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Recent progress on understanding the effects of rotation in models of sea straits

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Abstract

A review is given describing progress over the past decade in understanding the effects of rotation in sea straits through the use of models. Major areas of advancement include the hydraulics of rotating fluids (including hydraulic control and hydraulic jumps), upstream influence and interaction with upstream basins, estimates and bounds on transports, time-dependence, and climate monitoring. A number of field programs also have led to better observation of the effects of rotation, and these efforts are mentioned here, and elsewhere in this volume.

1. Introduction

Sea straits are dynamically and strategically important choke points in the general thermohaline (and in some cases the wind-driven) circulation of the world's oceans. Shallow sills and narrow widths confine otherwise broad currents, simplifying problems of measurement and monitoring. Straits and sills also exert long-range effects over the upstream and downstream flows. For example, upstream flows can be 'choked', leading to reduction in the volume transport that would occur if the ocean bottom were flat. An integral part of the choking process is the blocking of information in the form of wave propagation from one basin to another. The presence of deep straits and sills thus alters the routing of information through the ocean. When a choked flow passes over a sill and becomes hydraulically supercritical, entrainment is enhanced, the volume flow rate of the overflow plume is increased, and the temperature and salinity of the throughflow are modified. Many of these processes are difficult to resolve in numerical models of the general ocean circulation.

This review of recent progress in understanding the effects of rotation in sea straits will concentrate on work carried out during the past decade. Reviews of prior modeling and field work can be found in *The Physical Oceanography of Sea Straits* and a review of earlier progress in rotating hydraulics can be found in Pratt and Lundberg (1991). I will describe recent advances in the understanding of hydraulic effects in rotationally influenced overflows, such as those of the Denmark Strait or Faroe-Bank Channel. Particular aspects that have been elucidated by recent modeling or field works include hydraulic control, hydraulic jumps, and closed recirculations. There also have been efforts to understand the upstream

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effects of deep sills in the presence of more realistic 'reservoirs'. This topic is particularly relevant to long-term monitoring of deep overflows using measurements made in an upstream basin. The strategy is based on one's ability to establish a 'weir' formula in which the volume flux over the sill is determined by some readily available information about the state of the flow in the upstream basin. Recent progress in formulating such relations and bounds is also described.

There are several important topics that will not be mentioned here but are covered by others papers in this volume. One involves bottom and interfacial friction, along with Ekman layers and related secondary circulations (see paper by C. Garrett or recent publications by Astraldi et al., 2002; Jungclaus and Vanicek, 1999; Johnson and Ohlsen, 1994; Johnson and Sanford, 1992). Also, exciting new field programs have been carried out in places like the Denmark Strait, the Faroe-Bank Channel, the Baltic, and the Strait of Sicily. Much of this work is reviewed in papers by Lundberg and Borenas, Farmer, and Barringer et al. A subject that is covered only marginally in this article and volume is the outflow 'plume', the descending, entraining portion of an overflow that occurs well downstream of a sill. The reader may wish to consult Baringer and Price (1997), Jungclaus et al. (2001), Astraldi et al. (2002), and references contained therein for more information.

2. Governing equations for simple models

For the most part, simple models of hydraulically driven flows in rotating straits have been confined to inviscid, single-layer systems in which the overflow is confined to a deep, hydrostatic layer underlying a relatively thick and inactive upper layer (Fig. 1). The strait or channel is aligned in the y-direction and the bottom elevation and depth are denoted by $h(x,y)$ and $d(x,y,t)$. I will frequently refer to the 'left' and 'right' walls or edges of the flow, and these apply to an observer facing downstream, as in Fig. 1. If the along-channel (y) variations of the flow occur over a scale that is large compared to the channel width, the along-axis velocity v will be much greater than

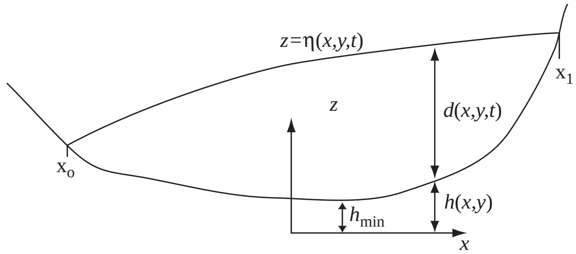

Fig. 1. Definition sketch showing cross-section of the rotating channel as seen by an observer facing downstream.

the cross-channel velocity u. A simple scaling argument then suggests that v will be in geostrophic balance:

$$fv = g'\frac{\partial d}{\partial x} + g'\frac{\partial h}{\partial x} \tag{1}$$

where g' is the reduced gravity and f the Coriolis parameter.

Under the same conditions, the shallow-water potential vorticity is approximated by

$$q = \frac{f + (\partial v/\partial x)}{d} = \frac{f}{d_\infty}. \tag{2}$$

As suggested by the second equality, q can be represented by the 'potential depth' d_∞, the thickness a fluid column must have in order that its relative vorticity be zero.

Eliminating v between (1) and (2) yields a second-order equation governing the cross-sectional structure of the flow:

$$\frac{\partial^2 d}{\partial x^2} - \frac{fq}{g'}d = -\frac{f^2}{g'} - \frac{\partial^2 h}{\partial x^2}. \tag{3}$$

Although v is geostrophic, the cross-channel velocity u is not and thus the flow is subject to the full momentum balance in the y-direction:

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu = -g'\frac{\partial d}{\partial y} - g'\frac{\partial h}{\partial y}. \tag{4}$$

Such a flow is sometimes called 'semigeostrophic'.

If the flow is steady, the potential vorticity is a function of the transport stream function ψ and is related to the Bernoulli function

$$B(\psi) = \frac{v^2}{2} + g'd + g'h \tag{5}$$

by the relation

$$\frac{dB}{d\psi} = q. \tag{6}$$

The main simplification afforded by the semigeostrophic approximation is that the cross-channel (x) structure of the flow can often be determined at the outset, without consideration of the y- (or t-) dependence. To do so one must specify the potential vorticity $q(\psi)$ and solve the second-order Eq. (3). The y-dependence of the solutions is contained in the two constants of integration and is determined from (4) or a derived relation. The early models of hydraulically driven, rotating flow are based on solutions to (3) with $q = 0$ (Whitehead et al., 1974, hereafter WLK) and $q = \text{const.} > 0$ (Gill, 1977).

A difficulty arising in the application of reduced-gravity models to deep straits and sills lies in the identification of the active layer. For example, the overflows in the Denmark Strait and Faroe-Bank Channel are composed of various deep and intermediate water masses with different origins and debate may arise as to whether all of these or some subset comprises the 'active' layer of a reduced-gravity system.

3. Flow criticality

Given the strait geometry and given some general statement about the flow in the upstream basin, it is possible to identify among the family of possible solutions certain ones that are subject to hydraulic (or 'critical') control. Such solutions are 'choked', meaning that the volume transport and other upstream properties are constrained by the geometry of the strait itself, usually the characteristics of the sill. Hydraulically controlled solutions pass through a subcritical-to-supercritical transition at a 'control' section and typically have the maximum transport of all members of the family of possible solutions (Killworth, 1995). As the control section is passed, the flow evolves from a subcritical state, in which long-wave propagation in the upstream direction is possible, to a supercritical state, in which propagation is only permitted in the downstream direction. This supercritical region blocks information generated in the downstream basin, preventing it from influencing conditions in the upstream basin. Instead, it is the geometrical characteristics of the flow at the critical section (often the sill) that influence upstream conditions. The upstream/downstream asymmetry associated with the hydraulic transition is manifested by the spillage of water across the sill (exemplified in Fig. 2 for the Denmark Strait). Flows that are not hydraulically controlled have an upstream–downstream symmetry with respect to the topography, at least when mixing and dissipation are negligible.

It is the 'overflow' character that has led investigators to believe that the deep flows in places like he Denmark Strait, the Faroe-Bank Channel, and the Vema Channel are hydraulically controlled. It is notable that in none of these cases has the criticality of the sill flow been verified by direct calculation of the long-wave speeds. If the flow was non-rotating, free of horizontal shear, and confined to a channel of horizontal section, then the speed of gravity waves attempting to propagate against the flow would be $v - (g'd)^{1/2}$

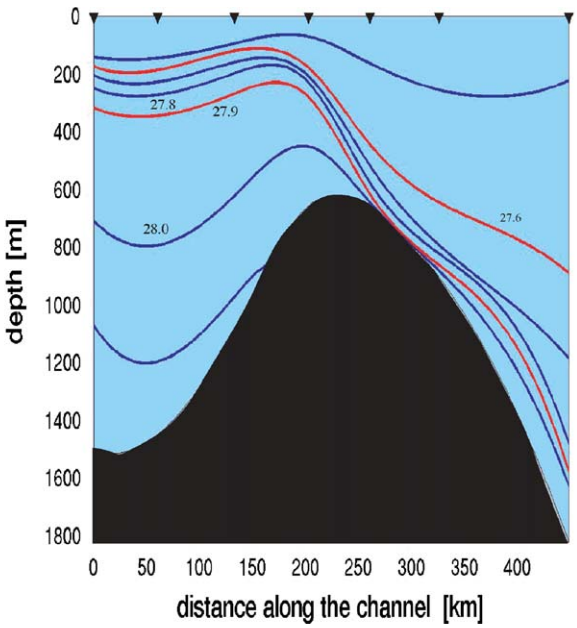

Fig. 2. Longitudinal density (σ_θ) section through the Denmark Strait (from Nikolopoulos et al., 2003).

and the flow would be supercritical when the conventional Froude number $F_d = v/(g'd)^{1/2} > 1$. Gravity continues to be of fundamental importance in rotating overflows and the wave of interest would therefore appear to be the Kelvin wave, or the generalization of the Kelvin wave when the background flow has strong shear and vanishing depth at its two edges.

Some progress on defining a generalized critical condition or Froude number for a rotating flow has been made in highly idealized systems. If the channel has a rectangular cross-section (with sidewalls at $x = 0$ and $x = w$), and v is unidirectional, it can be shown (Stern, 1974) that critical flow requires:

$$\int_0^w \frac{1}{dv^2}\left(1 - \frac{v^2}{g'd}\right)\partial x = 0. \tag{7}$$

The traditional Froude number at a critical section must therefore range above and below unity for hydraulic criticality. Since the potential vorticity can be non-uniform, it is possible that the flow might become critical with respect to a Rossby wave. It is surprising that such a state should depend on the behavior of the local Froude number; however, it is possible that the requirement of unidirectional flow might be restrictive.

Kaese et al. (2003) and J. Girton (pers. comm.) have calculated the value of $v/(g'd)^{1/2}$ across various sections at and downstream of the sill in the Denmark Strait overflow. Values less than unity are generally found at the sill, whereas values consistently greater than unity are found 60–70 km downstream. This would seem to suggest a critical section well downstream of the sill, though (7) is rendered strictly inapplicable by the assumption of rectangular geometry.

Another successful attempt to derive a generalized critical condition was made by Borenäs and Lundberg (1986, 1988), who considered a channel with a parabolic cross-section and uniform potential vorticity fluid. Killworth (1995) has shown that the occurrence of critical flow in more general settings is equivalent to the requirement that a certain homogeneous equation (whose coefficients depend on the flow) has a solution. That is, a flow state must be found such that the coefficients in the equation yield a non-trivial solution. In general, the method for doing so involves a numerical procedure that is equivalent in difficulty to the procedure for calculating the wave speeds of an observed flow numerically.

4. Upstream influence

In classical hydraulics, the upstream influence of a sill (or width contraction) is typically contained in a relation between the volume transport and the elevation Δz above the sill of the water surface in the upstream basin. If the sill is raised, as is possible in engineering applications, the transport and upstream surface elevation are altered. This alteration is caused by a blocking wave that is generated at the sill and moves upstream. In geophysical applications, the sill remains fixed and upstream influence occurs as a response to information generated upstream. Changes in the upstream basin are communicated to the sill in the form of incident waves and the choking effect is transmitted back into the basin by reflected waves (Pratt, 1984). In principle, this process could be used as the basis for parameterization of upstream influence in numerical models unable to resolve straits. The strait and sill system is replaced by a partially reflecting boundary that allows fluid to pass through. The reflection coefficient for an incident wave is calculated using a hydraulic relation (such as nine below). Pratt and Chechelnitsky (1997) present some examples of this procedure.

All simple models of rotational effects in straits employ idealized upstream conditions. The simplest situation, originally suggested by WLK, involves an infinitely deep upstream basin containing quiescent fluid. To pass over a shallow sill, a fluid column must be severely squashed and its absolute vorticity must become small in the sense

$$\frac{f + \partial v/\partial x}{f} = \frac{d_{\mathrm{sill}}}{d_\infty} \ll 1, \tag{8}$$

where d_∞ is the reservoir depth and d_{sill} is a scale for the depth at the sill. In this limit, sometimes referred to as ‘zero-potential vorticity’, the transport over a sill with a rectangular cross-section is given by

$$Q = \begin{cases} \left(\frac{2}{3}\right)^{3/2} w_s (g')^{1/2} [\Delta z - (w_s^2 f^2/8g')]^{3/2}, \\ (g'/2f)\,(\Delta z)^2 \quad \text{(separated)}, \end{cases} \tag{9}$$

where Δz is the elevation of the reservoir interface above the sill. The second expression holds when the sill flow becomes separated from the left wall (facing downstream), which occurs when the sill width w_s exceeds the value $(2g'\Delta z)^{1/2}/f$. Eq. (9) reduces to the non-rotating formula

$$Q = \left(\tfrac{2}{3}\right)^{3/2} w_s (g')^{1/2} (\Delta z)^{3/2} \tag{10}$$

in the limit $w^2 f^2/(g'\Delta z) \ll 1$. This last parameter provides a measure of the importance or rotation at the sill section.

It should be pointed out that the assumption of quiescent flow in the infinitely deep basin cannot be verified using the zero-potential vorticity model, since approximation (8) fails there. However, Borenäs and Pratt (1994) have verified the hypothetical upstream state can exist by integrating the equations for finite (but still uniform) potential vorticity.

A more sophisticated view (Gill, 1977) is that the upstream basin is infinitely wide but not necessarily deep. The potential vorticity is assumed uniform and this filters Rossby waves from the system, leaving just Kelvin waves to provide communication. The basin flow is confined to boundary layers with thickness $L_d = (g'd_\infty)/f$, where d_∞ is the depth in the quiescent interior of the basin. In addition to Δz, which is now interpreted as the elevation of the interface in the basin interior above the sill, the transport of a critically controlled state depends on an additional parameter. Gill chose the latter to be a measure of how the transport is partitioned between the boundary currents on the two sidewalls of the basin. The resulting transport relations involve solutions to higher-order polynomials and cannot be expressed as simply as (9). The transports are generally smaller than those that would be predicted by WLK for flow from an infinitely deep basin.

Killworth (1992) presents an alternative view of the flow in a wide upstream basin in which boundary layers are not present. The transport is contained in a weak flow that is smoothly distributed across the whole basin width. The basin bottom is assumed horizontal (with $h = 0$) and therefore the basin Bernoulli function is approximated by $g'd$. In addition, the transport stream function and depth are related by $\psi = (g'/2f)(d^2 + \text{const.})$, which follows from (1). The basin Bernoulli function is therefore given by $B(\psi) = g'[(2f\psi/g' - \text{const.}]^{1/2}$ and the corresponding potential vorticity $\mathrm{d}B/\mathrm{d}\psi$ is clearly non-uniform. In addition to the total transport, the upstream flow is completely specified by the constant, which can be related to the depth along one of the sidewalls. The basin flow therefore has one fewer degrees of freedom than in the Gill model. The transport relation for a critically controlled solution must be obtained numerically, and the fluxes are generally smaller than what is predicted by (9).

Among the many approximations that one might call into question is the conservative nature of the upstream flow. In the WLK model, for example, fluid is expected to rise from the bottom of a deep reservoir, crossing f/d contours without the aid dissipation, in order to pass over a shallow sill. The spatial scales of the motion in the basin are typically an order of magnitude or more greater than the length of the strait, giving forcing and dissipation more room to act. Pratt and Llwewllyn Smith (1997) and Pratt (1997) attempted to come to grips with some of these issues by matching an inviscid model for hydraulically controlled flow in a strait to a basin containing a nearly geostrophic flow with distributed sources of mass and with bottom friction. The basin topography is bowl-shaped with closed f/H contours (except for those contours leading into the strait) and $f = \text{const.}$ Fluid introduced into the basin through the sidewalls is fed directly into the strait through frictional boundary layers whose dynamics are equivalent to that of an arrested topographic wave. When fluid is fed into the basin from above, an anticyclonic circulation is set up causing the fluid columns to move in a widening spiral that eventually reaches the basin edge. From there, the fluid is channeled into boundary layers that feed into the strait.

In principle, one would like to use one or more of the above models to understand the effects of deep sills on the thermohaline circulation. A convenient thought experiment is to ask how the flux and the upstream state would alter in response to a small change in the elevation of the sill. In the WLK case, the interface level of the upstream basin would tend to rise and the transport would be reduced by some small amount. In the Gill model, the change in sill level would be transmitted by a Kelvin wave that would move into the basin along the Northern Hemisphere left wall (facing downstream). (If the basin were closed, this wave might circle the basin and interact again with the sill, resulting in the generation of a secondary upstream wave.) It is not clear if or how the interface level in the quiescent interior of the basin could be influenced by these changes, and this begs the question of how that interior level is established in the first place. In the Killworth (1992) model, the potential vorticity is non-uniform and signals presumably would be carried upstream by Rossby and Kelvin waves. In contrast to the Gill model, it is clear how information would propagate into the interior of the basin. However, it is not clear that the sluggish conditions envisioned by Killworth (with no boundary layers) would be maintained in the presence of Kelvin wave influence. Finally, the model proposed by Pratt (1997) involves an upstream basin whose flow is completely determined by linear dynamics. Thus, the circulation pattern remains independent of the magnitude of the flux across the sill. If the sill height is altered, the mean elevation of the interface in the basin rises but the streamline patterns remain fixed. This behavior is consistent with the results of numerical simulation by Helfrich and Pratt (2003), as discussed below.

As suggested by these last remarks, the necessity of tractability has resulted in the development of idealized models that are somewhat disjoint. Attempts to relate the hypothetical upstream states to the ocean have been frustrated by a lack of upstream observations. In particular, it has been difficult to trace the various deep and intermediate water masses that compose the Denmark Strait and Faroe-Bank Channel to specific upstream current systems or source regions.

Models with conservative upstream states are helpful in developing intuition but may be unrealistic. For example, the presence of deep western boundary layers in upstream basins suggests that friction cannot be ignored. In understanding an upstream circulation subject to forcing and dissipation, a helpful constraint can be developed in the form of Kelvin's circulation theorem. If the tangential component of the shallow-water momentum equation is integrated about the basin edge, and across the mouths of any straits leading into and out of the basin, it follows that:

$$\frac{\mathrm{d}}{\mathrm{d}t}\oint_c \mathbf{u}\cdot\mathbf{t}\,\mathrm{d}s = -\oint_c (f+\varsigma)\mathbf{u}\cdot\mathbf{n}\,\mathrm{d}s - r\oint_c \mathbf{u}\cdot\mathbf{t}\,\mathrm{d}s, \tag{11}$$

where ζ is the relative vorticity, $\mathbf{t}$ and $\mathbf{n}$ are unit tangent and outward normal vectors to the boundary, and r is a linear drag coefficient. Eq. (11) tells us that the rate of change of circulation about the basin edge (zero for steady flow) equals the flux of absolute vorticity through the basin edge due to inflows and outflows, plus the dissipation due to contact with the bottom. In Pratt's (1997) steady model, f is constant and $\gg\zeta$, so that $\oint_c f\mathbf{u}\cdot\mathbf{n}\,\mathrm{d}s = -r\oint_c \mathbf{u}\cdot\mathbf{t}\,\mathrm{d}s$. If fluid feeds into the interior of the basin and drains out through the strait, the flux of planetary vorticity $\oint_c f\mathbf{u}\cdot\mathbf{n}\,\mathrm{d}s$ into the strait is >0 and thus the rim circulation must be generally anticyclonic $r\oint_c \mathbf{u}\cdot\mathbf{t}\,\mathrm{d}s<0$ as observed. On the other hand, if fluid is fed into the basin entirely through a second strait (such as the Jan Mayan fracture zone in the Norwegian Sea) and if the inflows and outflows have the same velocity, then $\oint_c f\mathbf{u}\cdot\mathbf{n}\,\mathrm{d}s = 0$ and thus $\oint_c \mathbf{u}\cdot\mathbf{t}\,\mathrm{d}s = 0$. In this case, the inflow splits into two boundary currents that move around the edge of the basin. If the inflows and outflows have similar velocity but the value of f is significantly higher at the inflow, then $\oint_c \mathbf{u}\cdot\mathbf{t}\,\mathrm{d}s>0$, implying an intensification of the rim current on the western side of the basin. In fact, this argument can be used as an explanation for deep western boundary layers in the basin. Further applications of this constraint are illustrated in Yang and Price (2000) and Helfrich and Pratt (2003).

The interaction between the sill and the upstream basin is important from the standpoint of climate monitoring. In a recent study, Hansen et al. (2001) attempted to establish a relation between the directly measured transport Q of the Faroe-Bank Channel overflow and the upstream hydrography. They fitted a 6-month record of transport to $\gamma(\Delta z)^n$ ($n = 1$ or $3/2$), where Δz is the elevation above the Faroe-Bank sill of the upstream 'interface' or bounding isopycnal, here the $\sigma_t = 28.0$ surface. The upstream elevation of this surface has been monitored for over five decades at a station in the eastern Norwegian Sea by a weather ship. After estimating the constant γ based on the 6-month transport record, they applied the resulting model to the five-decade record of Δz and found a 20% decrease in transport from the mid-1990s to the present time.

How reasonable are the assumptions made by Hansen et al. (2001)? The $n = 3/2$ law agrees with the non-rotating limit of (11) of the flux relation but not with any power law relations derived in rotating hydraulic theory. The 3/2 law does, however, agree with findings based on a numerical simulation of a two-layer, rotating exchange flow (U. Riemanschneider and P. Killworth, pers. comm.). The $n = 1$ law does not agree with any hydraulic relation, but it might if the sill flow was frictionally dominated rather than hydraulically controlled. A more important question is whether the time-dependence of a single upstream station tells us anything about the time-dependence of the overflow. This is exactly the type of problem that models of upstream influence should be able to address. Interestingly, the Gill model requires *two* independent upstream parameters to predict Q. In addition, the upstream station in the case in point lies close to the Norwegian–Atlantic Current and therefore dynamics of the upper layer may be relevant.

Some of the problems that arise in remote monitoring of overflows have been discussed by Helfrich and Pratt (2003). Their numerical study considers the circulation that results when fluid is introduced into an upstream basin in various ways and is allowed to spill out of the basin through a strait containing a shallow sill. If the volume flux Q remains constant but the location of the source is changed the horizontal circulation in the basin undergoes dramatic changes, as suggested by (11). At the same time, the characteristics of the flow in the strait and sill remain remarkably fixed. In fact, the configuration of the exiting flow is such that the potential energy in the basin is maximized. From the standpoint of climate monitoring, the discouraging aspect of the study is that the depth and velocity at a fixed upstream location will vary as the location of the source is altered, even though Q remains fixed. However, this variation is minimized as one moves closer to the entrance of the strait, suggesting that upstream monitoring might work best if the monitoring instrument is placed near the entrance.

There is an even more basic question that one might ask about upstream effects. How shallow must a sill be in order to influence fluxes associated with the abyssal circulation? For guidance one might first consider the study of Long (1954), who was able to predict how large an obstacle must be to alter the upstream state of a moving stream. His calculations and laboratory experiments have served as a model for understanding upstream effects of obstacles in stratified flows. Baines (1995) is responsible for much of this work and the interested reader should consult his text. This approach also has been in connection with flows in rotating straits by Pratt et al. (2000), who used the Gill model to predict how large an obstacle height is required in order to establish hydraulic control over an initially steady, geostrophically balanced stream. The results are checked via numerical simulation and can be summarized by a regime diagram (Fig. 3) showing the outcome of the experiment given the dimensionless obstacle height and initial Froude number. The predictions are in fairly good agreement with numerical simulations of the adjustment process itself. The idealizations present make it difficult to extrapolate the results to the ocean, but the general approach might prove helpful when used with regional numerical models.

5. Application of simple models to specific straits

The application of simple models of rotating, hydraulically driven flow to deep straits is largely

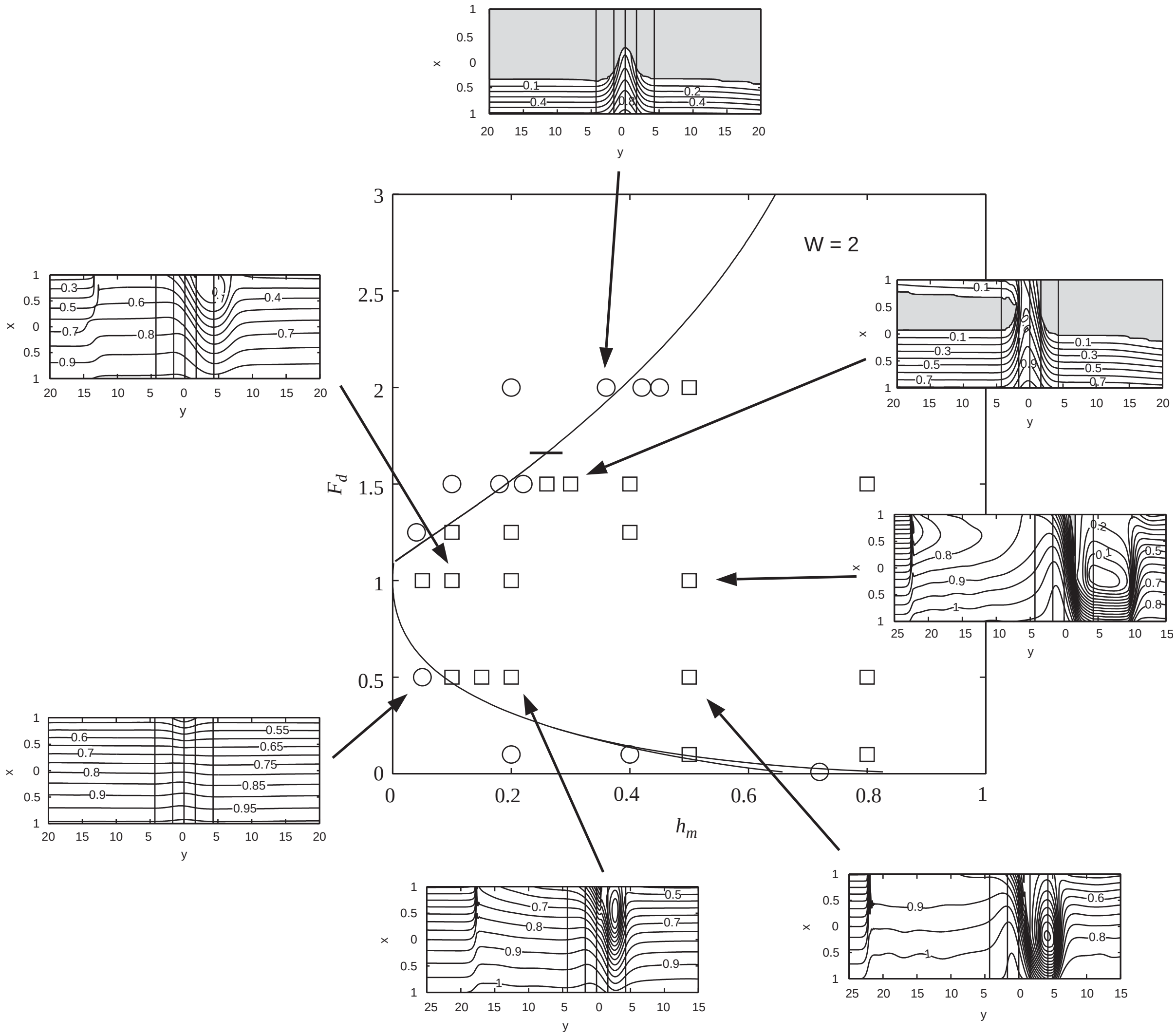

Fig. 3. Regime diagram showing outcome of numerical experiment in which an obstacle of dimensionless height h_m is placed in the path of a rotating-channel flow with Froude number F_d and with uniform potential vorticity. The potential vorticity of the initial state has uniform value f/d_∞ and the width of the rectangular channel is twice the Rossby radius of deformation based on d_∞. Hydraulically controlled states are predicted to the right of the solid or dashed curves. Squares indicate cases in which hydraulic control (and upstream influence) actually occurs while circles correspond to cases where the control does not occur. The thumbnail sketches show plan views of the final state that occurs after an upstream propagating bore has passed out of the numerical domain. In some cases the bore can be seen at the left edge of the domain. The initial flow is from left to right. Shaded areas of the channel bottom are dry.

limited to transport comparisons based on formulas such as (10) and to comparisons between observed outflow characteristics and plume models. Comparison between the flow upstream of the sill section and that predicted by models has been limited by the lack of upstream observations. However, one intriguing controversy recently brought to light involves the apparent approach of overflow water in the Denmark Strait along the eastern (Iceland) side of the channel (Jonsson and Valdimarsson, 2004; Nikolopoulos et al., 2003). Where would a boundary current of this type

Table 1
Overflow transport Q, observed and estimated from hydraulic theory

Q (Sv)	Denmark Strait	Faroe-Bank Channel	Ceara Rise	Vema Channel	Jungfern Passage
Observed	2.1–3.7[a]	1.5–3.1[b]	1.1–2.1	4.0	0.072–0.092[c]
Rectangular sill, zero pv[d]	3.8	3.6	4.6	16.0	
Parabolic sill with Killworth '92 model	0.6	0.53	2.1	4.5	
Parabolic sill with zero pv	0.5	0.5	1.4	2.9	
Actual sill geometry, zero pv	1.2–2.3[e]				0.079[f]
Actual sill geometry, zero pv, band of reverse flow excised	1.8–4.5[e]				0.085[f]
Actual sill geometry, finite but constant pv	1.3–2.5[e]				
Actual sill geometry, finite but constant pv, reverse flow excised	1.79–4.35[e]				

Note: All values are taken from Killworth (1992) unless otherwise indicated.
[a] Girton and Sanford (2001).
[b] Mauritzen et al. (2004 submitted).
[c] Fratantoni et al. (1997).
[d] For the first four straits, all of which are dynamically wide, the Killworth (1992) model and WLK (zero-potential vorticity model) produce the same Q in this case. The values quoted are therefore the same as those appearing in Whitehead (1989b).
[e] Nikolopoulos et al. (2003).
[f] Borenäs and Nikolopoulos (2000) and MacCready et al. (1999).

originate? Does it involve a crossing of boundary flow along the Greenland coast to the Iceland coast? Such crossing seems to be a feature of the solutions found by Pratt (1997) and also occurs in the laboratory experiments of Whitehead and Salzig (2002) and the numerical solutions of Helfrich and Pratt (2003). The crossing is due to the shoaling bottom topography that the approaching flow sees as it moves into the strait and toward the sill. The sloping bottom creates a topographic beta effect in which the Iceland coast becomes the dynamical western boundary. A current that flows southward along the Greenland coast and is deep enough to feel the shoaling topography would tend to cross the strait and collect in a 'western' boundary layer along the Iceland coast.

Returning to the subject of transport comparisons, Whitehead (1986, 1989a, b, 1998) has applied (9) to a number of well-known deep overflows and has found that the observed transport Q is overestimated by 160–400%. A number of complications including time-dependence, non-zero-potential vorticity, interactions with upper layers, non-rectangular geometry of the cross-section, and perhaps the outright lack of hydraulic control at the sill section might contribute to the error. For example, the work of Borenäs and Lundberg (1986, 1988, 1990) suggests that non-uniform cross-sections tend to reduce the transport below that of a rectangular geometry. Killworth (1992) furthers this point in the context of a dynamically wide sill. Using the actual geometry of the Jungfern Passage, Borenäs and Nikolopoulos (2000) present an estimate of the deep transport that is quite close to the observed value.

The results of all these calculations are presented in Table 1. A striking feature of the results is the extent to which Q is reduced when the sill geometry is changed from rectangular to parabolic. In the cases of the Ceara Rise and Vema Channel, the reduction brings the estimate into better agreement with the observations. For the Denmark Strait and Faroe-Bank Channel, the reduction lowers the predicted Q below that observed values.

A predicted feature of hydraulically controlled flows is that rounded cross-sections, particularly wide ones, tend to produce flow reversals. A common situation is that the predicted sill flow has a band of reverse flow along the Northern Hemisphere right wall (e.g., Borenäs and Lundberg, 1986; Killworth, 1992). Such reversals are sometimes regarded as violations of the assumptions underlying a theory based on specification of upstream conditions. Perhaps a more significant point is that strong flow reversals at sill sections are not generally observed in the ocean, in laboratory models (Borenäs and Whitehead, 1998), nor in numerical models of overflows (Helfrich and Pratt, 2003). For example, the Denmark Strait overflow is bounded on the right by a region of sluggish motion and level isopycnals (Nikolopoulos et al., 2003). It is in this region that inviscid hydraulic theory tends to predict flow reversals. Because of this behavior, it has been suggested that regions of reverse flow arising in models should be replaced, for purposes of estimation of Q, by stagnant regions. Table 1 indicates several instances where this replacement has been made, thus increasing the estimate of Q.

6. Transport bounds

Given the difficulty of establishing accurate transport relations, one might instead try to formulate an upper bound on Q. Killworth and McDonald (1993) and Killworth (1994) have done so, using conservation of mass, energy (and therefore potential vorticity), and the geostrophic relation as a foundation. As an example of the procedure and reasoning they use, consider the reduced-gravity flow at a section of channel with smoothly varying, arbitrary geometry (Fig. 1). If η denotes the elevation of the interface and x the cross-channel coordinate, the geostrophic transport is given by

$$Q = \frac{g'}{f}\int_{x_0}^{x_1}(\eta - h)\frac{\partial \eta}{\partial x}\,\mathrm{d}x = \frac{g'}{f}\left\{\tfrac{1}{2}(\eta_1^2 - \eta_0^2) - \int_{x_0}^{x_1} h\frac{\partial \eta}{\partial x}\,\mathrm{d}x\right\}, \qquad (12)$$

where x_0 and x_1 are the positions of the left and right edges of the current (where the depth vanishes).

An upper bound on Q can be formulated by modifying the profile in a way that only adds to the flux, then calculating the transport of the modified state. This procedure involves chopping off regions of negative velocity at the end points of the profile and replacing them with regions of positive flow in which the depth is smoothly brought to zero. A slightly more involved procedure is used to replace interior regions of negative flux by stagnant regions. The bound that results can be written as

$$Q \leqslant \frac{g'}{2f}(\eta_1 - h_{\min})^2, \qquad (13)$$

where $h_{\min}$ is the minimum elevation of the bottom elevation across the section. By choosing the sill (where $h_{\min}$ has its greatest value over all possible sections) as the cross-section, the upper bound is minimized. Also note that the bounding Q is exactly the geostrophic transport associated with a current in a rectangular channel if the flow is separated from the left wall (facing downstream) and if the depth along the right wall is $\eta_1 - h_{\min}$.

To write the bound in terms of the upstream flow, note that $g'\eta_1$ cannot be greater than the maximum value of the Bernoulli function over the cross-section in question, here the sill. If the latter is conserved following streamlines leading from the upstream basin into the channel and across the sill, then $g'\eta_1$ is bounded by the maximum value E of the Bernoulli function in the basin, at least for those streamlines that connect with the channel section in question. Thus

$$Q \leqslant \frac{g'}{2f}\left(\frac{E}{g'} - h_{\min}\right)^2. \qquad (14)$$

In the WLK ('zero-potential vorticity') model the upstream basin is assumed to be quiescent and therefore $E = g'(h_{\min} + \Delta z)$. Substitution into the above bound yields the WLK expression (9) for 'zero potential vorticity' flux for the case in which the flow is separated from the left wall of the rectangular channel. Thus, the bound is achievable. The connection between the bound and 'zero potential vorticity' flow lies in the fact that $B(\psi)$ is uniform when $q = 0$. Given an arbitrary flow with variable $B(\psi)$, one could pick out the maximum E of B and ask what the transport would be if each streamline had $B = E$ [zero-potential vorticity in view of (6)], if the elevation of the channel bottom had the uniform value $h_{\min}$, and if the flow depth went to zero at the left edge of the stream. This flux is exactly that given by (14).

7. Hydraulic jumps

Although hydraulic jumps have been detected in Gibraltar, Knight Inlet, (apparently) the Romanch Fracture Zone, and in other ocean straits, these are sites where the effects of rotation are weak. One of the most interesting aspects of rotationally dominated ocean overflows is that no direct observations of hydraulic jumps have been made. The strongest indirect evidence for a jump involves data from the Vema Channel (Hogg, 1983), suggesting a sharp decrease in the Bernoulli function over a short distance downstream of the sill. Such a decrease would be consistent with the strong energy dissipation that could occur in a fully developed jump.

Can hydraulic jumps exist in strongly rotating flows and what would they look like? For channel flows with rectangular cross-sections, different possibilities have been suggested by Pratt (1983, 1987), Nof (1986), and Pratt et al. (2000). The most robust version seems to be a transverse jump that occurs when the supercritical flow downstream of a sill becomes separated from one of the sidewalls. The jump consists of an abrupt widening and reattachment of the current (see Fig. 4 and feature shown near $y = 2$ in the $t = 70$ frame of Fig. 5). The abrupt depth change and strong vertical overturning characteristic of a non-rotating jump is suppressed. Turbulence generated in the jump appears to be concentrated in horizontal eddies. To date, no strong evidence of such a feature has been found in the Denmark Strait outflow, which hugs the right-hand boundary as it descends (e.g., Girton and Sanford, 2003). It is conceivable that something like transverse jumps could be integral to eddies that occur regularly in the strongly time-dependent plume. Or, the jump may be sublimated by friction and entrainment processes. The fact that rotating jumps have not been identified in other overflows, of course, could be due to a lack of observations in the right places.

One characteristic of all stationary, rotating jumps that arise in models is that the downstream

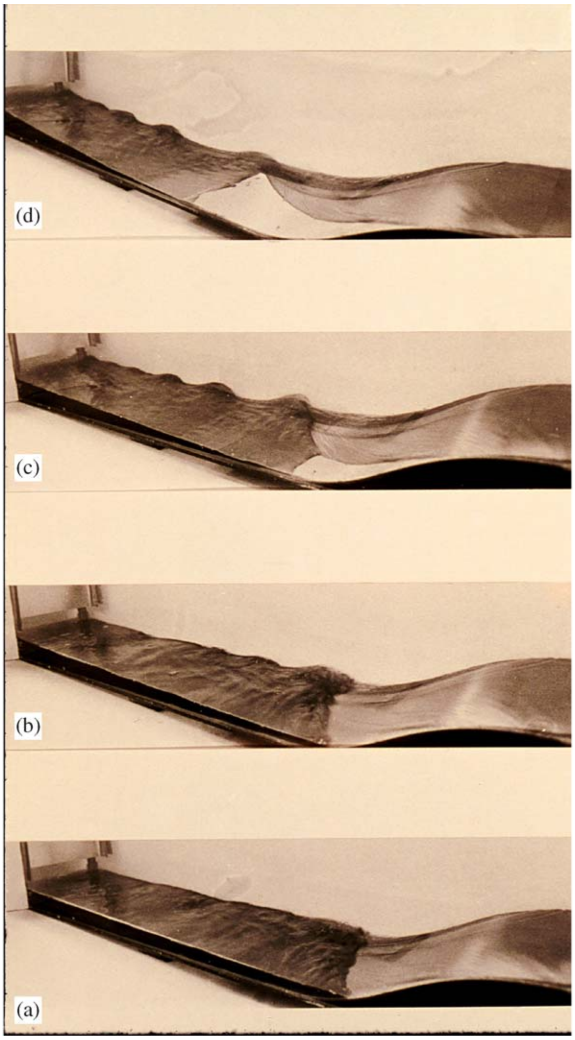

Fig. 4. Photo of rotating hydraulic jump in laboratory channel (Pratt, 1987). Fluid flows from right to left, spilling down an obstacle. At the base of the obstacle is a hydraulic jump, shown at increasing rotating rates from (a) to (d). In (c) and (d), where the spilling flow separates from the near wall, the jump is transverse.

(subcritical) end state must stay in contact the left channel wall. [Although shock structures are possible in flows that are completely separated from the left wall, they generally do not remain stationary (Nof, 1984).] It therefore would seem that any stationary hydraulic jump would have to occur within the confines of the deep strait in

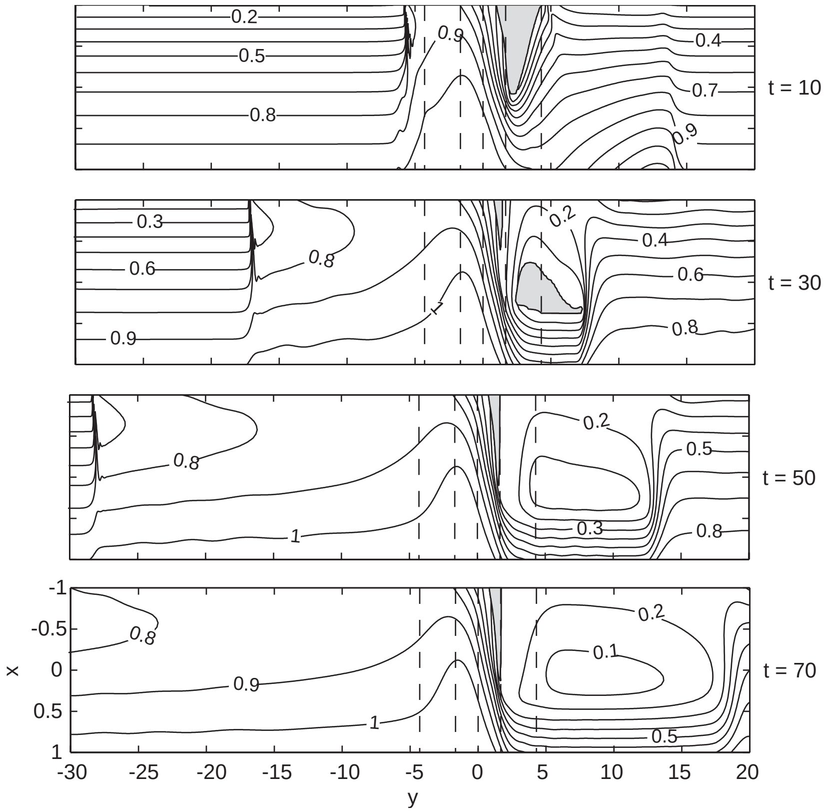

Fig. 5. Adjustment of a rotating-channel flow when an obstacle is placed in its path (from Pratt et al., 2000). The plan views show contours of free surface elevation at various dimensionless times after the introduction (at $t = 0$) of the obstacle. Shaded areas indicate 'dry' regions where the bottom has lost contact with the active layer. The position of the obstacle is indicated by the dashed lines. The first few frames show a Kelvin wave bore moving upstream (left) along one wall. A transverse hydraulic jump appears in the final two frames near $y = 2$.

question (before the strait broadens into the downstream basin).

8. Two-layer studies

The hydraulics of non-rotating, two-layer flow has been well studied, and the reader can refer to Baines (1995) for a review. The presence of a second layer provides an extra degree of freedom that is manifested by the presence of whole families of hydraulically controlled solutions for fixed topographic configurations. It is typical that within each such family, there exists a distinguished solution that has some sort of maximal properties. Such is the case in pure exchange flows, where the distinguished solution has the maximum volume exchange rate of all possible controlled solutions. In addition, the 'submaximal' solutions have a single critical section whereas the maximal solutions have two (which may coalesce if topographic variations are limited to a width contraction). These properties also have been identified in rotating, two-layer flows (Dalziel, 1988, 1990), though the rotating system is considerably complicated by the possibility of various types of layer separation from sidewalls and by at least one new type of control.

One of the physical mechanisms that can make two-layer, rotating flows differ from both single-layer, rotating systems and two-layer, non-rotating systems is the way in which the internal and external (or baroclinic and barotropic) motions interact. To illustrate this point, suppose that the potential vorticity has constant values $f/d_{1\infty}$ and $f/d_{2\infty}$ within the top and bottom layers (numbered 1 and 2). Then it can be shown (Pratt and Armi, 1990) that baroclinic motions are confined to the sidewalls of the channel within the internal Rossby radius of deformation

$$L_{\mathrm{I}} = f^{-1}\left[\frac{g' d_{1\infty} d_{2\infty}}{d_{1\infty} + d_{2\infty}}\right]^{1/2}$$

based on the potential depths $d_{1\infty}$ and $d_{2\infty}$. (For simplicity we assume here that both layers contact the sidewalls and that the channel cross-section is rectangular.) If the channel width is much larger than L_{I} then the channel interior will have a depth-independent velocity v_{b} with lateral shear $\partial v_{\mathrm{b}}/\partial x$ that can be shown to be independent of the cross-channel coordinate x. Motion in the channel interior therefore occurs in columns and the corresponding barotropic velocity profile can be written as $v_{\mathrm{b}} = \bar{v}(y) + \gamma(y)x$, where $x = 0$ lies along the channel centerline. If the baroclinic and barotropic velocities are of similar magnitude, then the total volume flux is dominated by the contribution from the barotropic velocity since it is felt all across the channel. With the channel walls positioned at $x = \pm w/2$, this flux equals $w(y)d(y)\bar{v}(y)$.

Motions within the baroclinic boundary layers depend in part on the imposed sidewall barotropic velocity $v_{\mathrm{b}}(\pm w) = \bar{v}(\pm w) \pm \gamma(y)w/2$, itself controlled by conservation of mass. As the fluid passes through a width contraction or over a sill, $\bar{v}(y)$ must increase to conserve mass. If fluid passes over a sill, vortex squashing will cause the horizontal shear $\gamma(y)$ to change. As it turns out, this indirect forcing of the baroclinic flow does not reach a peak at the usual topographic extrema (sills or minimum widths). The result is that points of hydraulic control for the baroclinic boundary layers may occur at remote locations (termed 'remote controls') by Pratt and Armi (1990). This type of control should be distinguished from the 'virtual' control that is well documented in two-layer, non-rotating systems.

When the channel width is comparable to L_{I}, it becomes less advantageous to decouple the barotropic and baroclinic parts of the flow. The few studies that have been carried out reveal behavior that is qualitatively similar to the non-rotating two-layer case. WLK derived a condition for critical control of a two-layer, zero-potential vorticity, pure exchange flow through a pure width contraction that is analogous to the maximal, non-rotating exchange solution. Dalziel (1988, 1990) explored rotating exchange flows under more general circumstances and showed that the WLK solution is indeed the maximal solution for the geometry in question. He also described maximal and submaximal solutions for pure exchange flow across a sill. Much of the work that has been done is restricted to cases in which the layers are attached to both sidewalls or which separate in limited ways.

There are several cases of observed phenomenon whose explanation requires something more sophisticated than a two-layer, inviscid model. One is the 'pinching' phenomenon in which the vertical separation between isopycnal surfaces is smaller on the (Northern Hemisphere) left wall of a deep channel. Pinching has been observed in both the Vema Channel (Hogg, 1983) and the Faroe-Bank Channel (Johnson and Sanford, 1992). A possible explanation (Johnson and Ohlsen, 1994) is that secondary circulations set up by Ekman layers on the bottom and about the strongly sheared interface lead to a convergence along the left wall that pushes isopycnals together. Alternatively, the pinching can be explained as a purely inertial phenomenon caused by changes in layer depth as the hydraulically driven flow passes over bottom topography. Hogg (1983) demonstrated this effect using a 2–1/2 layer model. It is not known which (if either) of these mechanisms dominates in the sites mentioned. Other cases where two-layer, inviscid dynamics appears inadequate include the prediction of certain dense outflows, where friction, entrainment, and interactions with multiple upper layers can be important. One example is the deep flow through the Strait of Sicily, which has been simulated by Astraldi et al. (2002) using a three-layer model with interfacial entrainment and bottom drag.

9. Time-dependence

A number of straits contain sill flows that are strongly time-dependent due to inherent instabilities or to external forcing such as tidal forcing or motions transmitted down through overlying fluid. Fluctuations in the flow at the sill can become rectified, leading to increased or decreased transport. It is unclear how well the traditional ideas about hydraulic control and upstream influence apply to such flows. These are difficult issues and it is not surprising that very little progress has been made on the subject (though Helfrich, 1995 has clarified some aspects of a non-rotating, two-layer case). On the other hand, a great deal is known about geostrophic adjustment in rotating channels, even in the strongly nonlinear regimes where hydraulically driven flows live. The initial-value problem typically involves the sudden removal of a barrier separating resting fluids of unequal depths or densities, or the sudden appearance of an obstacle in the path of a moving stream. In either case the study of the adjustment to a new steady state could be the first step toward reaching an understanding of flows with more complicated time-dependence. In addition, such problems provide a great deal of insight into fundamental processes such as upstream influence and formation of shocks and bores.

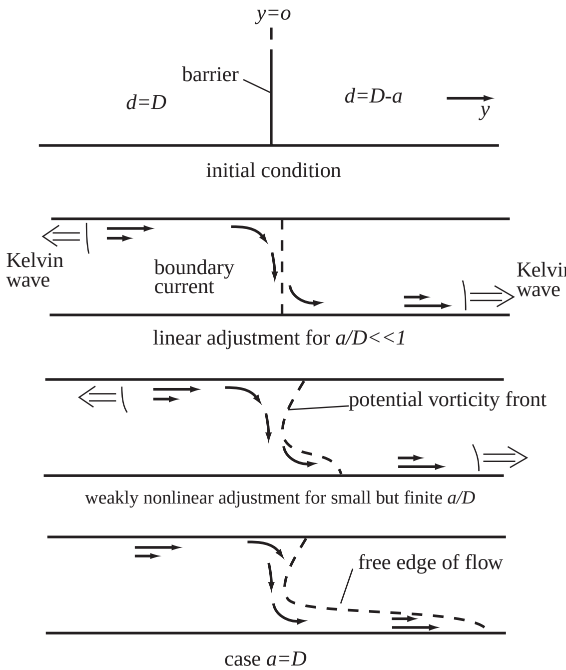

Fig. 6. Schematic views of the Rossby adjustment problem in the channel for increasingly nonlinear dynamics: (a) initial conditions, (b) linear adjustment according to the Gill (1976) solution, (c) weakly nonlinear case ($a/D = O(1)$); (d) the case of zero depth on one side of the barrier. Panels (c) and (d) show numerical solutions from Helfrich et al. (1999).

The first person to address the issue of Rossby adjustment in a rotating channel was Gill (1976), who solved for the motion that ensues after the destruction of a barrier separating fluids of depths D and $D - a$ (Fig. 6a). Gill considered the linear problem that arises when the initial depth difference is small ($a/D \ll 1$) and demonstrated the role of Kelvin and Poincaré waves in the adjustment. The removal of the barrier at $y = 0$ results in the generation of two Kelvin waves which move away on opposite sides of the channel as shown in Fig. 6b. These waves set up a boundary flow that approaches the barrier from the deeper water, crosses the channel at $y = 0$, and continues along the opposite wall. The crossing region is set up through the action of Poincaré waves. The transport Q of the adjusted state obeys

$$Q \leqslant g' Da/f, \tag{15}$$

where D is the mean initial depth and a is the initial depth difference across the barrier. In the limit of a dynamically wide channel ($wf/(g'D)^{1/2} \gg 1$) the transport approaches the expression given by the right-hand side of (15).

Toulany and Garrett (1984) have suggested that an inequality like (15) should govern any geostrophic flow between two wide basins in which the interior surface (or interface) elevations differ by amount a. The idea is that the flow in the basin with a higher interior elevation will move into the connecting strait along its 'left-hand' wall, cross the strait, and continue into the other basin along its 'right' wall, much as in the Gill problem. For example, assume that the bottom elevation in the system is uniform and that the interior depths in the two basins are d_1 and d_2. Then if the flow is geostrophically balanced, the flux in the current that crosses the strait separating the two basins is exactly

$$\frac{1}{2}g'f^{-1}[D^2-(D-a)^2], \tag{16}$$

which reduces to the right-hand side of (15) in the limit $a/D \ll 1$. The notion that (16) (or its generalization for basins of unequal bottom elevation) bounds Q is known as *geostrophic control*.

Counter examples of geostrophic control arise in hydraulic theory, where basin-to-basin flows do not necessarily cross the connecting strait (Pratt, 1991). Some intuition into how such flows are established can be gained by considering the Gill adjustment problem with finite a/D. The new feature of interest is that the potential vorticity difference between the fluids on either side of the barrier becomes important. After the barrier is destroyed, the two fluids remain separated by a front capable of supporting potential vorticity waves (Fig. 6c). For the case in which a/D is small but finite, Herman et al. (1989) have shown that the initial adjustment occurs much as in the Gill linear solution. Left- and right-wall boundary currents with a channel crossing at $y=0$ occur as before. This is followed by a much slower adjustment phase involving the dynamics of the potential vorticity front. As a result of the secondary adjustment the crossing point of the boundary layer flow moves downstream (toward positive y). The approaching boundary flow from the deeper end of the channel ceases to cross the channel at $y=0$ and remains attached to the left wall. Application of the idea of geostrophic control, which is based on the idea of channel crossing, becomes difficult under these conditions. However, the concept does remain valid in certain time-dependent systems in which a basin-to-basin flow occurs in response to a fluctuating forcing mechanism with a period short compared to the potential vorticity time scale (Hannah, 1992). In such cases, the potential vorticity dynamics do not have sufficient time to alter the crossing point.

The Gill adjustment problem has recently been explored under conditions of full nonlinearity ($a/D = O(1)$), including the case of zero depth on one side of the barrier. In such cases there is no time scale separation between the Kelvin wave and potential vorticity adjustment phases. The potential vorticity front lags the leading edge of the Kelvin wave intrusion but the two features

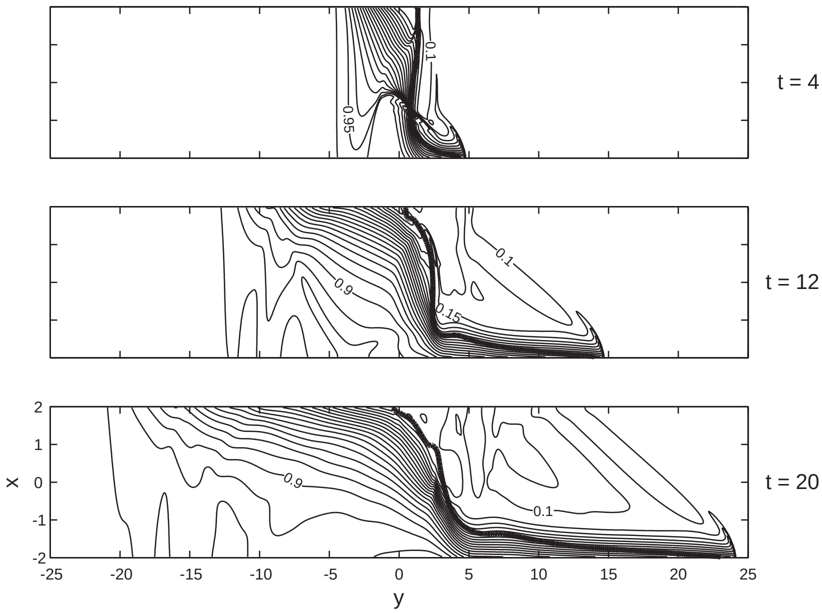

Fig. 7. Numerical solution of the Gill adjustment problem in a channel for the strongly nonlinear case $a = D/5$ (from Helfrich et al., 1999). The thin contours show free surface elevation and the thick contour is the potential vorticity front separating fluid initially on opposite sides of the barrier.

propagate at similar speeds. Fig. 7 shows a numerical solution for the case $a = D/5$ (from Helfrich et al., 1999). Because of its finite amplitude, the leading edge of the forward Kelvin wave intrusion can propagate more rapidly than the linear Kelvin wave speed, allowing it to match the phase speed of Poincaré modes (Tomasson and Melville, 1992). This situation can lead to the resonant excitation of Poincaré waves near the leading edge leading to an enhancement of small-scale motions there. In other cases, the leading edge may remain smooth and maintain a curved shape (as is the case in Fig. 7). Federov and Melville (1996) have devised a shock joining theory for the latter case.

In the limiting case $a = D$ the leading edge of the forward propagating Kelvin wave intrusion *is* the potential vorticity front (Fig. 6d). An approximate solution for this case can be obtained using the method of characteristics (Helfrich et al., 1999) and it can be shown that the flux Q of the adjusted state grows as w is increased, reaching the limiting value $\frac{1}{2}g'D^2$. Thus, the bound expected from geostrophic control theory remains in force, provided that the bound is based on the *initial* state. For all cases of finite w, however, the channel crossing of the boundary currents moves downstream and is eventually lost.

Initial-value problems involving the 'Long-type' adjustment of a steady, rotating-channel flow to an obstacle are also quite valuable in understanding how sill flows are established and how special features such as recirculations and hydraulic jumps arise. Fig. 5 (from Pratt et al., 2000) follows the adjustment of a subcritical flow in response to the sudden appearance of an obstacle. As in the Gill problem, the obstacle generates a Kelvin bore that moves upstream and alters the approaching flow. As the flow crosses the obstacle (demarcated by the dashed lines) the approaching boundary current crosses the channel, becomes supercritical, and detaches from the left wall. Slightly downstream (near $y = 2$ in the $t = 70$ frame) a transverse hydraulic jump forms and a circulation appears in its lee. Tracing the evolution of such flows can be quite helpful to the person who is trying to develop intuition into rotating hydraulics.

10. Potential vorticity hydraulics

The past 15 years have seen the development of a number of models of flows that exhibit hydraulic behavior with respect to potential vorticity waves. Such waves propagate much more slowly than the Kelvin waves (and their relatives), and it is questionable whether a flow that is strong enough to arrest Kelvin waves could become critical with respect to the slower Rossby waves. The major deep overflows therefore do not seem to be strong candidates for Rossby wave control. Applications are apparently limited to broader ocean currents (Armi, 1989; Woods, 1993) and coastal currents (e.g., Hughes, 1987). There have been a few studies of potential vorticity hydraulics in channel flow, including Pratt and Armi (1987), Haines et al. (1993), and Johnson and Clark (1999), but no concrete application to any oceanic flow has been established. One obvious candidate is the Drake Passage, in which the fluid velocities are the right magnitude and direction to arrest a long Rossby wave (Pratt, 1989). Johnson and Clark (2001) provide a comprehensive review of this subject.

A related body of work explores the blocking effects of straits on the wind-driven circulation in places such as the Indonesian archipelago and the Caribbean island chain. Although some form of Rossby wave hydraulics may occur in such applications, this aspect has not been emphasized. Instead, the dynamics of the throughflows are interpreted using circulation integrals of the form (12) but written down along a closed contour that circles the north, south and west sides of an island and extends to the eastern boundary of the basin. Godfrey (1989) was the first to point out the advantages of such a form, namely that it yields a simple approximation for the total meridional wind-driven flow to east of the island. Godfrey's "island rule" can be generalized to account for frictional effects in the narrow straits separating the islands (Wajsowicz 1993, 2002; Pratt and Pedlosky, 1998). The resulting formulas give generalized expressions for the flux in the straits that depend on friction coefficients. For barriers such as the mid-Atlantic Ridge that are breached by multiple gaps and straits, it is possible to combine multiple applications of the island rule thus creating a theory for the flow through the boundary. If the widths of the individual straits and 'islands' are reduced to infinitesimal values, the gappy ridge becomes a porous medium whose throughflow is described a differential equation (Pratt and Spall, 2003).

11. Discussion

A great deal of current attention is devoted to the problem of parameterizing the effects of deep straits in models of large-scale ocean circulation. At least two issues are involved. The first concerns the parameterization of mixing, entrainment, and friction in deep outflows. On this subject, I again refer the reader to other papers appearing in this volume. The second issue concerns the upstream effects of deep sills such as blocking, control over meridional overturning cells, and influence on upstream basin circulations. Only one work, that of Pratt and Chechelnitsky (1997), has appeared on this subject. The primary emphasis is on principles for parameterizing upstream effects and their application in idealized models. These ideas have yet to be applied to specific GCMs.

Otherwise, the major dynamical problems that need attention are the ones outlined above. They consist of using the well-established results on geostrophic adjustment to understand flows with more complicated time-dependence, furthering understanding of multi-layered system, continuing to broaden local models of straits and sills in order to understand far field effects, searching in data and models for features (such as Rossby wave control and transverse hydraulic jumps) which occur in idealized models or laboratory experiments but which have not been observed, developing more general criticality relations and weir formulas, and continuing to look for optimal ways of monitoring deep overflows.

Acknowledgements

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