

Evolution of oceanic circulation theory: From gyres to eddies*

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Abstract: Physical oceanography is now entering the eddy-resolving era. Eddies are commonly referred to the so-called mesoscale or submesoscale eddies; by definition, they have horizontal scales from 1 to 500 km and vertical scales from meters to hundreds of meters. In one word, the ocean is a turbulent environment; thus, eddy motions are one of the fundamental aspects of oceanic circulation. Studies of these eddies, including observations, theory, laboratory experiments, and parameterization in numerical models, will be the most productive research frontiers for the next 10 to 20 years. Although we have made great efforts to collect data about eddies in the ocean; thus far, we know very little about the three-dimensional structure of these eddies and their contributions to the oceanic general circulation and climate. Therefore, the most important breakthrough may come from observations and physical reasoning about the fundamental aspects of eddy structure and their contributions to ocean circulation and climate.

Key words: eddy; baroclinic instability; eddy parameterization

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1 Landscape of oceanic circulation

1.1 Wind-driven circulation vs thermohaline circulation

Wind-driven circulation is defined as the circulation driven by surface wind stress alone. Note that wind-driven circulation can exist in a homogeneous ocean. In such a conceptual model ocean, the wind-driven circulation is rather sluggish. In contrast, the thermohaline circulation exists in a stratified ocean only. There is a main thermocline in the world oceans, which is often treated as an interface between the warm/light water in the upper ocean and the cold/dense water in the deep ocean. The main thermocline is actually a zone of strong vertical density gradient, i.e., it is also a pycnocline. The main pycnocline in the oceans compresses the wind-driven circulation to the upper 500–800 m. As a result, the wind-driven circulation in the world ocean with a realistic stratification is a circulation

mostly confined in the upper ocean; thus, it is intensified and ten times stronger than that in an idealized homogeneous ocean.

There are numerous definitions for the thermohaline circulation. Literally, the name of thermohaline circulation means it is directly linked to the density difference due to surface thermal (heat flux) and haline (freshwater flux) forcing. Therefore, thermohaline circulation was interpreted as the circulation driven by surface buoyancy forcing. In textbooks and papers published before the end of last century, most theories of thermohaline circulation were based on this classical framework. In fact, most people believe that thermal forcing can drive thermohaline circulation. This framework may stem from the common wisdom that the ocean is subject to strong surface heating/cooling, and such surface heating/cooling provides a huge amount of thermal energy, which should be able to put the ocean in

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motions, very much like how a burner in the kitchen works. This line of reasoning was reflected by the parameterizations used in numerical models. In fact, a common practice in numerical modeling was specifying the vertical mixing coefficient and running the models for different climate conditions, completely ignoring the external sources of mechanical energy required for sustaining vertical mixing in a stratified ocean. However, such a framework is now challenged by a new paradigm based on mechanical energy balance, which we will discuss shortly. For an in-depth review of oceanic general circulation, the reader is referred to a recently published textbook^[1] and the references cited there.

1.2 Entering the regime of eddies

Motions in the ocean are composed of at least two major forms: waves and currents. One can also say that motions in the ocean are the manifestation of geostrophic turbulence, which consists of difference sizes of eddy. As far as 600 years ago, Da Vinci described motions in our world in forms of many large and small eddies^[2].

Since the world oceans have an extremely small aspect ratio (vertical depth vs the horizontal dimension), our description here is mostly focused on the horizontal dimension. The largest eddy in the world oceans is the Antarctic Circumpolar Current (ACC), existing in the form of the global artery with a horizontal dimension of 20,000 km. Except the ACC, currents in the world oceans exist in forms of different size of gyres and eddies (Fig. 1).

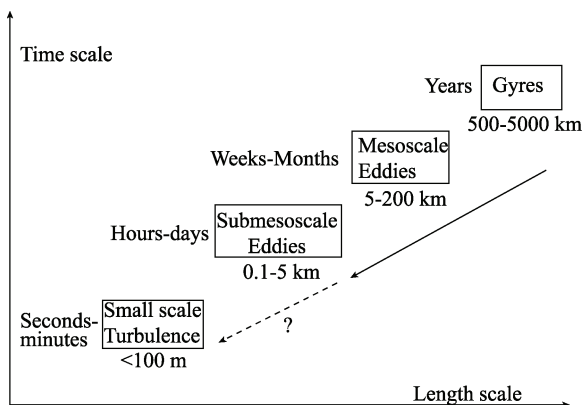


Fig. 1 Landscape of the oceanic circulation

The theory of the oceanic circulation was first

developed for the gyre-scale circulation, which has a typical horizontal scale of 500–5000 km and temporal scale on the order of one year or longer. Major wind-driven gyres exist in the world oceans, including the subtropical gyres in the North Atlantic, North Pacific, South Atlantic, South Pacific, and Indian Oceans, the subpolar gyres in the North Atlantic and North Pacific Oceans. The corresponding subpolar gyres in the Southern Hemisphere are confined to the edge of the Antarctica due to the existence of the strong ACC.

The historic development of theories and numerical models for basin-scale wind-driven circulation and thermohaline circulation is outlined on the left part of Fig. 2. At the end of the 1970s, our understanding of the wind-driven circulation in the oceans was based on simple two-dimensional models. A major breakthrough took place since the 1980s. With the potential vorticity homogenization by Rhines et al.^[3] and ventilated thermocline theory by Luyten et al.^[4], the framework of wind-driven gyres in the world oceans was completed. At the end of the 1980s, a major breakthrough took place along the line of thermohaline circulation. The possibility of multiple solution of thermohaline circulation was first discussed in the seminar work by Stommel^[5]; however, it took more than 20 years for the related research to become one of the main streams, and the major breakthrough is represented by the landmark work by Bryan^[6].

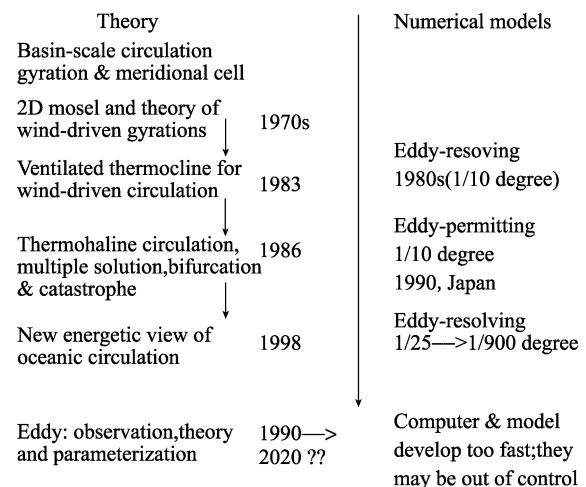


Fig. 2 Development of theories and models for the oceanic general circulation

One of the most important progresses along this line is the creation of new energetic view of oceanic general circulation, which took place at the end of last century. Over the long period of development, the thermohaline circulation was theorized as a circulation driven by surface buoyancy fluxes, including heat flux and freshwater flux. As it is well known, the atmospheric circulation is primarily driven by thermal forcing from the Sun. The great success in atmospheric dynamics led people to think the oceanic counterpart of circulation could be explained in a rather similar way, i.e., the oceanic circulation can be driven by surface thermohaline forcing. With a huge amount of thermal energy flux going through the air-sea interface, it seems rather a straightforward reasoning that such a huge amount of energy should be able to drive the oceanic general circulation.

Over the past 15 years, a new paradigm of thermohaline circulation emerged, e.g., Munk et al.^[7], and Huang^[8-9]. This paradigm is based on the second law of thermodynamics. The basic idea is that although there is a huge amount thermal energy flux going through the ocean surface, such energy cannot be efficiently converted into mechanical energy because the ocean is heated and cooled from about the same geopotential level. In order to overcome the friction associated with thermohaline circulation there is a need of external source of mechanical energy, which is due to wind stress and tidal dissipation. The major point of this new theory is that parameterization of sub-grid scale processes must be based on the constraint of mechanical energy balance. This new methodology has been applied in many recent studies of parameterization of the diapycnal diffusivity used in numerical models.

With the framework of the basin-scale circulation established, we are now entering the regime of eddies. Due to severe technical limitations of observations and computer power, our understating of the dynamics associated with eddies remains rather rudimentary. In 1980s, models with rather low resolution of 1/3 degree were called

eddy-resolving. However, it was soon realized that such models do not resolve eddies; instead, they should be called eddy-permitting model. For a long time, studies along this line were limited to such eddy-permitting models. With the rapid development of computer technology, eddy-resolving models with resolution of 0.1 degree have become a realistic tool for oceanography, as shown in the right part of Fig. 2. The major impetus of studying eddies in the ocean is due to the tremendous potential impact of eddies on the oceanic general circulation. As will be shown in the next section, eddy kinetic energy may consist more than 90% of the total kinetic energy in the ocean; thus, it is inconceivable that any theory about oceanic circulation without eddies can really work.

2 The new era of eddies

2.1 Energy scales associated with the wind-driven gyre

The importance of eddies in the oceanic general circulation can be explained in many different ways. The most straightforward way is to show how much of the total energy is contained by eddies. For the wind-driven gyre in the upper ocean of a semi-closed basin, this can be shown as follows. Note that, however, this argument may not apply to the Southern Ocean where no zonal boundary exists.

Omitting the small contribution associated with the vertical velocity, the density of total kinetic energy is

$$\begin{aligned} e_k &= \frac{1}{2} \overline{\rho h(u^2 + v^2)} = \frac{1}{2} \rho g'^2 \overline{h(h_x^2 + h_y^2)} / f^2 \\ &\approx \frac{1}{2} \rho g'^2 \overline{h} h'^2 / L_y^2 f^2, \end{aligned} \quad (1)$$

where the overbar indicates the horizontal average over the whole basin, u and v are the horizontal velocity, g' is the reduced gravity, f is the Coriolis parameter, ρ and h are the mean density and thickness of the upper layer where the wind-drive gyre is mostly confined, h' is the thickness perturbation, and $L_y=1000$ km is the north-south length scale of the wind-driven gyre. In this estimate, the contribution due to the east-west

gradient is small, so it is neglected; accordingly, the kinetic energy associated with the meridional velocity is omitted.

The density of available gravitational potential energy (GPE) is defined as

$$e_p = \frac{1}{2} \rho g' \left[\overline{h^2} - (\overline{h})^2 \right] \approx \frac{1}{2} \rho g' \overline{h'^2}. \quad (2)$$

Therefore, the ratio of these two types of energy is

$$\frac{E_p}{E_k} = \frac{e_p}{e_k} = \frac{L_y^2}{\lambda^2} \approx 1000, \quad (3)$$

where $\lambda = \sqrt{g'h}/f \approx 30\text{km}$ is the radius of deformation. Thus, the kinetic energy of the mean flow in a wind-driven gyre is associated with a huge amount of available potential energy^[10].

On the other hand, we can estimate eddy potential energy and kinetic energy. Assume that all available potential energy can be converted into eddy kinetic energy, then the total energy of eddies should approximately equal to the available potential energy, that is,

$$E_{\text{edd}} \approx E_{p,\text{mean}}. \quad (4)$$

The horizontal scale of meso-scale eddies is k^{-1} . In general, eddy scale is larger than the deformation radius, $k^{-1} > \lambda$; thus, most eddy energy is in the

form of eddy potential energy. Similar to the estimate of mean-flow kinetic energy, the eddy kinetic energy can be estimated as

$$e_{k,\text{eddy}} \approx \frac{1}{2} \rho g' \overline{hh'^2} k^2 / f^2. \quad (5)$$

Hence, the ratio of eddy potential energy and mean-flow potential energy is

$$\frac{E_{p,\text{eddy}}}{E_{k,\text{eddy}}} = \frac{e_{p,\text{eddy}}}{e_{k,\text{eddy}}} = \frac{k^{-2}}{\lambda^2} \approx 10. \quad (6)$$

The ratio of eddy kinetic energy to the mean-flow kinetic energy is

$$E_{k,\text{eddy}} / E_{k,\text{mean}} \approx (kL_y)^2 \approx 100. \quad (7)$$

Therefore, eddy kinetic energy is 100 times of the mean-flow kinetic energy.

With the advance of satellite altimetry, we have now a wealth of data collected over the global oceans. Most importantly, satellite observations provide a continuously monitored global view of the eddy activity on the sea surface. By comparing kinetic energy for the mean geostrophic flow and that for eddies inferred from satellite altimetry, a clear view of the relation between these two terms emerged. As shown in Fig. 3, eddy kinetic energy is about 100 times larger than the mean-flow kinetic energy.

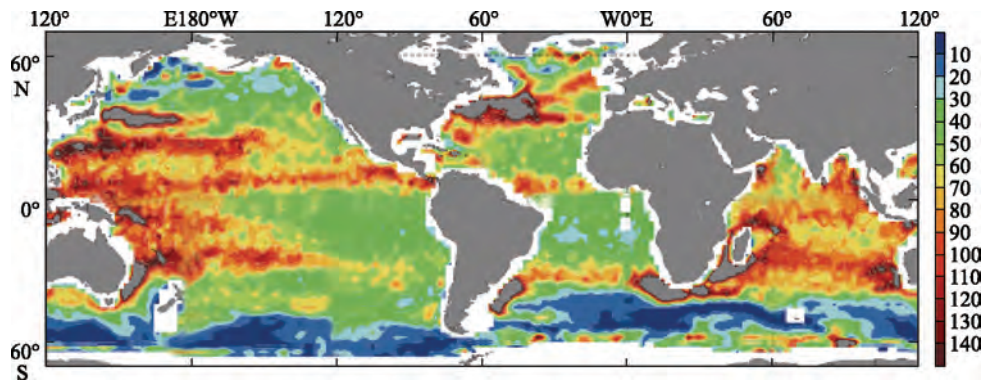


Fig. 3 Ratio of eddy kinetic energy and mean (geostrophic) flow kinetic energy. On average, this ratio is the lower bound because the kinetic energy of the mean flow may be somewhat overestimated due to inaccuracy of the geoid used in estimating the mean flow^[11]

However, the ratio of 100 : 1 is not valid everywhere in the world oceans. As seen from the lower part of Fig. 3, the corresponding ratio is on the order of 10–20 in the core of the ACC. It is clear that, due to the specific nature of a fast circumpolar current, the mean kinetic energy is rather large and

the argument listed above does not apply. In addition, although the big ratio is verified for most part of the surface ocean in the world, it is not clear whether or not such a ratio is valid for the subsurface ocean. Thus, much work remains to be done in the future to verify this energy ratio, maybe

using subsurface measurements obtained from ARGO and other instruments.

Eddies in the ocean can be described in terms of geostrophic turbulence; thus, common wisdom may suggest that eddies have no large-scale organized structure. However, recent studies indicate that eddies in the ocean can have some kind of large-scale structure. In fact, two-dimensional turbulence on a rotating sphere tends to organize itself into a zonal jet, as shown in both theoretical studies and idealized numerical simulations. Rhines^[12] postulated a theory of the beta-plane turbulence. He argued that turbulence on a beta-plane should organize itself into a zonal jet with the meridional scale of $L_{Rh} = \sqrt{U/\beta}$, where U is the root-mean-square velocity of the jet-like feature, and β is the meridional gradient of the Coriolis parameter. At middle latitude, $\beta \approx 2 \times 10^{-11} \text{ m}\cdot\text{s}^{-1}$. Assuming $U \approx 1 \text{ m}\cdot\text{s}^{-1}$, $L_{Rh} \approx 200 \text{ km}$,

this is sometimes called the Rhines scale.

There are now new developments along this line of thought. In particular, the so-called stationary striations have now generated great interest in the oceanographic community. As shown in Fig. 4, eddies with similar sign of zonal geostrophic velocity are organized into some kind of large-scale structure, called stationary striations. They can be seen in forms of zonal bands with wave lengths on the order of 300–500 km, and with sea-level amplitude on the order of 1 cm. These features can penetrate at least the upper 700 m. There are also time-variant striations, which can be detected from both altimetric sea-level anomaly and numerical simulations with high resolution. More updated descriptions can be found through web sites via searching with the key word “striation”. The nature of striation and its contribution to the oceanic general circulation remains unclear, and it is now a research frontier.

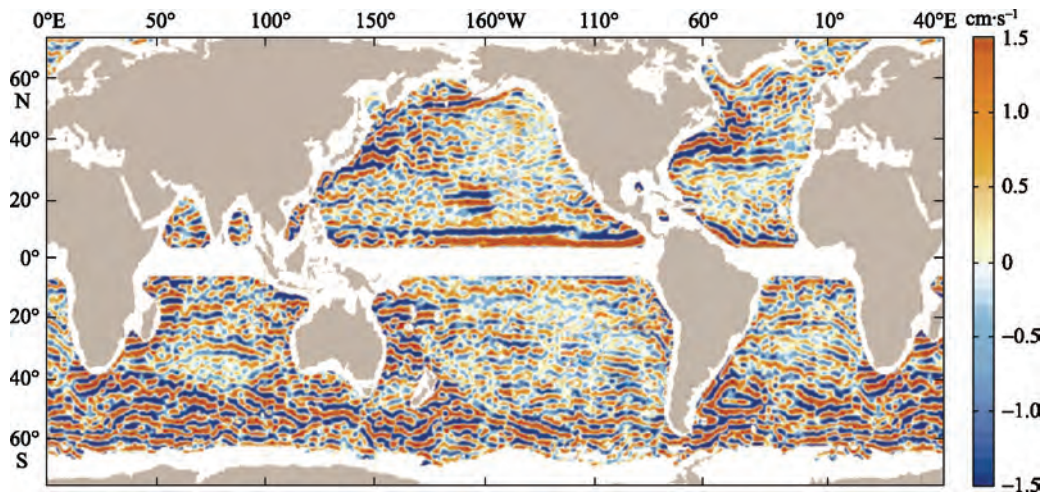


Fig. 4 Stationary striations calculated by high-pass-filtered zonal geostrophic velocity^[13]

2.2 Size of eddies in the ocean

A remark regarding the terminology used in atmospheric dynamics and oceanic dynamics: In atmospheric dynamics, perturbations (cyclones and anticyclones) have typical horizontal scale on the order of 1000 km, and are called synoptic-scale phenomena, for which the traditional quasi-geostrophic dynamics applies. The next order of horizontal scale is called meso-scale, which has a typical scale on the order of 100 km.

In dynamical oceanography, eddies at mid

latitudes have a typical horizontal scale of 5–200 km, and are called meso-scale eddies, for which the traditional quasi-geostrophic dynamics applies. The next order dynamics is called the submesoscale, which has a typical horizontal scale on the order of 0.1 to 5 km. At high latitudes, the corresponding deformation radius is much smaller, due to weak stratification; thus, the corresponding scales for meso-scale eddies and submesoscale eddies are smaller. Note that meso-scale eddies in the ocean are the dynamical equivalence of synoptic-scale

eddies in the atmosphere. Unfortunately, there is currently a confusion of terminology in atmospheric and oceanic dynamics.

In addition, there is a major difference in the nature of eddies in the atmosphere and in the ocean. Synoptic-scale eddies in the atmosphere are closely related to thermodynamic forcing; on the other hand, meso-scale eddies in the ocean are primarily dynamically controlled.

The most important horizontal scale often used in dynamic analysis is the Rossby radius of deformation, in particular the radius of deformation of the first baroclinic mode. A simple way to estimate this scale is based on the so-called reduced gravity model. As introduced in Eq. (3), the radius of deformation is defined as

$$\lambda = \sqrt{g' \bar{h}} / f, \quad (8)$$

where $g' = g \Delta \rho / \rho_0$ is called the reduced gravity, g is the gravity acceleration, $\Delta \rho$ is the density difference between the upper layer and the lower layer, ρ_0 is constant reference density, and \bar{h} is the mean thickness of the upper layer.

For a continuously stratified ocean, the corresponding value can be calculated by solving the following eigenvalue problem

$$\frac{d}{dz} \left(\frac{f^2}{N^2} \frac{dF_n}{dz} \right) + \lambda_n F_n = 0, \quad (9)$$

subject to boundary conditions: $dF_n/dz = 0$, $z = 0, -H$, where $N = \sqrt{-g \partial \rho_\theta / \partial z}$ is the buoyancy frequency, ρ_θ is potential density, F_n is the eigenfunction, and $z=0$ and $z=-H$ are the upper and lower boundaries of the ocean. The corresponding radius of deformation is defined as $R_{d,i} = \lambda_i^{-1/2}$.

Due to changes in the background stratification and the Coriolis parameter, the size of radius of deformation varies greatly over the world oceans. The global distribution of the radius of deformation of the first mode was discussed by Chelton et al.^[14]. At mid latitudes, the typical values for the first, second, and third modes are: 38 km, 18 km, and 11 km. However, near the equator, the radius of deformation for the first mode is about 240 km; thus, it is quite easy for a numerical model to resolve

eddies near the equator. On the other hand, the radius of deformation of the first mode is about 10 km at high latitudes, especially near the Antarctica. At such high latitudes, one degree in latitude (longitude) is about 110 km (60 km); thus, even a model with 1/10 degree resolution cannot really resolve eddies at all.

Although many eddies may be formed with their initial radius near the radius of the first baroclinic mode, through eddy-eddy interaction, their size grows. Since the oceanic general circulation takes place in a rather thin layer of fluid, with a typical aspect ratio of 1000 : 1, turbulence in such fluid environment tends to share some features common to two-dimensional turbulence. One of the major features of two-dimensional turbulence is the upscale transfer of energy. In fact, many eddies observed in the ocean are with size on the order of 100 km. This is sometimes called the upscale cascade of eddies, i.e., part of the eddy energy is transferred from small scales to large scales.

The smallest eddies in the ocean are on the Kolmogorov scale, which is defined as the scale at which energy dissipation is balanced by molecular dissipation. From dimensional analysis, the dimension of dissipation and viscosity is $[\varepsilon] = \text{m}^2 \cdot \text{s}^{-3}$, $[\nu] = \text{m}^2 \cdot \text{s}^{-1}$; thus, the dissipation scale l_k is $l_k = (\nu^3 / \varepsilon)^{1/4}$. For the world ocean, the total amount of external mechanical energy is $E = 0.1 - 0.8$ TW, and the total volume is $V = 1.3 \times 10^{18} \text{ m}^3$; so that the mean energy dissipation rate is about $\varepsilon = 1 \times 10^{-6} \text{ W} \cdot \text{m}^{-3}$. Since the molecular diffusivity is $\nu = 1 \times 10^{-6} \text{ m}^2 \cdot \text{s}^{-1}$, the corresponding Kolmogorov scale is about 1 cm. In zones of strong dissipation, such as the mixed layer, it is on the order of 1 mm.

How we should go along the line of sub-grid scale parameterization? Right now, most basin-scale models are running with horizontal resolution on the order of 10 km and vertical resolution of 10 m. In order to avoid parameterization of sub-grid scale turbulence, we may have to use grid as fine as defined by the Kolmogorov scale, i.e., one centimeter or less. Thus, we must increase the horizontal resolution 10^6 times and vertical

resolution 10^3 times. Higher horizontal resolution corresponding to smaller time step; thus, the total increase in computer power is about 10^{21} . According to the famous Moore's Law in computer industrial, computer power doubles every 18 months. As reported in the *Time* magazine published in February 2011, there is an indication that computer power increases with a double power law, something like $\exp[e^{0.0287(\text{yr}-1946)}]$; thus, a super computer in 2050 may be able to do such a super job. However, this may be a dream; and before this grand day arrives, we may still have to rely on sub-grid scale parameterization heavily.

3 Eddies or waves

There has been a commonly debated issue: What do we see in the ocean, eddies or waves? To answer this question, we can start from the basic equation that describes the movement of perturbations in the ocean. The most commonly used equation for describing perturbations is the potential vorticity equation, which can be written in the following form^[15]

$$\frac{\partial}{\partial t} \left[\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial}{\partial z} \left(\frac{f_0^2}{N^2} \frac{\partial \Psi}{\partial z} \right) \right] + \beta \frac{\partial \Psi}{\partial x} = 0 \quad (10)$$

where Ψ is the streamfunction, β is the meridional gradient of the Coriolis parameter. The suitable boundary conditions at the upper and lower boundaries are

$$\frac{\partial^2 \Psi}{\partial t \partial z} = 0, \text{ at } z = -H, \text{ and } z = 0. \quad (11)$$

This equation can be solved by separation of variables, and we obtain the classical wave solution that has a dispersion relation of

$$\sigma_n = -\frac{\beta k}{k^2 + l^2 + \lambda_n^{-2}} \quad (12)$$

where k and l are the horizontal wave numbers and λ_n is the n -th deformation radius. Therefore, what we call the Rossby waves are essentially the vorticity waves, i.e., waves in a rotating ocean is closely related to the vortex motion.

Of course, when such perturbations grow, they may eventually develop closed streamlines; and at this stage, they may well be called eddies. Strongly

nonlinear eddies are characterized by having enclosed streamlines; thus, parcels can be trapped inside eddies and move with eddies.

Another important feature of strongly nonlinear eddies is that they move faster than linear waves. From this point of view, there may not be a clear-cut boundary between eddies and waves in the ocean. In fact, even 300 years ago Newton came forth and made a brave statement that light can be treated either as waves or as particles. Thus, there should be no surprise that we can treat most perturbations observed in the ocean in terms of either waves or eddies. Further debate along this line can be found in a recent paper by Early et al.^[16]

The advantage of using wave dynamics is that there are many powerful tools for analyzing waves. In comparison, however, there is no systematic tool for analyzing eddies. Several books provide excellent review for eddy dynamics, including the most classical book by Tuesdell^[17] and the recent book by Wu et al.^[18]. In fact, in most cases we have to deal with eddies individually; unfortunately, this is so far the only way we can deal with eddies.

4 Isolated eddies

Lacking powerful tools for the description of eddies, the existing theory of eddies heavily relies on the study of individual eddies, in particular the so-called isolated eddies. The study of isolated eddies may provide insightful knowledge about eddies, their structure and their contribution to the oceanic general circulation.

The best known example of the so-called isolated eddies is Jupiter's Great Red Spot, which may have existed for at least 300 years. Blocking in the atmosphere is another well-known phenomenon. In oceanography, Gulf Stream warm/cold rings exist over a vast area; and they survive in the turbulent environment for about one year and carry their distinctive chemical and biological characters with them.

Some of the well-known examples of isolated eddies studied include: Modon, dipole, and heton. Modon is an isolated eddy in a simple analytical form. Dipole is a pair of eddies distributed

horizontally next to each other. Heton consists of a pair of vortexes with the same strength of potential vorticity, but with different signs and located at different depths; and they carry a portion of buoyancy anomaly. Heton was discussed in details by Hogg and Stommel^[19] and Gryanik et al.^[20]. Flierl^[21] presented a comprehensive review for isolated eddies in geophysical fluid dynamics. McWilliams^[22] also reviewed the dynamics of submesoscale eddies, in particular the so-called Coherent Vortices (SCV).

5 Tracking meso-scale eddies

Progress in understanding the roles of eddies in the ocean primarily comes from a wealth of data collected from the ocean. Among many different approaches, tracking meso-scale eddies through altimetry data is probably the most efficient way up till now. Much progress has been made along this line, as reported in the literature, e.g., Chelton et al.^[23-24]. Over the past several decades, satellite observations have provided continuously the state on the ocean surface in forms of sea-surface temperature (SST), ocean color, and altimetry data. The most updated progress is that now sea-surface salinity can be measured through satellite.

Other means of observing eddies in the ocean

include: current meter, floats, and in particular Argo. Tracking subsurface currents with drift buoys communicating with satellites and acoustical networks have provided some quite interesting results. Argo program is probably one of the most ambitious ocean observation systems. With up to more than 3000 Argo floats moving around the global oceans, this program can provide continuous coverage of temperature and salinity distributions within the upper two kilometers of the world oceans.

Tracing eddies through the combination of TOPEX/Poseidon satellite data and XBT data collected by commercial ships going along some repeated lines throughout the ocean is a very good way to collect information about the structure of eddies in the ocean. Roemmich and Gilson^[25] combined these two observation techniques and produced a composed map for the vertical structure of meso-scale eddies in the North Pacific Ocean (Fig. 5). This figure clearly shows the three-dimensional structure of the warm-core and cold-core eddies in the top 800 m of the ocean. One of the remarkable features is the westward tilting of eddies in the upper 400 m, which is consistent with the theoretical prediction that in order to transfer heat poleward, eddies must have the right tilt.

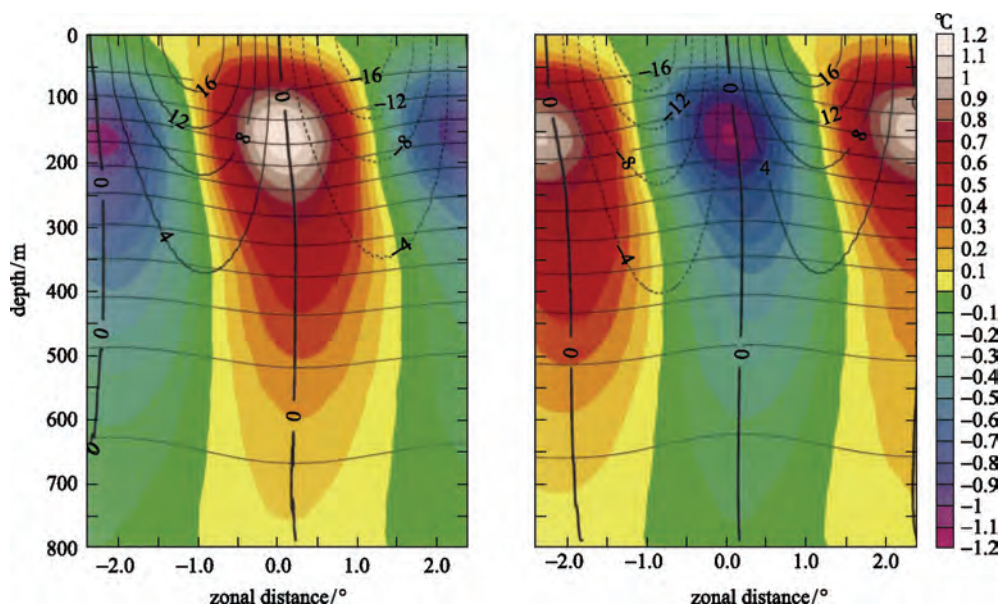


Fig. 5 Composite eddy: left panel for a warm-core eddy; right panel for a cold-core eddy. The nearly horizontal contours indicate the mean temperature (contour increment = 2°C, with the warmest contour of 26°C); the bowl-shaped contours indicate the meridional geostrophic velocity ($\text{cm}\cdot\text{s}^{-1}$); and color shading indicates temperature anomaly (in units of $^{\circ}\text{C}$)^[25]

Over the past 10 years, a new technique has emerged. Seismic oceanography may be a very good tool for observing the three-dimensional structure of eddies in the ocean^[26]. Most commonly used techniques cannot really provide an instantaneous three-dimensional structure of eddies. On the other hand, seismic oceanography may be able to provide us with a clear view of the three-dimensional structure of eddies in the oceans. Of course, such a technique is relatively new, and a lot of work has to be done in order to adapt and modify this technique to the need of scientific research in physical oceanography.

6 A mismatch of vertical fluxes

According to the traditional quasi-geostrophic theory, both rotation and stratification suppress vertical velocity and vertical exchange of tracer. As a result, for vast parts of the world oceans, vertical velocity is rather small, on the order of 10^{-7} – 10^{-6} $\text{m}\cdot\text{s}^{-1}$. For a long time, there is a puzzle that the new production rate inferred from oxygen utilization and cycling rate is about one order of magnitude larger than what is predicted by current basin-scale ocean general circulation models (OGCMs).

If the simulation of OGCMs were accurate, the

interior of subtropical gyres would be a desert of ecologic activity; however, satellite images reveal a wealthy of biological activity in the interior of subtropical gyres. Such phenomena pose a challenging question: What has gone wrong with the traditional theory and model simulation, when it applies to problems related to chemical and biological cycles in the ocean?

Recent studies on submesoscale phenomena may have provide an answer of this seemingly puzzling question. The essential point is that at horizontal scale of submesoscale eddies and fronts, the dynamical constraints on vertical velocity imposed by rotation and stratification may break down. As a result, the vertical velocity can grow much larger than the corresponding bounds set by large-scale dynamics.

Accordingly, the traditional quasi-geostrophic dynamics applies at horizontal spatial scale on the order of 50–100 km; these eddies are the commonly called meso-scale eddies and have vertical scale on the order of 500–1000 m and time scale of a month (Table 1).

At horizontal scale of 1 km, both the Rossby number and the Richardson number are of order one.

Tab. 1 Typical scales for eddies associated with baroclinic instability in the ocean

	Horizontal scale / km	Vertical scale / m	Time scale	Rossby number
QG Baroclinic instability	50	500	month	0.1
Ageostrophic baroclinic instability	1	50	day	1.0

This is the regime for submesoscale eddies, which has a vertical scale of 50 m and a time scale of one day. The dynamics of submesoscale regime is quite different from the quasi-geostrophic dynamics for the mesoscale eddies. Within the parameter range, there is ageostrophic baroclinic instability. Typical examples include mixed-layer instability and restratification. In fact, many outstanding features associated with small eddies, fronts and filaments identifiable from satellite observations, such as ocean color map and SST, can be classified into the categories of submesoscale phenomena.

There has been a misconception of vertical velocity associated with eddies. According to the

common wisdom, as often cited in published papers discussing ecology and other marine environmental issues, people tended to make simple statement regarding vertical velocity associated with eddies. For example, cyclonic (cold core) eddies used to be identified as equivalent to have upwelling; on the other hand, anticyclonic (warm core) eddies implies downwelling.

In reality, vertical velocity is a rather complicated issue needing careful examination. For most cases, vertical velocity of an eddy-like feature can change during its life cycle.

A more accurate approach of calculating vertical velocity is to apply the ω -equation based

on the Q -vector. Hoskins^[27] gave a brief introduction to the Q -vector method. Since then, there is much progress, especially in the atmospheric literatures. Recently, the ω -equation has been applied to the study of meso-scale eddies in the ocean. According to Giordani et al.^[28], a generalized ω -equation has the following form,

$$f^2 \frac{\partial^2 w}{\partial z^2} + \nabla_h (N^2 \cdot \nabla_h w) = \nabla \cdot Q \quad (13)$$

where w is the vertical velocity; the Q -vector is defined as follows,

$$Q = Q_{th} + Q_{dm} + Q_{tw} + Q_{dag} + Q_{dr} \quad (14)$$

where the subscripts th, dm, tw, dag, and dr indicate the contributions from:

- a) turbulent buoyancy forcing;
- b) turbulent momentum forcing;
- c) kinematic deformation;
- d) thermal wind imbalance deformation;
- e) thermal wind imbalance trend.

A typical case is shown in Fig. 6, which includes strong upwelling and downwelling associated with such eddies. For example, both upwelling and downwelling appear along the edge of eddy A1 (an anticyclonic eddy) and eddy C4 (a cyclonic eddy). As shown in this figure, the amplitude of vertical velocity can be on the order of $6 \text{ m}\cdot\text{d}^{-1}$.

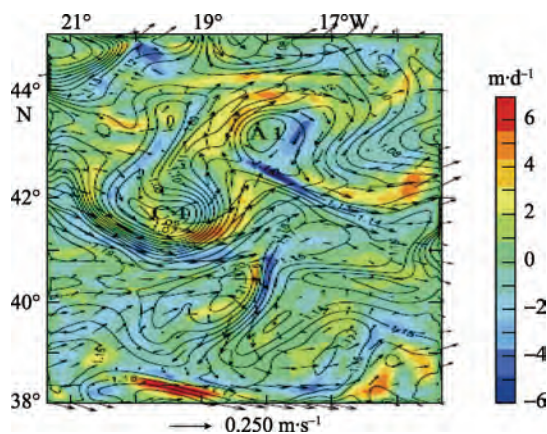


Fig. 6 Daily-mean vertical velocity (colorbar, in units of $\text{m}\cdot\text{d}^{-1}$) at 200 m, surface dynamic height (contours, in units of dynamic meter) and the daily-mean surface current (arrows, in units of $\text{m}\cdot\text{s}^{-1}$), during the POMME field experiment^[28]. A1 and C4 are anticyclonic and cyclonic eddies, respectively

In comparison, the typical scale of vertical

velocity in the ocean set by the Ekman pumping is on the order of $0.04 \text{ m}\cdot\text{s}^{-1}$ (that means the water parcel can move up and down for about $15 \text{ m}\cdot\text{a}^{-1}$). Thus, along the edge of eddies, the vertical velocity is two-order of magnitude larger than that of the mean vertical velocity, according to Giordani et al.^[28].

7 Baroclinic instability

Baroclinic instability theory has been developed over the past half century, and there are many classical papers and text books published along this line. The best known book is the *Geophysical Fluid Dynamics* by Pedlosky^[15]. In most cases, baroclinic instability was examined under the assumptions of steady flow and small Rossby number. Among many directions of instability study, there are currently two directions worth mentioning: the time-dependent baroclinic instability and the ageostrophic baroclinic instability.

7.1 Instability of time-dependent mean flow

Pedlosky and his colleagues turned their attention to the instability of time-dependent mean current. For example, Flierl et al.^[29] studied a modified Philips 2-layer model for a beta-plane channel. When the frequency of the basic flow matches the multiple of the characteristic frequency of the otherwise stable perturbations, the time-dependent flow becomes unstable, and the instability is similar to the case of resonance.

7.2 Mixed-layer baroclinic instability

Mixed-layer theories developed over the past were mostly focused on a quasi one-dimensional model; thus, lateral inhomogeneity plays only a minor role through interaction with currents and meso-scale eddies. Some of the recent new studies on mixed layer, however, focused on the submesoscale theory of the mixed layer, in which lateral density stratification gradient plays a critical role in driving the ageostrophic baroclinic instability in a weakly stratified fluid. Most importantly, this instability speeds up the restratification, affects the potential vorticity balance and enhances the transports of tracers

within the mixed layer.

Stability analysis for the ageostrophic baroclinic instability associated with the mixed layer was performed by Bocaletti et al.^[30]. Using mean profiles of stratification and zonal velocity taken from in-situ observations, they solved the eigenvalue problem associated with the instability of the model ocean. The growth rate curve obtained from the calculation is shown in Fig. 7.

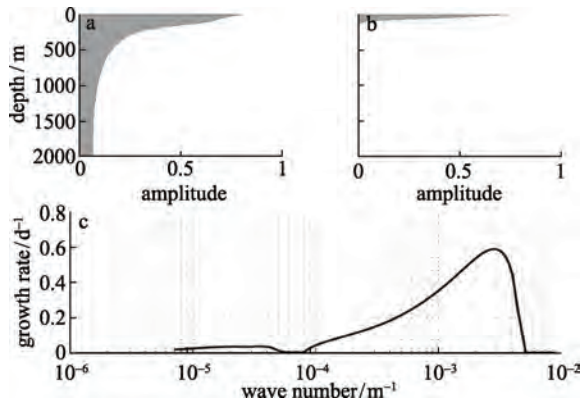


Fig. 7 Interior stability with a vertical structure spanned over the whole depth of the model ocean (a); mixed-layer stability with a vertical structure mostly confined within the upper 50 m of the model ocean (b); and growth rate as a function of wave number (c)^[30]

7.3 Restratification of mixed layer

Mixed-layer dynamics is one of the key components of oceanic circulation and climate. In most published studies, the mixed layer is treated in terms of quasi one-dimensional model, in which the lateral inhomogeneity only plays a minor role through currents and meso-scale eddies.

As we enter the regime of submesoscale eddies, there is a new impetus to improve the simulation of mixed-layer dynamics. Lateral density stratification gradient plays an important role in driving the ageostrophic baroclinic instability in weakly stratified fluid, which speeds up the restratification and enhances the mixed-layer balance in PV (potential vorticity)/tracer, including temperature and salinity (Fig. 8). Although recent progress along this line has shed light on the improvement of mixed-layer dynamics, further study is needed; in particular, energy from surface waves must be included in the model formulation.

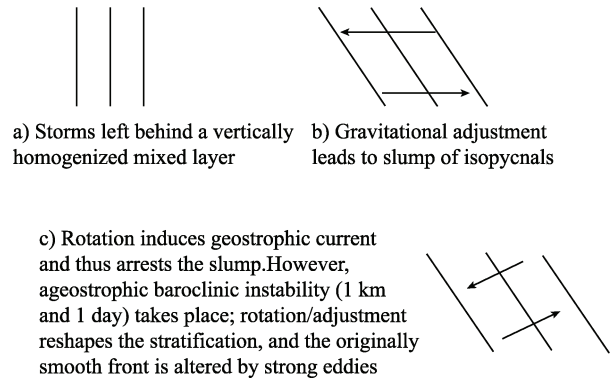


Fig. 8 Restratification of the mixed layer and ageostrophic baroclinic instability

8 Major challenges

8.1 Three-dimensional structure of eddies in the ocean

Since we have just entered the regime of eddy-resolving ocean, our knowledge about the structure of eddies remains preliminary. Thus, it is one of the top priorities for us to study the structure of eddies in the ocean. Most importantly, our study should start from the examination of individual eddies in the ocean, in particular their three-dimensional structure.

8.2 Observing and better parameterizing eddies

Historical development in meteorology told us that, every time model's resolution increases, the results obtained from the model subjected to the same parameterization used previously turn to be inaccurate. It is only after we have learned how to tune these parameterizations under the guidance of physics, the results of model simulation can be improved eventually.

With the rapid development of computer technology, numerical models are now running with finer and finer scales. Many current numerical simulations are running with horizontal resolution on the order of 1–10 km. (For example, see website: hycom.org). However, we do not have many observational data with such fine scales. Thus, although numerical models keep generating a wealthy of outputs with eddies, we are in a dilemma of not knowing what to do with such new numerical simulations, i.e., we cannot say what is correct or

wrong with these outputs. Eventually, numerical results can be accepted only if we have validated them with real observations.

8.3 Lateral eddy diffusion

At the early stage, most basin-scale numerical models run with quite low horizontal resolution, on the order of a few degrees. At such low resolution, most models have difficulty in resolving narrow boundary currents, such as the Gulf Stream, the Kuroshio, and the ACC. Thus, in order to resolve these narrow boundary currents with the low resolution used in these models, a common practice was to lower the horizontal diffusivity used in the simulation as much as possible.

As the computer power increases rapidly, more and more energetic eddies appear in model simulations. Apparently, due to the lack of any physical constraint, the parameterization of lateral tracer diffusion remains fairly subjective. As a continuation of the common practice, the same criterion of using the lowest possible lateral diffusivity has been applied in many numerical simulations with resolution high enough to resolve eddies. There is, however, indication that eddies seen in such high-resolution simulations may be too energetic. For example, the Kuroshio exhibits a bimodal behavior on irregular interannual to decadal time scale. A detailed PV analysis of such phenomenon indicates that the bimodal behavior is closely associated with the gain of positive potential vorticity, i.e., it is a way for the Kuroshio to get rid of the extra negative PV obtained from the basin-scale wind stress curl. Recently, high resolution numerical simulations with 0.1 by 0.1 degree resolution indicate that the frequency of such bimodal behavior seen in the model is much higher than the observed. Thus, it may indicate that the lateral diffusion parameterization used in such numerical simulations be too low, and the system tries to get rid of the extra negative PV by forming such large meandering more frequently than observed (Bo Qiu, personal communication).

A close re-examination reveals that transport of tracers consists of two components: advection and diffusion. Advection of tracers is directly linked to the velocity field averaged at the grid scale. Recent

progress along this line includes the introduction of eddy transport term, i.e., separating the advection velocity into mean-flow transport and eddy transport. If we know the mean-flow velocity, the contribution due to mean flow is relatively easy to calculate.

On the other hand, diffusion is directly linked to small-scale processes, including meso-scale eddies, turbulence and internal waves. It is obvious that such small-scale processes cannot be resolved in basin-scale numerical simulations.

To incorporate the dynamical effects of these sub-scale processes into numerical models, different kinds of sub-grid parameterization scheme have been postulated for momentum and tracers, including momentum dissipation in the vertical and quasi-horizontal directions and tracer diffusion in the vertical and quasi-horizontal directions.

These parameterizations are the most critical components for ocean modeling. Due to historical reasons, the parameterization of tracer diffusion has been treated as a subject disconnected with the balance of mechanical energy of the ocean. It is only in the last decade people came to realize that the ocean is not a heat engine and external sources of mechanical energy are required for sustaining the oceanic general circulation, including both the wind-driven circulation and thermohaline circulation.

In recent years, parameterization of vertical (diapycnal) diffusion of tracers in the ocean based on mechanical energy available from tides and wind stress has been advanced rapidly. It is obvious that sub-scale lateral eddy diffusion of tracers is also a key component in oceanic numerical simulation. However, the common practice of quasi-horizontal eddy diffusion of tracers in the ocean remains nearly unchanged. To my best knowledge, no paper linking quasi-horizontal eddy diffusion to GPE of the system has ever been published. A common approach in ocean modeling is to use harmonic or bi-harmonic diffusion for quasi-horizontal (horizontal, along-isopycnal and along-sigma-surface) eddy diffusion of tracers. In general, quasi-horizontal eddy diffusion is assumed to be spatially uniform and isotropic, and the corresponding diffusivity has been treated as an adjustable constant, which people can

choose within certain range. In fact, thus far there is no commonly accepted criterion, which can be used to judge whether a quasi-horizontal diffusion scheme and its corresponding diffusivity have been chosen appropriately.

According to the common wisdom, water parcels moving along isopycnal surfaces or along neutral surfaces do not experience buoyancy force. This is equivalent to the statement that isopycnal stirring is free from buoyancy work, and equivalent to zero GPE change. However, a close examination reveals that isopycnal stirring is associated with change of GPE due to the nonlinear equation of state for seawater. In addition, cabbeling associated with subgrid scale turbulent diffusion can release GPE from the mean state.

The amount of mechanical energy in connection with isopycnal diffusion of tracers is a substantial portion of the total mechanical energy budget for the world oceans. GPE releasing is highly non-uniform in space and highly non-isotropic. In fact, most of the energy releasing is concentrated in the Southern Ocean, in particular in connection with the quasi-stationary eddies in the core of the ACC. These features should be taken into consideration in the subgrid-scale parameterization of lateral diffusion of tracers in numerical models of the next generation.

To conclude this short review, it is clear that we are entering the era of eddy-resolving ocean. There are much more to learn before we can claim that we understand the oceanic circulation in this new regime.

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