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Buoyancy-Driven Instabilities of Low-Viscosity Zones as Models of Magma-Rich Zones

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The low Reynolds number dynamics of a thin layer of fluid bounded below by a flat horizontal boundary and above by a fluid of another viscosity and greater density is reported. Three distinct stages of growth were observed. The first stage is a Rayleigh-Taylor instability, in which disturbances of one specific wave number grow most rapidly. If ε is the ratio of the viscosity of the thick layer to the viscosity of the thin layer, fastest growth is for wave number $\varepsilon^{-1/3}$. In the second stage, distortion of the interface is large, and it is found experimentally that the fluid moves out of the thick layer as circular columns surrounded by relatively broad regions of descending material. In the third stage, fully matured structures are formed. If the upwelling material has less viscosity than the surrounding material, the structure develops a rim syncline and a pronounced overhang and eventually ascends as a spherical pocket of fluid fed by a conduit. Two applications to geophysics are given: The first application follows from the fact that a melt source must exist in the mantle below mid-ocean ridges. This source can be approximated as a cylindrical body with lower viscosity and density compared to the overlying mantle. If the cylinder develops a gravitational instability, it will develop regularly spaced vertical protrusions. Estimates of the spacing are compared to high-resolution segmentation data, and some constraints on viscosity of the mantle below spreading centers are made. These are that viscosity of the mantle is $10^{18\pm1}$ P, and the ratio of this viscosity to the viscosity of the cylinder is less than 100. In the second application, the upwelling conduits are measured experimentally and solitary waves are observed. A recently found analog with magma rising up through the pores of a viscous crystalline matrix is discussed.

1. INTRODUCTION

Volcanic systems are probably the largest chemically differentiating systems in the upper mantle. In addition to large and important chemical changes, they possess large-scale differential flow which results in displacements between melt and residue of the order of tens to hundreds of kilometers.

There has been much recent progress concerning the dynamics of these differentiating flows. The microscale initiation of melt and the leaching of the melt through a porous grain boundary matrix have recently been studied as a problem in compaction by *Sleep* [1974], *Stevenson* [1980], *Fowler* [1984], *McKenzie* [1984], *Scott and Stevenson* [this issue], and *Richter and McKenzie* [1984]. The general result depends on the observation that the network of melt is interconnected even when the proportion of melt is a few percent or less [*Waff and Bulau*, 1979; *Waff*, 1980; *Cooper and Kohlstedt*, 1984]. This implies that the process of compaction may dominate the early differentiation of melt from the mantle. Compaction is the viscous differentiation and buoyant sinking of the crystalline matrix in which the melt is embedded. Since the melt is lower in density than the crystals, it is buoyantly squeezed upward.

Whether melt rises by compaction or rises by some other method, we know little of the aggregation of the melt and the ponding of the melt in reservoirs. One possibility is that magma concentrates in solitary waves which have been found both analytically and numerically by *Scott and Stevenson* [this issue] and in a laboratory analog [*Scott et al.*, 1986]. However, these waves are only set up in a transient problem and go away over long time scales for a steady problem. Other possibilities are that a variation of resistance to flow with depth will lead to a region of high-melt concentration compared to the overlying mantle. This may occur from a change of the physical properties of either the crystals or the melt with depth through chemical or physical alteration. In any case, some important process surely concentrates magma at spreading centers, at island arcs, at numerous hot spots, and at isolated seamounts. Once that happens, a central feature of these systems is the huge change from a mantle viscosity of $10^{20\pm3}$ P to magma viscosities of 10^1 to 10^8 P.

The class of archetypical problems summarized here involves fluid flows where there is a large viscosity variation. They are problems in the gravitational stability of viscous fluids with large viscosity variations. They have been studied by laboratory experiments augmented by theoretical considerations.

The experiments described are relatively easy and inexpensive to do. Readers may want to reconstruct some of these for their own benefit or for that of students or colleagues. There is no substitute for seeing the real thing, neither photos, movies, nor data serve the same purpose.

2. OBSERVATION OF A GRAVITATIONALLY UNSTABLE LAYER

The first problem was stimulated by a lecture on salt domes. After the lecture, at the request of W. Chapple and D. Griggs, two immiscible viscous fluids were put into a cylindrical glass container. The denser fluid was less viscous and filled most of the jar. The lighter fluid lay overhead and was greater in viscosity. A plate of glass was then slid over the top.

The next morning the tank was inverted and a gravitational instability was observed. The lighter silicone oil floated up as a number of narrow columns. These exploratory experiments were not sealed, so the thin layer leaked and lost fluid. In order to make the experiments reproducible, two rectangular tanks were fabricated. Observations are reported by *Whitehead and Luther* [1975]. The first container was almost completely filled with glycerin with a kinematic viscosity of 14 cm^2/s and a density of 1.25 g/cm^2, to which a small amount of immiscible silicone oil with a viscosity of 600 cm^2/s and a density of 0.92 g/cm^3 was added. This formed a 5-mm-thick

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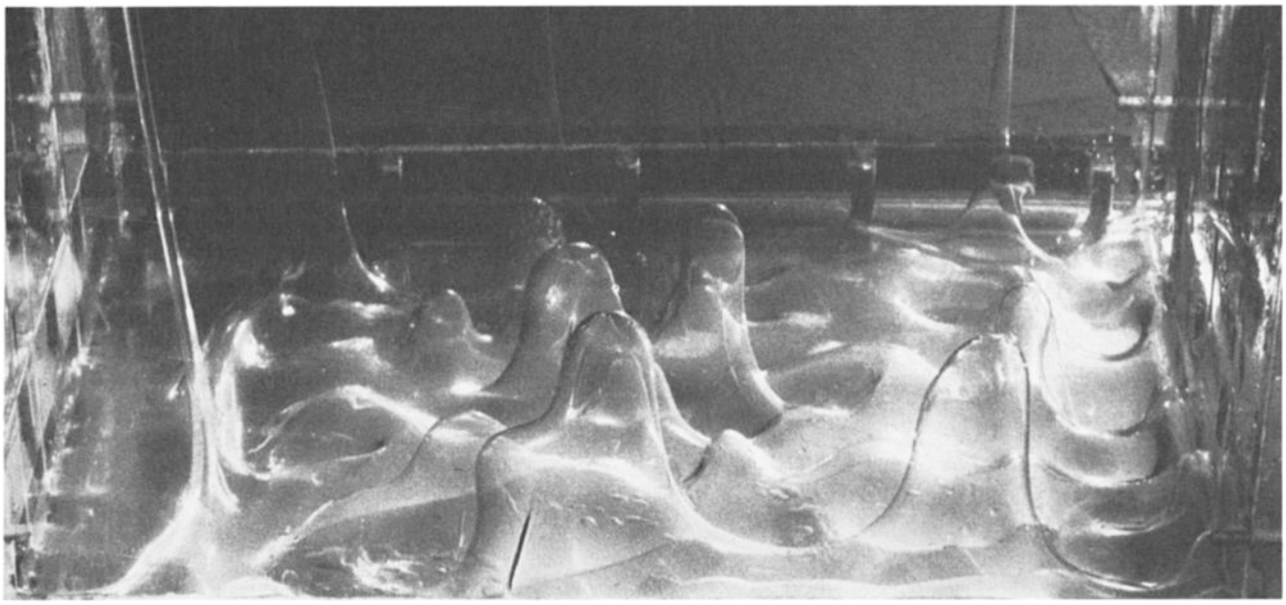

Fig. 1. A thin layer of silicone oil of viscosity 60,000 cSt protruding into clear glycerin of viscosity of approximately 1400 cSt.

layer of very viscous fluid on top of the glycerin. The container was then carefully covered, left overnight, and next day was inverted. In about 30 s the layer of viscous oil (which was on the bottom of the tank) was observed to develop protrusions that buoyantly pushed upward through the glycerin as long, buoyancy-driven columns. These are shown in the series of photographs in Figure 1. It was found that if the container was left carefully leveled for a number of days so that the oil interface was very flat before inversion, the columns that developed were spaced quite uniformly throughout the tank and were very nearly equal in size, volume, and growth rate. The wave number of the columns, defined as $2\pi H/L$, where L is the distance between columns and H is the depth of the thin layer, was 2.5.

The same experiment was performed in another container filled with the same fluids with opposite proportions. Thus the only difference with the preceding experiment was that the thickness of the layers was reversed. After the container was carefully leveled and left for a couple of days, a number of protrusions developed shortly after inversion. These protrusions also arranged themselves quite uniformly throughout the tank, but the wave number in this case was much less, 0.63. Hence the wavelength was much greater than before. The finite amplitude behavior of these protrusions was dramatically different from that of the previous case. The protrusions formed large spherical pockets of fluid that gradually developed a pronounced overhang to the point where the neck of fluid feeding these pockets almost pinched off and left a tiny pipe of fluid trailing the main pocket of fluid. The main pocket descended through the viscous fluid as almost perfect spheres. This sequence is shown in Figure 2. The pictures are inverted for clarity.

The unstable flows proceeded through three distinct physical flow stages, which will be described in the next two sections. Initially, the surface was nearly flat and small distortions to the interface grew by a "Rayleigh-Taylor" instability. This is an instability in which perturbations to a fluid with a density inversion in a field of gravity grow by creating a pressure field in phase with the velocity perturbations so that the velocity is amplified. During this stage the assumption of an almost flat surface allows the governing equations to be linearized. The wavelength of maximum growth rate and the exponential time constant of growth have been theoretically and numerically predicted for a number of geometries and boundary conditions [*Rayleigh*, 1900; *Dobrin*, 1941; *Chandrasekhar*, 1955, 1961; *Hide*, 1955; *Daneš*, 1964; *Selig*, 1965; *Ramberg*, 1963, 1967*a*, 1968*a*, *b*, *c*, 1970; *Biot*, 1966; *Biot and Ode*, 1965; *Berner et al.*, 1972; *Whitehead and Luther*, 1975]. Demonstration experiments with putty and other non-Newtonian fluids have been extensively photographed and compared to geological formations by *Nettleton* [1934, 1943], *Parker and McDowell* [1955], and *Ramberg* [1963, 1967*b*, 1970]. There was no intercomparison between the laboratory experiments and theory due to the unknown rheology of the laboratory materials.

The limit in which one layer is thin and the other much thicker is particularly relevant to the geophysical context here and will be reviewed in section 3. Various theoretical predictions are compared with observations of laboratory experiments.

The second stage occurs when the interface becomes distorted enough to violate the linearizing assumptions of the first stage. More complicated physical processes that generally have not been identified begin to occur at this stage. A few of the things that can happen are enhanced growth rate (superexponential), nonlinear interactions among the various growing modes, and a consequential narrowing of the class of the fastest growing modes. A three-dimensional theory with predictions that hexagonal planforms would develop first with spouts coming out of a thin layer is given by *Whitehead and Luther* [1975], who also give a general criterion for two fluids of finite depths.

In the third stage the interface is greatly distorted, and the intrusions have developed a matured structure. No known set of equations predict the structure of the flow, although two-dimensional numerical computations have been made. Unfortunately, experiments show that the planform is two-dimensional, so the flow is three-dimensional. The experiments

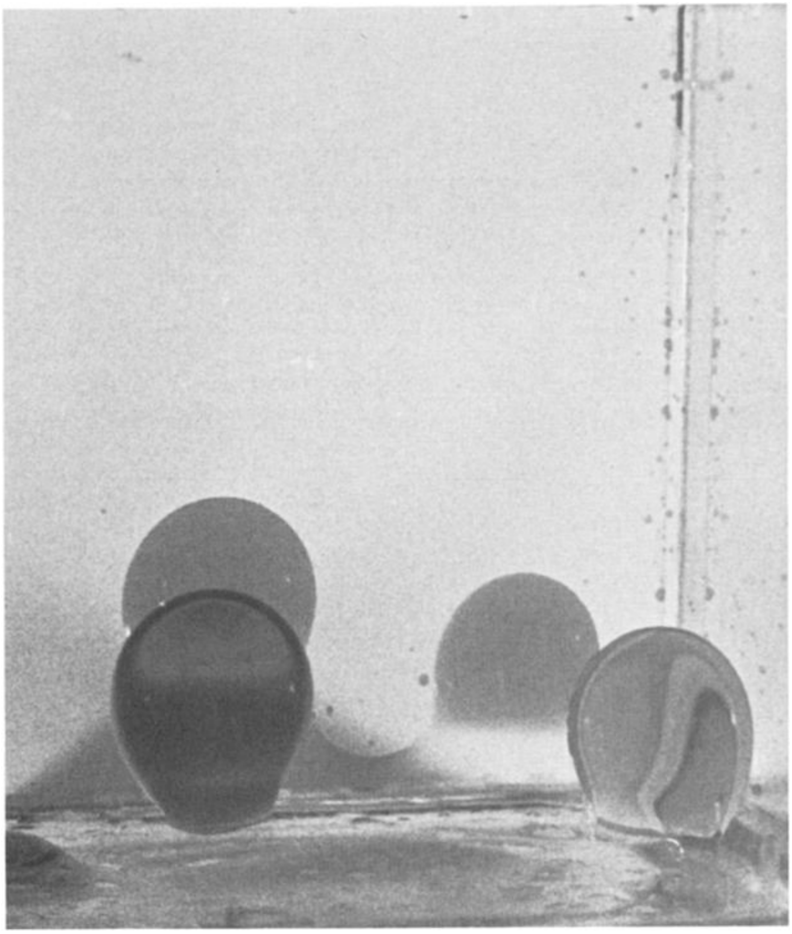

Fig. 2. A thin layer of glycerin protruding into clear silicone oil. Viscosities are as in Figure 1. These photographs are inverted for clarity, so the blobs of glycerin, which are normally descending through the less dense silicone oil appear to be rising.

indicated that the structures are a strong function of the ratios of the viscosities of the two fluids. Some cases will be discussed in section 4.

3. Rayleigh-Taylor Instabilities

Theory and Sealing

Discussion here will be limited to the case of a thin layer of fluid bounded by a very deep layer, which was theoretically analyzed by *Selig* [1965] and *Whitehead and Luther* [1975]. When the interface between a thin layer of fluid and an overlying region of denser fluid is slightly distorted, a small pressure gradient is created in the horizontal direction. A slow flow will thus be created no matter how small the distortion. To a first approximation, distortion of the interface arises from vertical movements; i.e., $\partial\eta/\partial t = w(0)$, where η is the interface that was originally at $z = 0$ and $w(0)$ is the vertical velocity at the plane $z = 0$. This flow is completely described theoretically in the above citations and will be summarized here. The solution to both regions is of the form

$$w = f(z)g(x, y)e^{nt} \tag{1}$$

where w is the velocity in the z (vertical) direction and $f(z)$ is the solution to the equation

$$(D^2 - k^2)^2 f(z) = 0 \tag{2}$$

where D is a derivative with respect to z. The function $g(x, y)$ satisfies the equation

$$\nabla_1{}^2 g(x, y) = -k^2 g(x, y) \tag{3}$$

where $\nabla_1{}^2$ is the Laplacian operator in x and y. Every solution with the same value of k has the same growth rate. The class of periodic functions that satisfy (3) is infinite, but some simple functions are sin $(\mathbf{k} \cdot \mathbf{x})$, cos $(\mathbf{k} \cdot \mathbf{x})$ (where $\mathbf{k}$ is a vector in some arbitrary horizontal direction), Bessel functions, and the sums or differences of such functions. Obviously this linearized Rayleigh-Taylor analysis is not specific about the x-y structure of the flow but can only predict that the flow is described by some horizontal wavelength $2\pi/k$. For given boundary conditions on $f(z)$, the growth rate n is a function of the magnitude of wave number k, and obtaining this dependence involves solving for the matching conditions at the interface. Solutions which neglect inertia are appropriate for geological applications, and the following have found $n(k)$ in this limit. *Selig* [1965] found solutions for a single layer penetrating into an infinite fluid. *Ramberg* [1968*a*, *b*] found solutions for many layers, and *Whitehead and Luther* [1975] solved equations for a single layer penetrating into an infinite fluid for all viscosities. Whitehead and Luther found analytical asymptotic approximations for when the viscosity of the thin

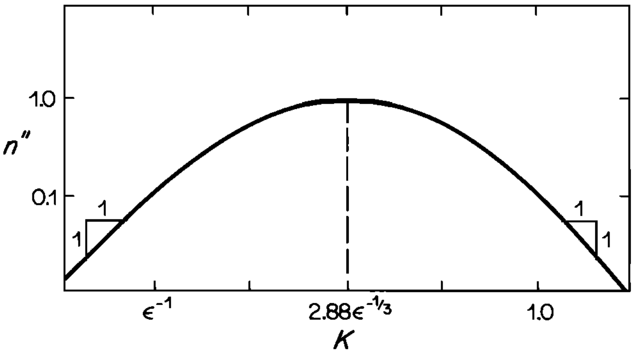

Fig. 3. Normalized growth rate $n'' = \mu_2 n/0.232\ g\Delta\rho h$ as a function of wave number $K = 4\pi h/\lambda$, where λ is the wavelength for the case when the thin layer is much less viscous than the deep fluid.

layer is very large compared to the infinite fluid. The wave number of fastest growth, defined by k_m, is

$$k_m = (180\varepsilon)^{1/5}/2h \qquad (4)$$

where h is the depth of the thin layer, $\varepsilon = \mu_2/\mu_1$, and μ_1 is the viscosity of the thin layer, while μ_2 is the viscosity of the infinitely deep layer. Growth rate is

$$n = (-g\Delta\rho h/4\mu_1)(1 - 0.443\varepsilon^{4/5}) \qquad (5)$$

where g is the acceleration due to gravity, h is depth of the thin layer, and $\Delta\rho$ is the density difference between the two fluids. Note that in (5), n is proportional almost entirely to μ_1, the large viscosity.

When the thin layer is lower in viscosity, the results are different as shown in Figure 3 for a free slip boundary below the thin layer. Wave number of fastest growth is

$$k_m = 1.44\ \varepsilon^{-1/3}/h \qquad (6a)$$

and the growth rate is

$$n = 0.232(g\Delta\rho h/\mu_2)\varepsilon^{1/3} \qquad (6b)$$

Selig [1965, equations (2.11)] has found if the thin layer has a no-slip boundary condition that

$$k_m = 2.15\ \varepsilon^{-1/3}/h \qquad (7a)$$

and

$$n = 0.153(g\Delta\rho h/\mu_2)\varepsilon^{1/3} \qquad (7b)$$

Equations (6*a*) and (7*a*) both have $k_m \simeq \varepsilon^{-1/3}$. Possibly this is a common scaling when ε is very large. If so, this scaling implies that when the viscosity of the thin layer is very low compared to the viscosity of the overlying fluid, the spacing of diapiric intrusions will be significantly greater than the thickness of the original layer. This fact may be of widespread application in magma formation problems, since a layer of magma or magma-source material should be many orders of magnitude lower in viscosity than the overlying material.

There may be a simple reason for the $\varepsilon^{-1/3}$ scaling. Possibly, it is more efficient for the low-viscosity fluid to flow large lateral distances up a gradual slope and to accumulate in massive diapirs. These have more buoyancy force with which to intrude up into the stiff overlying material than diapirs formed by shorter-wavelength instabilities. However, the $\varepsilon^{-1/3}$ is probably not universal. There is experimental evidence (R. Kerr, private communication, 1985) that the $\varepsilon^{-1/3}$ scaling is not found for one geometry.

Recent Applications of Rayleigh-Taylor Instability to Magma Genesis

The linearized Rayleigh-Taylor instability, when the thin layer is low in viscosity, provides a plausible mechanism for a horizontal layer of melt to form into a line of magma chambers. This idea in the context of island arc volcanism was originated by *Marsh* [1973] and followed up by *Marsh and Carmichael* [1974]. It was applied in detail by *March* [1979], who studied the instability of a ribbonlike or cylindrical body (with its axis horizontal) of low-viscosity material. The ribbonlike region was produced in the laboratory by allowing black oil or glycerin to rise from a slit in the bottom of a container of glycerin. It was convincingly suggested that this diapir geneis occurs under island arcs and is the mechanism that leads to the formation of the magma chambers for the island arc volcanoes. Noting the regular spacing of magmatic centers across Iceland, *Siggurdsson and Sparks* [1978] suggest that a similar instability exists at the base of the lithosphere in Iceland.

The global mid-ocean ridge system is the most extensive volcanic zone on earth. Regions of normal crust are separated by regions of crust-free ocean floor [*Whitehead et al.*, 1985]. Thus the volcanism is segmented. Abundant evidence for this segmentation is found in magnetic studies of older crust [*Schouten and Klitgord*, 1982, 1983]. The segmentation persists even if the offset of the transform fault zone is zero (Figure 4); hence magma pockets may be responsible for the segmentation. Since the source of magma is the underlying mantle, the magma must aggregate at some unknown depth and ascend to the crust due to its low density compared to the parent mantle rock. The melt must pool in crustal magma chambers: from whence it periodically erupts to the surface.

Schouten et al. [1985] have shown that the volcanism is regularly spaced in the only oceanic regions where data are extensive enough to resolve clearly the bathymetry for many segment lengths. The spacing of these centers l is systematically related to the effective spreading rate v_e, which is the magnitude of the vector difference in motion between the two plates. The relationship between l and v_e as fit by the logarithmic least squares is

$$l = 18.3 v_e^{0.27} \qquad (8)$$

which they postulate is represented by a model of the form [*Schouten et al.*, 1985, equation (7)]

$$l = Q v_e^{1/3} \qquad (9)$$

A least squares fit of (9) to their five data points gives $Q = 14.4$ km$^{2/3}$ m.y.$^{1/3}$.

Whitehead et al. [1985] postulate that a linear region of high-melt content exists in the mantle below the ridge where melt aggregates from the rising asthenosphere. This region can be approximated as a horizontal cylindrical body with relatively low viscosity and density compared to the overlying mantle. Under such circumstances, gravitational instability leads to regularly spaced vertical protrusions.

Some simple experiments show this. A water-glycerin mixture was quickly injected into glycerin along a horizontal line. Although this line will gradually rise because the water-glycerin mixture is less dense than the pure glycerin, an instability developed as shown in Figure 5 and led to semispherical pockets. It is reasonable to expect that a linear region of partially molten mantle in the earth will behave in a similar manner and will lead to fairly regularly spaced protrusions

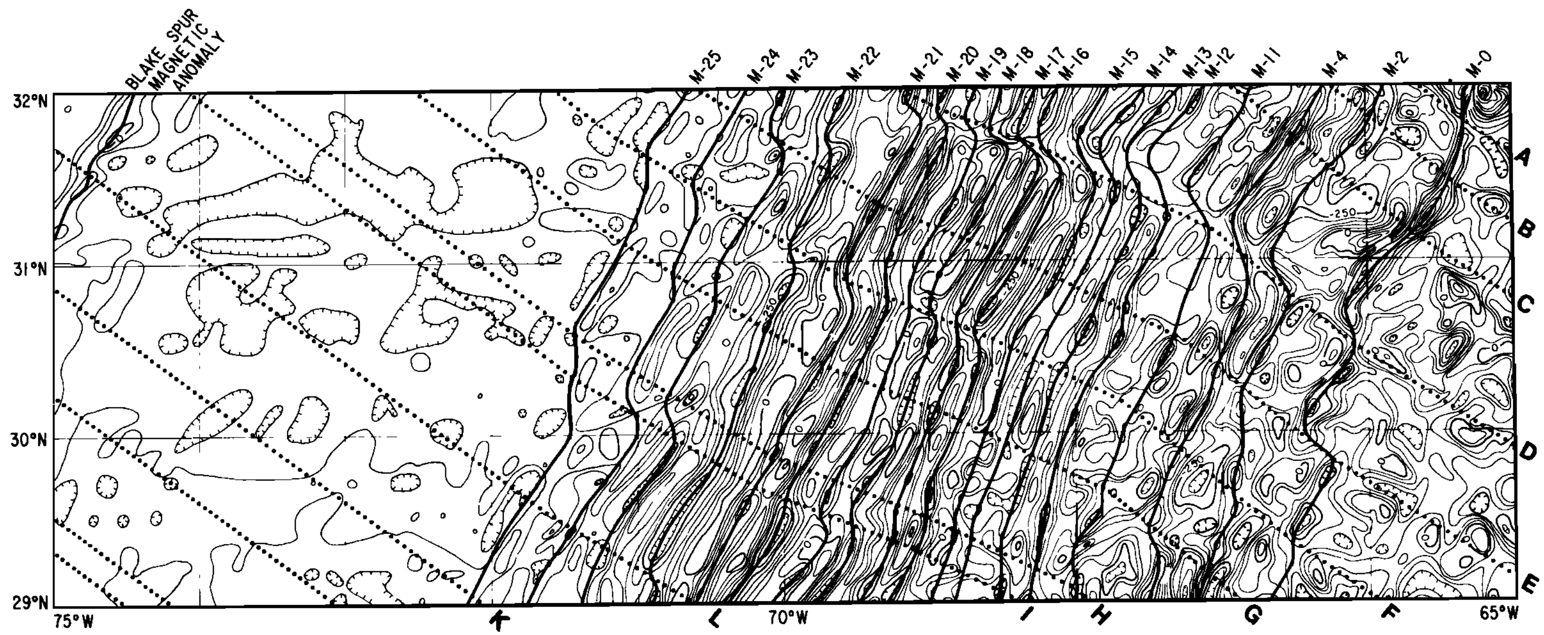

Fig. 4. Magnetic anomalies in contour interval of 50 nT from project Magnet [*Schouten and Klitgord*, 1982]. The anomalies of the seafloor reveal a characteristic cellular structure. Many strips of oceanic crust are separated by narrow zones of anomalous crust. Between the cells lie the old fracture zones, manifested by off sets in the magnetic isochrons (heavy lines). The reconstructed flow trajectories of some fracture zones are shown as dotted lines. The persistence of cellular structure during intervals of zero offset is consistent with diapirism under the plates in the crust-forming zone of the mantle.

Fig. 5. Instability of a horizontal tube of water-glycerin mixture injected into pure glycerin.

from which the melt will ascend to form magma chambers at spreading ridges.

An important prediction for magmatic systems is given in (6*a*) and (7*a*). The wavelength of fastest growth is proportional to $\varepsilon^{1/3}$, so as the viscosity of the thin layer gets small, the wavelength gets larger than the depth of the thin layer.

Calculations by *Schouten et al.* [1985] used equation (7*a*) (their equation (4)) in conjunction with the growth time

$$\tau = \frac{1}{0.153}\left(\frac{\mu_2}{g\Delta\rho d}\right)\left(\frac{\mu_2}{\mu_1}\right)^{-1/3} \tag{10}$$

to generate the relationship between the segmentation spacing λ_c and spreading rate U

$$\lambda_c = \frac{1}{0.169}\left(\frac{\mu_2}{g\Delta\rho}\right)^{1/3}\left(\frac{\mu_2}{\mu_1}\right)^{2/9} U^{1/3} \tag{11}$$

where U is the two-dimensional volumetric production rate of the low viscosity and density region. To derive this, the theoretical assumption as first put forward by *Howard* [1966] was used. It is that the boundary layer grows until the growth time is less than the time which has elapsed since the growing began. If U is proportional to v_e, which is consistent with the fact that oceanic crust thickness is independent of spreading rate, equation (11) has the same power law for U as (9) does for v_e. Thus if (9) is equated to (11), constraints on physical properties (density difference and viscosity) of the region below spreading centers will be generated, subject to the assumption that the mechanism of the model is correct. These constraints lead to $\mu_2 = 10^{18\pm1}$ P. Consideration of the time scale prediction (10) with geological estimates of the time scale of 20,000 years of the FAMOUS area lead to the constraint $\mu_2/\mu_1 \leq 10^2$.

4. Finite Amplitude Effects

The linear theory clearly predicts the spacing of intrusions, and section 3 outlines a theory based upon one geometry. Any detailed finite amplitude evolution of the structure in Rayleigh-Taylor instability has continued to defy precise theoretical prediction, although a theory of *Whitehead and Luther* [1975] predicts a hexagonal planform with spouts coming out of the thin layer. Figures 1 and 2 clearly show that a later structural form exists which appears to be in equilibrium. If fluid were continuously injected over some horizontal plane, it is logical to suppose that it would continue to flow in that form. Definitive experiments to test this have not yet been done.

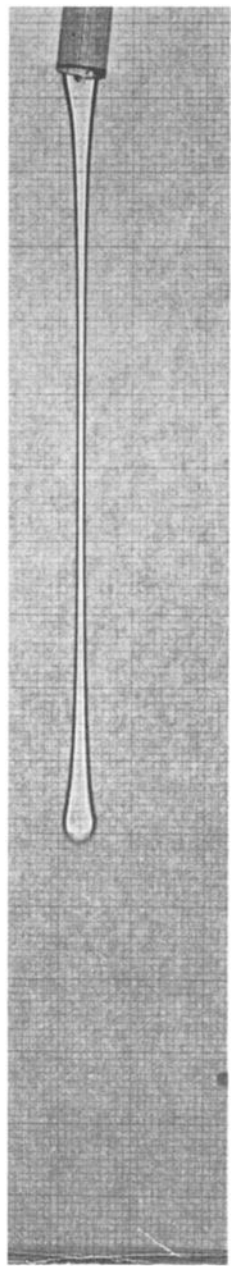

Fig. 6. Silicone oil of viscosity 60,000 cSt falling through oil of viscosity 10 cSt.

Fig. 7. A conduit of silicone oil with viscosity 10 cSt rising through oil with viscosity 60,000 cSt. The conduit was initiated by a spherical cavity.

The properties of individual structures can be observed by steadily injecting fluid from a local source. First, the case where the upwelling fluid is much more viscous than the fluid being penetrated with be reviewed (Figure 1). This may be the case for rhyolitic intrusions into more basaltic melts. The force balance is between forces due to pressures generated by density differences and forces due to viscous deformation in the upwelling fluid.

The matured jet is an upside down version of a viscous fluid being poured from a spout, like syrup being poured from a pitcher. A steady viscous jet is shown in Figure 6 in which silicone oil, with a viscosity of 60,000 cSt, drops through silicone oil with a viscosity of 10 cSt. The similarity between the structures in Figures 1 and 6 is obvious.

The theoretical equation governing the flow of the jet is

$$\rho\nu\ \partial^2 w/\partial z^2 = \partial p/\partial z$$

where $p = g\Delta\rho z$ and w is velocity in the direction of gravity z. This integrates to $w = (g\Delta\rho/2\rho\nu)\ z^2 + Az + B$. Far downstream, but not farther than the viscous buoyancy boundary layer length $(g\Delta\rho/\rho\nu^2)^{1/3}$, continuity can be used so that the radius of the spout is

$$r = c/z \tag{12}$$

where c is a constant.

There is little tendency for a very viscous dome to develop a plumelike structure with rim synclines that neck off. The nose of the dome can experience a small amount of local widening, which is produced as the dome pushes its way upward through the lower-viscosity material. In the experiments this dome has been seen to be approximately twice as wide as the viscous pipe that follows it.

When the upwelling fluid is much less viscous than the fluid being penetrated, the structure evolves along a very different sequence. As before, significant impediment is associated with the fluid of greater viscosity. As shown in Figure 2, the jet pushes outward into this viscous material, and its nose is subjected to a gradually increasing normal stress that ultimately provides most of the drag equal and opposite to the buoyancy force. By the time the jet has made an order one indentation into the viscous fluid, most of the material from the layer has filled a rather large cavity with a circular horizontal cross section. As the pocket of fluid begins to rise, a pronounced rim syncline is produced around the cavity and it necks off. In Figure 2 there is little fluid left to fill the neck. The bulk of the fluid in the pocket then rises as an almost perfect sphere, which is a solution to the flow equation of a fluid with small viscosity rising through a fluid of very much larger viscosity [*Batchelor*, 1970]. At the same time, a small pipe or "conduit" trailing the spherical cavity brings remaining fluid upward.

This structure looks very much like some of the simpler salt

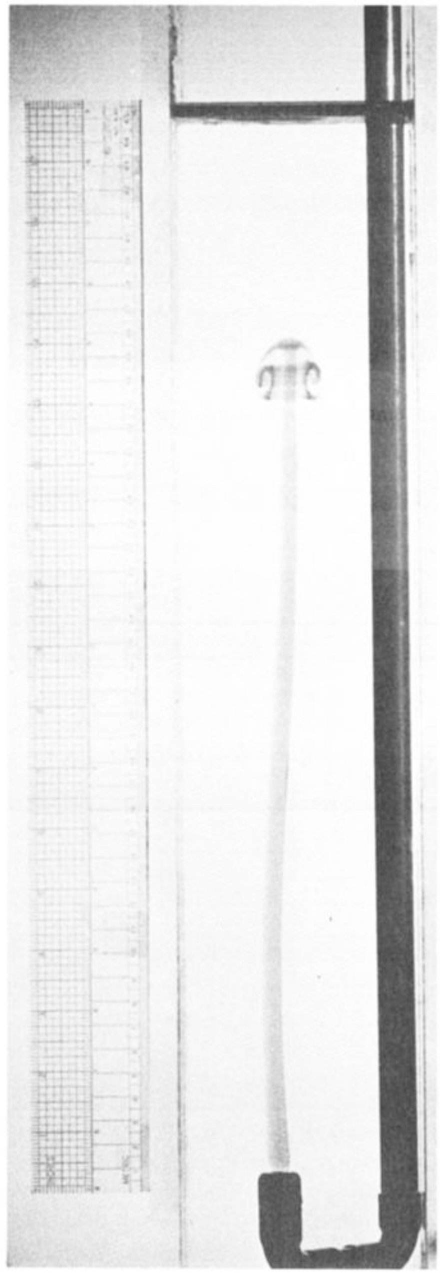

Fig. 8. A conduit with two fluids of almost the same viscosity.

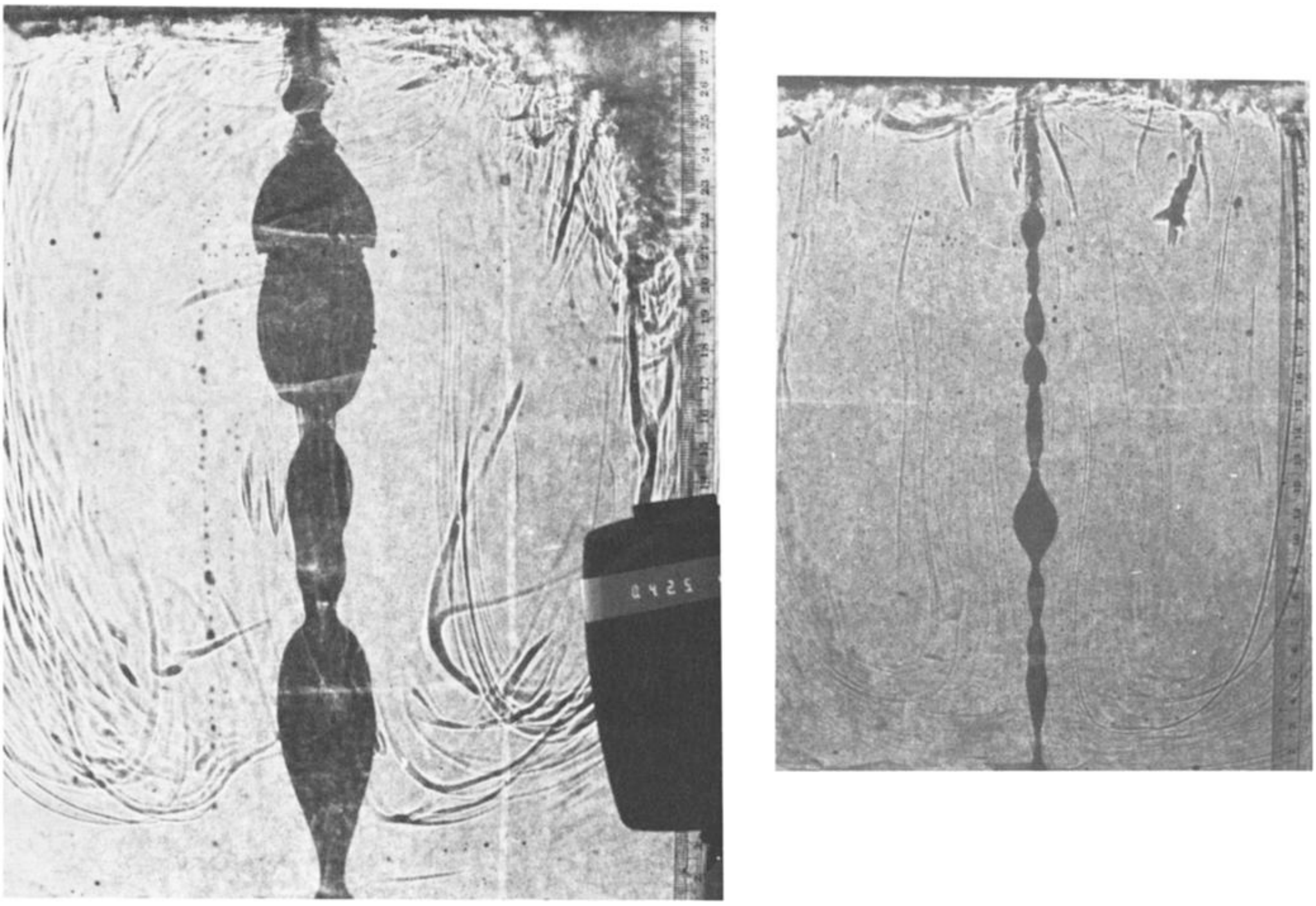

Fig. 9. The wavy walls of two conduits with large Reynolds number of the interior fluid.

domes. It has been suggested that in the case of salt domes the upwelling salt necks off because the source strata of salt is depleted. We believe that this is not the case and that necking is a universal feature of flows where the upwelling fluid is much lower in density. Figure 7 shows water being fed at a steady rate into a denser corn syrup with a viscosity of 1.2 $\times 10^5$ cSt. A pocket of fluid forms at the spout and grows until it attains a diameter great enough to buoyantly rise more rapidly than the rate of growth of the radius. The cavity of fluid then rises away, leaving the neck. The neck grows vertically into what we have previously called a conduit or small pipe that continues to feed the parent cavity.

If the flow from the source were to be completely stopped, a neck consisting of small amounts of the lower-viscosity fluid trapped by friction remains long after the initial starting plume has passed. If the flow is reinitiated before this neck completely diffuses away, the old neck offers a path of least resistance so that the fluid travels up the old neck faster than the initial dome with a nonspherical shape.

Motor oil with a viscosity of 3.2 cm^2/s and a density of 0.86 and silicone oil with a viscosity of 3.7 cm^2/s and a density of 0.96 have also been used to study the flow that results when viscosities are nearly equal. These two fluids are not miscible but appear to have low interfacial tension. The nose of the resulting spout developed a flow resembling a vortex ring, as is shown in Figure 8. This flow entrained surrounding fluid and is particularly interesting because it exhibited entrainment at a Reynolds number of approximately 2.

5. THEORY OF LOW-VISCOSITY DOMES AND CONDUITS

Whitehead and Luther [1975] and *Marsh* [1979] have used the Stokes approximation for a sphere to describe the rate of ascent v of the pocket of radius a, density $\rho \pm \Delta\rho$, and viscosity μ in a fluid of density ρ and viscosity μ_2 [*Batchelor*, 1970, pp. 236–238]:

$$v = \frac{1}{3}\frac{a^2 g}{\mu}(\Delta\rho)\left[\frac{\mu_2 + \mu_1}{\mu_2 + \frac{3}{2}\mu_1}\right] \tag{13}$$

which, for $\mu_1 \ll \mu_2$, reduces to

$$v = \tfrac{1}{3}(a^2 g \Delta\rho/\mu_2) \tag{14}$$

Note that our subscript convention is the reverse of Marsh's.

If a source feeds fluid at a constant rate Q and if originally the sphere is small enough that $da/dt > v$, it will continue to stay near the source and grow at the rate

$$da/dt = Q/4\pi a^2 \tag{15}$$

The sphere can be expected to rise away from the source after v has become equal to da/dt. The radius a_s at this time can be determined by equating (14) and (15):

$$a_s = (3Q\mu_2/4\pi g\Delta\rho)^{1/4} \tag{16}$$

B. D. Marsh (private communication, 1985) has pointed out that in many cases in the earth, Q is fed by a pipe of radius a

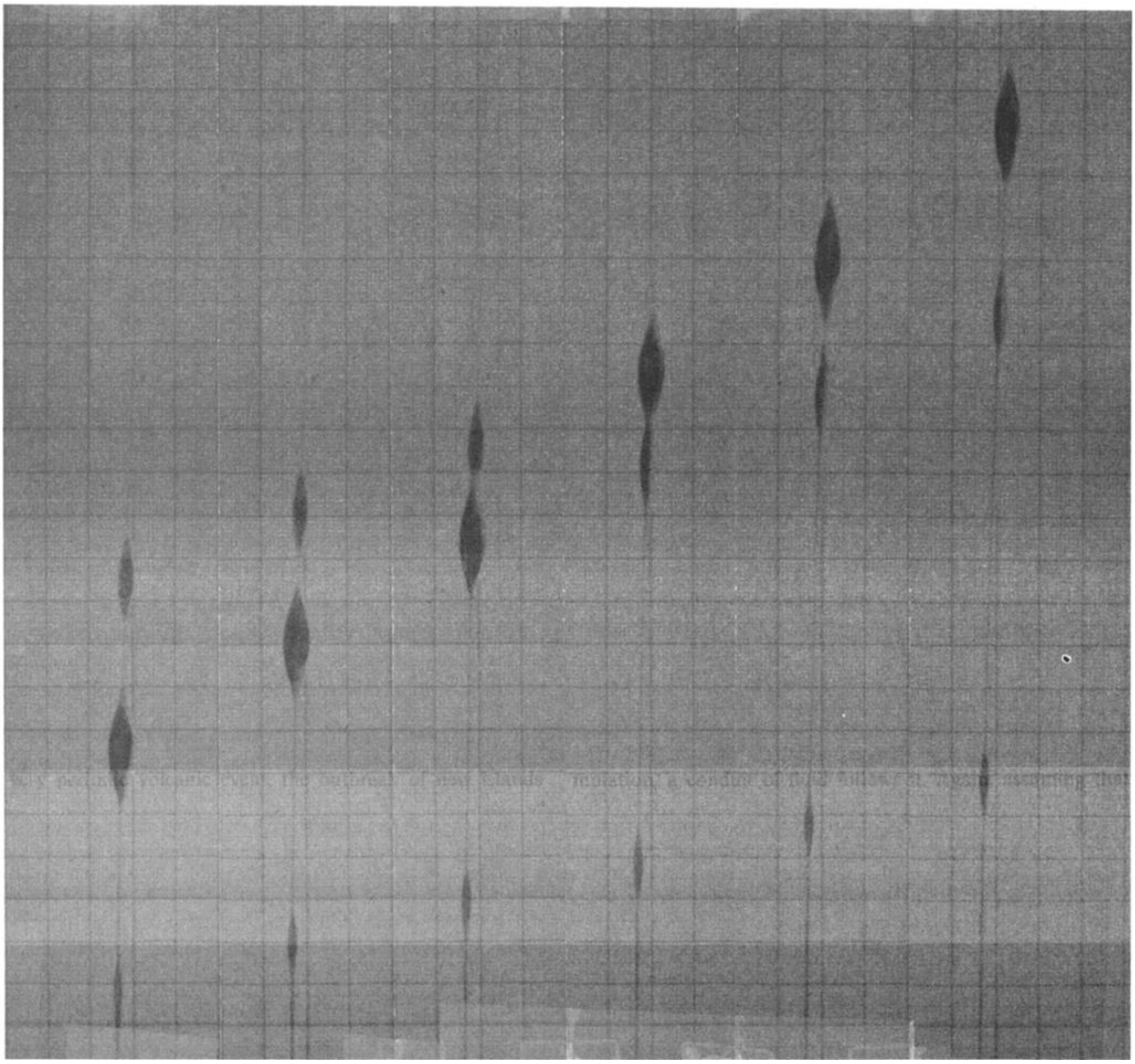

Fig. 10. Two solitary waves imposed on a conduit. They collide like solitons. Time between frames is 5 s.

and angle θ. With $Q = \pi \Delta \rho g a^4 \sin \theta / 8\mu_1$ [*Marsh*, 1979, equation (7)],

$$\frac{a_s}{a} = \left(\frac{3}{32} \sin \theta \frac{\mu_2}{\mu_1}\right)^{1/4} \tag{17}$$

This fundamental relationship between the size and viscosity of a diapir and its source geometry and viscosity was discovered by *Marsh* [1979]. The time it takes to form this sphere is given by $4\pi a_0{}^3/3Q$, which is

$$t_s = [4\pi v_2{}^3/3Q(g\Delta\rho)^3]^{1/4} \tag{18}$$

These equations will be useful if the surrounding region of magma accumulation is Newtonian and viscous. Many suggested mechanisms for upward magma migration involve a brittle, nonfluid region. To see if numbers from the above theories make any sense at all, let us take some parameters for a very periodic volcanic event, the outbreak of new islands along the Hawaiian-Emperor chain which has been operating more or less regularly at a rate of one island per million years for at least 80 m.y. We use $v = 10^{22}$ cm^2/s as a surrounding mantle viscosity from Fennoscandia uplift [*Peltier*, 1976], $Q = 10^6$ cm^3/s [*Whitehead and Luther*, 1975], and $g^* = 100$ cm/s^{-2}. With these numbers,

$$t_s = 4.1 \times 10^{13} \text{ s} \approx 1 \text{ m.y.}$$

This is the approximate period of new islands in the chain. If the viscosity under Hawaii is guessed to be significantly smaller because of the well-known large thickness of the low-velocity zone, $v = 10^{20}$ and $t_s = 30{,}000$ years. This is the eruptive period of individual volcanoes, and it suggests that such a mechanism, operating in the mantle below the plates, is crudely consistent with the parameters of the Hawaiian chain.

After the dome escapes upward from the depth of lava accumulation, a conduit of fluid follows it. Again, assuming that

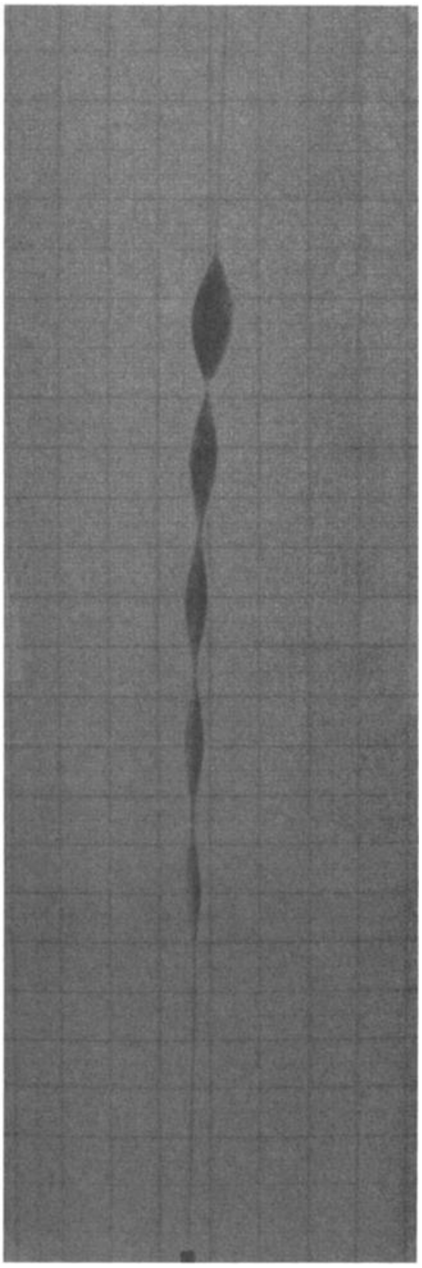

Fig. 11. A packet of solitary waves in a conduit.

lava has much lower viscosity than the host fluid, the equation of laminar pipe flow is valid in the conduit; this equation is

$$w = \frac{1}{4\mu_1} g\Delta\rho(r_0^2 - r^2)$$

where r_0 is the outside radius of the conduit and r is the distance from the center. Mass flux is found by integration to be

$$Q = (\pi/8\mu_1)g\Delta\rho r_0^4 \tag{19}$$

and (19) can be inverted to give r_0 as a function of Q:

$$r_0 = (8\mu_1 Q/\pi g\Delta\rho)^{1/4} \tag{20}$$

Reynolds number Re of the flow in the conduit is

$$Re = wr_0/\nu_1 = 0.50\left[\frac{g^*Q^3}{\nu_1^5}\right]^{1/4} \tag{21}$$

Such conduits or pipes have now been studied in various geological contexts. In one set of laboratory and theoretical studies [*Skilbeck and Whitehead*, 1978; *Whitehead*, 1982], the stability of conduits to tilting was investigated. In mind was a model of the effect of subplate shear on hot spots. An outcome of the study was to postulate the formation of discrete islands in island chains such as the Hawaiian-Emperor seamount chain and to suggest a new method by which the variation of shear with depth in the mantle could be inferred. Laboratory conduits were made by injecting oil below a more viscous and denser oil. At first, the growing chamber of lower-viscosity fluid formed near the injector, but when the chamber got sufficiently large, it rose as a buoyant spheroid. Behind this trailed the vertical cylindrical conduit through which fluid could continue to rise to the surface as long as the source continued. In some experiments, the conduit and host fluid have been rotated laterally [*Skilbeck and Whitehead*, 1978], while in others the more viscous fluid was sheared laterally [*Whitehead*, 1982]. In both cases, the conduit was gradually rotated to a more horizontal position, and when the conduit was tilted to more than 60° with the vertical, it began to go unstable by developing bumps which ultimately initiated a new chamber which rose to a new spot. The possibility that this mechanism operates under the Pacific plate was raised. If shear under the Pacific plate has to tilt buoyant mantle plumes to as much as 60° to form the relatively regular island chains associated with hot spots, most of the shear would be found in a zone with a vertical extent of less than 200 km.

An important second feature of conduits are disturbances that the walls develop. According to *Huppert et al.* [1986], as the Reynolds number of the flow in the conduit becomes greater than ~ 10, the waves develop an axisymmetric, upward propagating wave. This results in the mixing coefficient between the conduit and the surrounding fluid being enhanced. At progressively greater Re the waviness and mixing become greater. Two such examples are shown in Figure 9.

The waves that lie on an otherwise laminar conduit, i.e., $Re \leq 10$ are particularly interesting. Waves have been imposed upon the conduit by imposing a gradual change in mass flux into the bottom by *Scott et al.* [1986]. An immediate conclusion one forms if one pulses the mass flux, so that the flux increases and then decreases, is that solitary waves can form. Figure 10 shows two solitary waves that were produced in this way in a tank of corn syrup. In the case of the first solitary wave, we increased the source strength for 5 s. In the case of the second solitary wave which was larger than the first, the source was increased much more strongly for five seconds. The larger wave traveled faster than the smaller one and caught up with it. Then there was a collision between the two waves, at which time there was fluid exchanged from the larger to the smaller one. Subsequently, the larger one which was now on the top broke away from the smaller one. Since the waves approximately preserved their amplitude after collision, they collide like solitons.

It is also possible to make a group of waves that disperse. Figure 11 shows such a packet, which was generated by uniformly raising the source vessel at approximately 5 cm/s from an initial height equal to the free surface of the syrup to a final height 1 m above the free surface of the syrup. After this the flow was stopped. The leading solitary wave was initially a slight amount larger than the ones trailing it, but as time progressed, the leading wave grew even larger, probably because the conduit through which it traveled was very tiny, whereas the pinch between the lead wave and the second one admitted a flux of material upward to the lead wave. It rapidly got bigger and accelerated. The second grew also but accelerated less rapidly because fluid was bled to the lead wave above, even though the second wave got some fluid fed up from below as well. The third grew less rapidly and so on. Thus the string of waves (Figure 11) gradually dispersed and traveled upward, not only because of the initial size distribution, but because the wave-wave interaction caused the lead wave to grow. Such a wave train would also be expected if the solitary waves are solitons, since one would expect that the initial conditions would be represented as a cluster of solitons which would gradually disperse so that the fastest (largest) solition would take the lead, the second fastest would follow and so forth.

The dispersion properties of waves and some exact solutions have recently been published by *Olson and Christensen* [1986]. Recently, and as a direct result of thoughts which originated at this meeting, *Scott et al.* [1986] reported that this system is analogous to a number of recently discussed compaction models of melt rising through a viscous deformable matrix. In this analog, the syrup rising through the conduit represents melt, the outer viscous flux represents the matrix and the radius of the conduit represents the porosity of the matrix. The governing conduit equations are shown by *Scott et al.* [1986], and are, in our notation (where r_0 is now a time and space varying conduit radius), the equation of pressure in the outer (very viscous) fluid

$$p = \rho g + 2\mu_2 \frac{\partial r_0}{\partial t} \tag{22}$$

where ρ is density of the outer fluid; continuity:

$$\frac{\partial Q}{\partial z} = -\frac{\partial}{\partial t}\pi r_0{}^2 \tag{23}$$

and the balance between buoyancy pressure and volumetric flow in the conduit

$$Q = \frac{\pi r_0{}^4}{8\mu_1}\left[\frac{\partial p}{\partial z} + (\rho - \Delta\rho)g\right] \tag{24}$$

Equation (22) is substituted into (24) and (23) to get

$$Q = \frac{\pi r_0{}^4}{8\mu_1}\left[g\Delta\rho + \mu_2\frac{\partial}{\partial t}\left(\frac{1}{\pi r_0{}^2}\frac{\partial Q}{\partial z}\right)\right] \tag{25}$$

Equations (23) and (25) are the same as those which govern the rise of melt through a viscous matrix [*Scott and Stevenson*, this issue].

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REFERENCES

Batchelor, G. K., *An Introduction to Fluid Mechanics*, pp. 236–238, Cambridge University Press, New York, 1970.

Berner, H., H. Ramberg, and O. Stephansson, Diapirism in theory and experiment, *Tectonophysics*, *15*, 197–218, 1972.

Biot, M. A., Three-dimensional gravity instability derived from two-dimensional solutions, *Geophysics*, *31*, 153–166, 1966.

Biot, M. A., and H. Ode, Theory of gravity instability with variable overburden and compaction, *Geophysics*, *30*, 213–227, 1965.

Chandrasekhar, S., The character of the equilibrium of an incompressible heavy viscous fluid of variable density, *Proc. Cambridge Philos. Soc.*, *51*, 162–178, 1955.

Chandrasekhar, S., *Hydrodynamics and Hydromagnetic Stability*, Oxford University Press, New York, 1961.

Cooper, R. F., and D. L. Kohlstedt, Solution-precipitation enhanced diffusional creep of partially molten olivine-basalt aggregates, *Eos Trans. AGU*, *65*, 280, 1984.

Daneš, Z. F., Mathematical formulation of salt dome dynamics, *Geophysics*, *29*, 414–424, 1964.

Dobrin, M. B., Some quantitative experiments on a fluid salt dome model and their geological implications, *Eos. Trans. AGU*, *22*, 528–542, 1941.

Fowler, A. C., On the transport of moisture in polythermal glaciers, *Geophys. Astrophys. Fluid Dyn.*, *28*, 99–140, 1984.

Hide, R., The character of the equilibrium of an incompressible heavy viscous fluid of variable density: an approximate theory, *Proc. Cambridge Philos. Soc.*, *51*, 179–201, 1955.

Howard, L. N., Convection at high Rayleigh number, in *Proceedings of the Eleventh International Congress of Applied Mechanics, Munich (Germany)*, pp. 1109–1115, edited by H. Gortler, Springer-Verlag, New York, 1966.

Huppert, H. E., R. S. J. Sparks, J. A. Whitehead, and M. A. Hallworth, The replenishment of magma chambers by light inputs, *J. Geophys. Res.*, in press, 1986.

Marsh, B. D., Island arc magmatism (abstract), *Eos Trans. AGU*, *54*, 1205, 1973.

Marsh, B. D., Island arc development: Some observations, experiments and speculations, *J. Geol.*, *87*, 687–713, 1979.

Marsh, B. D., and I. S. E. Carmichael, Benioff zone magmatism, *J. Geophys. Res.*, *79*, 1196–1206, 1974.

McKenzie, D. P., The generation and compaction of partial melts, *J. Petrol.*, *25*, 713–765, 1984.

Nettleton, L. L., Fluid mechanics of salt domes, *Am. Assoc. Pet. Geol. Bull.*, *18*, 1175–1204, 1934.

Nettleton, L. L., Recent experimental and geophysical evidence of mechanics of salt-dome formation, *Am. Assoc. Pet. Geol. Bull.*, *27*, 51–63, 1943.

Olsen, P., and U. Christensen, Solitary wave propagation in a fluid conduit within a viscous matrix, *J. Geophys. Res.*, in press, 1986.

Parker, T. J., and A. N. McDowell, Model studies of salt-dome tectonics, *Am. Assoc. Pet. Geol. Bull.*, *39*, 2384–2470, 1955.

Peltier, W. R., Glacial-isostatic adjustment, II, The inverse problem, *Geophys. J. R. Astron. Soc.*, *47*, 669–705, 1976.

Ramberg, H. Experimental study of gravity tectonics by means of centrifuged models, *Bull. Geol. Inst. Univ. Uppsala*, *42*, 1–97, 1963.

Ramberg, H. Model experimentation of the effect of gravity on tectonic processes. *Geophys. J. R. Astron. Soc.*, *14*, 307–329, 1967*a*.

Ramberg, H., *Gravity Deformation and the Earth's Crust as Studied by Centrifuged Models*, 224 pp., Academic, Orlando, Fla., 1967*b*.

Ramberg, H., Fluid dynamics of layered systems in a field of gravity: A theoretical basis for certain global structures and isostatic adjustment. *Phys. Earth Planet. Inter.*, *1*, 63–87, 1968*a*.

Ramberg, H., Instability of layered systems in a field of gravity, 1, 2, *Phys. Earth Planet. Inter.*, *1*, 427–474, 1968*b*.

Ramberg, H., Mantle diapirism and its tectonic and magmagenetic consequences, *Phys. Earth Planet. Inter.*, *5*, 45–60, 1968*c*.

Ramberg, H., Model studies in relation to intrusion of plutonic bodies, *Geol. J. Spec. Issue*, *2*, 261–286, 1970.

Rayleigh, Lord (J. W. Strutt), Investigation of the character of the equilibrium of an incompressible heavy fluid of variable density. *Sci. Pap.*, *2*, 200–207, 1900.

Richter, F. M., and D. P. McKenzie, Dynamical models for melt segregation from a deformable matrix, *J. Geol.*, *92*, 729–740, 1984.

Schouten, H., and K. D. Klitgord, The memory of the accreting plate boundary and the continuity of fracture zones, *Earth Planet. Sci. Lett.*, *59*, 255–266, 1982.

Schouten, H., and K. D. Klitgord, Overlapping spreading centers on East Pacific Rise, Nature, *303*, 459–550, 1983.

Schouten, H., K. D. Klitgord, and J. A. Whitehead, Jr., Segmentation of mid-ocean ridges, *Nature*, *317*, 225–229, 1985.

Scott, D. R., and D. J. Stevenson, Magma ascent by porous flow, *J. Geophys. Res.*, this issue.

Scott, D. R., D. J. Stevenson and J. A. Whitehead, Observations of solitary waves in a viscously deformable pipe, *Nature*, *319*, 759–761, 1986.

Selig, F., A theoretical prediction of salt dome patterns, *Geophysics*, *30*, 633–643, 1965.

Siggurdsson, H., and R. S. J. Sparks, Lateral magma flow within a rifted Icelandic crust, *Nature*, *274*, 126–130, 1978.

Skilbeck, J. N., and J. A. Whitehead, Jr. Formation of discrete islands in linear island chains, *Nature*, 272, 499–501, 1978.

Sleep, N. H., Segregation of a magma from a mostly crystalline mush, *Geol. Soc. Am. Bull.*, *85*, 1225–1232, 1974.

Stevenson, D. J., Self-regulation and melt migration (can magma oceans exist?) *Eos Trans. AGU*, *61*, 1021, 1980.

Waff, H. S., Effects of the gravitational field on liquid distribution in partial melts within the upper mantle, *J. Geophys. Res.*, *85*, 1815–1825, 1980.

Waff, H. S., and J. R. Bulau, Equilibrium in fluid distribution in partial melts under hydrostatic stress conditions, *J. Geophys. Res.*, *84*, 6109–6114, 1979.

Whitehead, J. A., Jr., Instabilities of fluid conduits in a flowing earth—are plates lubricated by the asthenosphere?, *Geophys. J. R. Astron. Soc.*, *70*, 415–433, 1982.

Whitehead, J. A., Jr., and D. S. Luther, Dynamics of laboratory diapir and plume models, *J. Geophys. Res.*, *80*, 705–717, 1975.

Whitehead, J. A., Jr., H. J. B. Dick, and H. Schouten, A mechanism for magmatic accretion under spreading centers, *Nature*, *312*, 146–148, 1985.

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