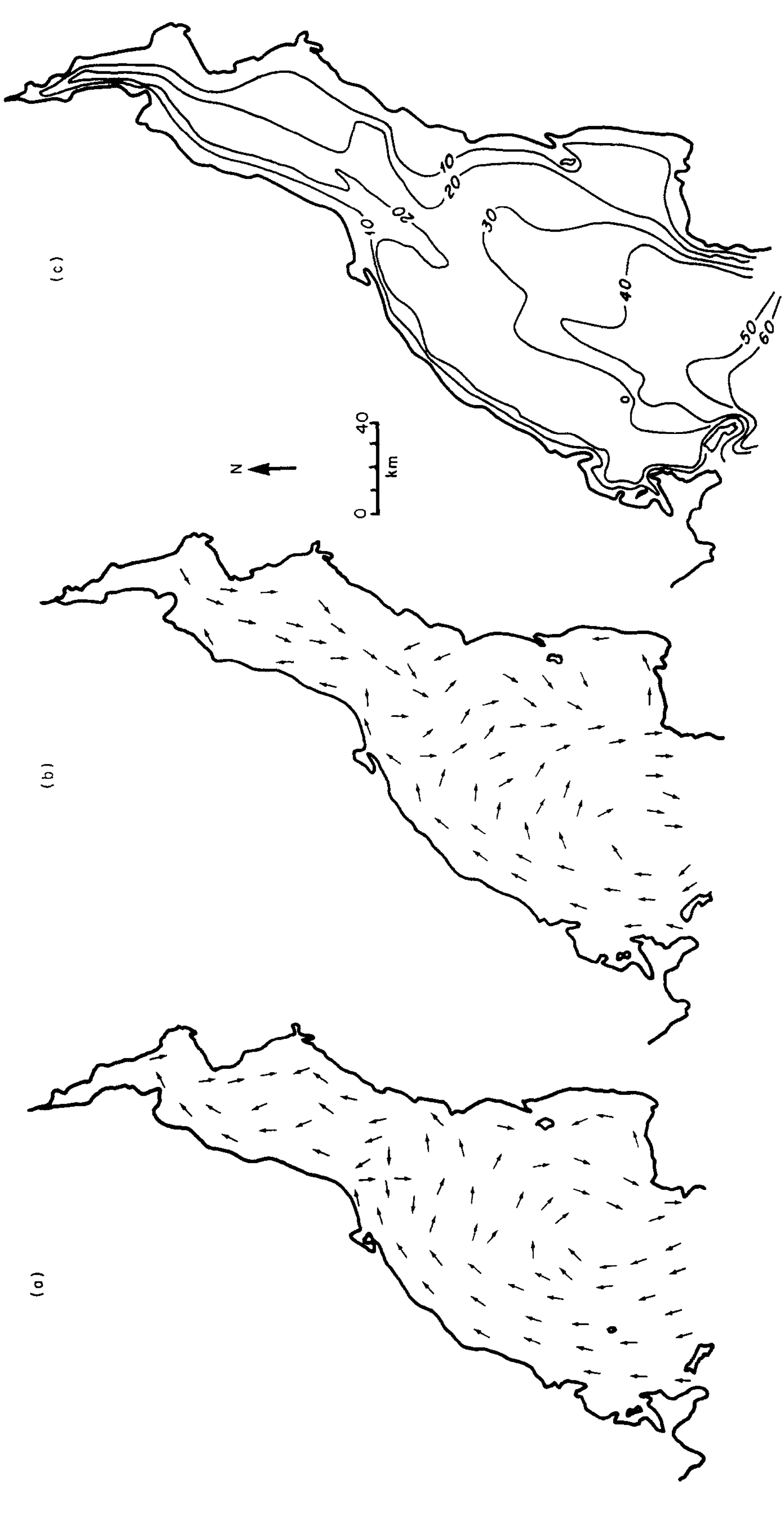

Figure 2. Flows associated with the observed density distribution in Spencer Gulf, as calculated by Bullock (1975). (a) Surface currents; (b) bottom currents; (c) bathymetry of the gulf.

where Q is volume flux and S is salinity per unit volume, and where subscripts i, o and e represent into, out of and evaporated from the Spencer Gulf side of the model, respectively. We presume here that Q_e is fixed by climatological factors and will therefore be specified. Equations (1) and (2) can be combined to give

$$Q_o \Delta S = S_i Q_e \tag{3}$$

where $\Delta S = S_o - S_i$. Since S_i and Q_e are regarded as fixed, we have two unknowns, Q_o and ΔS.

A dynamic condition is used from Whitehead, Leetmaa & Knox (1974), Equation 6.10; it reads

$$Q_o = \frac{1}{6}\frac{g\Delta\rho}{\rho}\frac{H^2}{|f|} = \frac{1}{6}\frac{g\beta\Delta S}{\rho}\frac{H^2}{|f|}, \tag{4}$$

where β is the coefficient of density change due to a salinity change, H is the depth of the mouth, S is salinity, ρ is average density of the fluid, $\Delta\rho$ is difference in density between the two water masses, and $f = 2\Omega$ where Ω is rotation rate of the earth times sine of the latitude. This formula is strictly valid in a long channel [longer than $\left(\frac{g\Delta\rho H}{\rho f^2}\right)^{\frac{1}{2}}$], connecting two very deep basins filled with motionless homogeneous fluid. It would appear that the mouth of Spencer Gulf is long enough to satisfy the length criterion. Bullock (1975) reports velocities in the gulf sufficient to make it difficult to justify the condition of no motion, while Spencer Gulf is not significantly deeper than the 40-m depth near its mouth. We have also assumed in using Equation (4) that $Q_o \approx Q_i$ and $S_o \approx S_i$, as was assumed by Whitehead, Leetmaa & Knox (1974).

Equations (3) and (4) can be combined to eliminate Q_o and the salinity difference can be calculated to be

$$(\Delta S)^2 = \frac{6|f|S_i Q_e}{g\beta H^2}\rho. \tag{5}$$

Using $g = 9{\cdot}8$ m/s^2, $H = 40$ m, $\beta = 0{\cdot}71 \times 10^{-3}$ kg/l/‰, $f = -0{\cdot}8 \times 10^{-4}$ s^{-1}, $S_i = 35{\cdot}9$‰, $\rho = 1{\cdot}027$ kg/l, and $Q_e = 6{\cdot}0 \times 10^2$ m^3/s [corresponding to a net evaporation of water of 5 mm/day, which is approximately that observed in several adjacent cities (Hounam 1961)], we find

$$\Delta S = 0{\cdot}96‰$$

therefore,

$$\Delta\rho = 0{\cdot}68 \times 10^{-3} \text{ kg/l}, \tag{6}$$

which is somewhat below the observational value of approximately 1·5‰ which is the extreme difference actually observed at the mouth, but is close to the salinity jump seen across the sharp jump in density observed in the middle of the mouth. Using this value of salinity change, the mass flux and velocity profile can be calculated. Putting the value (6) into equation (4) yields a mass flux,

$$Q_o = 2{\cdot}16 \times 10^4 \text{ m}^3\text{/s}. \tag{7}$$

Lastly, velocity and interface height vary according to the formula

$$h = \frac{H}{2}(1 + x/x_o) = 20(1 + x/3{\cdot}18)\text{m} \tag{8}$$

$$v = \frac{1}{2}\sqrt{\left(\frac{g\beta\Delta SH}{\rho}\right)}(1 \pm x/x_0) = 0{\cdot}255\ (1 \pm x/3{\cdot}18)\mathrm{m/s} \tag{9}$$

$x_0 = \frac{1}{2}\sqrt{\left(\frac{g\beta\Delta SH}{\rho f^2}\right)}$ = the Rossby radius of deformation times $\frac{1}{2}$, x is the lateral distance across the channel in km and the plus and minus velocities refer to the upper and lower fluid, respectively. The predicted density difference and interface tilt are shown in Figure 3 in comparison with a section across the mouth. The model interface tilts approximately as much as the observed true tilt of the 'front'.

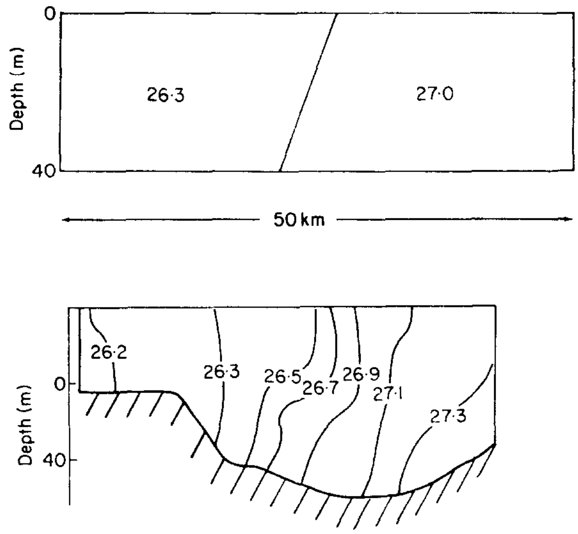

Figure 3. Sketch of the interface of the idealized two-fluid model with the calculated density difference, compared with surfaces of constant σ_t observed across the mouth of Spencer Gulf (from Bullock, 1975). The section is located along the dashed line in Figure 1.

It should be noted that the model provides for the most intense flows in a constricted region of width 2 x_0 somewhere inside the mouth. The fact that the flow in Spencer Gulf possesses this qualitative structure may be regarded as evidence of confirmation of the model. The rough quantitative agreement of the angle of tilt is as good as could be hoped for due to the fact that the model ignores the true bottom topography, the effects of winds, tides, waves and turbulence. It would appear that a buoyancy–inertial current of a steady nature may well be the principal flushing mechanism in Spencer Gulf.

Acknowledgement

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