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Flow of a Homogeneous Rotating Fluid Through Straits

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A formulation is presented for barotropic, steady, inviscid flow through a rectangular opening by a rotating fluid. The magnitude of the flow depends on the difference in free surface height $\Delta\zeta$, force of gravity g, Coriolis parameter f, and opening width w. It is not necessary for the channel to approach the Rossby radius in width for important rotational effects to be felt. When $W/w<1$ (the ratio of a novel width scale $W=(2g\Delta\zeta)^{1/2}/f$ to channel width), the Garrett and Toulany (1982) prediction for mean velocity of $\bar{v}=g\Delta\zeta/fw$ is found. In the converse limit, $\bar{v}=(2g\Delta\zeta)^{1/2}-\frac{1}{2}fw$. Both are confirmed by laboratory experiment. The laboratory flow approximately matches the assumption that relative vorticity is $-f$. There are straits in the oceans and flows through mountain passages which may be governed by the laws discussed here.

KEY WORDS: Homogeneous rotating flow, straits, barotropic flow.

1. INTRODUCTION

In a recent series of studies, the limit of flow through a strait connecting two neighboring basins has been analyzed (Garrett, 1983; Garrett and Majaess, 1984; Toulany and Garrett, 1984) using the theoretical model of Garrett and Toulany (1982). This model assumes there is a flow through a strait with across-strait geostrophic balance and along-strait balance between friction, temporal acceleration, and the along-strait pressure gradient. It is asserted that the sea level difference across a strait due to geostrophic setup *cannot be greater* than the sea level difference ($\Delta\zeta$) between connected bodies.

In other words, low frequency (i.e. quasi-steady) flow through a strait of rectangular cross-section cannot be more than a "geostrophic control" flow given by the formula

$$u=g(\Delta\zeta)/fw. \tag{1}$$

Here g is acceleration due to gravity, f is the Coriolis parameter, and w is the width of the Strait.

The purpose here is to generalize the calculation to arbitrary f. One motivation is that the statement "cannot be more" is correct but useless in the limit $f=0$. Using a rotating hydraulics formulation, the equations of motion for steady flow through a channel in a rotating fluid will be solved. Flow will be less than Eq. (1) in a limit which appears to be realizable by many oceanic strait flows and close to Eq. (1) in another limit. In the calculations presented here, an across-strait geostrophic balance exists as it did in Garrett and Toulany but, in addition, the free surface in the channel is allowed to adjust itself in accordance with conserved properties along streamlines. In the results, the calculations are valid for all rotation rates in rectangular channels.

2. THEORY

Our formulation involves only a slight modification of an existing theory. This theory is for the problem of baroclinic strait flow (Whitehead, Leetmaa and Knox, 1974). The new formulae involve a simple prediction of the speed of barotropic flow through a strait of arbitrary width w given a sea level difference $\Delta\zeta$ between two connected basins in a rotating system.

Calculations for a very deep upstream basin will be presented here. This is the simplest because the potential vorticity is then zero. As a consequence, the Bernoulli potential is constant for every streamline. The results are piecewise valid algebraic formulae which will agree with (1) in one limit and reproduce a well-known non-rotating hydraulics relation in the other limit.

Calculations for finite potential vorticity [as done by Gill (1977) for a constant depth upstream reservoir] might appear to be more generally applicable and thus have greater appeal. Arguing against

constant potential vorticity is that the solutions are probably far more complicated and usually require invoking an additional assumption about an upstream current. In any case, such calculations have yet to be successfully made. In addition, zero potential vorticity is the simplest flow, the only one possible with a motionless upstream fluid and they clearly should be done first.

The starting equations are almost identical to Eqs. (2.7)–(2.9) of Whitehead *et al.* They were derived from the fully non-linear, depth-averaged, inviscid, steady Navier-Stokes equations. Although they were derived in Whitehead *et al.* for a fluid layer of density ρ below a deep motionless fluid of density $\rho-\Delta\rho$, we will use them here with the upper fluid having zero density. The co-ordinate system and geometry is sketched in Figure 1. The first equation [which corresponds to (2.7)] has a geostrophic balance across the strait

$$-fv+g\,\partial\zeta/\partial x=0, \tag{2}$$

where v is the along-strait flow direction and x is the across-strait coordinate. Here we have used $g\zeta$ as the normalized pressure due to the height of the surface ζ. The second [which corresponds to (2.8)] is Bernoulli's law from the upstream basin to the strait. This is

$$\tfrac{1}{2}v^2+g\zeta=g\zeta_1, \tag{3}$$

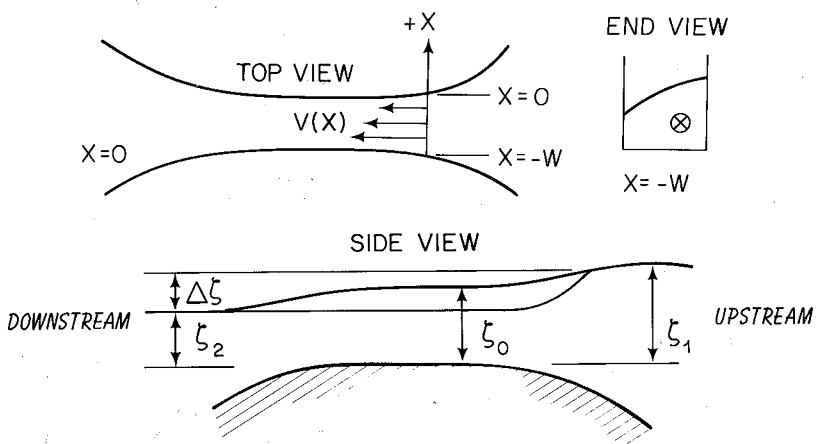


FIGURE 1. Sketch of physical problem and definition of heights and coordinates.

where $g\zeta_1$ is upstream pressure (constant everywhere). The third equation is the zero potential vorticity statement

$$v_x + f = 0, \tag{4}$$

which is valid because the vorticity in the upstream basin is f/H, which is approximately zero when the upstream H becomes very large.

To solve, (4) is integrated

$$v = -fx + v_0 \tag{5}$$

to obtain a velocity profile, and (2) is used to determine the free surface profile across the channel,

$$\zeta = -\tfrac{1}{2}f^2g^{-1}x^2 + fg^{-1}v_0x + \zeta_0, \quad \text{where} \quad -w < x < 0. \tag{6}$$

ζ_0 is the height of the free surface above the sill at the origin. There are two constants of integration v_0 and ζ_0. Equation (3) can be used to relate v_0 to ζ_0, i.e.

$$v_0 = [2g(\zeta_1 - \zeta_0)]^{1/2}. \tag{7}$$

Let ζ_0 and hence v_0 be determined by the sea level of the downstream basin, i.e. let $\zeta = \zeta_2$ at the postion of fastest velocity which is at $x = -w$. In this case Bernoulli's law gives

$$v_0 = [2g(\Delta\zeta)]^{1/2} - fw, \tag{8}$$

where $\Delta\zeta = \zeta_1 - \zeta_2$.

Setting $\zeta = \zeta_2$ at $x = -w$ is possibly suspect since it clearly cannot be proven. A precedent for doing this was set by Toulany and Garrett. They also argue that ζ can locally be less than ζ_2. We feel such a possibility is unlikely. It would lead to an adverse pressure gradient in the sidewall boundary layer of the downstream basin which should lead to separation and relief of the adverse gradient. Such separation on the other wall (corresponding to $x = 0$) has been observed by Whitehead and Miller (1979) in a baroclinic flow when curvature of the wall was more abrupt than a Rossby radius of deformation.

We also note that setting the height equal to the downstream height is commonly done in non-rotating hydraulics (Prandtl and Tietjens, 1957, p. 243; Rouse, 1961, p. 257). However, since this is the only assumption in addition to zero potential vorticity, it can equally be regarded as an assumption that is tested by our laboratory experiment in the next section.

The analysis is now essentially complete and v and ζ can be calculated as a function of $\Delta\zeta$. There are two cases, corresponding to two possible signs for v_0. If v_0 is positive, there is unidirectional flow through the channel. If it is negative, there would be a reverse flow in the channel which would be inconsistent with the fluid source being upstream. In that case, we must set velocity equal to zero everywhere to the right of the point where $v = 0$. This point is found by setting (5) equal to zero and using (8). It is

$$x_p = -\alpha = v_0/f = \{[2g(\Delta\zeta)]^{1/2} - fw\}/f. \tag{9}$$

If x_p is positive, the average velocity $\bar{v}$ is found by integrating (5) across the entire channel, so

$$\bar{v} = [2g(\Delta\zeta)]^{1/2} - \tfrac{1}{2}fw \quad \text{when} \quad [2g(\Delta\zeta)]^{1/2}/fw > 1. \tag{10}$$

If x_p is negative, we integrate across the channel from $x = -w$ to $x_p = -\alpha$, in which case

$$\bar{v} = g(\Delta\zeta)/fw \quad \text{when} \quad [2g(\Delta\zeta)]^{1/2}/fw < 1. \tag{11}$$

The formula (11) is identical to that by Toulany and Garrett, and Eq. (10) is the classical hydraulic relation for the velocity of flow through a channel with a second negative term which arises from lateral shear due to frame rotation. The important point is that the magnitude of the current as a function of the sea level difference is given by (10), which looks like a modified Bernoulli's law, for $[2g(\Delta\zeta)]^{1/2} > fw$ and by (11), which looks like a geostrophic balance and the Garrett and Toulany formula in the converse limit. Note that the latter limit is valid for large $\Delta\zeta$, rather than small $\Delta\zeta$. If f is fixed in a "run" down problem, the rotational effects come in more strongly with time. This was also experimentally observed by Whitehead, Leetmaa and Knox (1974). Note also that when $[2g(\Delta\zeta)]^{1/2} = fw$, the two results are the same.

Both the concept of a geostrophic balance and a Bernoulli's law balance is well established. What is new here is how to formulate the problem so both limits are linked. A new result is that the criterion for transition form one to the other is the ratio of a new width scale

$$W=(2g\Delta\zeta)^{1/2}/f \tag{12}$$

to channel width w. W could be called a Rossby radius except that Eq. (12) contains no absolute depth as usual formulations for the Rossby radius do (Gill, 1982; Pedlosky, 1979). Instead, W is proportional to the square root of the change in height between basins. Thus, this width has a different physical interpretation than the Rossby radius. Instead of W being a measure of the deviation of local vorticity from upstream vorticity, it is a measure of the velocity due to Bernoulli's law times the inverse of the cross-current shear, which is the inertial time scale f^{-1}.

Finally, we will remark that the formulae here can be reconciled with the formulae in the earlier rotating hydraulics studies, even though the new width scale W was not found in those studies. In comparing these predictions to experiments in the next sections, the assumption will be made that $\Delta\zeta/\zeta$ is small. If that assumption is not made and the consequences of $\Delta\zeta/\zeta$ being order one are investigated, the predictions of Whitehead *et al.* are recovered.

3. A LABORATORY EXPERIMENT

A laboratory experiment was conducted in an attempt to test the predictions. A false bottomed sill 15 cm wide and 20 cm long was placed 10 cm above the bottom of a 120 cm × 30 cm × 30 cm Plexiglas tank, as sketched in Figure 2. Water was put at $H=1$ and $H=4$ cm above sill depths (in two different sets of experiments), and a submersible pump was placed in the basin downstream of the sill. The pump with a volumetric flow of $Q=500$ cc/sec ± 30 cc/sec pumped water to an upstream reservoir which was filled with crushed rocks of roughly 1 cm diameter, and the reservoir was separated from a second upstream basin by a 5 cm thick piece of artificial sponge rubber. The tank was mounted on a 1 meter diameter leveled turntable which could rotate at periods of approximately

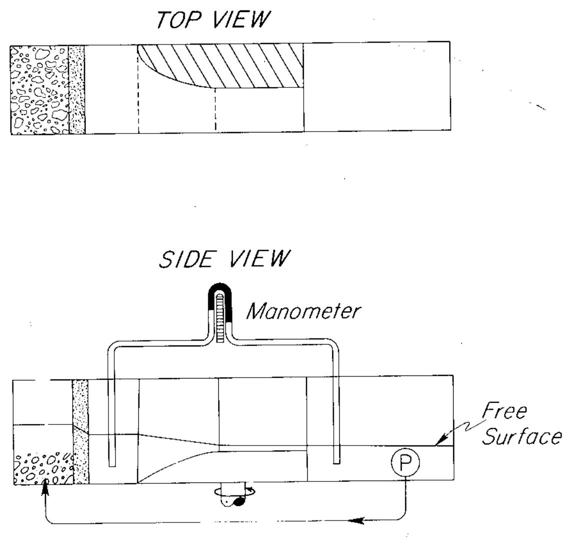


FIGURE 2 Sketch of laboratory apparatus, not to scale.

200 seconds down to periods of five seconds, which was the limit allowed by centrifugal distortion of the free surface. Measurement was made of the difference in surface elevation between the upstream and downstream basins (corresponding to $\Delta\zeta$ in the theory) by constructing a differential manometer filled with silicon oil of density 0.940. The two ends of the manometer were connected to the upstream and downstream basins, respectively, so that a gain of 16:1 was realized, i.e. a reading of 16 cm in the manometer matched a difference in surface elevation $\Delta\zeta$ of 1 cm in the basins. The manometer was mounted so that any centrifugal effects on the interface were cancelled. To do this to sufficient accuracy, the two manometer tubes were equidistant from the axis of rotation of the tank to within 3 mm, which is consistent with a maximum error of 0.6 mm.

The largest source of reading error was due to hysteresis of the oil/water interface in the manometer. Manometer readings were reported to the closest mm, but were clearly not unambiguously readable due to distortions of the interface by surface tension and hysteresis of the glass-oil-water contact angle. We estimate that readings can be relied upon with a ± 1 mm accuracy. This yields an error estimate of ± 0.006 cm, and this error bar is shown in Figure 3.

The data for $H=4\,\text{cm}$ and both theoretical curves are shown in Figure 3. There is clearly good agreement with both the Toulany and Garrett limit (11) and the rotationally modified hydraulic limit (10). The transition from one to the other, found by equating (10) to (11) is at $f=2Q/w^2H=0.88$, as indicated by an arrow. We thus see excellent agreement between the data and the unified theory.

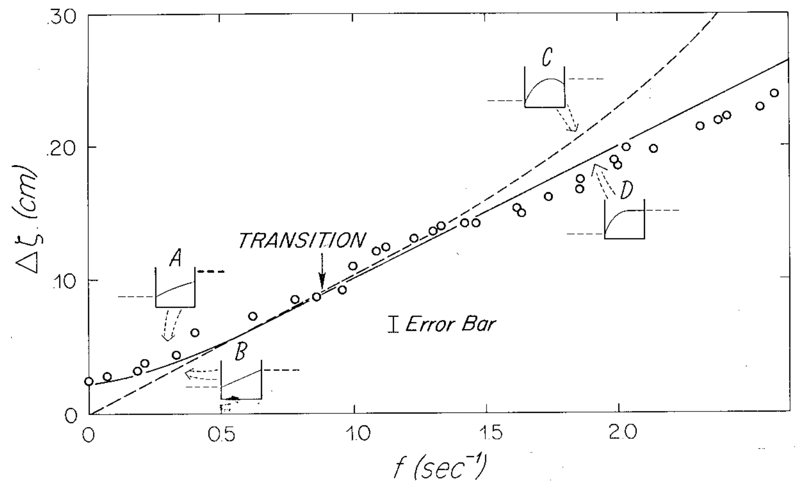


FIGURE 3 Measurement of $\Delta\zeta$ as a function of f for experiments with $H=4\,\text{cm}$. The solid curve to the left of the transition arrow is Eq. (10) and the solid line to the right (11). The formulae in the incorrect limits are also shown as a dashed curve for (10) and a dashed line for (11). Four cartoons of cross-strait sea surface and its relation to the upstream and downstream heights (dashed lines to the right and left of the channel, respectively) are shown. (a) Correct nonlinear theory. There is a lateral shear and the right-hand height cannot match upstream height and still be consistent with Bernoulli's law. (b) Incorrect balance, based on geostrophy alone. The heights match upstream and downstream, but the flow is faster than Bernoulli's law allows. (c) Incorrect nonlinear theory. The channel is wider than W [Eq. (12)] and there is a return flow which must be excluded. (d) Correct geostrophic balance. There is a confined jet and the other region is stagnant.

To clarify why each limit is not valid on the other side of the transition point, extrapolation of the curves are shown as dashed lines. A cartoon of the physical situation of each limit is also sketched and explained in the caption.

Also supporting (10) are data from experiments conducted at

$H=1\,\text{cm}$, as shown in Figure 4. Observations could only be made in the hydraulics limit because at rotations greater than $f=1.7\,\text{s}^{-1}$ there was a clear formation of a lee-jump downstream of the sill. Again, the agreement is good.

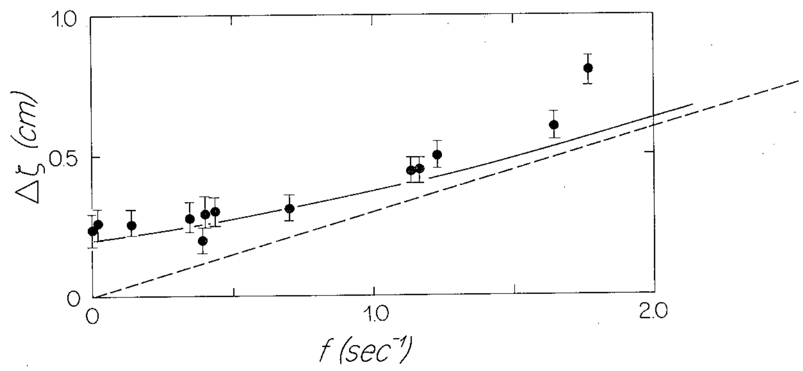


FIGURE 4 Measurement of $\Delta\zeta$ for $H=1\,\text{cm}$. The dashed line is Eq. (10) and the solid line and curve are Eqs. (10) and (11).

A central starting assumption of this theory and other rotating hydraulic theories (Whitehead *et al.*, 1974; Shen, 1981) is that the potential vorticity is zero. In this experiment the vorticity in the channel could be directly measured. To do this, polyethylene pellets were scattered upstream of the channel and streak photographs were taken of pellets by a half-second time exposure as they moved through the channel. Figure 5 shows two such photographs. In practice, many photographs were taken of each run. To determine vorticity, the velocity vectors were obtained by plotting the streaks on a master chart. From this the vectors on Figure 6 were drawn. Next the equivalent of the left-hand edge of the jet was plotted (shown by the dashed lines in Figure 6). The flow did not go straight along the channel but was tilted with respect to the channel axis. The velocity along the flow axis was then determined (Figure 7). A regression analysis was conducted of the least squares fit of each of the five data sets to uniform shear velocities. The results are shown in Figure 7. Shears for the four rotating cases were of size $-0.36f$, $-0.39f$, $-0.59f$ and $-0.72f$. Thus the vorticity was negative and its magnitude was measured to be a significant fraction of f.

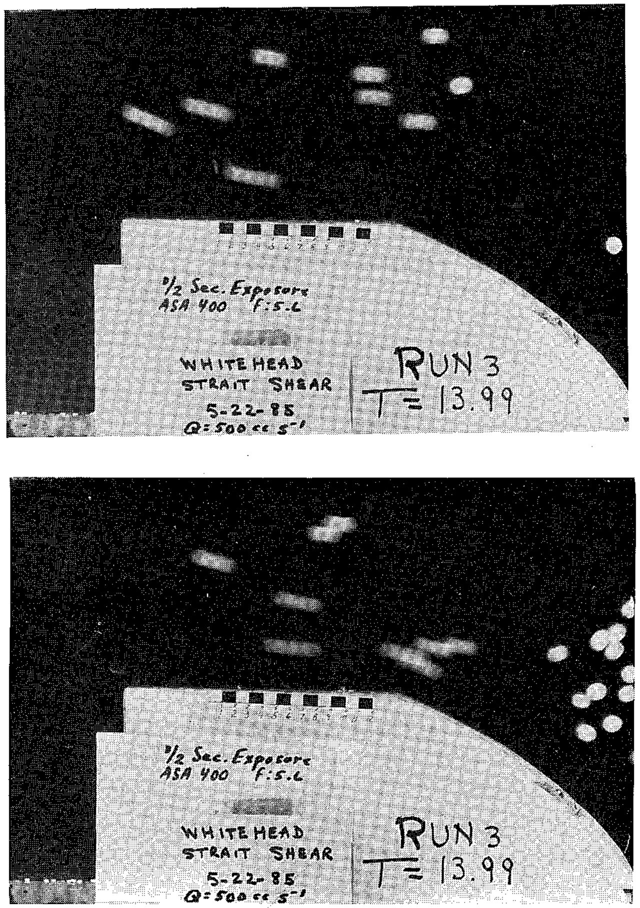


FIGURE 5 Two time exposures of pellets being swept along the laboratory channel.

The current may sharpen as it leaves the channel, and if so, this is not encompassed in the theory. However, there are really very few vectors downstream of the channel. The apparent intensification of the jet as it exits the channel in Figure 6 may be more an artifact of the drafting (the vectors represent velocities at their base) than any real continued sharpening of the current. Also, there is some intense cyclogenesis of the current in the downstream basin due to evacuation of water by the pump which should distort any flows there.

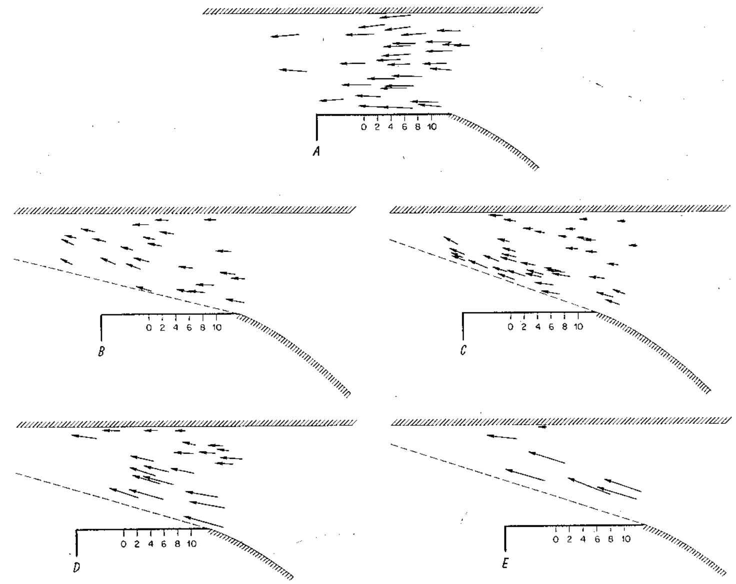


FIGURE 6 Velocity vectors from the time exposures. Periods of rotation for A→E are ∞, 24.6, 13.98, 8.82 and 6.17 seconds, respectively. Note how the flow separates from the left-hand wall (looking in the direction of the current). The theory predicts that configuration D in Figure 3 (a narrow jet) is appropriate for cases D ($W=11.4$ cm) and E ($W=9.8$ cm). We see that in fact the jet is tilted also when W is greater than the channel thickness of 15 cm.

Although the angle of the velocity vectors with respect to the axis of the channel may seem surprising, it is not unusual to see a jet at an angle to a channel. It is actually consistent with sketch D in Figure 3, in which a narrow jet occupies only a part of the channel. We observe that in reality the jet crosses the channel at an acute angle and thus will allow the two boundary conditions to be met at each edge of the jet.

4. DISCUSSION

In what circumstances does one or the other limit of Eqs. (10) and (11) apply to an oceanic problem? Let us look at the ratio of (10)

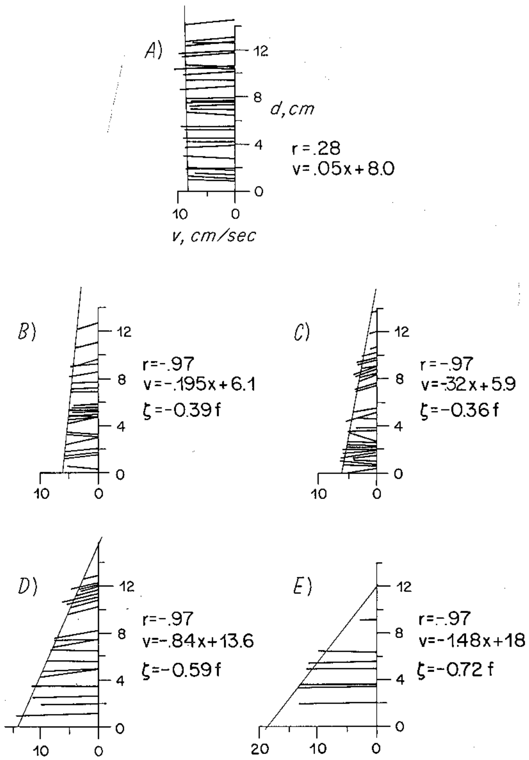


FIGURE 7 Stick diagrams of velocity along the jet as a function of a cross-jet distance. The best fit linear regression line is shown with its regression coefficient r. The measured absolute vorticity is given as a percentage of the basin vorticity f.

and (11),

$$\lambda=\{[2g(\Delta\zeta)]^{1/2}-\tfrac{1}{2}fw\}/g(\Delta\zeta)(fw)^{-1}. \qquad (13)$$

Let us speculate upon the flow of Atlantic water through the Strait of Gibraltar. It is well known that Atlantic water flows into the Mediterranean over the outflow (albeit time varying) of saltier and very much denser Mediterranean water. The Mediterranean water flowing out will be regarded as part of the bottom over which the Atlantic water flows. Thus, the interface between the Atlantic and

Mediterranean water will be regarded as the bottom and we interpret the flow of Atlantic water over this bottom as a barotropic flow.

The parameters are $g=10^3$, $\Delta\zeta=40\,\text{cm}$, $f=10^{-4}\,\text{s}^{-1}$ and $w=10^6\,\text{cm}$ (10 km). Since we assume that Atlantic water is flowing barotropically, we use the full value of g rather than reduced gravity. Thus, $(2g\Delta\zeta)^{1/2}/fw=2.83$ and Eq. (10) is appropriate. The inverse of Eq. (13) shows how much Eq. (11) overestimates the flow, i.e. λ^{-1} is 1.72, so a calculation for Gibraltar using (11) [the geostrophic estimate used by Garrett and Toulany (1982)] is 1.7 times larger than what we suggest here to be a more appropriate limit. This is certainly within the limits of error of many oceanographic measurements and the ability of simple models to accurately mimic complex geometries and hydrographies of real straits. Even though λ is considerably smaller than one, it may be even larger or smaller for other straits.

Using the above values, the width scale W from Eq. (12) is 28 km. Thus w/W is 0.35, which implies that the geostrophic constraint should be less important than Bernoulli's law but only marginally so. In contrast, the usual measure for the importance of rotation for the barotropic current is the ratio w/R. Here R is the barotropic Rossby radius of deformation $R=(g\zeta)^{1/2}/f=547\,\text{km}$ (using $g=10\,\text{m}\,\text{s}^{-1}$ and $\zeta=300\,\text{m}$). Thus w/R is 0.018, which is a very small number that would have led one (and did lead this author and others) to expect that rotational effects on the current were almost completely negligible.

Clearly, the central thrust of Toulany and Garrett is to extend the ideas of linearized time-dependent adjustment through straits with time-dependent (wave) adjustments in the two basins. Linearized flux relations through straits are convenient for this purpose. Moreover, linearization can, in principle, be used for regions where geostrophic balances are more significant than inertial balances. However, the geostrophic Eqs. (10) and (11), with the included limits, can more generally be used.

An important assumption of the formulation developed here is that of zero potential vorticity. Some discussion of this seems necessary. It is the simplest assumption and has the pleasing feature that the fluid in the upstream basin is stagnant. It leads to a lateral shear of size $-f$ in the channel. Although this seems at first like a drastic assumption, the laboratory measurements indicated that the vorticity in the channel was less than $|-f|$, but was certainly of the same order and sign. Although the assumption could not be regarded as verified with any precision, it would seem that it is good to a crude first approximation. A considerably more precise laboratory test is planned.

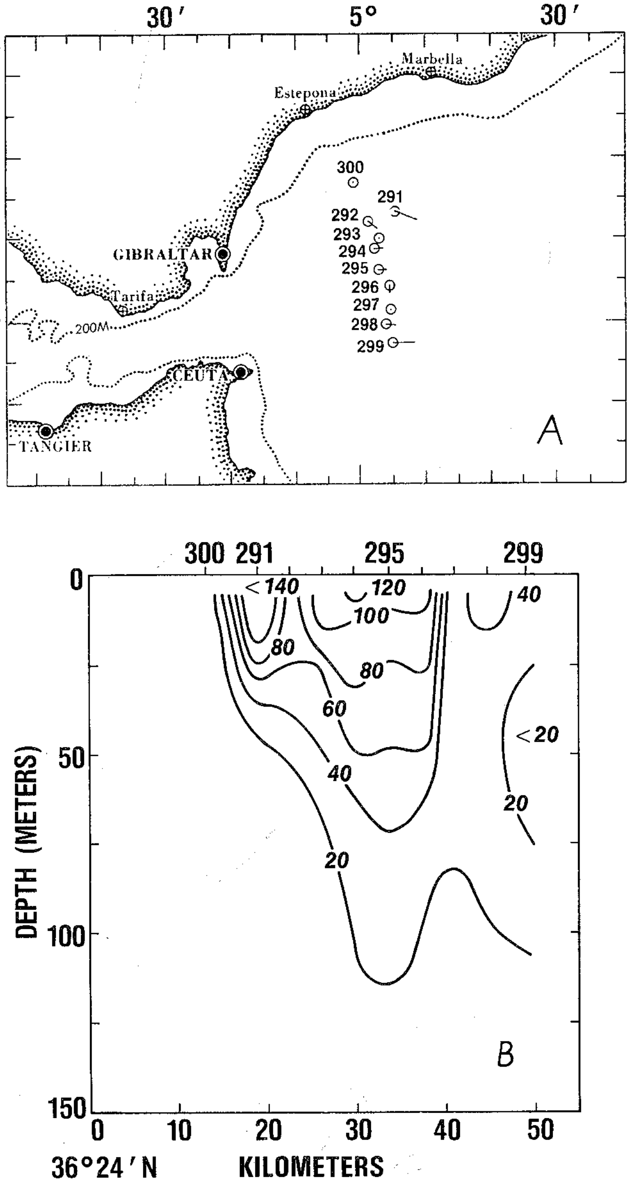
It has been pointed out that the measured values of $\Delta\zeta$ in Figures 3 and 4 are systematically higher than the predictions below the transition point. Possibly this is due to the finite potential vorticity effect. In any event, the flux in the channel will always obey geostrophy irrespective of the vorticity (which merely alters the velocity profile in the channel), and other shears should not alter the flow balances to any great degree. Within this whole context, it is important to remember that drastic alterations to these velocity profiles could produce unstable currents which would not be realized.

Finally, in defense of the zero potential vorticity assumption, it should be noted that the constant potential vorticity solutions of Gill (1977) produce results rather similar to the zero potential vorticity results of Whitehead, Leetmaa and Knox (1974), and that the splitting properties of jets also seem to be similar for the two cases (Whitehead, 1985a).

With the approximately ten experiments which have been done to date in rotating hydraulics, we have found that it is easy in the laboratory to make what in concept should be close to a zero potential vorticity upstream condition for a steady experiment in the laboratory, and very difficult to make anything else. Say we attempt a constant potential vorticity upstream condition by constructing a constant depth upstream basin. As the experiment starts, a Kelvin wave propagates into the basin and makes a constant potential vorticity current on the right-hand wall (looking upstream). In a short time the wave will have propagated around the periphery of the basin a few times and led to a gyre in the upstream basin. This gyre, is progressively more anticyclonic with time and the depth and hence potential vorticity tends to zero at large time. In practice, the potential vorticity is not strictly zero, but it very rapidly becomes of *order* zero. Now if we resupply the basin with new fluid in order to generate a steady state, we could conceivably pump in fluid with a constant velocity profile at exactly the original depth. To our knowledge, this has not been done. Some of the fluid will flux in through the bottom Ekman layer and provide Ekman convergence in the interior which will enhance the anticyclogenesis. Most frequently, fluid is pumped in through a porous bottom (usually constructed of gravel, foam rubber, or some packing material) and this very strongly generates an anticyclonic gyre in the upstream basin.

Is potential vorticity on the order of zero at oceanic straits? There is some supporting evidence. The Atlantic water which flows into Gibraltar is not fed by any known coastal current with the Rossby radius constant, as constant potential vorticity would require. In addition, at many oceanic straits, water masses are drawn up hundreds of meters by the suction of the strait flow. This vertical compression should produce close to zero potential vorticity. Probably the best known example is the upwelling of deep Mediterranean water (Stommel, Bryden and Mangelsdorf, 1973) into the Strait of Gibraltar which has been modeled both in the laboratory (Whitehead, 1985b) and theoretically (Bryden and Stommel, 1982; Hogg, 1984). Since the magnitude of flow velocities is the same on the Atlantic side of the Strait of Gibraltar as on the Mediterranean side, it is reasonable to expect that deeper Atlantic water is also sucked up to sill depth. However, this is not clearly established by the existing data. Gascard and Richez (1985) give evidence for some North Atlantic central water of salinity less than 36.0‰, $\theta = 13.5°C$ and $\sigma = 27.00$ being in the Strait of Gibraltar at 50 m depth. This water lies at approximately 220 m in the North Atlantic. Such water is also found in the Strait of Gibraltar at a depth of 7–150 m in Figure 9 of Lacomb and Richez (1961). If vertical columns of water are drawn from 200 m depth to 100 m depth and potential vorticity is conserved, the vorticity would already be $-0.5f$.

What is the magnitude of the lateral shear in actual oceanic straits? Although such measurements would be extremely interesting, there are not many measurements with enough resolution to discriminate the shear inside a Strait. One exception is a section taken by Perkins and Saunders (1984) which captures the entire jet of Atlantic water as it enters the Mediterranean Sea. Their "Estepona" section (Figure 8) is located approximately twenty kilometers east of Gibraltar. A velocity maximum is located at the surface at station 294 and is of magnitude 1.34 m/s. The velocity at 297 approximately 10 km south of 294 has dropped to 0.30 ± 0.10 m/s. Using these values, one estimates a shear of -10.4×10^{-5} seconds^{-1}. This is



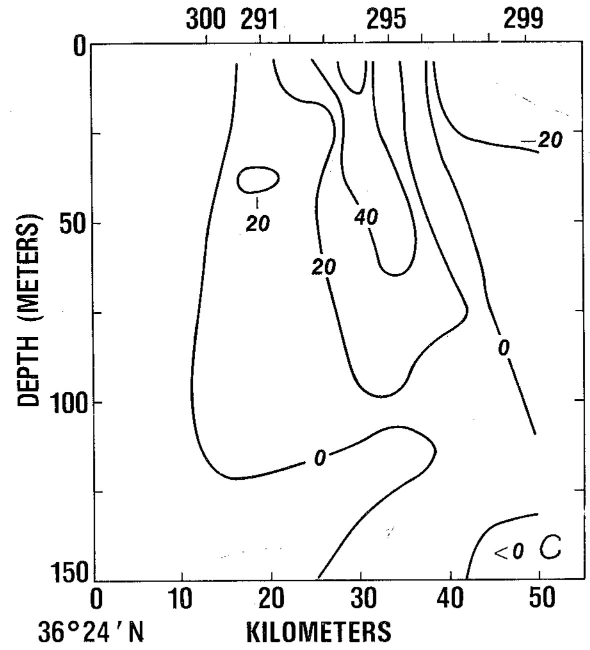


FIGURE 8 Velocity measurements by Perkins and Saunders (1984). Oceanographic section in the northwestern Alboran Sea. (a) Location of the moorings, (b) the East-West velocity, and (c) the North-South velocity from their Figure 4.

quite close to minus the local Coriolis parameter, which at 36°N is approximately 12×10^{-5}.

Of course, there are many corrections which could be made to this crude estimate. There is a second maximum in East-West velocity at station 291, 10 km north of station 294. Also there is a broad region further south which is roughly 10 km or more in width and which has an eastward velocity of 40 cm/sec. If such measurements were incorporated, the magnitude of the estimated shear would be one-half to one-third the present estimate. Also, the jet may have adjusted its width since emanating from the confined strait. There are also the chronic oceanographic temporal sampling problems in view of the fact that this is a two-day data set. Thus, the crude estimate of $-10.4 \times 10^{-5}\,\mathrm{s}^{-1}$ is probably larger than the true vorticity, but its magnitude and sign compared to the local Coriolis parameter is promising.

In conclusion, the lateral shear in the laboratory and ocean seems to be only crudely consistent with the zero potential vorticity assumption. However, oceanic waters are generally drawn up into straits and sills, and the zero potential vorticity calculations are the easiest to do. Both laboratory investigators and physical oceanographers alike could well consider obtaining more measurements to see what the actual vorticity is, and to see if the simple flux laws found here are obeyed.

This problem involves a flow rate which is sensitive to the statement of the downstream depth at the exit. This inevitably raises the question of separation of current as it leaves the channel and enters the downstream reservoir. The details of the separation could be vital in a smoothly varying geometry (which is beyond the scope of this study), where separation does not occur at a well-known point. Here we assume that separation occurs at the narrowest and shallowest point, i.e. the sharp edge of the channel. For a non-rotating channel, if the channel diverges with an angle less than 10°, experiments in Venturi tubes indicate that separation may not occur (Rouse, 1961, p. 265). In that case, an interesting Bernoulli set down (Toulany and Garrett, 1984) (which has rarely been observed) could occur. With rotation, barotropic studies do not exist. Baroclinic experiments (Whitehead and Miller, 1979) indicate separation from the right-hand wall will occur if the radius of curvature is greater than the baroclinic Rossby radius of deformation. If this criterion is true for the baroclinic problem, separation would always occur in experiments such as we have seen here. However, the matching conditions $\zeta=\zeta_2$ is not involved on this wall, hence separation there might not matter. In any case, the practical aspects of calculating fluxes through more realistic openings remain uninvestigated and are important topics for future studies.

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References

Bryden, H. L. and Stommel, H., "Origins of the Mediterranean outflow", *J. Mar. Res.* **40** (Suppl.), 55–71 (1982).

Garrett, C. J. R. and Toulany, B., "Sea level variability due to meteorological forcing in the northeast Gulf of St. Lawrence", *J. Geophys. Res.* **87,** 1968–1978 (1982).

Garrett, C. J. R., "Variable sea level and strait flows in the Mediterranean: a theoretical study of the response to meteorological forcing", *Oceanol. Acta* **6,** 79–87 (1983).

Garrett, C. J. R. and Majaess, F., "Non-isostatic response of sea level to atmospheric pressure in the eastern Mediterranean", *J. Phys. Oceanogr.* **14,** 656–665 (1984).

Gascard, J. C. and Richez, C., "Water masses and circulation in the western Alboran Sea and in the Strait of Gibraltar", *Prog. Oceanogr.* **15,** 157–216 (1985).

Gill, A. E., "The hydraulics of rotating channel flow", *J. Fluid Mech.* **80,** 641–671 (1977).

Gill, A. E., *Atmospheric-Ocean Dynamics*, Academic Press, New York, 662 pp. (1982).

Hogg, N. G., "Multilayer hydraulics control with application to the Alboran Sea circulation", *J. Phys. Oceanogr.* **14,** 454–466 (1984).

Lacomb and Richez, "Contribution a l'etude du detroit de Gibraltar. II. Etude Hydrologique", *Cah. Oceanogr.* **5,** 276–292 (1961).

Pedlosky, J., *Geophysical Fluid Dynamics*, Springer Verlag, New York, 624 pp. (1979).

Perkins, H. and Saunders, K. D., "Sections of current, salinity and temperature in the northwestern Alboran Sea, October 18", Preliminary Results of the Donde Va Meeting in Fuengirolo, Melaga, Spain (ed. G. Parrilla), *Informes Technicas* **24,** 142–149 (1984).

Prandtl, L. and Tietjens, O. G., *Fundamentals of Hydro- and Aero-mechanics*, Dover Publications, Inc., New York, 270 pp (1957).

Rouse, H., *Fluid Mechanics for Hydraulic Engineers*, Dover Publications, Inc., New York, 422 pp (1961).

Shen, C. Y., "The rotating hydraulics of the open channel flow between two basins", *J. Fluid Mech.* **112,** 161–188 (1981).

Stommel, H. M., Bryden, H. and Mangelsdorf, P., "Does some of the Mediterranean outflow come from great depth? *Pure & Appl. Geophys.* **105,** 874–889 (1973).

Toulany, B. and Garrett, C., "Geostrophic control of fluctuating barotropic flow through straits", *J. Phys. Oceanogr.* **14,** 649–655 (1984).

Whitehead, J. A., Leetmaa, A. and Knox, R. A., "Rotating hydraulics of strait and sill flows", *Geophys. Fluid Dyn.* **6,** 101–125 (1974).

Whitehead, J. A. and Miller, A. R., "Lavoratory simulation of the gyre in the Alboran Sea", *J. Geophys. Res.* **84,** 3733–3742 (1979).

Whitehead, J. A., "The deflection of a baroclinic jet by a wall in a rotating fluid", *J. Fluid Mech.* **157,** 79–93 (1985a).

Whitehead, J. A., "A laboratory study of gyres and uplift near the Strait of Gibraltar", *J. Geophys. Res.* **90,** 7045–7060 (corrected plates 12,011-12,013) (1985b).