1.8 Wave reflections and upstream influence in time-dependent flows.

Although Long's experiment provides intuition into blocking and upstream influence, it does not tell us how these processes occur in the real ocean or atmosphere. There the heights of the sills are fixed and adjustments occur in response to temporally varying water mass formation and other time-variable forcing. To gain some perspective, consider a channel flow that is established and that is subject to slow variations in the upstream or downstream state. If the flow is hydraulically controlled, it will be immune to disturbances generated downstream of the controlling sill or narrows as long as those disturbances remain small, and we will therefore concentrate on disturbances generated upstream. How is hydraulic control manifested in such a situation? The guiding principle here is that *control* establishes a relationship between the parameters determining the upstream flow and those describing the channel geometry at the critical section. If we choose the flow rate Q and depth d_o to represent the upstream flow, and w is constant, then this relationship is given by (1.4.11), which links Q and d_o to the sill height h_c. In a laboratory experiment with fixed h_m, one would be free to vary Q alone or d_o alone, but not both.

It is natural to ask what would happen if the upstream flow was altered so as to violate this relationship. A numerical simulation along these lines begins with a steady, hydraulically controlled solution (Figure 1.8.1a). The upstream depth d_o is then increased to a new value d_1 (Figure 1.8.1b) creating a wave of elevation that approaches the obstacle from upstream. The new values of d_1 and Q (to the left of the wave) do not satisfy the relationship (1.4.11) required by critical control. The subsequent evolution is shown in Figures 1.8.1c and 1.8.1d. The incident wave strikes the obstacle generating a reflected wave that moves upstream and establishes a new steady flow of depth $d_2>d_1$ and a new Q. These new values satisfy (1.4.11) and thus the reflection process re-establishes the essential relationship between the upstream variables.

If the initial flow in the above experiment is *not* hydraulically controlled, the outcome is quite different, as shown in Figure 1.8.2. Here the reflected wave is isolated and does not alter the new steady state established by the disturbance. Thus the final upstream depth is d_1 rather than d_2. In this case one is clearly free to vary the upstream parameters independently.

The above experiments shows how hydraulic control is exercised and suggests a means of distinguishing controlled from uncontrolled flows using data time series at a fixed instrument. Figure 1.8.3 shows the difference in the time histories of d measured at a fixed location upstream of the obstacle. In the uncontrolled flow the reflected disturbance results in only a temporary change in d, while the controlled case gives a permanent change in d.

Some of these ideas can be exploited in order to parameterize the upstream effects of a sill or width contraction in a numerical model. The grid scale of such models is often too course to resolve the controlling topographic feature. Since the upstream effect of the sill or narrows is communicated by a reflected wave, it may be sufficient to know the reflection coefficient. For the incident wave shown in Figure 1.8.1b, which has an height amplitude d_1-d_o, the ultimate upstream depth d_2 established after wave reflection would be given by the reflection coefficient (d_2-d_o)(d_1-d_o). Consider a linearized version of this problem in which the incident wave has the form

$$\eta_I = f_I(y - c_+ t), \quad \text{and } v_I' = \left(\frac{g}{d_o}\right)^{1/2} f_I(y - c_+ t)$$

where $c_+ = v_o + (gd_o)^{1/2}$ and v_o and d_o are the undisturbed upstream flow. The reflected wave is of the form

$$\eta_R = f_R(y - c_- t), \quad \text{and } v_R' = -\left(\frac{g}{d_o}\right)^{1/2} f_R(y - c_- t),$$

with , $c_- = v_o - (gd_o)^{1/2}$.

If the wave length is much longer than the length of the obstacle, then the flow over the obstacle can be approximated as steady at any given instant. Thus the relationship (1.4.11) holds at any instant even though the flow itself is evolving in time. In the present context this relationship can be written

$$\frac{v^2(0,t)}{2} - \frac{3}{2}\left(\frac{gv(0,t)d(0,t)w(0)}{w_s}\right)^{2/3} + gd(0,t) = gh_m \tag{1.8.1}$$

where $v(0,t)$, $d(0,t)$ and $w(0)$ are the velocity, depth and width at the upstream edge (here y=0) of the obstacle, h_m is the obstacle height and w_s is the minimum width (here assumed to coincide with the sill).

If the values $d_o + f_I(-c_+ t) + f_R(-c_- t)$ and $v_o + (g/d_o)^{1/2}(f_I(-c_+ t) - f_R(-c_- t))$ are substituted for $d(0,t)$ and $v(0,t)$ in (1.8.1) and the resulting equation linearized, it follows that

$$R_c = \frac{\eta_R(0,t)}{\eta_I(0,t)} = \frac{(1+F_d)\left[1 - F_d^{1/3}(w(0)/w_s)^{2/3}\right]}{(1-F_d)\left[1 + F_d^{1/3}(w(0)/w_s)^{2/3}\right]}. \tag{1.8.2}$$

where $F_d = v_o/(gd_o)^{1/2}$

In order to apply this relation, suppose that an upstream disturbance is created in which the free surface level is raised from the value d_o to d_o+a. The disturbance propagates downstream and eventually reaches the obstacle where it is reflected with amplitude aR_c, with R_c given by (1.8.2). The reflected disturbance travels upstream and established a new state with depth d_o+aR_c and velocity v_o-$(g/d)^{1/2}aR_c$. This new state is guaranteed to satisfy the upstream conditions consistent with a hydraulically controlled flow, at least to O(a/d_o)^2.

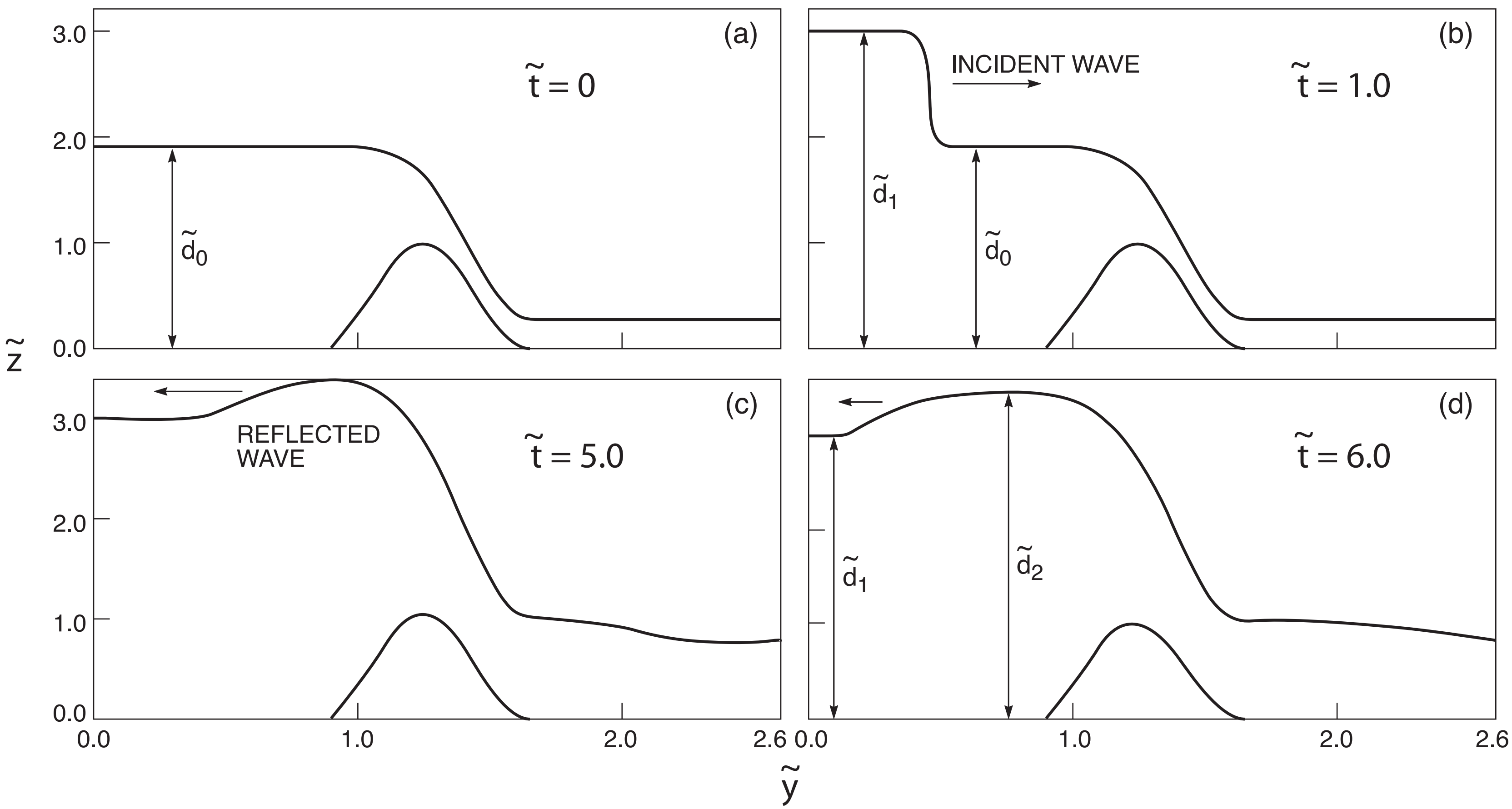
Hydraulic control is often equated exclusively with regulation of the flow rate Q, but this this is an oversimplification. Suppose that the drain in a kitchen sink is closed and the faucet is left running, causing the sink to fill up and water to spill out onto the floor. At the lip of the sink the flow will be critical and the flow will therefore be hydraulically controlled. However Q in this case is set by the faucet and is independent of the height of the lip or sill. In this case it is the depth of water in the sink (d_0) that is controlled: Q and h_m are set and d_o is then determined by something like (1.4.11).

Figure Captions

Figure 1.8.1 The wave reflection process for a hydraulically controlled flow. The nondimensional quantities shown are $(\tilde{d},\tilde{z}) = (d,z)/h_o, \tilde{v} = v/\sqrt{gd_o}$ and $\tilde{t} = t\sqrt{gd_o}/L$ where d_o is the initial upstream depth, h_o is the height of the obstacle, and L is the obstacle length. (from Pratt 1984b)

Figure 1.8.2 The wave reflection process for a purely subcritical flow. The notation is as in Figure 1.8.1. (from Pratt 1984b)

Figure 1.8.3 The time history of the upstream surface level during the wave reflection process for the controlled flow of Figure 1.8.1 and the subcritical (uncontrolled) flow of Figure 1.8.2. (from Pratt 1984b)



(a)
$\tilde{t} = 0$
$\tilde{d}_0$
(b)
INCIDENT WAVE
$\tilde{t} = 1.0$
$\tilde{d}_1$
$\tilde{d}_0$
(c)
REFLECTED WAVE
$\tilde{t} = 5.0$
(d)
$\tilde{t} = 6.0$
$\tilde{d}_1$
$\tilde{d}_2$
3.0
2.0
1.0
0.0
$\tilde{z}$
0.0
1.0
2.0
2.6
$\tilde{y}$

Figure 1.8.1

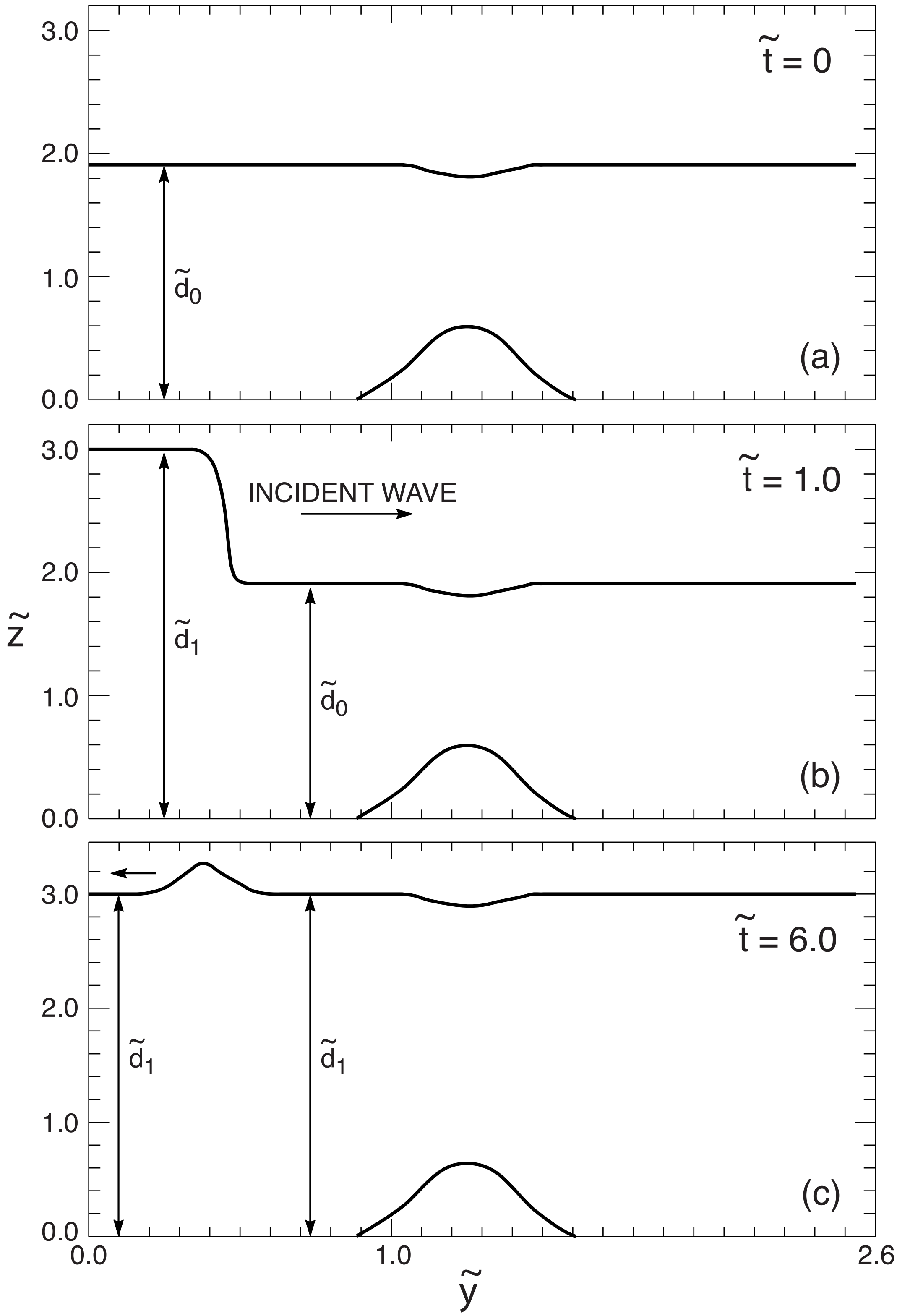
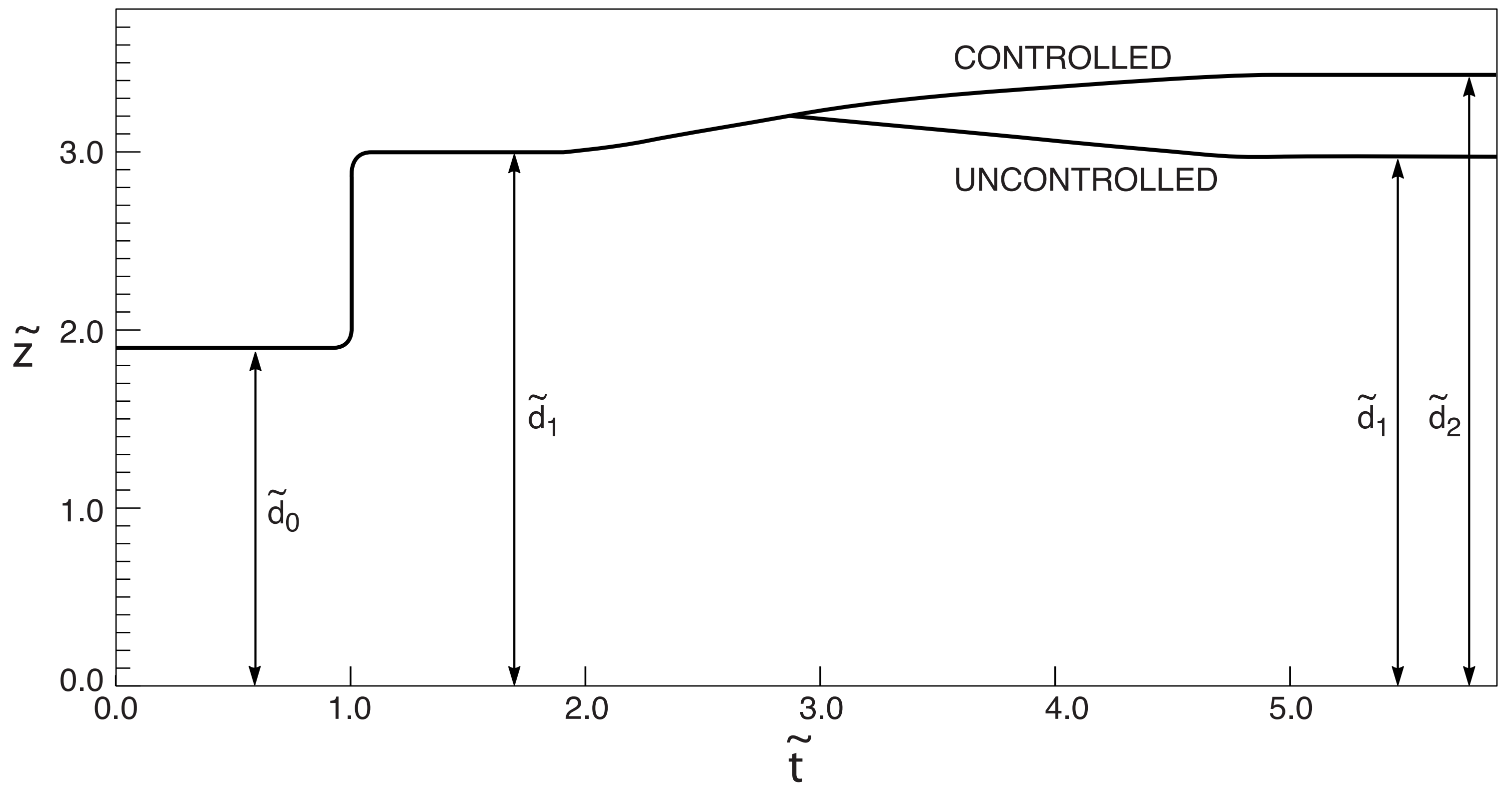

$\tilde{t} = 0$
$\tilde{d}_0$
(a)
$\tilde{t} = 1.0$
INCIDENT WAVE
$\tilde{d}_1$
$\tilde{d}_0$
(b)
$\tilde{t} = 6.0$
$\tilde{d}_1$
$\tilde{d}_1$
(c)
$\tilde{z}$
$\tilde{y}$
0.0
1.0
2.0
3.0
1.0
2.6

Figure 1.8.2



CONTROLLED
UNCONTROLLED
3.0
2.0
1.0
0.0
$\tilde{z}$
$\tilde{d}_0$
$\tilde{d}_1$
$\tilde{d}_1$
$\tilde{d}_2$
0.0
1.0
2.0
3.0
4.0
5.0
$\tilde{t}$

Figure 1.8.3