Gravitational Potential Energy Sinks in the Oceans

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Abstract. Gravitational potential energy (GPE) is lost during convective adjustment. Using monthly mean climatological datasets, the annual mean GPE loss due to convective adjustment in the world oceans is estimated at 0.24 TW. Conversion from the mean-state GPE to eddies GPE and kinetic energy (KE) is also estimated, using the commonly accepted Gent-McWilliams scheme. Our estimate is that about 1.1TW is converted from mean state into eddies.

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Introduction

For the study of climate on Earth, the oceanic flux of heat carried by the wind-driven circulation and thermohaline circulation play a vital role. Although most previous studies of the thermohaline circulation have paid attention to the link between the circulation and its energetics, recent studies have pointed out the important role of the mechanical energy in controlling the strength of the thermohaline circulation. In fact, it turns out that thermohaline circulation is not driven by surface thermohaline forcing; instead, it is driven by mechanical energy sources, such as wind stress and tides. Therefore, we will use the following working definition: thermohaline circulation is a circulation driven by external sources of mechanical energy that transports heat and freshwater fluxes. Note that surface thermohaline forcings are preconditions for the thermohaline circulation through the setting up of the horizontal density difference, but the circulation is not thermohaline-driven.

The pioneering work by *Sandstrom* (1908, 1916) pointed out the subtle role of thermal forcing in the thermohaline circulation. A concise description of the Sandstrom work can be found in textbooks, such as *Defant* (1961). Sandstrom postulated that a system in motion requires sources of mechanical energy to overcome friction and he discussed two cases with heating and cooling sources placed at horizontally and vertically separated locations. He argued that for a pure thermally forced system, there is strong circulation if the heating source is located below the level of the cooling source (Case 1), and there is no circulation if the heating source is located above the level of the cooling source (Case 2). Sandstrom carried out laboratory experiments for which the results are consistent with his argument.

Note that due to the horizontal density difference existing in Case 2, there is circulation, as argued by *Jeffreys* (1925); however, it is extremely weak because the only mechanical energy source supporting the circulation is due to the extremely low conversion rate from internal energy to GPE through molecular diffusion.

More relevant to the ocean is a situation (Case 3) when the heating and cooling sources are both located on the upper surface. Whether thermal forcing in Case 3 can drive a strong circulation is somewhat controversial, and there has been continuous debate over past decades. Recently, *Paparella and Young* (2002) discussed an anti-turbulent theorem for Case 3: in the limit $\kappa \to 0$, with ν / κ fixed, the motion in the fluid disappears.

The Sandstrom theorem can be explained in terms of GPE generated by heating/cooling sources in the ocean. Using Eq. (A.2) and assuming α is constant, a positive source of GPE can be generated through heating/cooling sources placed in the ocean interior, only if the heating source is located below the cooling source. For Case 3, thermal forcing does not generate GPE because the depth is zero; thus leaving the thermal diffusion as the only source of GPE, as discussed by Paparella and Young (2002).

Eq. (A.2) also poses the following puzzle for the case when α is not constant: If $\alpha(T_{cold})$ is close to zero, then $\Delta E_P \approx (gQ/C_P)\alpha(T_{hot})h_{hot}$. Assuming, $\alpha(T_{hot}) > 0$ and $h_{hot} > 0$, a non-zero heat flux Q may drive a detectable circulation, regardless of whether h_{hot} is smaller than h_{cold}. Whether the Sandstrom theory needs a modification remains unclear, and it is currently under study.

Strong thermohaline circulation exists in the world oceans, although thermal forcing is applied at virtually the same level—the sea surface. This seemingly controversial issue, raised by the Sandstrom theorem, can be explained by the fact that Sandstrom did not consider the possible role of mixing driven by external sources of mechanical energy. In fact, the strong thermohaline circulation in the oceans is driven by the external sources of mechanical energy, including work done by wind stress and tidal dissipation (*Toggweiler and Samuels*, 1998; *Munk and Wunsch*, 1998; *Wunsch*, 1998; *Huang*, 1999).

Munk and Wunsch (1998) postulated that the total amount of mechanical energy required for sustaining the stratification in the open oceans, excluding the marginal seas, is about 2.1 *TW* (1 $TW = 10^{12}$ W). Their estimate was based on a quasi one-dimensional model with vertical advection balanced by vertical diffusion. Due to the simplicity of their model formulation other mechanisms involved in the mechanical energy balance are omitted. Many questions remained unanswered: Are there other important sources of GPE excluded in their model? How is GPE dissipated in the oceans?

There is a large uncertainty in the sources and sinks of mechanical energy in the oceans. At this time we do not even have a crude balance of the mechanical energy in the oceans, and many important terms remain uncertain, including the wind contribution through surface and internal waves. In this study we will explore the role of convective adjustment and baroclinic instability in mean-state GPE dissipation. In the following discussion, GPE is always referred to as the mean-state GPE, unless otherwise specifically indicated. Both of these processes are omitted in the simple quasi one-dimensional model used by *Munk and Wunsch* (1998). However, these processes turn out to be extremely important components of the GPE balance in the world oceans. In this study we will extend their work and focus on the sinks of GPE of the mean state in the oceans, and our discussion will be confined to the climatological mean state with a grid size on the order of 100 km.

Convective adjustment as a sink of GPE

When the source of cooling is at the same level as that of heating, there is no thermally-forced circulation. The impossibility of circulation is due to the fact that, when heating and cooling is at the same level, there is no GPE generated by thermal forcing (*Huang*, 1998a). Without energy source, it is impossible to overcome the friction or dissipation associated with the circulation, and this is essentially consistent with the Sandstrom theory.

1) GPE loss due to convective adjustment

Under surface heating (cooling), density of a thin layer on the sea surface declines (increases). Since the total mass of this thin layer remains unchanged, changes in GPE is negligible. Therefore, surface thermal forcing does not create or destroy GPE. A close examination reveals that when cooling and heating are at the sea surface, there is actually an energy sink due to the convective adjustment.

With a finite amount of heat loss due to cooling, water in a thin layer on the top of the mixed layer becomes heavier than water below. This unstable stratification leads to a convective adjustment during which GPE is transformed into energy in small scales and most of this energy is eventually dissipated into thermal energy. This energy loss is probably one of the most important components of the energetic balance in world oceans.

With surface heating, water in a thin top layer becomes lighter than water below. This is a very stable stratification; thus, without external mechanical stirring, GPE remains unchanged. However, wind stress and other sources of mechanical stirring drive mixing in the upper ocean, and lead to a layer of uniform density. Since the center of mass is moved upward, the total GPE is increased. However, the increase of GPE in this case is entirely due to the external source of mechanical energy; while heating itself makes no direct contribution to the increase of GPE.

GPE loss during idealized situations is analyzed for the cases under thermal forcing in the Appendix. These formulas can be easily extended to the general formula for the annual-mean loss of GPE due to buoyancy flux out of the ocean

$$\dot{e}_{p,loss} = c\overline{\int_{B>0} ghBdt} \qquad (1)$$

where integration counts the contribution whenever the buoyancy flux is out of the ocean, the bar indicates an annual mean, h is the depth of the convective penetration and

$$B = -\frac{\alpha Q}{C_p} + \frac{\beta(E-P)S}{1-s/1000} \qquad (2)$$

is the buoyancy flux out of the ocean (*Schmitt et al*. 1989), Q is the net heat flux into the ocean, and E - P is the net freshwater flux out of the ocean.

The constant c in Eq. (1) reflects how convective adjustment is accomplished (discussed in the Appendix). For the case of pure convection, dense water formed at the surface directly sinks to a depth h without mixing with the environment; thus it is equivalent to place the cooling source at depth h, so $c \approx 1$. If dense water formed at the surface is completely mixed with the whole layer of water, $c \approx 0.5$ is a suitable choice. For the extreme case when the mixed layer starts from a zero depth, $c \approx 0.33$ is a suitable choice.

Similarly, the formula for the annual-mean gain of GPE during the phase of buoyancy flux into the ocean is

$$\dot{e}_{p,gain} = c\overline{\int_{B<0} ghBdt}. \qquad (3)$$

According to Eq. (3), when the ocean is heated, GPE is increased. The rate of GPE generation is proportional to the depth of the water column directly affected by heating. If the water column height h goes to zero, the rate of GPE generated by heating becomes infinitesimal.

Note that surface thermal forcing on models based on the Boussinesq approximations can induce artificial source/sink of GPE. In such models, mass conservation is replaced by the volume conservation; thus, cooling (heating) induces density increase (decrease), but with no volume change. As a result, cooling (heating) generates (reduces) GPE. Similarly, freshwater flux through sea surface is replaced by a virtual salt flux, with no volume change. Consequently, evaporation in such models induces a gain of GPE. In the oceans, evaporation is associated with loss of freshwater to the atmosphere, and thus a loss of GPE. Sources (sinks) of GPE induced by thermohaline forcing in the Boussinesq models are the artifacts of the models, and thus caution should be taken in analyzing the GPE balance diagnosed from such models (*Huang*, 1998b).

Although many existing oceanic general circulation models are supposed to conserve energy, they do not. For example, most models do not track the GPE gain through diapycnal mixing, nor the GPE loss due to convective adjustment. In addition, the leap-frog scheme used in time stepping in most models renders the energy conservation impossible. As a matter of fact, energy conservation has not been the main concern in designing oceanic circulation models. To the contrary, oceanic circulation models rely on semi-empirical parameterizations and surface relaxation technique. Whether such models can truthfully simulate the circulation under different climate is an open question. It is only recently that our community began to realize the importance of mechanical energy balance for understanding the oceanic circulation. An upcoming review by *Wunsch and Ferrari* (2003) offers a comprehensive survey related to these issues.

The time evolution of the stratification in the upper ocean is rather complex. This part of the ocean is commonly described in terms of the mixed layer. The mixed layer has an annual and a seasonal cycle. Although density in the mixed layer can be considered as roughly homogenized in low-resolution models, there are many complicated processes taking place in this layer. For example, convection in the open ocean can directly penetrate the water column and reach to a great depth, without completely mixing with the environment on its way, and for such a phenomenon the reader is referred to a recent review by *Marshall and Schott* (1999).

Our calculation of GPE loss is based on the monthly-mean climatology, and our discussion is focused on the annual cycle only. Note that the mixed layer depth in the ocean is determined by complicated three-dimensional processes; thus, continuous cooling for a given month does not necessarily mean the mixed layer depth should increase. Assuming that in the monthly time scale, change in the mixed layer depth is much smaller than the depth itself, $c = 0.5$ is a suitable choice.

2) GPE loss/gain in the world oceans

The calculation of GPE loss/gain in the oceans requires the mixed layer depth and heat flux through the air-sea interface. These data are available in existing climatological datasets. The monthly-mean mixed layer depth is from the Lamont Earth Observatory website ingrid.ldeo.columbia.edu/SOURCES. The mixed layer depth is defined as the depth where the potential density (referred to the sea surface) is $0.125\sigma_\theta$ larger than the surface density, based on the Levitus 1994 data (*Levitus and Boyer*, 1994; *Levitus et al.*, 1994). The surface heat flux is based on the da *Silva et al*. (1994) data.

In the oceans, solar insulation can penetrate to a depth on the order of 10 m, so heating does generate a small amount of GPE, on the order of 0.01 *TW*, which we neglect. Furthermore, turbulent mixing in the upper ocean gives rise to the mixed layer with a depth on the order of 30 to 50 m. As a result, GPE increases during the phase of heating. However, this source of GPE is primarily due to external sources of mechanical energy, such as wind stress, which supports mixing in the stratified ocean. During the cooling phase, the ocean loses GPE. Since wind stirring contributes mechanical energy, it could not directly contribute to the loss of GPE in this phase.

Based on the da Silva data and using (1), the global annual-mean rate of GPE loss is about 0.24 *TW*. Using (3), the global rate of GPE gain during the phase of buoyancy increase is about 0.13 *TW*.

This GPE loss is mostly confined to a few sites where the late winter cooling and evaporation are strong, such as the recirculation regimes of the Gulf Stream and Kuroshio, the Norwegian and Greenland Seas, and the Labrador Sea (see Fig. 1). In addition, there is a relatively strong energy loss within the Southern Oceans, which appears in the form of patches. Note that we define GPE loss as positive and gain as negative.

On the other hand, GPE gain through the heating (buoyancy gain) period is much smaller than the loss because the mixed layer penetrates much deeper during the cooling phase. The sites of GPE gain are primarily confined to both the equatorial ocean and the Southern Ocean. The strong source of GPE in the Southern Ocean is apparently associated with strong precipitation there.

It should be noted that there are a few areas where the GPE gain or loss is set to zero, including the Ross Sea, the Weddell Sea and a small part of the Arctic where the ice covers the ocean. In fact, the climatological dataset used in this study does not have reliable data for these areas, therefore the GPE calculation could not be made. It is speculated that the formation of Antarctic Bottom Water may increase our estimate of GPE loss due to convective adjustment.

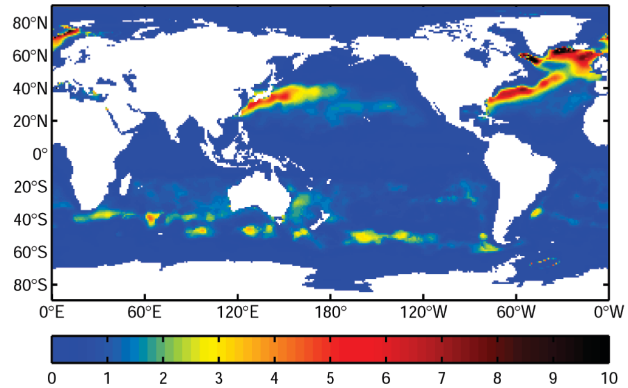

Figure 1. Annual-mean GPE loss due to convective adjustment (in mW m^2)

Although the GPE loss and gain are primarily controlled by thermal forcing, the freshwater flux through the air-sea interface also contributes. At subtropical latitudes, evaporation induces a buoyancy loss, thus contributes to the GPE loss. On the other hand, net precipitation near 10°*N* and at high latitudes contributes to buoyancy gain, and thus a gain in GPE. In particular, the precipitation band in the Southern Ocean seems too strong. As a result, there is a band of GPE gain in the southern part of the Southern Ocean. Such a strong band of precipitation may be an artifact in the da Silva dataset.

Using monthly mean datasets of mixed layer depth and air-sea flux is likely to underestimate the GPE loss due to the convective adjustment. From the definition itself, GPE loss is the annual-mean product of buoyancy flux and mixed layer depth. Since strong buoyancy loss during a few short periods is directly responsible for mixed layer deepening, using the data averaged over a long period will give rise to a lower estimate. As an example, if we use the annual mean buoyancy flux and annual mean mixed layer depth, the GPE loss due to convective adjustment is greatly reduced. Thus, we expect the annual mean GPE loss due to convective adjustment, calculated from either numerical model or better observational datasets, might be larger than the value we calculated based on the monthly mean dataset.

The North Atlantic is the main site of GPE loss under current climate conditions, with a total of 63 GW. The two peaks of energy loss correspond to the deep water formation at the high latitudes and to the subtropical mode water formation at mid-latitudes. In the North Pacific, there is no deep water formation, so GPE loss at high latitudes is small. However, there is a major sink of GPE around 30º*N*, which corresponds to the subtropical mode water formation in the Kuroshio recirculation regime.

In the Southern Hemisphere, the Pacific and Indian Oceans are the primary sites of GPE loss; while the Atlantic Ocean contributes to a rather small amount.

During the phase of heating (buoyancy gain), GPE is generated primarily near the equator and the southern half of the Southern ocean. The Pacific and Indian Oceans are major sites of GPE generation, while the Atlantic Ocean contributes a relatively small amount. Thus, the Atlantic and Pacific Oceans are major contributors to GPE loss, while the Indian Ocean has a very small contribution. Due to the small area covered by the North Atlantic, the intensity of GPE loss is at the global maximal, and this is obviously linked to the deep and intermediate water formation under the present-day climate condition.

Conversion from the mean-state GPE to eddy GPE/KE

Baroclinic instability is one of the most important mechanisms for energy transformation from the mean-state GPE to eddy GPE/KE, and parameterization of eddies has been a major focus over the past decade. A parameterization proposed by *Gent and McWilliams* (1990) has been widely used with some success in non-eddy-resolving numerical simulations. The basic idea is to parameterize the baroclinic instability in terms of isopycnal layer thickness diffusion. Their formulation can be better explained in terms of the energy conversion, and the reader interested in the detail of this parameterization is referred to *Gent et al.* (1995) and the references therein. Accordingly, the conversion rate of mean-state GPE to eddy GPE/KE is governed by the following equation

$$\frac{D^*}{Dt}(g\rho z) = \nabla\cdot(g\rho\kappa_{th}\mathbf{L}) + g\rho\omega + g\kappa_{th}\frac{\nabla\rho\cdot\nabla\rho}{\rho_z} \quad (4)$$

where D^*/Dt is the substantial derivative that advects with the effective transport velocity, including the Eulerian mean velocity and the eddy transport velocity; κ_{th} is the thickness diffusivity; $\mathbf{L} = -\nabla\rho/\rho_z$, see Eq. (19) in *Gent et al.* (1995). Since the pressure effect on density has no dynamic result on mixing, ρ in this formula should be interpreted as the potential density. The first term on the right-hand side integrates to zero, so it does not contribute. The second term is the conversion from the mean-state KE to the mean-state GPE. The third term is a sink because $\rho_z < 0$, and this is the term associated with the eddies. Accordingly, the conversion rate is

$$\dot{p}_{e,bi} = -g\kappa_{th}\frac{\nabla\rho\cdot\nabla\rho}{\rho_z}. \quad (5)$$

The potential density gradient is calculated using the center of each layer as the reference level for the potential density, and a central difference scheme in space.

As suggested by *Gent et al*. (1995), we have chosen κ_{th} = 1000 $m^2 s^{-1}$, and the density field is calculated from the World Ocean 1998 Atlas. GM90 parameterization applies to the case of a relatively small isopycnal slope. In order to deal with large slopes appearing in the ocean, especially near the sea surface, different tapering techniques have been suggested:

a) The Cox scheme

$$\dot{p}_{e,bi} = C_{Cox}g\kappa_{th}\left|\nabla\rho\right|S; \quad (6)$$

where S is the isopycnal slope, $C_{Cox} = 1$ if $S \le S_0$; $C_{Cox} = S_{0/S}$ if $S > S_0$, and S_0 = 0.01 is the commonly used value.

b) The Gerdes et al. (1991) scheme

$$\dot{p}_{e,bi} = C_{GKW}g\kappa_{th}\left|\nabla\rho\right|S; \quad (7)$$

where C_{GKW} = 1 if $S \le S_0$; $C_{GKW} = S_0^2 / S^2$ if $S > S_0$ is set to 0.01.

c) The Danabasoglu and McWilliams (1995) scheme

$$\dot{p}_{e,bi} = C_{DM}\, g \kappa_{th} |\nabla \rho| S; \qquad (8)$$

where $C_{DM} = 0.5 \times \left[1 + \tanh\left(\frac{S_c - |S|}{S_d} \right) \right]$, S_c = 0.004, and S_d = 0.001 are the recommended parameters.

stimates based on Eqs. 6, 7, and 8 are listed in Table 1. However, whatever we can compute from such simple parameterizations should be interpreted with caution. The theory of the baroclinic instability has been developed over the past half century. In particular, the theory of the baroclinic instability of time-independent zonal flows is now on firm ground. For standard problems, there are many existing books, e.g., *Pedlosky* (1987). Although the GM90 scheme provides a short cut to the conversion rate calculation, there is no reason, *a priori*, to believe that such a simple parameterization can capture the complicated baroclinic instability in the oceans.

Table 1. Conversion rate of mean GPE to eddy GPE/KE, according to different formula.

Formular	6	7	8	9	10
Total Rate (TW)	1.74	1.61	1.10	1.11	0.50

For example, the GM90 scheme parameterizes the baroclinic instability in terms of purely local properties, with an eddy transfer coefficient constant in time and space. *Visbeck et al*. (1997) combined the approaches of *Green* (1970) and *Stone* (1972) with the GM90 scheme and proposed a scheme, in which the eddy mixing coefficient κ_{th} is a function of space and time. Combining this scheme with the tapering techniques discussed above gives rise to the following formulas:

$$\dot{p}_{e,bc} = C_{Cox}\, g \kappa_{th,Vis} |\nabla| S; \qquad (9)$$

$$\dot{p}_{e,bc} = C_{DM}\, g \kappa_{th,Vis} |\nabla| S; \qquad (10)$$

where $\kappa_{th,Vis} = aSNl^2$; , a=0.015, $N = \sqrt{\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}}$ is the buoyancy frequency, and l is a measure of the eddy transfer scale.

For idealized cases, this formulation performs better than the GM90 scheme (*Visbeck et al*., 1997). One major uncertainty in applying this scheme is the choice of l, the eddy scale. For the global climatological dataset with 1°×1° horizontal resolution, the choice of $y \approx 111$ km seems a suitable choice. For different choices of tapering, the global conversion rate is 1.11 *TW* or 0.5 *TW* (Table 1).

Judging from this sensitivity study, we believe that the total energy transfer rate through baroclinic instability is in the middle range listed in Table 1. For the following discussion, we chose the formula by *Danabasoglu and McWilliams* (1995), which gives the total energy transfer rate as 1.1 *TW*, based on the monthly mean climatology of the World Ocean 1998 Atlas. (If the annual mean data are used, the corresponding rate is 0.9 *TW*.)

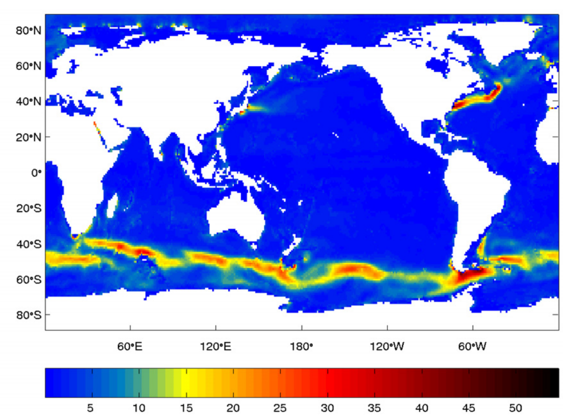

Figure 2. Conversion rate of mean GPE to eddy GPE/KE through baroclinic instability based on the empirical eddy parameterization of *Gent and Williams* (1990) in units of mW/m^2.

The horizontal distribution of this GPE conversion rate is shown in Fig. 2. It is clear that most energy transformation takes place in the Antarctic Circumpolar Current, the Gulf Stream, and to a much smaller degree in the Kuroshio. The Southern Ocean is clearly a major site of baroclinic instability and conversion from the mean-state GPE to eddy

GPE/KE; this has been discussed by many investigators, such as *Bryden* (1979).

Similarity is noted in the spatial distribution of wind stress energy input through the surface geostrophic current and the energy conversion from the mean-state GPE to eddy GPE/KE. As discussed by *Wunsch* (1998), the total amount of wind energy input to the surface geostrophic current is about 1 *TW*, with a value of 0.88 *TW* based on satellite altimeter data and excluding the equatorial band and a value of 1.3 *TW* based on a numerical model. The major portion of this wind energy input is primarily concentrated in the Southern Ocean, and this is quite similar to the distribution of the eddy conversion rate shown in Fig. 2. Apparently, there is a direct connection between the wind energy input to the geostrophic current and the energy conversion from the mean-state GPE to eddy GPE/KE through baroclinic eddy transfer.

The conversion from mean-state GPE to eddy GPE/KE takes place within the entire depth of the ocean. Although the energy conversion rate (per unit thickness) is the highest within the main thermocline for the depth range of 200-800 meters, the contribution from the middle depth of the ocean also plays an important role. Thus, eddy activity at the middle depth of the ocean is probably as important as that in the upper ocean.

The pattern of the meridional GPE flux diagnosed from our study is rather similar to the direction of barotropic tidal energy flux, as described by *Munk and Wunsch* (1998, their figure 5). To illustrate this point, we define the northward transport of mechanical energy

$$E_{flux} = \int_{\theta}^{90^\circ} r^2 \cos\theta d\theta \int_{\phi w}^{\phi_e} d\phi \left(\dot{e}_{p,loss} + \dot{e}_{p,gain} + \dot{p}_{e,bc} - \dot{w}_w\right) \qquad (11)$$

where ϕw and ϕe are the western and eastern boundaries of the basin. This energy flux includes the contribution from the GPE loss $\dot{e}_{p,loss}$ and gain $\dot{e}_{p,gain}$ in the mixed layer and the eddy conversion through baroclinic instability $\dot{p}_{e,bc}$ discussed in this study, subtracting the wind energy input through surface geostrophic current $\dot{w}_{wind}$ based on a numerical model (*Wunsch*, 1998). Although many other terms of mechanical energy sources/sinks are excluded in this flux, it is rather interesting to examine what this flux can explain.

The most striking feature in this figure is that in order to balance the mechanical energy, there should be a large northward GPE/KE flux for the North Atlantic Ocean. In addition, there is also a small northward GPE/KE flux in the North Pacific, Fig. 3.

It is speculated that the pattern of the tidal energy flow in each basin, as described by *Munk and Wunsch's* (1998) figure 5, is not entirely independent of the general circulation. To the contrary, barotropic tidal energy flux is being linked to the thermohaline circulation in each basin. For example, in the present-day climate configuration there is a strong thermohaline cell in the North Atlantic. As a result, water is warm and salty in the North Atlantic, and there is strong cooling and evaporation associated with the deep mixed layer and mode water and deepwater formations. Therefore, there are strong northward GPE/KE and heat fluxes in the modern North Atlantic. On the other hand, there are relatively small northward GPE/KE and heat fluxes in the North Pacific. It is speculated that if the cooling-induced convection in the North Atlantic is interrupted, both the Northward GPE/KE and heat fluxes would be substantially reduced, and the tidal energy flux map may look different.

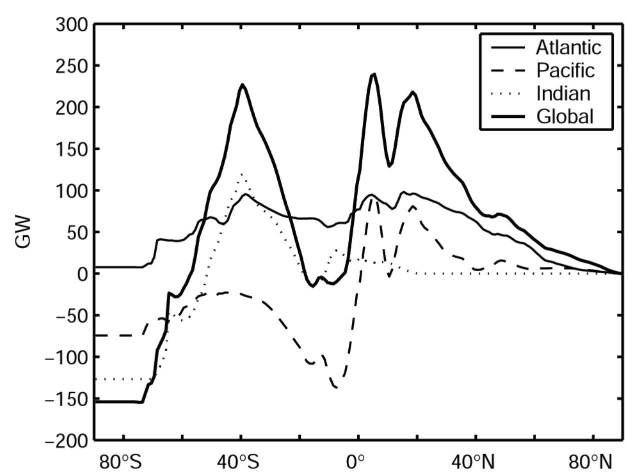

Figure 3. Northward mechanical energy transport in the world oceans.

Discussion

Munk and Wunsch (1998) estimated that the wind stress may contribute to 1-1.3 *TW* mechanical energy to the ocean interior, in addition to the 0.9 *TW* due to tidal dissipation in the deep ocean. Recent studies challenged their numbers and claimed that Munk and Wunsch overestimated the mechanical energy required to sustain ocean stratification. However, our study suggests that we are far away from being able to claim any kind of balance of mechanical energy in the world oceans.

In fact, many important sources and sinks of mechanical energy have not been studied carefully. Thus, Munk and Wunsch might have substantially underestimated the total amount of mechanical energy required to sustain oceanic stratification.

Since convective adjustment gives rise to a sink of GPE, an external source of mechanical energy is required to maintain the circulation. Therefore, we have extended the classical Sandstrom theorem: cooling and heating the ocean at the sea surface does not create GPE. To the contrary, cooling and heating creates a sink of GPE due to the convective adjustment immediately after cooling. Thus, without the external mechanical energy input, there would be no thermal circulation in the ocean.

Our discussion of GPE loss is based on a simple formula and the monthly mean climatology. The formula we used can provide only rough estimates because of many idealizations used in deriving the formula. The assumption of a symmetric mixed layer structure during the cooling and heating phases in a diurnal cycle implies strong sources of external mechanical energy to support mixing, which is not realistic. In fact, due to the limited amount of energy available for mixing, a GPE increase in the heating phase may be substantially smaller than that lost during the cooling phase. It is speculated that our monthly mean calculation may underestimate the GPE loss due to cooling.

Appendix A. Gravitational potential energy generated by heating/cooling

Heating in the ocean induces an expansion, so GPE can be increased. Energy balance for a water column of unit area is $Q = \Delta E_p + \Delta U$, where Q is the heat energy applied to an element at depth h and with a thickness of h; heat expansion pushes the whole water column upward, so $\Delta E_p = p\delta h = pa\Delta\delta T$ is the increase of GPE, and $p \approx p_0 gh$ is the hydrostatic pressure at depth h, δT is the temperature increase due to heating; ΔU is the change in internal energy of the element heated.

In general, the increase in GPE is a tiny fraction of the total energy, so it can be neglected, i.e., ΔU=Q; thus, temperature increase is $\delta T = Q / c_p \rho_0 \Delta h$. The corresponding increase of GPE is

$$\Delta E_p = \frac{g\alpha hQ}{c_p} . \qquad \text{(A.1)}$$

For a pair of heating/cooling sources at temperatures T_{hot} and T_{cold}, and depths h_{hot} and h_{cold} the rate of GPE generation is

$$\Delta E_p = \frac{gQ}{c_p}\left[\alpha(T_{hot})h_{hot} - \alpha(T_{cold})h_{cold}\right] \qquad \text{(A.2)}$$

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