The Mediterranean Outflow—A Simple Advection-Diffusion Model[1]

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ABSTRACT

The influence of the subtropical gyre on the spread of Mediterranean Water in the Atlantic is discussed in terms of a simple horizontal advection-diffusion model. The northern, southern and western boundaries of a rectangular ocean are treated as salt sinks while the distribution of salinity on the east coast representing the highly saline Mediterranean Water is a sine curve. The velocity distribution for the subtropical gyre is that given by Stommel and includes westward intensification. Salinity distributions are calculated for various values of the Peclet number, and for oceanographically reasonable values they indicate that the gyre passes through the high-salinity tongue and advects it toward the south and west. The model is consistent with the observed salinity distribution of the mid-layers of the North Atlantic.

1. Introduction

Frequently the observed distribution of properties in the ocean is used to infer water movement and currents. To do this, as in the Kernschicht-methods of Wust (1935), the effects of diffusion are assumed to be small and advection is considered to play the dominant role in transporting properties. When diffusion is important, however, as it probably is in the ocean, one cannot accurately infer water movement from property distributions alone. The effects of diffusion must be recognized.

The observed distributions can also be used to evaluate circulation models. To do this a diffusion-advection equation is used to predict the distribution of properties which would result from a given circulation scheme; this can then be compared to the actual distributions. Kuo and Veronis (1970, 1973), for example, found the abyssal circulation model of Stommel (1958) to be broadly consistent with the observed distribution of oxygen in abyssal waters.

Recently the question of whether the subtropical gyre in the North Atlantic can pass through the Mediterranean Water tongue has been raised by Worthington (1975). The Mediterranean Water (MW) is a wedge of highly saline water which overflows the Straits of Gibraltar into the North Atlantic (Fig. 1). Although its transport is relatively small, estimated by Lacombe (1971) to be 1.2×10^6 m^3 s^{-1}, its influence on the water of the North Atlantic is large because of its large salinity input: it has been traced by its anomalously high salinity across the North Atlantic to the region near Bermuda. Its influence is strongest in the mid-thermocline (7–12°C) but it can be seen down to 2.6°C (Worthington, 1975; Worthington and Wright, 1970). Worthington (1975) argues that the presence of this tongue is diagnostic evidence that the circulatory gyre of the western North Atlantic cannot extend into the eastern Atlantic. It is desirable to know whether his argument is sound, or whether the gyre can pass through the tongue, as implied by Iselin (1936), Sverdrup *et al.* (1942) and Stommel (1965).

The purpose of this study is to investigate the relative roles of advective and diffusive processes in the mid-layers of the North Atlantic by comparing the observed distribution of the MW with results of a model utilizing the salt diffusion-advection equation together with Stommel's (1948) analytical circulation pattern for a rectangular ocean. The major finding is that with moderate diffusion, the tongue can indeed penetrate into the gyre. Features of the model are consistent with the salinity distribution in the North Atlantic, although this simple calculation is intended only to illustrate an effect rather than to reproduce observation in detail.

2. The model

a. The diffusion-advection equation

The equation governing the horizontal changes of a conservative property such as salinity is

$$\frac{\partial S}{\partial t}+u\frac{\partial S}{\partial x}+v\frac{\partial S}{\partial y}=K\left(\frac{\partial^2 S}{\partial x^2}+\frac{\partial^2 S}{\partial y^2}\right), \qquad (1)$$

where S is salinity (or other conservative property),

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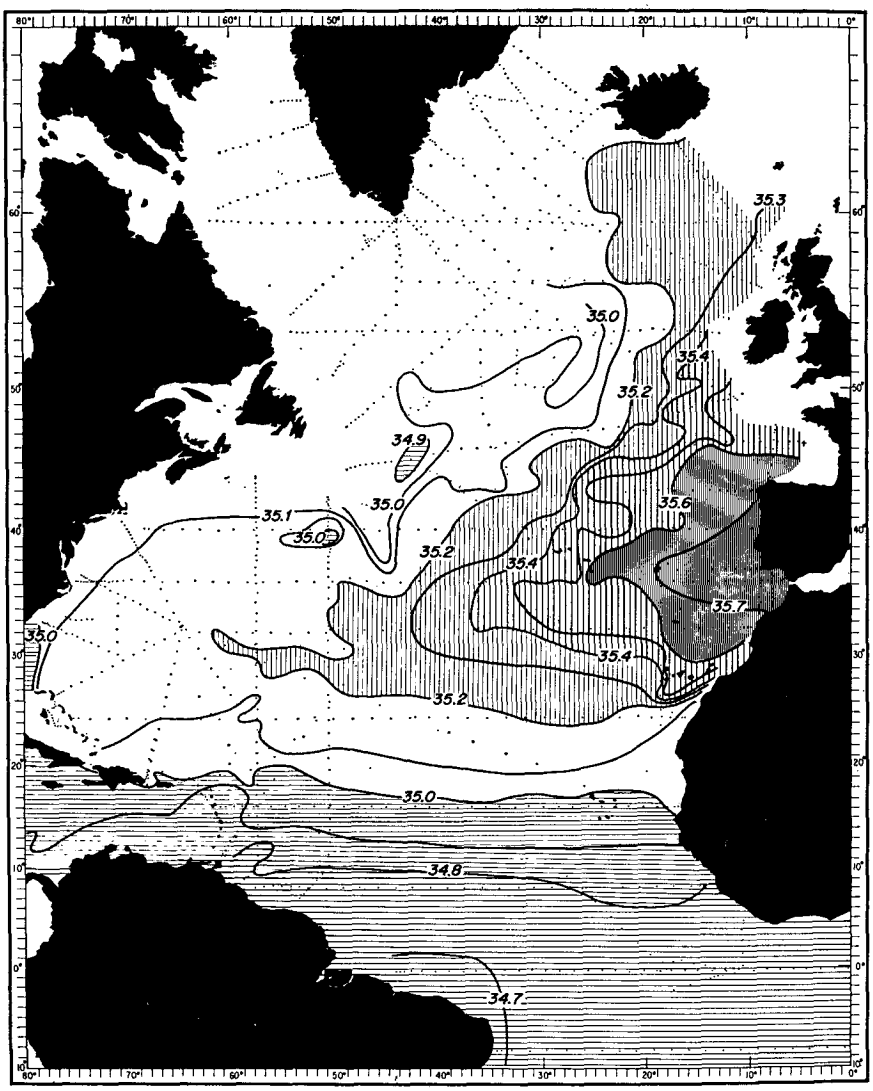

FIG. 1. Salinity (‰) at the 8°C isothermal surface in the North Atlantic (after Worthington, 1970).

u the east (x) velocity component, v the north (y) velocity component, and K the coefficient of horizontal eddy diffusion. The effects of vertical advection and diffusion have been neglected, and K is assumed constant.

Let us define new variables:

$$S^* \equiv S - \bar{S} \qquad t^* \equiv (V/L)t$$
$$x^* \equiv x/L \qquad u^* \equiv u/V$$
$$y^* \equiv y/L \qquad v^* \equiv v/V$$

where L is a characteristic horizontal dimension and V a characteristic horizontal velocity. The quantity S^* is the salinity anomaly and represents the departure in salinity at a given temperature of the high salinity MW from the mean salinity ($\bar{S}$) obtained from the mean T/S curve. The conservation equation can then be rewritten

$$\frac{\partial S^*}{\partial t^*} + u^* \frac{\partial S^*}{\partial x^*} + v^* \frac{\partial S^*}{\partial y^*} = \frac{1}{P}\left(\frac{\partial^2 S^*}{\partial x^{*2}} + \frac{\partial^2 S^*}{\partial y^{*2}}\right), \tag{2}$$

where

$$\mathrm{P} \equiv VL/K. \tag{3}$$

The quantity P is called the Peclet number and is a measure of the relative effects of advection to diffusion. As will be seen the numerical value of the Peclet number determines the salinity distribution in the model.

b. The basin and velocity field

The circulation scheme given by Stommel (1948) was used to model the subtropical gyre. The streamline

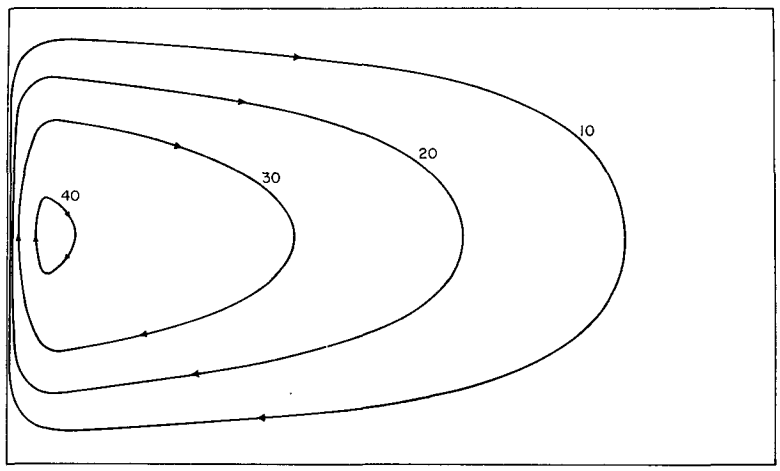

FIG. 2. Streamlines calculated from Stommel's (1948) circulation model when β is included. The values represent transport in units of 10^6 m^3 s^{-1}. For the dimensions that Stommel used (width 6283 km, length 10,000 km, depth 200 m) the western boundary current has speeds up to 240 cm s^{-1} and the broad return flow in the interior has speeds of 2.5 cm s^{-1}.

pattern for this model is shown in Fig. 2. This gyre is symmetrical in the north-south direction and has westward intensification which approximates the Gulf Stream.

As can be seen in Fig. 2 the velocity distribution is strongly a function of horizontal coordinates. The circulation over the eastern basin, however, is the wind-driven Sverdrup transport (Sverdrup, 1947). Its maximum value in Stommel's model, divided by depth, can be used to give a characteristic velocity of the basin. This velocity is

$$V=-\frac{F\pi}{\beta db}, \tag{4}$$

where F is the wind stress, β the variation of Coriolis parameter with latitude, d the depth, and b the north-south dimension of the basin.

Using (4), the Peclet number becomes

$$\mathrm{P}=\frac{F\pi}{\beta dK}, \tag{5}$$

where the north-south dimension is taken as the characteristic length scale. The same parameters used by Stommel were used in the calculations here and K was varied. The result is a series of different salinity distributions showing the effects of both advection and diffusion.

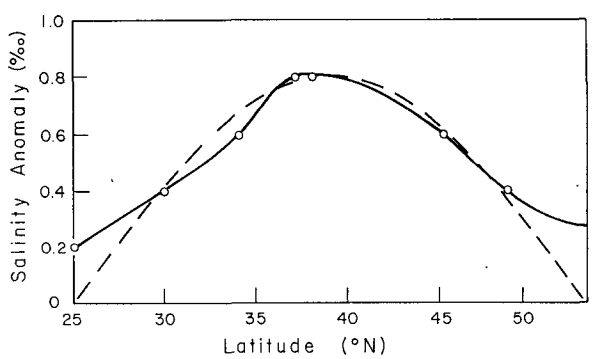

FIG. 3. Comparison of observed salinity anomaly in the core of the Mediterranean Outflow Water near the eastern boundary of the North Atlantic Ocean with a sine function. Values were taken from Worthington (1975, Fig. 18) and represent the departure in salinity at a given temperature from the value obtained from a standard T/S curve. The dashed curve is given by $S_0 \sin \pi y/b$, where $S_0=0.8‰$, $b=3000$ km and y is distance north of 25°N.

c. Boundary conditions

The model provides for a source of salt on the east and a sink on the north, south and west. The distribution of salinity anomaly on the east coast used to approximate the MW salt source is a sine curve which models the measured distribution off Gibraltar quite well (Fig. 3). The symmetrical distribution was used so that any final asymmetries could be clearly attributed to advection. When the real distribution was used, however, negligibly small differences in salinity patterns were observed.

A salinity sink around the west, north and south sides was chosen to approximate the mixing of MW with fresher water (Fig. 1). Water to the south of MW is dominated by South Atlantic Water characterized by low salinity values, while water to the north consists of Sub-Arctic Intermediate Water, the Labrador Current and Slope Water, and is also characterized by low salinities.[3] Water on the west is a mixture of water

[3] For a more complete discussion of the water masses in the North Atlantic see Wright and Worthington (1970), Worthington and Wright (1970) and Worthington (1975).

from northern and southern sources and of MW. It appears that the zero salinity-anomaly boundary condition on the west is not very important as the salinity gradients are very small in this region. Numerical experiments were run with the boundary condition of no salt flux through the western side; the only noticeable effect was to shift the westward extension of the MW tongue slightly to the north and west.

d. Computations

The salinity distributions were obtained by beginning with a uniform field of zero salinity anomaly except at $x=r$ where the eastern boundary condition was imposed and by using (2) to step ahead in time until the salinity distribution approached steady state over the entire basin. A finite-difference form (Ketter and Sherwood, 1969) of (2) and a coarse grid of 23 points in the east-west direction and 13 points in the north-south direction was used with a denser spacing near the western boundary where large velocity gradients are found (Fig. 4). Final salinity distributions were checked by using a denser grid spacing with the densest area of points located in the western and southern regions.

3. Results

Salinity anomaly distributions were calculated for a range of Peclet numbers, beginning with zero (pure diffusion), and ending with 31, in which advection is becoming dominant. The distributions are shown in Fig. 5. For simplicity $S_0=1.0‰$ in the computations. The exact steady-state solution to (2) for the case of pure diffusion is easily obtained (see Pipes, 1958):

$$S^*=S_0\frac{\sinh\pi x^*}{\sinh\pi r/b}\sin\pi y^*, \tag{6}$$

where the north-south dimension of the basin was chosen as the characteristic horizontal length. The exact solution was used to check the model results for the case of zero advection. Fig. 5 (P=0) clearly shows the diffusion of salt away from the eastern boundary toward the west resulting in a symmetrical north-south pattern. For the other extreme of pure advection (P=∞) there is no diffusion of salt across streamlines and the resulting S^* pattern is zero everywhere but at $x=r$, where it jumps discontinuously to the boundary condition values.

As the Peclet number is increased the effect of advection becomes more important (Fig. 5). Fig. 6 shows a superposition of the 5% isohaline for Peclet numbers of 0 to 31, and clearly illustrates the importance of advection on the shape of the MW tongue, which shifts toward the south and west and becomes narrower. For P=31 the 5% isohaline extends about 37% farther to the west than for P=0, and the tongue has become quite narrow in the westward extension.

The main result of this simple model is that for small Peclet numbers the salt tongue penetrates far into the gyre. Because salt is diffused away from the tongue toward the southern boundary, the closed streamlines of the gyre pass through the tongue and

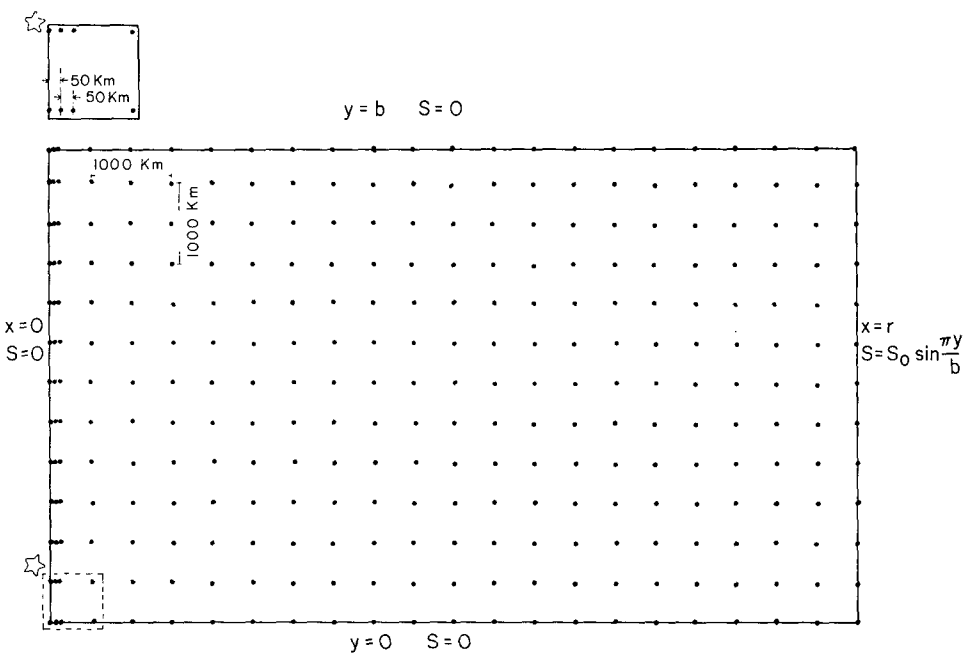

FIG. 4. The boundary conditions and grid used to calculate salinity distributions. For simplicity $S_0=1.0‰$ in the calculations.

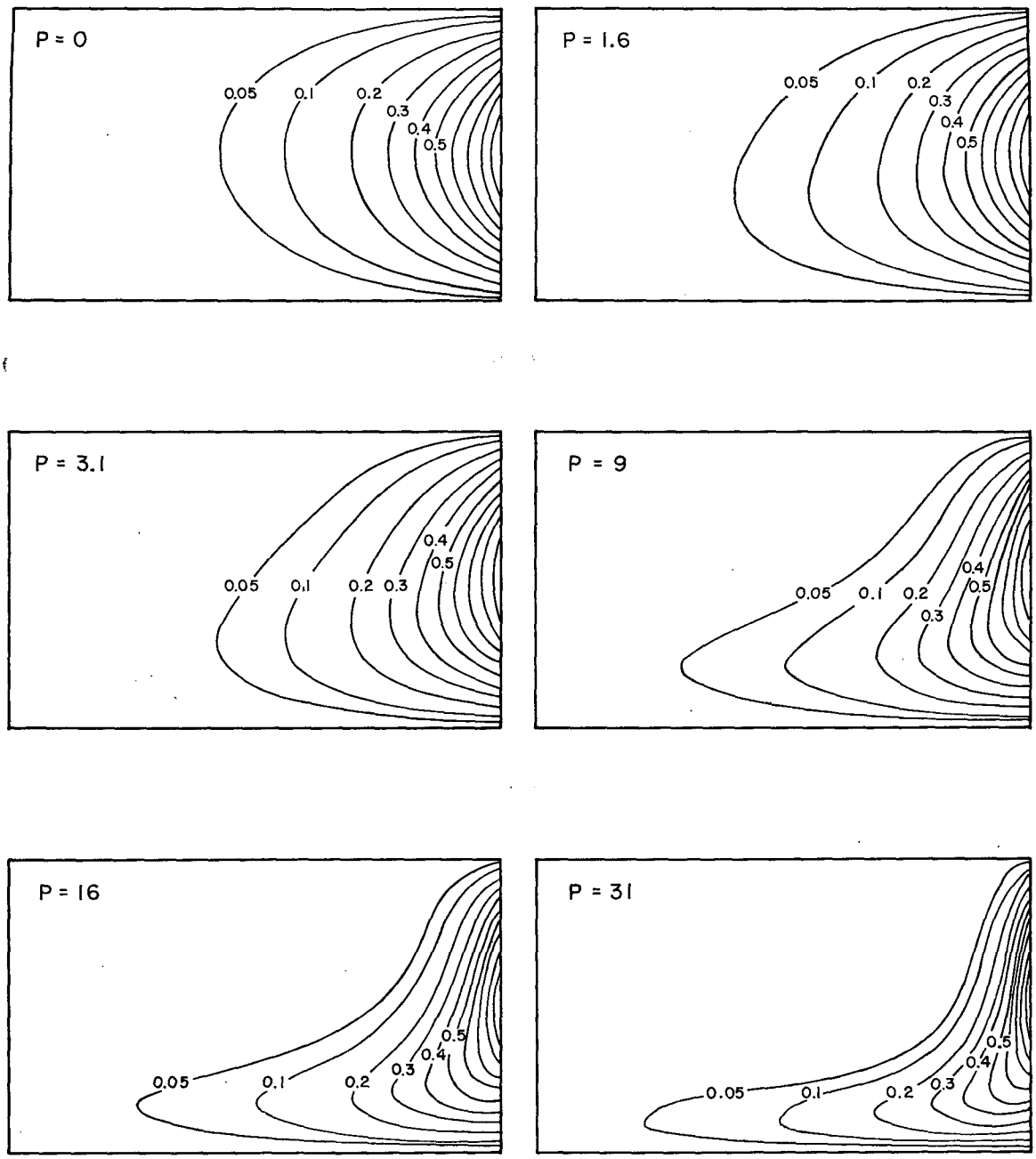

FIG. 5. The calculated distribution of salinity anomaly for different values of the Peclet number $P = VL/K$ (the ratio of advection to diffusion). The case of pure diffusion ($P = 0$), is shown in the upper left panel. The boundary conditions are shown in Fig. 4.

emerge on the other side without any net gain in salinity. As the Peclet number is increased the streamlines continue to pass through the tongue but advect it toward the south and west.

In all the examples shown the western boundary current has little effect on the salinity distribution in the western part of the basin. Although the velocities are 100 times the eastern velocities, the salinity concentrations there are so small that very little salt is advected by the boundary current. As the Peclet number is increased above 31 the tongue is advected further toward the west and finally salt is advected through the western boundary current and back into the interior of the basin.

4. Discussion

In order to make a comparison between the ocean and the model the Peclet number for the North Atlantic

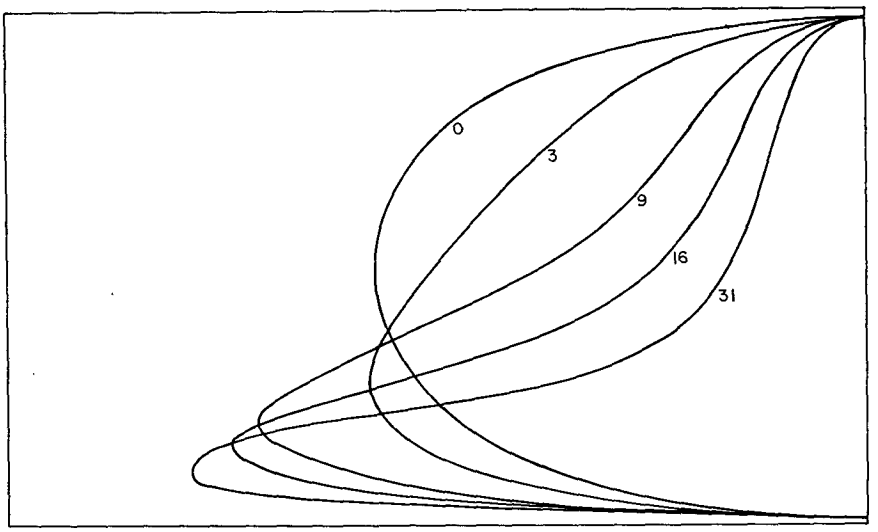

FIG. 6. Superposition of 0.05 isohalines for different Peclet numbers.

was estimated using (3); its value ranges from 3 to 30. These values were determined using values of the eddy diffusion coefficient representative of large-scale oceanic mixing [6×10^6 cm^2 s^{-1} (Kuo and Veronis, 1973); 5.5×10^7 cm^2 s^{-1} (Defant, 1955)]. The horizontal length scale used was 3×10^3 km and the characteristic velocity, estimated by redistributing the transport of the appropriate layers of the Gulf Stream over the width of the North Atlantic, was 0.6 cm s^{-1}. The implication is that the salinity anomaly distributions for P = 3 to 31 (Fig. 5) agree with the range in P values for the ocean.

In the model the MW tongue was advected toward the south and west and narrowed toward the west (Fig. 5). The actual MW tongue in the North Atlantic shows similar features. For example, the salinity distribution along the 8°C isothermal surface (Fig. 1) indicates a tongue of MW extending from the Straits of Gibraltar at 36°N toward the west, while at the same time extending south to around 30°N and becoming narrower. North-south salinity anomaly sections (Worthington, 1975) also indicate that the MW tongue is displaced to the south in the west: the high salinity core is at 38°N in the eastern region but at 28°N in the west.

5. Conclusions

Salinity distributions were computed for a variety of Peclet numbers, $P = VL/K$, in a horizontal advection-diffusion model of the Mediterranean high-salinity water. The circulation scheme of Stommel (1948) was used for the velocity field. For oceanographically reasonable values of P the gyre passes through the Mediterranean Water tongue and advects it toward the south and west. The calculated salinity distributions are in agreement with the main features of the measured salinity distribution in the North Atlantic. Based on this model the existence of an oceanwide subtropical gyre in the North Atlantic is consistent with the observed Mediterranean Water salinity tongue.

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