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Key Points:

- Spicity together with density constructs an orthogonal coordinate, in which the signals can be separated into two independent components
- Based on the orthogonal coordinate, distance can be introduced to trace water mass both along and cross isopycnal surfaces
- Spicity can serve as an appropriate tool to extract water mass properties supplementary to density

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Quantifying Climate Signals: Spicity, Orthogonality, and Distance

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Abstract The variability of water masses has been analyzed using temperature and salinity or density and various forms of spicity/spiciness. The objective way of comparing these spicity/spiciness functions is not the difference between these definitions; instead, it is through the application of these functions in tracing water masses and climate signal analysis. The major advantage of the spicity function introduced by Huang et al. Journal of Geophysical Research: Oceans, is that this function in combination with density gives rise to an orthogonal coordinate, in which the signals are separated into two independent components. In addition, a new quantity, distance, is introduced, that can be used to trace water masses both along and across isopycnal surfaces. Spicity can serve as an appropriate tool to extract globally varying water mass properties as a supplement to variations of density alone.

Plain Language Summary Spicity in combination with density leads to a new mean of tracing water masses and climate signals. In particular, the difference between two water parcels induced by different temperature and salinity can be quantified in terms of a single quantity, distance, defined in terms of spicity and density. Through concrete examples, we demonstrate how to use spicity and distance in the study of water mass and double diffusive instability. Most importantly, these new tools can provide a clear separation of climate signals through two independent components without signal contamination.

1. Introduction

We thank McDougall, Baker and Stanley's (hereafter MBS20) interest in our work and their comments (McDougall et al., 2021), which prompt us to present the prominent features of potential spicity defined in Huang et al. (2018, hereafter HYZ18) and its difference from spiciness defined in McDougall and Krzysik (2015, hereafter MK15). In physical oceanography, a long-standing question is: how does one quantify the difference between any two water masses? This situation is deeply rooted with the fact that there is no accurate definition of a distance-like index between water masses based on the traditional temperature (T)-salinity (S) diagram. When a water parcel is heated/cooled or becomes fresher/saltier, the corresponding climate variabilities can be quantified by the change in temperature or salinity alone. When both temperature and salinity are changed, density varies in general. However, the density variability alone cannot completely describe the change in the thermodynamic state of sea water induced by the variances in both T and S. As a result, an additional variable that is independent of density is needed.

Stommel (1962) first introduced such a thermodynamic function namely $q = \alpha T + \beta S$, which is linearly independent of the density function $p = -\alpha T + \beta S$, where α and β are the thermal expansion coefficient and saline contraction coefficient, respectively. Furthermore, Stommel used a figure to show his basic idea that the contours of p and q are orthogonal to each other, so that the quantity q does not affect the density. Veronis (1972) pursued Stommel's idea, and proposed a function whose contours are nearly orthogonal to those of density. Many studies afterward pursued a different approach by defining a spiciness function (η) which is not orthogonal to density, such as, Flament (2002) and Jackett and McDougall (1985, hereafter JM85), and McDougall and Krzysik (2015). Recently, Huang et al. (2018) proposed a potential spicity function (π) by a least squares method, which is almost perfectly orthogonal to the potential density, with the root-mean-square of angle deviation from orthogonality in the range of 0.0001°. Therefore, combining density and spicity gives rise to an orthogonal coordinate system.

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As described in HYZ18, temperature and salinity have different dimensions; furthermore, values of thermal expansion coefficient (α) and saline contraction coefficient (β) vary widely over the parameter domain. In order to define any variable like spicity which contours are orthogonal to those of density, we must make sure that both axes of the T-S diagram have the same dimension and the same axis lengths in an equivalent grid. We first define the normalized coordinates with two variables *x* and *y*, with their units the same as potential density: $x = \rho_0 \beta_0 S$ and $y = \rho_0 \alpha_0 T$, where $\rho_0 = \overline{\rho}, \alpha_0 = \overline{\alpha}$ and $\beta_0 = \overline{\beta}$ are the mean values of density, α and β averaged over the entire domain of the parameter space T = [-2,40] (°C) and S = [10,40] (psu). A similar normalization is also applied to the Conservative Temperature-Absolute Salinity ($\Theta - S_A$) diagram defined by the equation of state UNESCO TOES_10. Finally, we select a fixed aspect ratio for these two axes, so that the T-S plane is mapped into a Euclidean plane with equal horizontal and vertical dimensions. Therefore, many commonly used geometric measures can be defined, such as the angle, slope, curvature, and area.

MBS20 repeated their statement that using a fixed aspect ratio is an artificial choice in HYZ18. This is true, but of course it is also true of any selection process. For example, both JM85 and MK15 adopted a scaling factor $k = \beta/\alpha$ to make sure that along the 20°C isotherm the slopes of isopycnals ($k^{-1}\delta T/\delta S$) are equal to 1, that is, the isopycnals are subtended 45° with the salinity axis. By means of this scaling factor, the defined spiciness is perpendicular to the density along this isotherm. In addition, selecting the 20°C isotherm is also an artificial choice.

Similar to Veronis (1972), JM85 mentioned a method by seeking a suitable function a (*T*,*S*), that could satisfy the orthogonality constraint between isolines of density and spiciness; however, they made the following statement "A moment's reflection, however, reveals that implementation of this method is not trivial." Therefore, they abandoned the orthogonality constraint and constructed a spiciness function. Using a different approach based on a least squares method, HYZ18 found a spicity function that satisfies the orthogonality constraint almost perfectly everywhere.

It is expected that density and spicity/spiciness play different roles in the description of sea water dynamics. In large-scale processes, the horizontal velocity is driven by the horizontal density gradient through geostrophy. The stratification and its stability are regulated by the vertical density gradient. Thus, density signals are often called dynamical in the large-scale sense. Since the spicity signals do not play a direct role in large-scale dynamics, they can be regarded as dynamically passive (in the large-scale sense) (Veronis, 1972) even though small-scale processes could also affect large-scale dynamics through cross-scale interactions. In the abstract of MK15, it is stated that "the passive nature of variations of any thermodynamic variable is gained by evaluating those variations along isopycnals." This statement is clearly fine although seems to involve a self-enclosed logic because if one stays on the same isopycnal surface, any other signal behaves as passive because there are no density anomaly signals. This note shows, however, that when water masses move across isopycnal surfaces, spiciness variations may contain density signals, hence they are no longer passive.

Combining density and spicity gives rise to an orthogonal coordinate system and the advantage is that the climate signals could be clearly separated into two independent components: the dynamical component associated with density signals and the passive component containing no density signals. In addition, the distance between two water parcels can be exactly quantified. On the other hand, using a nonorthogonal (or the skew) coordinate system may lead to signal contamination so that distances might not be as meaningful. An example of this is shown below.

Naturally, if one does not reinforce the orthogonality constraint, there are many choices for coordinates. Temperature is definitely not a good choice (Section 1) because it contains density signals in the form of state is $\delta \sigma = \rho_0 (-\alpha \delta T + \beta \delta S)$. Still, one can also use the salinity as the second coordinate. Such a density-salinity coordinate is quite useful for some special cases. Probably the best example is the Antarctic Intermediate Water (AAIW) that is defined mostly by the subsurface low salinity tongue extending northward near the northern edge of the Antarctic Circumpolar Current. Such a salinity-weighted index was shown to be useful in the study of AAIW. However, the density-salinity coordinate may have problems similar to the case associated with temperature coordinate.



In this note, we use some concrete examples to demonstrate the prominent characters of spicity and the difference between spicity and spiciness in the description of sea water. In Section 1, we explain that if we use the temperature coordinate as the coordinate supplemented to the density coordinate, density signal contamination can happen. In Section 2, we compare the $\sigma - \pi$ coordinate defined by HYZ18 and the $\sigma - \eta$ coordinate defined by MK15. In Section 3, spicity/spiciness is applied to the water mass analysis in the low temperature and low salinity regime. In Section 4, we demonstrate how to trace the AAIW in terms of the spicity, distance or spiciness. In Section 5, density ratios inferred from the spicity and spiciness are compared, and the application of spicity/spiciness in double diffusive instability is explored. We show that spicity function gives some insight that spiciness does not. Section 6 is for conclusion and remarks.

2. Climate Signals Decomposition Based on Temperature Coordinate

In the new normalized coordinates defined above, we can separate the contributions due to a temperature anomaly into two parts: the isopycnal component and the residual component

$$\Delta T = \Delta T_{\sigma} + \Delta T_{\pi}.$$
 (1)

The gradient of the potential density vector is

$$\nabla \sigma = \sigma_x \vec{i} + \sigma_y \vec{j} = \frac{\partial \sigma}{\rho_0 \beta_0 \partial S} \vec{i} + \frac{\partial \sigma}{\rho_0 \alpha_0 \partial T} \vec{j} = \frac{\beta}{\beta_0} \vec{i} - \frac{\alpha}{\alpha_0} \vec{j}, \tag{2}$$

where (\vec{i}, \vec{j}) are the unit vectors in the horizontal/vertical directions.

We begin with a simple case on the T-S (or the $\Theta - S_A$) diagram, Figure 1a; the initial state and the perturbed state are marked by points O and A, and the corresponding temperature and salinity perturbations are ΔT and ΔS , depicted by the segments of OD and DA. We separate the signals as follows. First, using the gradient vector starting from point O we define the minimum values of temperature and salinity perturbations marked by point C, which gives rise to the density increment $\Delta \sigma$. According to Equation 2, the angle between the density gradient and the horizontal axis is

$$\phi = \tan^{-1} \left(\frac{\alpha / \alpha_0}{\beta / \beta_0} \right) = \tan^{-1} \left(\frac{\alpha \beta_0}{\alpha_0 \beta} \right).$$
(3)

According to Figure 1a, the isopycnal components of temperature and salinity anomalies are

$$\Delta S_{\sigma} = \frac{\Delta \sigma \cos \phi}{\rho_0 \beta_0}, \Delta T_{\sigma} = -\frac{\Delta \sigma \sin \phi}{\rho_0 \alpha_0}.$$
 (4)

The corresponding residual T and S signals from point C to point A contributing to the spicity anomaly are.

$$\Delta S_{\pi} = \Delta S - \Delta S_{\sigma}, \Delta T_{\pi} = \Delta T - \Delta T_{\sigma}.$$
(5)

As shown in Equation 4, the isopycnal component of temperature anomaly is proportional to $\Delta \sigma sin \phi$. Thus, this projection component is zero for two special cases:

- (a) $\Delta \sigma = 0$, that is, the perturbations remain on the original density surface. Thus, any spiciness function of (T, S) will lead to a result of no density signal noise, and the perturbations are dynamically inert.
- (b) $\phi = 0^{\circ} \rightarrow \sin \phi = 0$, i.e., the temperature coordinate is orthogonal to the density coordinate in the local sense. As a result, there is no density change for the signal component projected onto temperature coordinate.

Except for these trivial cases, signals projected onto the temperature coordinate are contaminated by the density perturbations; hence, if we desire to make a diagram with density as one coordinate and something else as the second one, the temperature coordinate is not a suitable choice. Similarly, using any other non-orthogonal coordinate, such as salinity or even spiciness, also induces perturbation-produced signal contamination, as shown in the next section.





Figure 1. (a) A T-S diagram illustrating the relation between temperature anomaly and density anomaly; (b) the relation between the σ - π and σ - η coordinates; (c) Potential density and potential spiciness contours based on MK15; (d) Potential density and potential spiciness contours based on HYZ18. The green line indicates the $\alpha = 0/^{\circ}$ C contour line; Panel c and d are based on reference pressure of $p_r = 0$.

3. Density-Spicity versus Density-Spiciness Coordinate

Figure 1b shows the difference between the $\sigma-\pi$ coordinate defined by HYZ18 and the $\sigma-\eta$ coordinate defined by MK15. Both coordinate systems share the same σ -axis. The π -axis is orthogonal to the σ -axis, and the η -axis is slightly skewed at an angle ϕ , indicating the nonorthogonality of $\sigma-\eta$ coordinate.

We examine a simple case with both the density and spiciness anomalies, $\Delta\sigma$ and $\Delta\eta$, equal to 1 kg/m³. When $\phi = 0^{\circ}$, the spiciness axis is along the same direction with that of spicity; the anomaly is marked by point A. When $\phi > 0$, the state should lie on the dashed blue line perpendicular to OB (the spiciness anomaly $\Delta\eta$ vector) and it is represented by point D, where the corresponding spicity perturbation in $\sigma - \pi$ coordinate is $\Delta\pi = \overline{DE}$. A simple geometric relation leads to

$$\Delta \eta = \Delta \sigma \sin \phi + \Delta \pi \cos \phi. \tag{6}$$



Obviously, the spiciness variation contains some density variations.

We can define the distance in both the $\sigma - \pi$ and $\sigma - \eta$ spaces. For the skewed coordinate $\sigma - \eta$, the distance is defined as

$$d_{\eta}^{2} = \left(\Delta\sigma\right)^{2} + \left(\Delta\eta\right)^{2} + 2\Delta\sigma\Delta\eta\sin\phi.$$
⁽⁷⁾

The practical application of Equation 7 is somewhat confusing, because the $\sigma - \eta$ system is nonorthogonal and curvilinear. The angle ϕ varies greatly in the parameter space, so that it is cumbersome to calculate the distance exactly following Equation 7. Instead, one may ignore the metric term associated with the angle ϕ , that is, use a simplified form that is not exact

$$d_{\eta}^{\prime 2} = \left(\Delta\sigma\right)^2 + \left(\Delta\eta\right)^2. \tag{8}$$

In particular, as will be shown below, the quantity calculated by Equation 8 is not exactly a distance; it is more reasonable to call it a distance-like index.

On the other hand, since the $\sigma - \pi$ coordinate system is orthogonal, the exact formula of the distance is reduced to

$$d_{\pi}^{2} = \left(\Delta\sigma\right)^{2} + \left(\Delta\pi\right)^{2} = \left(\Delta\sigma\right)^{2} + \left(\overline{DE}\right)^{2}$$
(9)

When $\phi = 0^\circ$, the results from Equations 7–9 are the same as $\sqrt{2}$. As $\phi \rightarrow \pi/2$, the distances calculated by Equations 7–9 approach to 2, $\sqrt{2}$ and 1, respectively. Therefore, using an orthogonal coordinate, in which the spicity variation contains no density signals, to define the distance is the best approach.

4. Water Properties in the Low Temperature and Salinity Regime

In a low salinity/temperature region, the thermal expansion coefficient α is very small or even negative. Based on the definition of MK15, it is obvious that the σ_0 contours and the η_0 contours can intersect at two points in the low temperature and salinity regime, see the blue points A (0°C, 10.103653 g/kg) and B (3.8033°C, 10.10400 g/kg) in Figure 1c, where density and spiciness are exactly the same. Thus, in any cases with some definitions of distance based on the $\sigma_0 - \eta_0$ coordinate, the distance between this pair of points is identically zero. This is a violation of the basic requirement of a distance. Thus, here it is impossible to define a distance using the $\sigma_0 - \eta_0$ coordinate. However, in the $\sigma_0 - \pi_0$ coordinate defined by HYZ18 (Figure 1d), the result is totally different. Although the difference in density between these two points is zero, the difference in spicity has a small, but nonzero value of 0.0028 (kg/m³). Thus, the corresponding distance between A and B is 0.0028 (kg/m³).

We notice that there exists a relatively large temperature difference between two water parcels A and B in Figures 1c and 1d, while the dynamic distance based on the $\sigma - \pi$ coordinate is quite small. This small distance is closely linked to the small or even negative value of α in the vicinity.

To illustrate the point, we define the ratio of spicity gradient versus density gradient $G = |\nabla \pi| / |\nabla \sigma|$, which shows the spicity variation to density variation in θ - S_A space. For the entire domain of definition, the range of *G* is 0.0028–4.3552. On the other hand, the spiciness by MK15 is defined by integration along isopycnals, and the corresponding ratio is close to $G = |\nabla \eta| / |\nabla \sigma| \sim 1$ as argued by MBS20. However, using the range of *G* alone to judge the advantages/disadvantages of spicity/spiciness might not be objective. As argued above, the small spicity difference of 0.0028 (kg/m³) reflects the actual physical situation in this vicinity and its small value is meaningful.

We now turn our focus on points C (1.61°C, 11.3 g/kg) and D (1.61°C, 12.3 g/kg) in Figures 1c and 1d. These two points lie in the vicinity of the contour of $\alpha = 0/°$ C (green line). The density and spiciness increments are $\Delta \sigma = 0.7973 (\text{kg} / \text{m}^3)$ and $\Delta \eta = 0.6858 (\text{kg} / \text{m}^3)$. However, in terms of the $\sigma - \pi$ coordinate, the spicity increment is completely negligible with the value of 0.0000273 (kg/m³). The corresponding angle



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Figure 2. A meridional section of the upper ocean through 20°W in the South Atlantic Ocean based on WOA18: (a) salinity; (b) spiciness from MK15; (c) distance to the origin (Equation 9'); (d) spicity from HYZ18. The respective isolines are shown in white contours, overlaid by potential density contours in black.

between the local π axis and η axis is close to $\phi \to \pi / 2$, as defined in Figure 1b. According to Equation 6, the large spiciness variation is mostly due to the contamination from density signal. In this case, we are dealing with finite variance, so that Equation 6 derived for infinitesimal variance is not exactly accurate. Nevertheless, since the angle between C-D and density change is close to 90° in this vicinity, the density anomaly contributes to the major part of the spiciness signal in Figure 1c, whereas there is almost none for spicity. We conclude that for this case, the $\sigma - \pi$ coordinate is a better tool for the description of variability than the $\sigma - \eta$ coordinate.

5. Tracing the AAIW With Different Indexes

AAIW is defined by the low salinity tongue, and spiciness is defined by salinity anomaly along isopycnal surfaces, so that salinity or spiciness could be used to trace AAIW. Using spicity defined by contributions from both Θ and S_A , the distance introduced in the $\sigma - \pi$ coordinate can be used as a tracer to trace water mass in the three-dimensional space both along and across isopycnal surfaces.

The World Ocean Atlas 2018 (WOA18) database is used to calculate such a distance and the results are shown in Figure 2. One can trace AAIW with different tracers, including the absolute salinity (panel a), the spiciness (panel b), the distance (panel c) and the spicity (panel d). The distance of each grid point *i* from the origin (defined by the temperature and salinity at $(30^{\circ}W, 52.6^{\circ}S)$ and 50 m below the sea surface) is defined as

$$d_{i} = \sqrt{\left(\sigma_{0,i} - \sigma_{0,0}\right)^{2} + \left(\pi_{0,i} - \pi_{0,0}\right)^{2}},\tag{9'}$$

where $\sigma_{0,0}$ and $\pi_{0,0}$ are calculated from the temperature and salinity at the origin defined above using sea surface as the reference pressure; $\sigma_{0,i}$ and $\pi_{0,i}$ are calculated from the temperature and salinity at each grid



point. Apparently, using spicity alone as the tracer gives a different result that is not so good. Using spiciness as the tracer the result is better. On the other hand, when using distance as the tracer, the result is quite comparable with the result based on using salinity alone as the tracer. Note that using different indexes to trace can lead to slightly different results. For example, using distance to trace AAIW gives rise to a slightly deeper penetrating tongue compared with using salinity, which reflects different aspects of movement and mixing of water masses.

In addition, the concept of distance can be used in many other situations. In particular, if the definition of water masses involves both temperature and salinity, the concept of distance has the potential to be a more accurate tool for applications.

6. Density Ratio Inferred From Spicity and Spiciness

Historically, the density ratio, R_{ρ} , was proposed as the vertical gradient ratio of temperature and salinity by Turner (1965) to show the relative strength in diffusive convection.

$$R_{\rho} = \frac{\alpha \partial \Theta / \partial z}{\beta \partial S_A / \partial z},\tag{10}$$

where z is depth. Density ratio became one of the key parameters to characterize the structures and processes in the double diffusive convection (e.g., Kelley et al., 2003; Schmitt, 1994). Gargett and Schmitt (1982) used R_{ρ} in analyzing salt fingers associated with the Central Water in the Atlantic Ocean, You (2002) also used R_{ρ} as a stability parameter in analyzing water masses in the world oceans.

In the definition of spicity (see Equation 9 in HYZ18), the ratio $G = |\nabla \pi| / |\nabla \sigma|$ is needed. It must be considered when one examines the relationship between the slope of a curve for a vertical profile ($k_H = -\Delta \sigma / \Delta \pi$) and density ratio $R_{\rho H}$ in the $\sigma - \pi$ diagram, where the subscript H of R_{ρ} and k refers to HYZ18. Based on the definition of spicity and the above Equation 10, the relationship between $R_{\rho H}$ and the slope k_H is estimated as

$$R_{\rho H} = \frac{1 + G\alpha' k_H / \beta'}{1 - G\beta' k_H / \alpha'},\tag{11}$$

where *G* is the gradient ratio defined above, $\alpha' = \alpha / \alpha_0$, $\beta' = \beta / \beta_0$, α and β are values at local temperature and salinity, respectively. Detailed derivation of Equation 11 is found in Huang (2020).

In the case of spiciness function defined by MBS20, one can also calculate the slope of the $\sigma - \eta$ curve as

$$k_M = -\Delta\sigma \,/\, \Delta\eta,\tag{12}$$

where the subscript *M* refers to MK15 hereafter, As suggested in Equation 8 in MBS20, the corresponding density ratio $R_{\rho M}$ is derived as

$$R_{\rho M} = \frac{1 + k_M \left[1 + (\Theta - 20) h(\Theta, S_A) \right]}{1 - k_M \left[1 + (\Theta - 20) h(\Theta, S_A) \right]},$$
(13)

 $h(\Theta, S_A)$ is the general function defined in Equation 6 of MBS20, which is approximated as

$$h(\Theta, S_A) \approx 0.0056 + 1.0 \times 10^{-5} (\Theta - 20) - 1.2 \times 10^{-5} (S_A - 35.16540)$$
 (14)

We will examine both Equations 11 and 13 inferred from potential spicity and spiciness in the water columns in two regions.

6.1. The Central Water in the Atlantic Ocean

As observed in Schmitt (1981), the density ratios R_{ρ} were 1.89 and 1.95 for the South and North Atlantic Central Water stations. Using the World Ocean Circulation Experiment (WOCE) database, we find two







Figure 3. (a) Data in Θ -*S*_A diagram: South (red) and North (blue) Atlantic stations, and Arctic Ocean station (green). Profiles of density ratio R_{ρ} from its original definition (black), and inferred from HYZ18 (red) and from MK15 (blue) at the (b) South Atlantic, (c) North Atlantic, and (d) Arctic Ocean stations. Insets in (b and c) are the zoom-in plots.

stations: (17.4°S, 30.8°W) in Section A17 and (24.5°N, 36.7°W) in Section A05-2, nearest to those in Schmitt (1981). The corresponding hydrographical data in the depth range of 200–700 db are shown in the Θ -*S*_A diagram (Figure 3a). The water mass properties of central water remain stable, indicated by the two similar curves in the Θ -*S*_A diagram.

The density ratio R_{ρ} generally increases with depth at the South and North Atlantic stations, Figures 3b and 3c. The respective mean values of R_{ρ} are 2.2 and 1.9, close to those in Schmitt (1981). R_{ρ} inferred from Equations 11 and 13 gives rise to similar results, and result calculated from spicity seems slightly closer to the original R_{ρ} , as shown in the insets of Figure 3.

6.2. Upper Arctic Ocean

In the Arctic Ocean, we used temperature and salinity data acquired with the Ice-Tethered Profiler (ITP) that, with the vertical resolution of 0.25 db, nicely resolves the small-scale structures, that is, diffusive convection staircases. The ITP4 station is located at (79.0°N, 142.5°W) in the Canada Basin, where a stack of staircases exists in the upper Arctic Ocean (Figure 3a) (Padman & Dillon, 1987).

As shown in Figure 3d, at the depth range of 180–420 db, R_{ρ} varies from 0.02 to 0.37 averaging approximately 0.24, consistent with the observations ($R_{\rho} \sim 0.17-0.25$) in Padman and Dillon (1987). R_{ρ} inferred from spicity or spiciness is rather close to the actual one. Comparatively, R_{ρ} from spicity seems have less deviation than that from spiciness.

It was expected that spicity/spiciness might serve as a potential measure of the turbulent mixing (Olbers et al., 1985; Veronis, 1972). Turbulent mixing is dominated by various dynamic processes within the small-scale structures. This suggested that a look at the comparative fine structures for temperature, salinity, density, spicity and spiciness might be useful. Zooming into the ITP data at the depth range of 302–355 db (Figure 4), three staircases are clearly identified in the temperature and salinity profiles, where the well-mixed layers of homogenous temperature and salinity are separated by thin and strongly stratified interfaces. However, the variances of the temperature and salinity are not exactly similar when considering the fine





Figure 4. (a) Θ and S_A profiles in the pressure range from 300 to 355 db. (b) The corresponding profiles of σ_0 , π_0 , and η_0 ; σ_0 is shift as $\sigma_0 - 27.98$ kg/m³ and π_0 is shifted as $\pi_0 + 2.8$ kg/m³ for clear demonstration.

structures. For example, at the depth range of 314-317 db there is an obvious salinity inversion, but temperature increases monotonously with depth. A coherent parameter $C_o = \frac{\langle XY \rangle}{X_{\delta}Y_{\delta}}$ is defined to examine the similarity between any two variables X and Y, where $\langle \rangle$ denotes the ensemble average and the subscript δ means the root-mean-square of the variable. Here, we focus on a thick mixed layer (depth of, 318–336 db) as an example. Figure 4a shows that Θ and S_A are less similar, confirmed with low coherence $C_o = 0.42$. Density variance looks like that of S_A rather than Θ with $C_o = 0.99$ and 0.39, respectively. This result implies that density variance is dominated by the contribution of salinity, and the temperature variance is less contained in that of density (due to the small thermal expansion coefficient α). However, the variance of spicity is more like variance of Θ , not of S_A , with the two being $C_o = 0.91$ and 0.41, respectively. That is, spicity π_0 can well provide the thermodynamic information of temperature, so that it plays a complemental role to density. The spiciness variance is very similar to those of density, and salinity with $C_o = 0.93$ and 0.97, respectively. η_0 also less includes the information of temperature ($C_0 = 0.44$). Both η_0 and σ_0 have almost similar behaviors, which implies that spiciness contains density information and it is unable to supplement density here. In contrast, spicity is independent of the density and it includes the information not contained in the density, which enables it to be exactly dynamically passive (Veronis, 1972). Our results suggest that spicity is a better tool in the potential applications in the study of small scale processes, e.g. turbulent mixing.

7. Conclusion and Remarks

Stommel (1962) pioneered the study of spicity. His basic idea and the application of the orthogonal function have not been thoroughly pursued. Munk (1981) gave the name of spiciness which has been widely used. However, another important idea for quantifying the thermohaline intrusion that he proposed has not been carefully pursued until now.

There are many potential ways to define the spicity/spiciness function, which can either satisfy the orthogonality constraint or not. Each definition may have its own advantages and disadvantages. HYZ18 introduced a potential spicity function which is practically orthogonal to the density function in the least squares sense. Defining the spicity function is only the first step, and many more studies are required to explore its



applications to water mass analysis and climate study. Possible applications include defining the radius of signal, radius of climate, and quantifying thermohaline intrusions from observations. Furthermore, water parcel properties can change, including density and spicity. When water parcels move across density surfaces, a clear separation of density signals and nondensity signals is very important.

In addition, with the help of the orthogonal coordinate the classical isopycnal analysis can be further extended and refined. Due to limitations of space, the details on these topics are omitted, and the readers are referred to the book by Huang (2020). We believe that further improvement of the spicity function and its application are an exciting frontier in the Physical Oceanography and Climate Dynamics.

Data Availability Statement

The main data sets supporting the results are WOA18 database downloaded from https://www.nodc.noaa. gov/OC5/woa18/woa18data.html, WOCE database downloaded from https://cchdo.ucsd.edu/, and ITP database downloaded from https://www.whoi.edu/page.do?pid = 28866.

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