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On Balance of Energy in Oceanic General Circulation

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The energy sources and sinks of the oceanic general circulation include tidal forces, sea surface atmospheric pressure variations, wind stress, heating and cooling at the upper surface, freshwater flux, and geothermal heat flux. Since turbulence and waves in the oceans cannot possibly be resolved even if we use most powerful computers that may be available in the future, sub–grid scale parameterization must be used. As a result, theories and numerical models of oceanic general circulation must be built on a theoretical framework that includes the parameterization of sub–grid scale motions. Especially, these models require an "external" source of mechanical energy to support mixing in the stratified oceans. Such "external" energy is, of course, not really external. In fact, this "external" energy is the result of a cascade of energy from basin–scale to small scale. In a model where the sub–grid scale parameterization is specified this energy appears as external. Most importantly, the amount of energy available for mixing controls the oceanic circulation. Thus, the conservation of energy may be one of the most important constraints for oceanic general circulation models.

Key words: oceanic general circulation; energy balance.

The climate system, including both the atmosphere and oceans, is primarily driven by solar radiation. Therefore, the balance of energy is one of the most important constraints to the climate system. In meteorological studies or climate modeling, thus, the balance of energy has been one of the most important topics, receiving much attention. There have been many classical theories related to the energy cycle in atmospheric circulation and climate.

For example, the concept of available potential energy has been widely used, especially the approximate definition of available potential energy first introduced by Lorenz[1]. A more comprehensive study of the energy balance must include all the major energy cycles in the atmosphere, and it has been done recently by Peixoto and Oort in their book about climate dynamics[2].

On the other hand, the energy balance of oceanic general circulation does not seem to receive the attention it deserves. On the basin to global scales, the energy balance of oceanic circulation has been discussed in only very few papers or books, e.g., Faller[3], Kamenkovich[4], Lueck and Reid[5]. In this study, the author will try to form a theoretical framework for the energy balance in oceanic circulation. Since this problem has been more or less overlooked for so long, we do not have even a rough estimation of many important source and sink terms.

1. ENERGY SOURCES AND SINKS OF OCEANIC GENERAL CIRCULATION

The major forms of energy in oceans include gravitational potential energy, internal energy, kinetic energy, and the chemical potential associated with salts dissolved in sea water. Note that the energy balance in the oceans is quite different from that of the atmosphere. The atmospheric circulation is primarily driven by heating from below and cooling from above (or from the middle level) while oceans are heated and cooled from above. As a result, the atmosphere can be treated as a heat engine; however, oceans are not heat engine at all. This essential difference between the atmospheric circulation and oceanic circulation can be illustrated using a simple tube model[6]. In stratified oceans, mixing is what really controls the strength of the circulation; thus, the amount mechanical energy that cascades down to small scale is the key element in the energy cycle in oceans (Fig. 1).

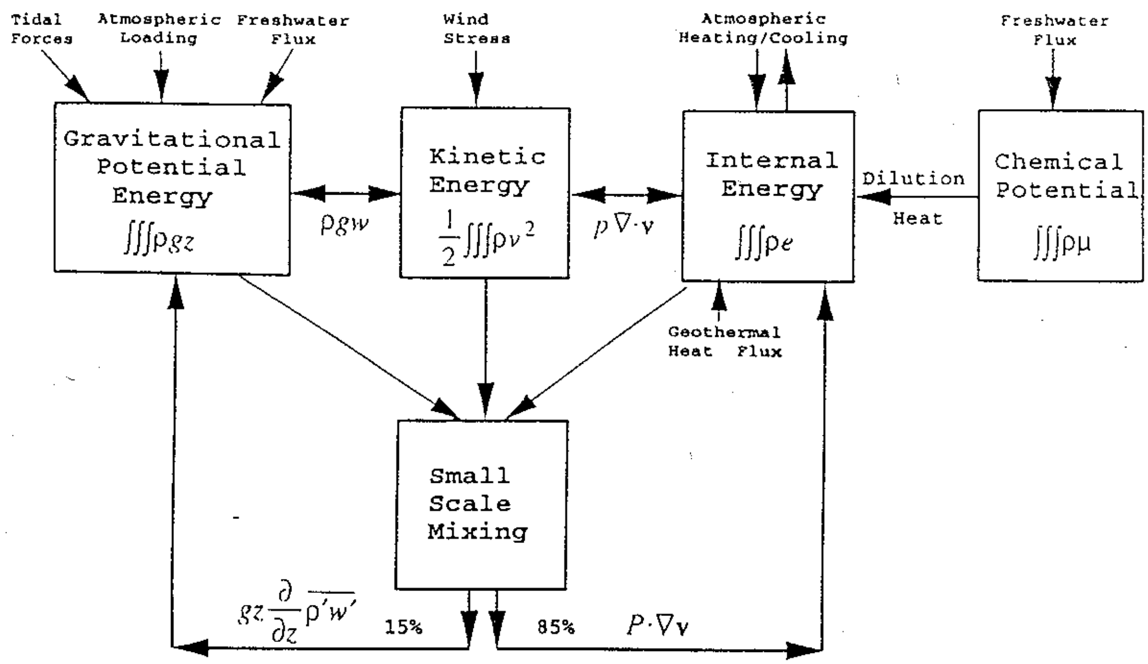

FIGURE 1. Balance of energy in the oceanic general circulation.

The direct sources of gravitational potential energy include the tidal forces due to the gravitational forces of the sun and the moon, atmospheric loading, and freshwater flux. The tidal forces are due to the difference in the gravitational forces between the earth itself and the sun and moon. Tidal currents in oceans have been studied extensively. If the tides were in exact equilibrium with the tidal forces, the tidal forces would not do work, so the tidal forces would contribute nothing to the oceanic circulation. However, tides always fall behind the tidal forces due to friction. As a result, the phase lags between the tidal forces and hence the tides give rise to tidal currents. And the tidal contribution to the gravitational potential energy of the oceans is due to the tidal dissipation, which is determined by the scalar product of the gradient of the tidal potential with the tidal currents. The most reliable estimation of a tidal dissipation rate comes from astronomical observations; it is about 3.17×10^{12} W for the lunar tides[7]. The total amount of oceanic tidal dissipation rate is 3.5×10^{12} W [8].

Atmospheric loading is very similar to tidal forces, and it is due to the fact that sea surface atmospheric pressure varies with time and space, with an amplitude of $\pm$ (10 — 20) hPa. The major difference between tidal forces and atmospheric loading is that tidal forces are very regular in time and space, but atmospheric loading varies with the state of the atmospheric circulation, so it is aperiodic. Otherwise, these two types of forcing are rather similar in nature. Up to now, our knowledge of the global oceanic response to atmospheric loading remains rudimentary. The global contribution to the gravitational potential energy in oceans from the atmospheric loading is unknown. See Wunsch and Stammer[9] for a comprehensive review on this topic.

The freshwater flux through the air–sea interface (in the form of evaporation, precipitation, and river runoff) contributes to the gravitational potential energy because precipitation and evaporation happen at slightly different levels. The global contribution to the gravitational potential energy due to freshwater flux is very small. Let us assume that the precipitation rate is 1 m / yr and the free surface elevation difference is 1 m. Since the total area of the oceans is about 3.6×10^{14} m^2, the rate of potential energy generated by precipitation is about 10^{11} W. By using satellite altimeter data and global evaporation and precipitation data from Baumgartner and Reichel[10], the rate of potential energy generated by evaporation and precipitation is about -6.7×10^9 W. The negative sign means that it is an energy sink. In oceans the free surface elevation is mostly controlled by wind–driven circulation and thermal circulation. A high free surface is often correlated with evaporation, so sea surface elevation and precipitation are negatively correlated.

Wind stress on the sea surface is a direct source of kinetic energy, which can be calculated in terms of $\tau \cdot \mathbf{v}_{surf}$, where τ is the surface wind stress and $\mathbf{v}_{surf}$ is velocity on the sea surface. Although this is probably the largest mechanical energy source for the oceans, most of this input energy is lost in strong surface waves and turbulent motions in the upper ocean. For the study of large scale geostrophic currents in the oceans, the useful part of the mechanical energy input from the surface wind stress can be estimated in terms of $\tau \cdot \mathbf{v}_{geo}$, where $\mathbf{v}_{geo}$ is the velocity of the subsurface geostrophic currents. Using global wind stress data and the surface geostrophic velocity inferred from satellite altimeter data, Wunsch[11] estimated that the total amount of mechanical energy input is about 8.8×10^{11} W. Estimation based on the results from a high–resolution numerical model is basically the same.

Note that the mechanical energy input through the surface geostrophic velocity is only a small part of the total mechanical energy input due to the surface wind stress. The downward atmospheric flux of kinetic energy is about 5×10^{14} W; however, only a small portion of this energy, about 2%—10%, actually enters the ocean[15]. Therefore, the total amount of energy input due to surface wind stress is at least ten times larger than that associated with the surface geostrophic currents. Better estimation of this energy and its distribution in the ocean interior requires careful study of energy transform through surface and internal waves.

The direct source of internal energy in oceans comes from heating and cooling at the air–sea interface. Although heating is ultimately due to solar isolation, we combine all forms of heat fluxes through the air–sea interface in terms of atmospheric heating / cooling. It is estimated that the internal energy source due to air–sea interaction is about 2×10^{15} W; this is roughly the same as the poleward heat flux in oceans. It is well known that the oceanic poleward heat flux constitutes about 50% of the poleward heat flux in the climate system of our earth; thus, the dynamic role of oceans in the climate system is of primary importance.

In addition, the geothermal heat flux through the ocean floor contributes about 3.2×10^{13} W to the internal energy of the oceans. Although this is less than 2% of the surface heat flux, it may play a very important role in driving the circulation, especially in the abyssal ocean. Its contribution to the oceanic circulation can be estimated in terms of the equivalent gravitational potential energy generated by the geothermal heating. Assuming a Boussinesq fluid, the gravitational potential energy generated by the geothermal heat flux can be estimated as

$$E^{p}_{\text{geo}} = \iint g \frac{\alpha \bar{\rho} \dot{Q}}{c_p} H \mathrm{d}x\mathrm{d}y \ , \tag{1}$$

where α is the heat expansion coefficient, $\dot{Q}$ is the heat flux rate through the sea floor, c_p is the heat capacity of sea water under constant pressure, and H is the depth of the sea floor. By using the empirical formula of geothermal heat flux and the NOAA $1^\circ \times 1^\circ$ topographic data set, the global contribution of geothermal heat flux is about 0.5×10^{12} W [6].

In addition, the freshwater flux through the air–sea interface also carries a certain amount of chemical potential, whose existence is due to the difference in concentration of dissolved salts. The chemical potential difference between fresh water and salty water can be expressed in terms of electric voltage difference or osmotic pressure difference between fresh water and salty water. The osmotic pressure of sea water of salinity 35 psu and 20°C is about 24.8 atmospheric pressures. Thus, the heat of dilution released by mixing one liter of fresh water with seawater is about 256.3 J. The global evaporation rate is about $4.24 \times 10^{14}\ \mathrm{m^3/yr}$ [10], thus, the rate of chemical potential generated by evaporation / precipitation is about 3.5×10^{12} W. Although a large portion of this energy is lost within the mixed layer in the upper ocean right after rainfall enters the sea, the remaining portion is what really drives the salt mixing at the level of molecular mixing.

Note that although the oceans receive energy from different sources, there is only one energy sink, i.e., all input energy eventually becomes internal energy and is exported in forms of atmospheric cooling.

These four types of energy are mutually transferable. For example, gravitational potential energy and kinetic energy are both mechanical energy, and they are interchangeable through the vertical motion term $\rho g w$. On the other hand, the exchange between kinetic energy and internal energy is through the pressure work during water parcel expansion and compression, $p \nabla \cdot \mathrm{v}$. However, the conversion rate must obey the second law of thermodynamics. Accordingly, kinetic energy can be converted into internal energy completely, but internal energy cannot be converted to mechanical energy completely, and the maximum efficiency is determined by the so–called Carnot efficiency.

Chemical potential is the force driving the salt diffusion (at the small scale of molecular diffusion) in oceans. The conversion from chemical potential to other forms of energy in oceans is not yet clear. Probably, most of the chemical potential is converted into internal energy as heat of dilution.

Energy in all these major forms is converted into energy supporting small scale waves and turbulent motion through the energy cascade from large scales to small scales. Turbulent motion generally leads to energy dissipation, as is true for a homogeneous fluid system. However, in a stratified fluid turbulent motion also leads to increase of gravitational potential energy of the large scale organized motion because mixing pushes dense water upward and light water downward, and thus raises the center of mass. Consequently, energy supporting small scale turbulent motion in a stratified ocean does not all become heat (or the wasted mechanical energy). See Appendix for the details.

According to theories of small scale mixing in a stratified fluid environment[12], about 15% of this energy ($gz\partial\overline{\rho' w'} / \partial z$, where ρ' and w' are the density and vertical velocity perturbation) is converted into the gravitational potential energy of large scale organized motion. The remaining, 85% energy ($P \cdot \dot{V}$, where P is the viscous stress tensor) is converted into heat or internal energy. The 15% of the energy that is given back to the large–scale potential energy is the most important source of mechanical energy supporting stratification and controlling the thermohaline circulation.

2. THEORETICAL FRAMEWORK OF OCEANIC CIRCULATION

Oceanic circulation encompasses currents, waves and turbulent motionon extremely broad spatial and temporal scales; thus, resolving motion on all these scales in a single numerical model is almost impossible even in the future. Historically, theories and numerical models of oceanic circulation have been evolved gradually by treating these dynamic processes separately. The advantage of such a separation of dynamics is, of course, its great simplicity and the solvability of each separate component. However, one potentially serious flaw of such an approach is that some interactions between these sub–systems may be overlooked.

So far, in most theories and numerical models the tidal calculation is separated from the large–scale current calculation. The oceanic circulation is treated in three sub–systems: First, the oceanic general circulation model (OGCM) is forced by upper boundary conditions of wind stress, heat flux, and evaporation minus precipitation. Second, the tidal calculation is carried out separately in the oceanic tide model (OTM). Atmospheric loading has not been studied carefully, but this can be carried out similarly to the tidal calculations, or it can be treated as a part of the forcing condition on the upper boundary of the OGCM. The baroclinic tide calculation requires stratification of the ocean, as can be provided from the OGCM. Third, the sub–scale parameterization of mixing due to turbulence and internal waves is the critical part.

Since we are unable to resolve the sub–grid scale waves and turbulence, sub–grid scale parameterization is used to simulate the contribution of these sub–grid scale motion. Thus, our theoretical framework must include the parameterization of sub–grid scale motion or the oceanic mixing parameterization model (OMPM). From observations or the output of the OGCM and OTM, the OMPM provides all the sub–grid scale parameterization, such as the isopycnal / diapycnal eddy mixing coefficients of momentum and tracers. Thus, our current theoretical framework of oceanic general circulation includes three components: OGCM, OTM, and OMPM (Fig. 2).

Both the OGCM and OTM feed the energy to OMPM through the energy cascade from large to small scales. There is, of course, up–scale energy transfer as well. The energy of the oceanic circulation comes from the large scale energy sources, but dissipation takes place through the small scale, so the general trend of the energy cascade is inevitable.

The important point is that when climate changes, the parameterization of these sub–grid scale motion should change in response. However, many sub–grid scale parameterizations used in existing numerical models are not explicitly related to the climate conditions. Ironically, when ocean–atmosphere coupled models are used to forecast climate change, the sub–grid scale parameterization does not change even when the system drifts into a totally different climate. Apparently, such an approach may not be consistent with the physics of the oceanic circulation, and it is hoped that in the near future our understanding of the sub–scale motion will allow us to use parameterizations that are explicitly dependent on the climate state.

Separating the oceanic circulation into these three sub–systems is for both historical and technical reasons. Tidal observations and theories are probably of the oldest science and technology; they have gradually evolved to a quite mature stature over the past. Based on harmonic analysis, we can predict tidal currents to quite high accuracy in most cases. With the advance of satellite altimeter data collection, numerical models of barotropic tides over the global oceans have also been greatly improved, and they can deliver solutions with high accuracy.

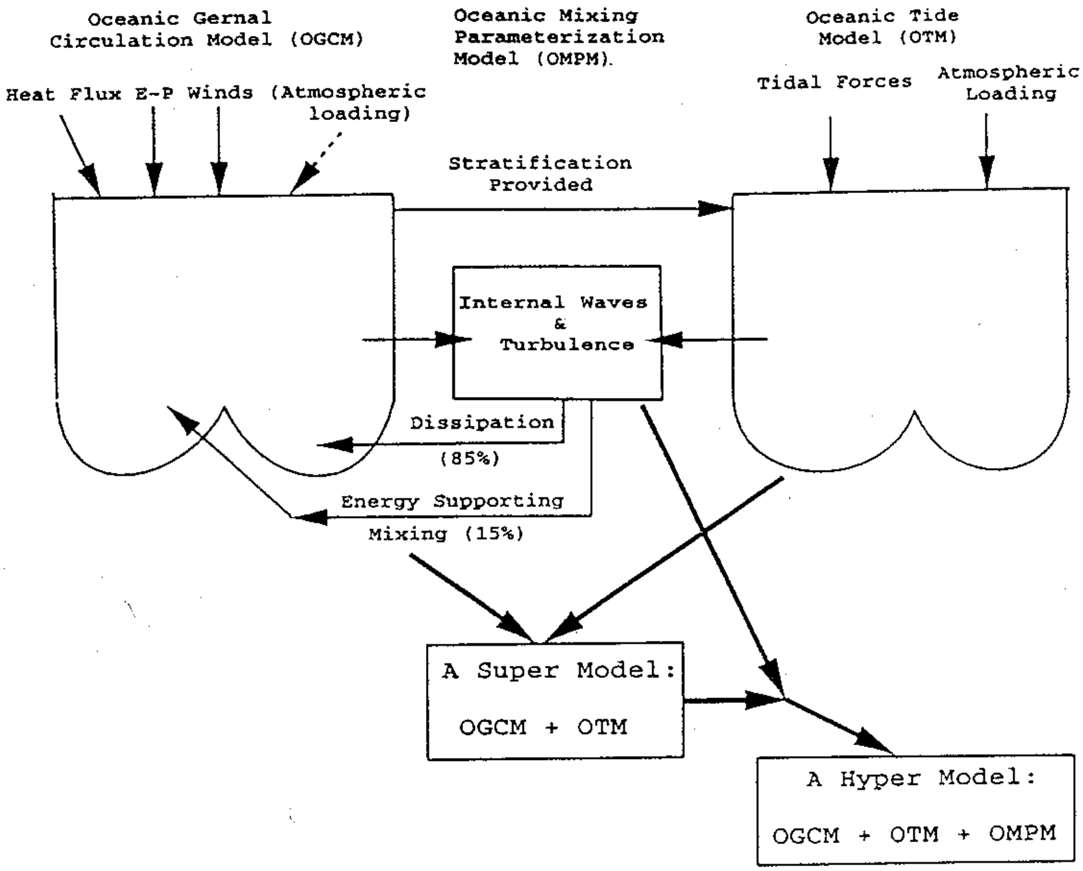

FIGURE 2. Fundamental structure of oceanic circulation models.

On the other hand, the baroclinic tidal calculations in the stratified oceans are still in its preliminary stage. One of the major technical difficulties in the baroclinic tidal calculations is our lack of understanding of the dissipation and mixing mechanism, which controls the sub–scale motions related to the tidal motion and the thermohaline circulation alike.

Tidal calculations are quite different from those of the wind–driven and thermohaline–driven circulation because the spatial and temporal scales involved are totally different. For the thermohaline circulation related to climate study, one has to integrate the system for more than several thousand years in order to study the climate change. Thus, we have to use models with very coarse resolution, generally on the order of 100 km in the horizontal and tens of meters in the vertical, and time steps on order of days or even longer. For such coarse resolution, the hydrostatic approximation is used, which greatly simplifies the dynamic calculations.

On the other hand, baroclinic tidal calculations, especially for the deep ocean, require much higher spatial resolution. For example, the baroclinic tidal current in the deep ocean has a velocity of 1 cm s^{-1} or less. The daily excursion of the currents driven by the semi–diurnal tide is about 400 m or less. Thus, in order to resolve the baroclinic tides in deep oceans, the numerical model must have a horizontal resolution on the order of tens of meters or finer, and time steps of no more than an hour. For such small scales, the hydrostatic approximation used in most OGCMs can no longer apply. The advantage of the tidal calculations is, of course, the periodic nature of the problems. Most important tidal cycles have periods on the order of days or less. Thus, tidal calculations require very fine spatial grids and small time steps, but the total duration of integration is relatively short.

Due to these differences in calculations, the traditional approach is still to keep the tides separate from the wind–driven and thermohaline circulations. The philosophy is to complete these two calculations separately, and then use some sub–grid scale parameterizations to provide the sub–grid scale paramerterizations used in both the OGCM and OTM. However, there might be a major flaw in separating OGCM and OTM because such a separation automatically excludes the possible large–scale interaction between the tidal motion and the wind–driven and thermohaline–driven circulation. Study of such interactions remains a challenge physical oceanographers.

One may imagine a super model, in which the OGCM and OTM are combined into a single model. In such a model the pitfalls of the artificial separation will be overcome naturally.

One may even dream about a hyper model, in which all the turbulence and waves are resolved; thus, the OGCM, OTM, and OMPM are all included in a single model. However, such a model can never be realized. In order to describe turbulence in complete detail, we have to resolve the small eddies whose scale is determined by the Kolmogorov micro–scale

$$\eta = (\mu^3 / \varepsilon)^{1/4}, \tag{2}$$

where μ is the molecular viscosity and ε is the rate of energy dissipation. Because the micro–scale determined by this scaling law is so small, resolution of all the turbulent motion in the oceans down to such a scale seems impossible even for the most powerful computers of the future. Therefore, our theories about oceanic circulation must be based on a certain parameterization of the turbulent motion in oceans.

It is important to note that as long as the sub–scale turbulent motion is not explicitly resolved, we are forced to parameterize the turbulent motion in some simple ways, and the model will always require a certain amount of "external" mechanical energy sources to support diapycnal mixing, as will be shown in the next section. Such "external" sources of energy are not really external as they come from the energy cascade from the large–scale energy sources imposed on the upper boundary and lower boundaries of oceans. The main function of the OMPM is to provide such an "external" energy source from given circulation and stratification obtained from the OGCM and the OTM.

The commonly used approach now is to use some extremely simple parameterizations in OGCM. For example, a one–line statement, such as $\kappa = 1.0\ \mathrm{cm}^2\ \mathrm{s}^{-1}$, has been widely used. In other cases, simple turbulence parameterization has been used. It is expected that as our understanding of the turbulence and internal waves improved, the OMPMs will become much more sophisticated. Maybe, five or ten years down the road we will be able to run OGCMs, coupled with OTMs and OMPMs with several thousand lines of FORTRAN code.

In theory, if one is able to resolve all the turbulence and waves down to the Kolmogorov scale, no eddy parameterization would be necessary, and such "external" sources of mechanical energy for mixing are unnecessary in the theory and in numerical models. Instead, the model will be driven solely by the forcing conditions illustrated in Fig. 1, and the momentum and tracer mixing will be controlled by the molecular viscosity and diffusion. However, we will probably never be able to reach such a goal, and for practical purposes, eddy parameterization and the associated "external" sources of mechanical energy for mixing will remain the cornerstone of oceanic circulation theory and numerical models.

3. AVAILABLE POTENTIAL ENERGY

The gravitational potential energy (GPE) can be reversibly transferred into kinetic energy, as shown in Fig. 1. Although one may think that a large amount of GPE implies large circulations, there is no such a simple link. What really matters is not the amount of GPE, but the amount of GPE that can potentially be transferred into kinetic energy, the so–called available potential energy (APE).

Lorenz[11] first introduced a quasi–geostrophic approximation form of APE for atmospheric circulations. His definition has since been extended to the studies of oceanic circulation[13,14]. In those studies of oceanic circulation, the exact definitions of APE and its source have been replaced by the following approximations

$$E_a = -\frac{g}{2}\iiint_v (\rho - \bar{\rho})^2\left(\frac{\partial \bar{\rho}_\theta}{\partial z}\right)^{-1} \mathrm{d}v, \tag{3}$$

$$\Phi_s = -g\iiint_v (\rho - \bar{\rho})\dot{\rho}\left(\frac{\partial \bar{\rho}_\theta}{\partial z}\right)^{-1} \mathrm{d}v. \tag{4}$$

Since these definitions are based on the quasi–geostrophic approximation, their application to basin–scale circulation may lead to erroneous results. The major problem associated with these definitions is that they are based on the quasi–geostrophic (QG) approximation. Since the basic stratification is assumed in the QG approximation, the maintenance of the basic stratification is beyond the scope of the QG. However, as will be shown shortly, "external" mechanical energy supporting the basic stratification is the most important energy source supporting the thermohaline circulation. (This mechanical energy is "external" because we have to parameterize the sub–grid scale turbulence and waves, as discussed in Section 1. In reality, this energy is, at least partially, the end product of the energy cascade from the large–scale energy sources for the oceanic circulation down to the small scale for mixing.) Thus, a theory of the energy balance in the ocean without including the energy supporting mixing is similar to drawing an elephant without legs.

According to the classic definition, however, APE is the difference in potential energy between the physical state and the reference state:

$$E_a = E_p - E_r = g\iiint_v \rho z \mathrm{d}v - g\iiint_v \rho_r Z_r \mathrm{d}v, \tag{5}$$

where (ρ, z) and (ρ_r, Z_r) are the density and the vertical coordinates in the physical and reference states, respectively.

The time rate of change of APE is[15]

$$\frac{\mathrm{d}}{\mathrm{d}t}E_a = \Phi_s + \Phi_{me} - \Phi_{mr} - \Phi_{pk}, \tag{6}$$

where

$$\Phi_s = \kappa g\iint_s (h_{m/2} - Z_r)\frac{\mathrm{d}\rho}{\mathrm{d}z}\mathrm{d}x\mathrm{d}y \tag{7}$$

is the source of APE due to surface heating / cooling, where $h_{m/2}$ is half of the mixed layer depth. In this definition, the energy lost during the convective adjustment is included, so that in the equilibrium state, the four terms on the right–hand side of Eq. (6) are exactly balanced.

$$\Phi_{me} = \kappa g\iint_s (\rho_b - \rho_s)\mathrm{d}x\mathrm{d}y \tag{8}$$

is the rate of potential energy increase due to mixing supported by the "external" mechanical energy source, and ρ_b and ρ_s are the densities at the bottom and the upper surface, respectively.

$$\Phi_{mr} = g\kappa\iiint_v \left(-\frac{\mathrm{d}Z_r}{\mathrm{d}\rho}\right)\left(\frac{\partial \rho}{\partial z}\right)^2 \mathrm{d}v + g\kappa_h\left(-\frac{\mathrm{d}Z_r}{\mathrm{d}\rho}\right)\left[\left(\frac{\partial \rho}{\partial x}\right)^2 + \left(\frac{\partial \rho}{\partial y}\right)^2\right]\mathrm{d}v \tag{9}$$

is the rate of potential energy increase in the reference state due to mixing.

$$\Phi_{pk} = -g\iiint \rho w dv \tag{10}$$

is the rate of conversion from potential to kinetic energy.

In order to show the substantial differences between these two definitions of APE, numerical experiments have been carried out using a three–dimensional primitive equation model by Cox[16]. The model ocean is a $60° \times 60°$ square basin, with a constant depth of 5.7 km. The horizontal resolution is $4° \times 4°$, and there are 15 layers vertically, with top layer of 30 m thick. The model is driven by a relaxation boundary condition for temperature only, with a reference temperature that is 25°C at the equator and decreases linearly to 0°C at the northern boundary. A simplified equation of state is used

$$\rho = 0.7948S_0 - 0.05968T - 0.0063T^2 + 3.7315\times 10^{-5}T^3, \tag{11}$$

where $S_0 = 35$ PSU is the mean salinity in the ocean.

There is neither wind stress nor freshwater forcing in the model. The coefficients of horizontal momentum dissipation and of tracer mixing are $A_h = 2.5\times 10^5\ \mathrm{m^2\,s^{-1}}$ and $\kappa_h = 10^3\ \mathrm{m^2 s^{-1}}$, respectively; the vertical momentum dissipation coefficient is $A_v = 10^{-4}\ \mathrm{m^2\,s^{-1}}$. The vertical tracer mixing coefficient is $\kappa_v = 10^{-4}\ \mathrm{m^2\,s^{-1}}$ in the first experiment, but it will be changed to $10^{-5}\ \mathrm{m^2\,s^{-1}}$ and $10^{-3}\ \mathrm{m^2\,s^{-1}}$ for two additional experiments to be discussed shortly.

The heat loss to the atmosphere reaches its maximum (60 W $\mathrm{m^{-2}}$) in the middle of the western boundary. However, this local feature does not appear as a maximum on the map of APE sources. Instead, the region of strong source of APE is located much farther poleward because the depth factor $h_{m/2} - z$ dominates the strength of the APE source, as indicated in Eq. (7) (Fig. 3). Although there is strong heat flux into the ocean along the equatorial edge of the basin, its contribution to APE is rather small because $h_{m/2} - z$ is very small near the equator.

For comparison, the source of APE due to surface forcing based on the Q–G approximation (3) indicates a strong source of APE in the equatorial region (the thin curve labelled $\varphi_{s,QG}$ in Fig. 3), similar to the results of Oort et al.[17]. According to such a definition, most of

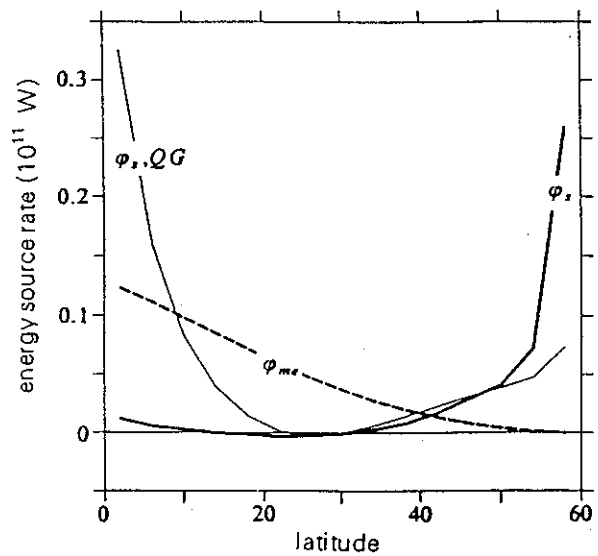

FIGURE 3. APE sources (integrated over each 4° latitude band) due to surface forcing and interior mixing. φ_s is the APE source due to surface forcing based on the exact definition, $\varphi_{s,QG}$ is the APE source due to surface forcing under the Q–G approximation, φ_{me} is the APE source due to external mechanical energy sustaining vertical mixing.

the APE would be generated near the equator, instead of in the polar basin. This seems to be in contradiction to the real physical process happening in the ocean where potential energy is released when dense water sinks into the deep ocean.

Most importantly, the exact definition of the source of APE includes the contribution due to mixing driven by the mechanical energy source, as indicated by the dashed line in Fig. 3. As is well known, mixing raises the center of mass against gravity, so mixing requires an external source of mechanical energy. Since mixing is essential in setting up vertical stratification and circulation in the ocean, the external energy supporting mixing is of vital importance for the oceanic circulation. In the equatorial part of the basin, stratification is strong, so the external energy source supporting mixing is also very strong (see Fig. 3).

The global balance of APE and total kinetic energy (TKE) is shown in Fig. 4, where results from three numerical experiments are included with $\kappa_v = 10^{-5}$, 10^{-4}, and $10^{-3}\ m^2\,s^{-1}$. Note that these three experiments were all carried out under the identical relaxation condition for temperature, with the only difference being in the vertical tracer mixing parameter. As κ_v increases by ten times, all quantities increase by about tenfold, including the total amount of APE and TKE, and all fluxes.

In all the cases, the source of APE due to mixing driven by an external energy source is about the same as the source of APE due to surface thermal forcing. Thermal forcing alone cannot determine the strength of the APE source due to surface forcing, the total amount of APE, the strength of the meridional overturning rate (MOR) or the poleward heat flux (PHF). In contrary, under a given surface thermal forcing condition, the amount of energy available for mixing controls the stratification and thus the meridional pressure gradient, which in turn controls the strength of the meridional overturning, the poleward heat flux, and even the APE source rate due to surface thermal forcing.

For example, when $\kappa_v = 10^{-4}\ m^2\,s^{-1}$, the external energy required for sustaining the mixing is about $0.338 \times 10^{-6}\ W\,m^{-3}$, which is larger than the APE source due to surface thermal forcing of $0.21 \times 10^{-6}\ W\,m^{-3}$. The net APE source due to mixing is equal to the APE source of mixing sustained by external energy minus the potential energy increase in the reference state due to mixing in the physical state, so it is $-0.183 \times 10^{-6}\ W\,m^{-3} < 0$. However, it would be a mistake to ignore the source of mechanical energy required to sustain mixing and to claim that mixing only dissipates APE. As discussed above, the mechanical energy to sustain mixing contributes about half of the APE source. In addition, mixing driven by the external energy source also controls the strength of the APE source due to surface thermal forcing. Furthermore, the actual amount of external energy required for sustaining the mixing is about seven times larger than the value of $0.338 \times 10^{-6}\ W\,m^{-3}$ because the efficiency of mixing is only about 15%, as suggested by Osborn[12]. This large amount of energy required for mixing may come from geothermal heat flux, tidal dissipation, internal wave breaking, and wind stress input.

In many existing text books and papers, the oceanic circulation has been compared with other heat engines. However, there is a big difference between oceanic circulation and other heat engines. As shown above, in order to put the oceanic engine in motion, the mechanical energy required for sustaining the background mixing is much larger than the amount of energy converted from potential energy to kinetic energy. The ratio is about 10. If we consider the efficiency of 15% suggested by Osborn[12], this energy ratio will be on the order of 100. Therefore, the oceanic circulation cannot export mechanical energy through the conversion of thermal to mechanical energy. On contrary, the maintainance of the oceanic circulation requires external mechanical energy source. In other words, the so-called "oceanic heat engine" has a negative heat efficiency. Thus, within the region of realistic parameters, the oceanic circulation is not a heat engine! In fact, it is more appropriate to treat the oceanic circulation as

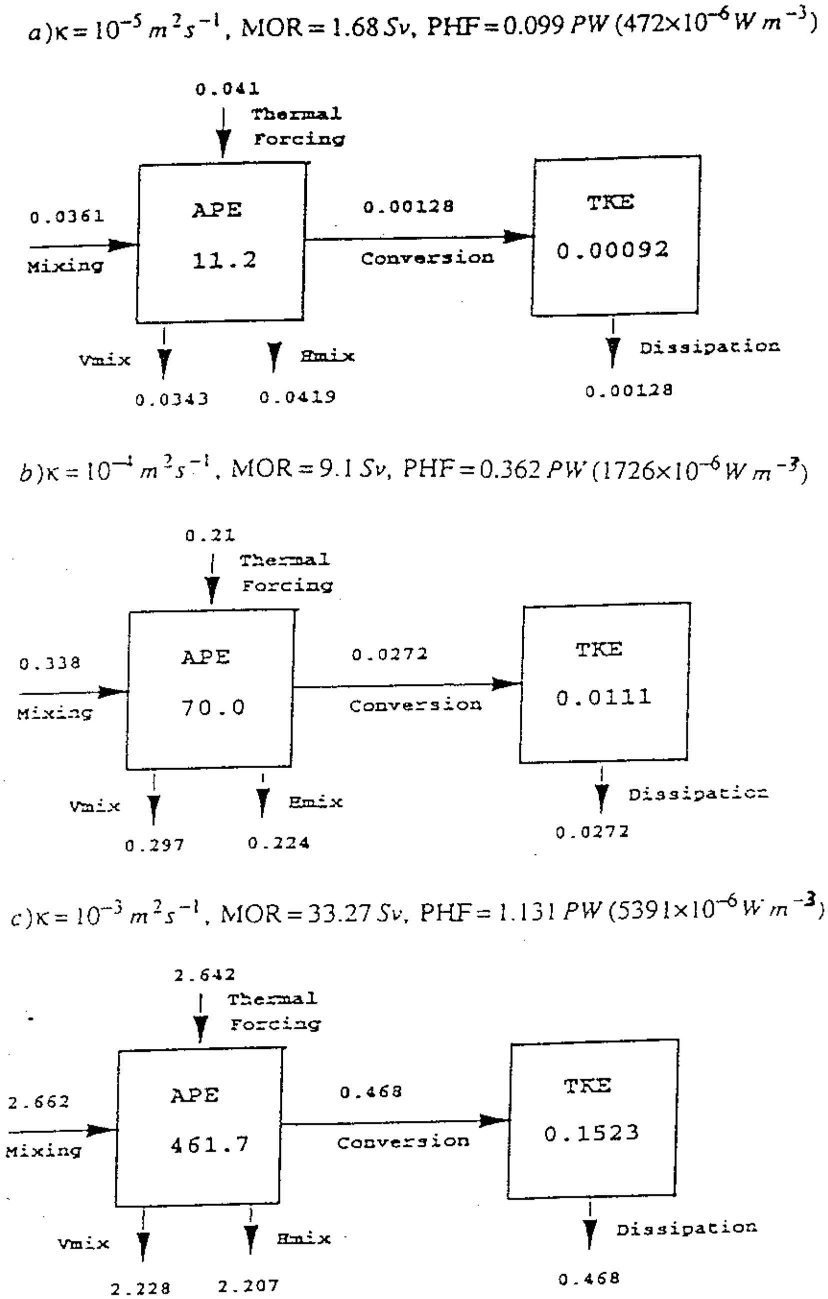

FIGURE 4. Balance of APE and TKE (Total Kinetic Energy) for three cases with $\kappa_v = 10^{-5}$, 10^{-4}, 10^{-3} m^2 s^{-1}. MOR is the Meridional Overturning Rate, in 10^6 m^3 s^{-1}; PHF is the Poleward Heat Flux, in 10^{15} W; V_{mix} and H_{mix} indicate the potential energy increase in the reference state due to vertical and horizontal mixing. Both APE and TKE are in J m^{-3}, while all flux terms are in 10^{-6} W m^{-3}.

a conveyor belt for transporting heat and fresh water.

However, there is a special region where very weak molecular diffusion, on the order of 10^{-8} m^2 s^{-1}, can alone drive the oceanic circulation. Since the diffusivity is about 1000 times smaller than that in observation, the meridional overturning rate and poleward heat flux will be reduced to about 0.1 Sv and 0.01 PW. This region is, of course, only an idealized case existing in theory, and it is totally unrelated to the real oceans.

4. PITFALLS OF GRAVITATIONAL POTENTIAL ENERGY IN MODELS BASED ON THE BOUSSINESQ APPROXIMATION

Since energy is ultimately the driving force for the atmospheric circulation, energy conservation has been one of the major constraints used in designing atmospheric general circulation models. On the other hand, energy conservation has seldom been taken as one of the basic principles in designing oceanic circulation models. Most OGCMs are based on the Boussinesq approximation, and one of its major assumptions is that the velocity field is nondivergent. The Boussinesq approximation has been extensively used in theories and numerical models of oceanic circulation. Although the errors involved in this approximation seem rather small, we will show that the gravitational potential energy balance in a Boussinesq model is greatly distorted.

For example, heating a grid box on the upper surface causes water to expand and the density to decrease. In the real ocean, the center of mass moves up, so potential energy increases due to surface heating. However, a numerical model based on the Boussinesq approximation conserves only volume, but not mass. The total mass in this surface grid box decreases because the density decreases. As a result, the amount of potential energy in this grid box decreases after heating. Therefore, the sign of potential energy source due to surface heating is wrong in the Boussinesq model. Furthermore, by simple algebraic manipulation, one can see that the magnitude of the source term due to heating is several hundred times larger than its value in the real ocean.

A similar error associated with cabbeling also appears. In real oceans, mixing of two adjacent water parcels gives rise to water parcels with higher density due to the nonlinearity of the equation of the state; this phenomenon is called cabbeling. Since the newly formed water mass is denser, the center of mass moves down, thus cabbeling in the real oceans releases potential energy, a process that is at least partially self–energized. However, in a Boussinesq model, the total mass in these two grid boxes increases because the density increases. As a result, the potential energy in these two grid boxes increases. Thus, cabbeling in a Boussinesq model requires external mechanical energy for support.

As an example, the energy balance of a numerical experiment based on the Bryan–Cox model is shown in Fig. 5, Huang[18]. The model is forced by a relaxation condition for temperature and a natural boundary condition for salinity, but there is no wind stress. The model formulation was given by Huang[19]. The gravitational potential energy balance is diagnosed after the model reaches an equilibrium state. The total amount of geopotential energy, using the bottom as a reference level for the gravitational potential, is 5.95×10^{24} J (2.837×10^{7} J m^{-3}), and the total kinetic energy is 3.95 PJ (10^{12} J) (0.0188 J m^{-3}). Although the total amount of potential energy is tremendously large, most of this energy is not usable, and only a very small portion, called the available potential energy, can be converted into kinetic energy. On the basis of a computational algorithm outlined by Huang[15], the corresponding available potential energy in the GFDL model based on the Boussinesq approximation is about 53500 PJ (2554 J m^{-3}). The rate of energy conversion from potential to kinetic energy is rather small, about 0.012 TW.

We have calculated the energy balance in two ways; first, the energy balance for the Bryan–Cox model[16,18], and second, the energy balance assuming a " real" ocean with fully compressible water, but with the same temperature and salinity at the same grids.

The main mechanical energy of the circulation comes from vertical mixing, which is divided into two parts: First, the " linear" diffusion is defined as that part of mixing without

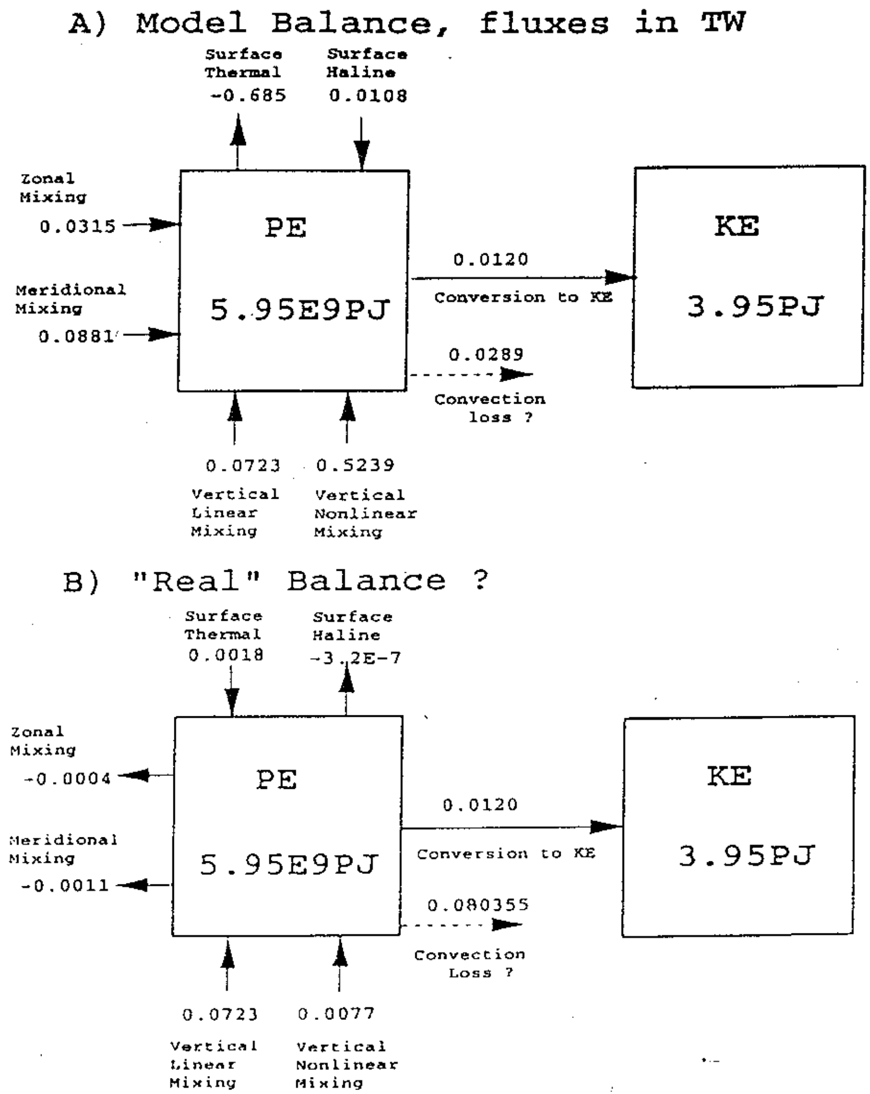

FIGURE 5. Mechanical energy balance diagnosed from the model run, all fluxes in TW (10^{12} W). The source of energy due to vertical mixing is divided into two parts: the linear mixing without the cabbeling and thermobaric effects and the nonlinear mixing, which includes cabbeling and thermobaric effects.

nonlinear effects, such as cabbeling and thermobaric effects. Second, the " nonlinear" mixing is defined as the total mixing minus the " linear" mixing. The potential energy due to nonlinear mixing diagnosed from the Bryan–Cox model is about 60 times larger than the contribution from the linear term. As discussed above, such a huge positive potential energy source due to nonlinear mixing is an artifact of the model.

The potential energy generated from surface thermal forcing has a wrong sign compared with what happens in the real oceans. The magnitude of the source terms is about several hundred times larger than the real terms. In addition, the contribution to potential energy due to lateral mixing of tracers, diagnosed from the numerical model, has a wrong sign and wrong magnitude, as discussed above.

The residual of this balance is considered to be the mechanical energy loss due to the convective overturning at high latitudes. The loss to convective overturning diagnosed from the numerical model is 0.0289 TW, which is much smaller than all source and sink terms.

On the other hand, the energy balance diagnosed from the " real" ocean model seems more reasonable. For example, both the surface thermal forcing and freshwater forcing work as small sources of potential energy. Zonal and meridional mixing induce small potential energy sinks, indicating that mixing due to cabbeling and thermobaric effects can be self–energized. The residual term diagnosed from the " real" balance is 0.0803 TW; thus, it is the dominant term in the energy balance. Accordingly, the supply of mechanical energy comes primarily from the linear vertical mixing. Most of the mechanical energy is lost through convective overturning plus a small amount of energy that is transferred to kinetic energy; while other dynamic processes, such as cabbeling, thermobaric effect, and surface thermohaline forcing, play minor roles in the mechanical energy balance in the " real" ocean model.

In summary, the balance of mechanical energy diagnosed from a well–known traditional oceanic circulation model indicates that most source and sink terms have the wrong sign and wrong magnitude. Since sources and sinks of energy comprise one of the most important aspects of oceanic circulation, such problems associated with the energetics of oceanic circulation must be studied more carefully. It is speculated that although models based on the Boussinesq approximation may be able to reproduce the current climatological circulation after suitably adjusting different parameters in the models, they may not be able to simulate the adjustment processes from one climate state to another. Since conservation of energy is one of the main physical laws governing the oceanic circulation, numerical models that conserve the total mechanical energy may be a better tool for the study of climate changes. However, we have been unable to prove this speculation in terms of a simple theory or numerical model, because such numerical models do not yet exist. Much study remains to be done, and this may be one of the most exciting fronts for theoreticians and numerical modelers.

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APPENDIX

The mean turbulent kinetic energy, mean internal energy, and mean potential energy in the ocean satisfy[19,21,22]

$$\frac{\partial \overline{K}}{\partial t} + \frac{\partial}{\partial x_\alpha}[\overline{K}\overline{u}_\alpha + \overline{\rho}\overline{u}_\beta \overline{u'_\alpha u'_\beta} + \frac{1}{2}\overline{u}_\beta \overline{u}_\beta \overline{\rho' u'_\alpha} + \overline{u}_\beta \overline{\rho' u'_\beta u'_\alpha} + \frac{1}{2}\overline{(\overline{\rho} + \rho')u'_\beta u'_\beta u'_\alpha}$$

$$+ \overline{p u_\alpha} - \overline{u_\beta \sigma_{\beta\alpha}} = \overline{\rho u_\alpha X_\alpha} + \overline{p \frac{\partial u_\alpha}{\partial x_\alpha}} - \overline{\sigma_{\beta\alpha}\frac{\partial u_\beta}{\partial x_\alpha}}, \tag{A1}$$

$$\frac{\overline{\partial \rho e}}{\partial t} + \frac{\overline{\partial \rho u_\alpha e}}{\partial x_\alpha} = -\overline{p\frac{\partial u_\alpha}{\partial x_\alpha}} + \overline{\sigma_{\beta\alpha}\frac{\partial u_\beta}{\partial x_\alpha}} - \frac{\partial \overline{F_\alpha}}{\partial x_\alpha}, \tag{A2}$$

$$\frac{\partial \overline{\rho \Phi}}{\partial t} + \frac{\partial \overline{\rho u_\alpha \Phi}}{\partial x_\alpha} = -\overline{\rho u_\alpha X_\alpha} + \overline{\rho\frac{\partial \Phi}{\partial t}}, \tag{A3}$$

where $\overline{K} = \frac{1}{2}\overline{\rho u_\alpha u_\alpha}$ is the mean kinetic energy, $\overline{\rho u_\alpha X_\alpha}$ is the rate of potential and kinetic energy conversion, $\overline{p\frac{\partial u_\alpha}{\partial x_\alpha}}$ is the rate of kinetic and internal energy conversion, and $\overline{\sigma_{\beta\alpha}\frac{\partial u_\beta}{\partial x_\alpha}}$ is the rate of dissipation due to viscosity, F_α is the internal energy flux, and $\overline{\rho(\partial\Phi / \partial t)}$ represents the energy source due to the time–variable component of the gravitational force, such as the tidal forces. The energy exchange between the potential, internal, and kinetic energy balance equations exactly cancels each other. The energy sources come from the surface stress term in the turbulent kinetic energy balance, the internal energy flux term in the internal energy balance, and the $\overline{\rho(\partial\Phi / \partial t)}$ term in the potential energy balance.

Thus, in a turbulent ocean the conversion from kinetic to potential energy is represented by a term $-\overline{\rho u_\alpha X_\alpha}$, or $\overline{\rho w g}$ in a local Cartesian coordinates. This exchange term can be separated into several terms

$$\overline{\rho w g} = \bar{\rho}\bar{w}\bar{g} + \overline{\rho' w'}\bar{g} + \overline{g'(\rho' w' + \rho' \bar{w} + \bar{\rho} w')}. \tag{A4}$$

The first term on the right–hand side represents the kinetic to potential energy conversion associated with the ensemble mean density and velocity fields. The second term on the right–hand side represents the contribution due to turbulent mixing. In a stratified fluid, upward motion is usually correlated with positive density anomaly, so turbulent mixing in a stratified ocean requires a positive conversion from kinetic to potential energy.

The rest terms on the right–hand side represent the potential to kinetic energy conversion due to the time–dependent component of the gravitational force, such as the tidal forces. Clearly, a mixing parameterization excluding tidal mixing is incomplete.

Since $\overline{\rho w g}$ is a reversible conversion between kinetic and potential energy and $\overline{p\frac{\partial u_\alpha}{\partial x_\alpha}}$ is a reversible conversion between kinetic and internal energy, the apparent turbulent kinetic energy "dissipation" rate would be

$$AD = -\overline{\rho' w'}\bar{g} - \overline{g'(\rho' w' + \rho' \bar{w} + \bar{\rho} w')} - \overline{\sigma_\beta \alpha \frac{\partial u_\beta}{\partial x_\alpha}}. \tag{A5}$$

Note that the last term is the real dissipation due to molecular viscosity, while the other terms represent the conversion from kinetic energy to potential energy. Thus, the decrease in turbulent kinetic energy does not entirely become "wasted" heat. In fact, a small percentage of the turbulent kinetic energy "dissipation", represented by the empirical coefficient 15%, is actually transformed to the large–scale potential energy. This energy is the energy required to sustain mixing, and it is the most important dynamic factor controlling the meridional overturning rate.

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