A Time-Dependent Aspect of Hydraulic Control in Straits

L. J. PRATT

Graduate School of Oceanography, University of Rhode Island, Kingston, RI 02881

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ABSTRACT

The concept of hydraulic control by a sill is discussed in terms of its consequences for the upstream flow. Based on observations of the upstream flow alone, "control" is shown to be distinguishable from "noncontrol" only if the flow is unsteady. This result is demonstrated numerically using a model in which controlled flow is distinguished from noncontrolled flow by the time-dependent response to forcing upstream of the sill.

1. Introduction

In his textbook on hydraulics Chow (1959) states: "The control of flow in an open channel has been defined loosely in many ways". The concept of hydraulic control remains misunderstood by many and has been misapplied by others. The confusion is due in part to the tendency of authors to explain the concept using only local criteria such as Froude number or symmetry conditions that apply at the controlling sill (or side contraction), thereby deemphasizing important ramifications for the upstream flow. For example, consider the hypothetical basin with connected strait and sill sketched in Fig. 1. The strait may contain a simple outflow driven by river runoff into the basin or a two-layer exchange flow driven by imbalances in evaporation and precipitation. According to existing hydraulic models that might be applied (e.g., Stommel and Farmer, 1952; Gill, 1977) the outflow is controlled only if conditions at the sill are critical with respect to some Froude-number criterion, the physical significance being that the phase velocity of a long wave attempting to enter the basin is zero at the sill. Although this yields local information about the flow near the sill, little is said about the consequences for the upstream basin, other than that certain types of disturbances generated in the open ocean are unable to reach the basin. One might wish to know, for example, how control by the sill might be reflected in observational data taken *in the basin* (at the point P, say).

Some authors (e.g., Chow, 1959; Whitehead, *et al.*, 1974; Stern, 1974) describe the hydraulically controlled state in terms of variational principles. Although they give some insight into the relationship between the sill and the upstream basin, these principles are often applied in a way that renders the definition of control inoperable; i.e., the definition can never be applied to geophysical data. For example, Stern (1974) defines control as occurring when the "free discharge . . . cannot remain unaltered when the width of the channel is reduced or the height of the channel floor raised." (This definition is associated with a variational principle that requires the fluid passing the sill or contraction to possess an energy minimum.) Although it is convenient to change the width or bottom elevation in laboratory situations (Long, 1954: Baines and Davies, 1980) or in numerical experiments (Pratt, 1983a,b), such changes do not often occur in geophysical situations.

The situation has been further confused by investigators who apply the term "control" to processes such as friction or mixing which play no role in classical hydraulic theory. The term "hydraulically controlled by" is not the same as "limited by" or "affected by."

The purpose of this note is to put forth a sufficient condition for hydraulic control by a sill (or side contraction) which: 1) emphasizes the relationship of the sill with the upstream flow fields; 2) is operational, and can in principle be applied to geophysical data taken upstream of the sill; 3) is independent of the geometrical details of the basin and strait; and 4) is to a great extent independent of the dynamical details of the flow. The definition is motivated by Long's (1954) hypotheses (discussed below) and is consistent with, but not dependent upon, existing hydraulic theory.

2. Hydraulic control and Long's hypothesis

One of the first to study the phenomenon of blocking in a stratified fluid, Long (1954) was faced early in his investigation by the following question: Given an obstacle of certain dimensions, mounted on the bottom of a flat channel, is it possible to specify the flow far upstream independently of the obstacle's height? The assumption that such a speci-

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fication can be made is known as "Long's hypothesis of no upstream influence," and has been shown by many investigators to be valid for sufficiently small obstacles (Baines and Davies, 1980). For sufficiently large obstacles, however, a functional relationship can be shown to exist between the obstacle's height and the upstream fields. In this case Long's hypothesis fails.

Reconsider the basin flow system sketched in Fig. 1. The flow through the basin may be characterized, say, by the flow rate Q, the stratification N, the fluid depth D, and any number of other dependent variables, each of which may vary with location x, y or z but not time. Our requirement for control of this flow by the sill is that Long's hypothesis is violated, i.e., that a functional relationship exists between the dependent flow variables and the geometry of the sill

$$F(Q, N, D, \ldots, b_{\max}) = 0, \tag{2.1}$$

where $b_{\max}$ is the elevation of the sill. The functional relation may involve other geometric characteristics of the sill as well (Pratt, 1984). Equation (2.1) implies that Q, N, D, $b_{\max}$, etc., may not be varied independently of each other. If the flow is not controlled by the sill (Long's hypothesis is satisfied) then any of Q, N, D, ... , may be varied independently of each other. Note that Stern's definition is implicit in (2.1).

3. Some simple examples

Suppose that the basin in Fig. 2 is shallow and has constant width. Suppose also that the outflow consists of a deep, irrotational stream of constant density ρ_2 flowing with thickness $h(x)$ and velocity $u(x)$ over a bottom of elevation $b(x)$ and beneath an inactive upper layer of density $\rho_1 < \rho_2$. Then classical hydraulic theory (e.g., Chow, 1959) equates control of the outflow by the sill with the critical condition

$$F_d = u_c/(g'h_c)^{1/2} = 1, \tag{3.1}$$

where u_c and h_c are the velocity and depth of the deep outflow at the sill and $g' = g(\rho_2 - \rho_1)/\rho_2$. The Froude-number condition (3.1) may be connected to the deep flow in the upstream basin if it is noted that the flow rate $Q = uh$ and energy (Bernoulli head) $B = \frac{1}{2}u^2 + g(h + b)$ must be constant for all x. Combining these relationships with (3.1) gives

$$\frac{B}{b_{\max}} - 3\frac{Q^{2/3}}{2g'^{1/3}b_{\max}} = 1, \tag{3.2}$$

where $b_{\max}$ is the elevation of the sill. Equation (3.2), a special case of (2.1), demands that the energy B and flow rate Q be related in a certain way that depends upon the sill elevation.

If the elevation of the bottom is constant and the width w of the strait is allowed to vary, then the counterpart of (3.2) is

$$B - 3\frac{Q^{2/3}g^{2/3}}{2w_{\min}{}^{2/3}} = 0, \tag{3.3}$$

where $w_{\min}$ is the minimum width of the strait.

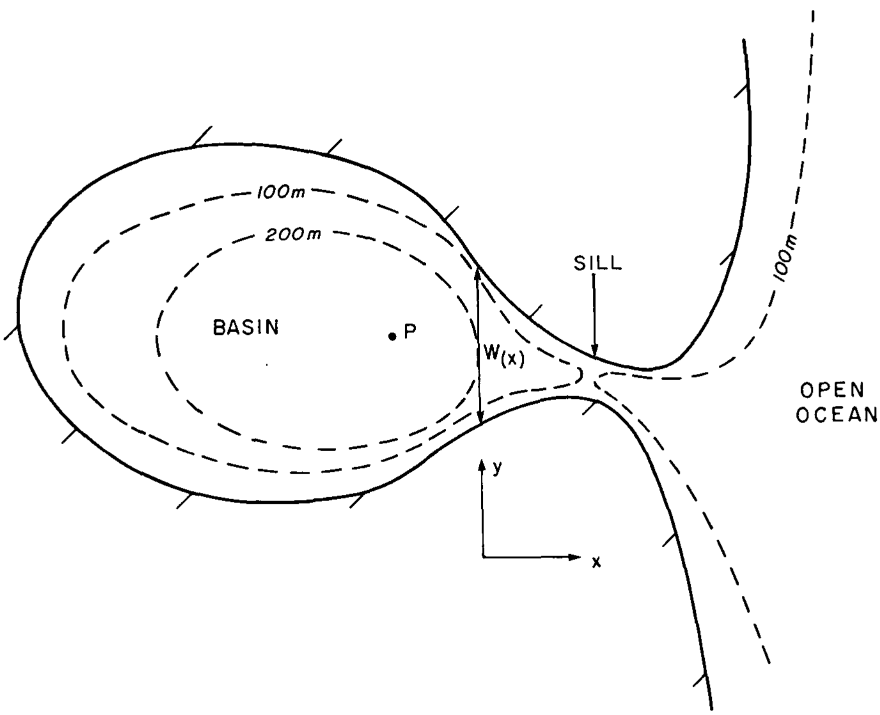

FIG. 1. Hypothetical basin connected to the ocean by a strait.

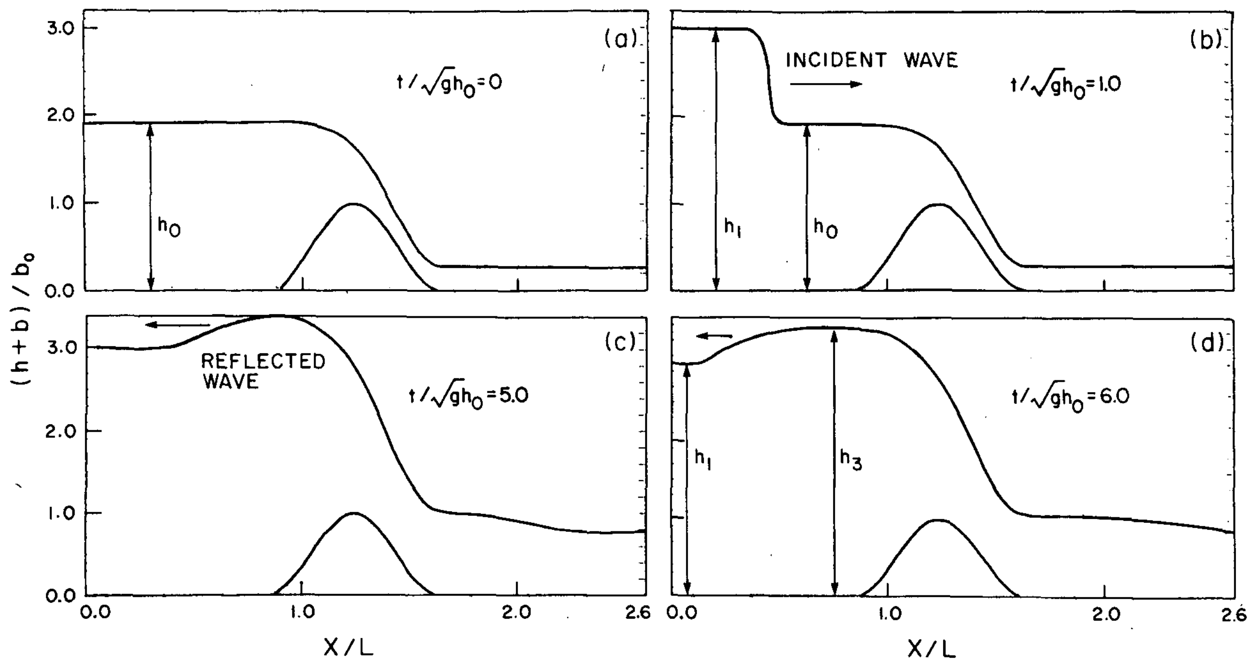

FIG. 2. Response of a controlled flow to an increase in upstream depth: (a) initial flow, characterized by $B/b_{\max} = 1.94$ and $Q^{2/3}/(g^{1/3}b_{\max}) = 0.62$; (b) wave of elevation approaches the sill; (c) and (d) reflected bore moves upstream. The final values of $Q^{2/3}/(g^{1/2}b_{\max})$ and $B/b_{\max}$ are 1.83 and 3.74, respectively. The initial upstream state is identical to that of Fig. 1.

Finally, suppose that the basin rotates at rate $f/2$ and that the outflow has constant potential vorticity ϕ. If the upper layer is again inactive, the Froude-number criterion for control by the sill (Gill, 1977) is

$$F_d = -\phi\delta h_c \bar{h}_c^{-1/2} f^{-1/2} \tanh(w/2L_d) \times [1 - \tanh^2(w/2L_d)(1 - \phi f^{-1}\bar{h}_c)]^{-1/2}, \quad (3.4)$$

where $\bar{h}_c$ and δh_c are half the sum and difference of the sidewall depths of the lower layer at the sill and L_d is the internal Rossby radius of deformation. Equation (3.4) can again be connected to the upstream basin through conservation of both the flow rate Q and a sidewall averaged Bernoulli head $\bar{B}$. The details of this calculation are given by Pratt (1983b) and the result is the following pair of equations:

$$\bar{h}_c^4 + f\phi^{-1}[\tanh(w/2L_d) - 1]\bar{h}_c^3 - (2g')^{-2}\tanh^{-4}(w/2L_d)Q^2 = 0, \quad (3.5a)$$

$$\bar{B}/g' - \frac{1}{2}\phi f^{-1}(f\phi^{-1} - \bar{h}_c)(f\phi^{-1} - 2\bar{h}_c) - \frac{3}{2}\bar{h}_c - b_{\max} = 0. \quad (3.5b)$$

In principle $\bar{h}_c$, the average depth at the sill, can be eliminated from (3.5a) and (3.5b) to obtain a relation of the form (2.1).

4. A time-dependent signature of control

Imagine now that one can obtain data from a fixed instrument at point P within the basin. The data may be in the form of velocity and/or density information. How can one deduce from such information whether or not the flow is controlled at the sill? The obvious answer is to test the data to see whether it satisfies the appropriate form of Eq. (2.1). However, these relationships describe highly idealized flows and the appropriate form of (2.1) may not be known in practice. Even if the correct form of (2.1) is known, the amount of data necessary to verify it may be unrealistically large. Verification of (3.5), for example, would require knowledge of the lower-layer Bernoulli head averaged over each side of the basin, the sill depth, the potential vorticity and the flow rate.

Apparently, a judgement based on measurements of the *steady* flow in the basin is difficult. If the flow is unsteady, however, a remedy may exist. Suppose, for example, that the flow at P is initially steady and controlled, but is altered in a time-dependent manner. This alteration might occur through the action of a storm over the basin or a seasonal weather change. At first, the change in forcing will result in a local violation of (2.1) at P, since a finite amount of time is required for communication between the flow field at P and the sill. This communication will occur via two waves: the first carrying information concerning the violation of (2.1) from P to the sill, and the second a return wave which tries to reestablish (2.1).

It is the general character of this return wave that determines whether or not the flow is controlled and, if so, where the flow is controlled.

As an example, suppose that the basin and strait in Fig. 1 have the same constant width w and that the bottom elevation b varies in the x-direction (along the strait) alone, forming a sill of elevation b_{max} near the mouth. If the outflow from the strait is barotropic and frictional effects can be neglected, the system is described by classical, open-channel hydraulic theory (Chow, 1959). The theory also applies if the outflow is restricted to a homogeneous, deep layer flowing beneath an inactive upper layer. Consider the two possible steady states sketched in Figs. 2a and 3a. The sketches show the free surface (or interface in the equivalent barotropic case) in longitudinal section. The second flow (Fig. 3a) is uncontrolled and the fluid depth is the same on either side of the sill. The first flow (Fig. 2a) is controlled, and the upstream depth h_0 is greater than the downstream depth. Fluid spilling over the sill is at the critical state (3.1) and the flow rate Q and Bernoulli function B satisfy (3.2) with g' replaced by g.

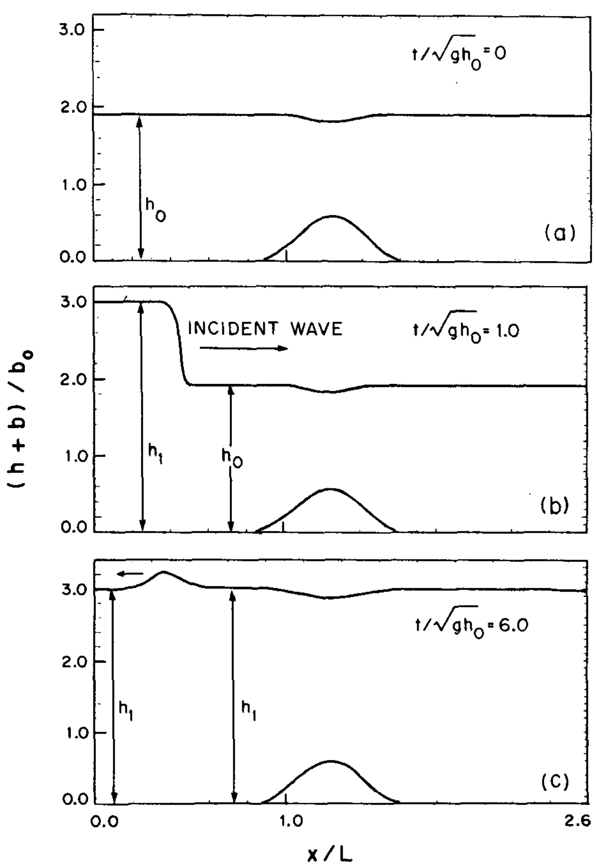

FIG. 3. Response of an uncontrolled flow to an increase in upstream depth: (a) the initial flow characterized by $B/b_{max} = 3.23$ and $Q^{2/3}/g^{1/3}b_{max} = 1.03$; (b) wave of elevation approaches the sill; (c) reflected, compact wave moves back upstream. The length of the channel (x-domain) is effectively infinite.

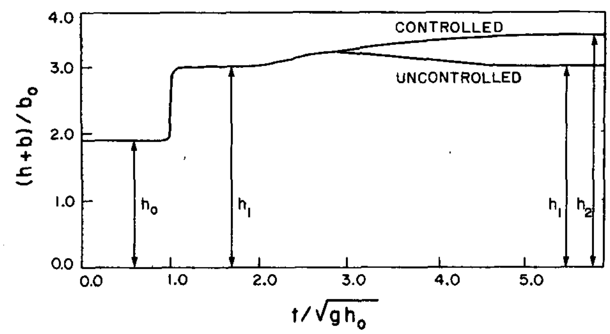

FIG. 4. Time history of upstream depth measured by an instrument upstream of the sill in both controlled and uncontrolled cases (cf. Figs. 2 and 3).

Suppose now that a seasonal increase in runoff occurs, causing the fluid depth far upstream of the sill to increase from h_0 to a new steady value h_1. The resulting motion has been computed numerically using a Lax–Wendroff (1960) scheme identical to the one used by Pratt (1983a). The increase in depth propagates toward the sill as a wave of elevation (Figs. 2b and 3b). Upon reaching the sill, a reflected wave is sent back upstream. In the uncontrolled case, the reflected wave is a compact disturbance which effects no permanent change in the upstream depth h_1 as shown in Fig. 3c. The upstream depth remains h_1 until further changes in the runoff rate occur. In the controlled case, however, the reflected wave establishes a new depth $h_2 > h_1$, as shown in Figs. 2c and 2d. Reflected waves in the uncontrolled flow can thus be loosely described as "compact" while reflected waves in the controlled flow can be described as "bore-like".

Figure 4 contains a time history of the depth h measured by an instrument placed upstream of the sill. In the uncontrolled case the depth rises from h_0 to h_1, is temporarily disturbed by the reflected wave, and returns to h_1. In the controlled case, the depth changes from h_0 to h_1, and then from h_1 to h_2.

The behavior of the reflected waves can easily be explained using Eq. (3.2). In the controlled case, the initial values $B/b_{max} = 1.93$ and $Q^{2/3}/(g^{1/3}b_{max}) = 0.62$ satisfy (3.2). However, the change in runoff produces new values $B/b_{max} = 3.70$ and $Q^{2/3}/(g^{1/3}b_{max}) = 1.97$ which violate (3.2). A bore-like return wave is therefore required to reestablish (3.2). In the uncontrolled case (3.2) does not apply and B/b_{max} and $Q^{2/3}/(g^{1/3}b_{max})$ can be varied independently. After the initial adjustment to the change in runoff is complete, no further adjustment is necessary.

It should be cautioned that a bore-like reflected wave is sufficient (but not necessary) evidence that the flow is controlled. For example, the change in runoff might contrive to change B and Q in a way consistent with (3.2), so that no secondary adjustment would be necessary. It is also possible that large changes in runoff might cause a transition from a controlled to an uncontrolled state (or vice versa), the magnitude of the required change depending in a complicated way upon conditions downstream of the sill (Pratt, 1982). Here, it is assumed that the upstream forcing is mild enough to preclude such changes.

Another complication comes into play in systems where multiple control points are present. This situation can arise when several active density layers are present (Wood and Lai, 1972), in which case a critical condition may exist for the external and each of the internal modes. Since these critical conditions would occur at different locations, the transient response to forcing would contain a number of bore-like return waves. The wave reflected from the external control would find expression primarily in the free-surface elevation, while those reflected from internal controls would find expression in the elevation of subsurface isopycnals.

5. Discussion

It has been argued that the wave response to the forcing of a controlled flow has a distinct signature: namely, the bore-like (as opposed to compact) nature of waves reflected from the controlling sill or side contraction. This response is due to the inability of upstream forcing to vary the dynamical fields of a controlled flow independently of one another.

The primary importance of the above principle is the intuitive information that is given concerning communication between the sill and basin. This description of the global, rather than local, implications of control is independent of particular models. The principle may or may not be useful when applied to geophysical data. The advantages are several. First, the appropriate form of the Froude number for the local geometry and stratification may not be known. Second, the location of the control may not be known in advance. This is particularly true in straits (such as the Strait of Gibraltar) containing more than one sill or side contraction. (In principle, the measurements in Fig. 4 give the position of the control.) Finally, application of the principle requires measurement of only a single property at an arbitrary location; the velocity or flow rate could have been used in Fig. 4 in place of the depth.

The greatest value of the method may lie in interpretating time-dependent changes in coherent structures, such as the Gulf-of-Mexico Loop Current or the Alboran Gyre, which are linked to straits (in this case the Florida Strait and the Strait of Gibraltar). Such features can be easily monitored using satellite imagery. It may be asked, for example, whether the Florida Strait acts as a control for the Loop Current. If so, upstream changes in transport through the Yucatan Passage should excite a reflected wave of the general form discussed above. This wave may show up as some distinct response in the structure of the Loop Current.

The method also has a number of practical drawbacks. If the width or depth of the basin in which the instrument is placed is large compared with the width or depth of the control, the reflected wave amplitude will undergo a proportional attenuation and may be difficult to detect. This problem will be corrected by placing the instrument nearer the control. On the other hand, if the instrument is placed too near the control, the reflected wave will return before the incident wave has completely passed and a continuous (rather than stepwise) change in h will be seen locally.

A more severe restriction is placed on the method by the requirement that the incident wave be of the form shown in Figs. 2 and 3. More typically, the disturbance will consist of a continuous wave train, periodic or otherwise, forcing the controlling sill to continually adjust the upstream fluid to an ever-changing signal. How the return wave train differs from the return wave train that would occur in the uncontrolled case is unknown. Control may only be detectable on a time scale that is long compared to the typical period of the waves, with the dependent variables, h_1, h_2 and so forth, representing values time-averaged over several wave periods.

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