On salinity errors induced in volume-conserved numerical models

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Abstract

Surface salinity boundary conditions used in numerical models based on the volume conservation are re-examined. It is shown that such boundary conditions can introduce significant errors in salinity simulation. In addition, volume conserving models used in data assimilation may induce additional errors. Routine salinity observations are mostly limited to the upper ocean; assimilating of such spatially non-uniformly sampled salinity into a model based on the volume conservation approximation can lead to a gradual decline of salinity in the model ocean.

The global mean salinity diagnosed from ECCO2 and SODA data indicates that there might be some serious problems in the salinity simulated from these models. Much more careful examination may be required for a better simulation and understanding of the salinity distribution in the world oceans.

1. Introduction

Salinity is one of the most important quantities of sea water, and it plays a role as important as temperature. Due to technical difficulties, however, routine salinity measurements over the global oceans have not been available. It is only since the beginning of the ARGO project, a global observation net of salinity in the upper 2 km of the world ocean is now available. There is still a long way to go before we really understand the roles played by salinity in the world ocean circulation and climate. Our concern in this study is some of the seemingly small errors associated with salinity simulation in the oceanic general circulation models. In order to appreciate the importance of accurately simulating the salinity, we start with some rough estimates of salinity error and signals associated with global sea level changes.

Since salinity sensors based on CTD have been widely used, accuracy of salinity measurements is approximately 0.001. How this accuracy is compared with the signals associated with sea level changes in the world oceans? It is estimated that the rate of global sea level change is on the order to $2.8 \pm 0.4 mm / yr$ (Leutiette et al. 2004; Cazenave and Nerem 2004). The major contributor to the global sea level rise is due to fresh water from land-based glaciers melting. Let us take the rate of global sea level rise due to glaciers melting as 2mm/yr. Then the corresponding annual-mean global mean salinity change is estimated as

$\delta S = \overline{S}\delta h / H \simeq 35\times 0.002/3670 \simeq 2\times 10^{-5}$. Thus, in order to extract annual-mean global sea level change signal from in-situ salinity measurements, the global-mean salinity should have an accuracy of 10^{-5}, which is about 100 times higher than the accuracy of in-situ measurements. Even if we are talking about decadal variability of sea level, the required accuracy of salinity is 10 times higher than the individual in-situ measurements. This is apparently a grand question whether salinity measurements with such a relatively low accuracy can be used to yield a result with such high accuracy, and this question has been posted in previous publications, e.g., Wunsch et al. (2007).

Assuming the error of individual salinity measurements is 0.001, in the worst case, the global mean salinity has an error of 0.001. However, the situation in the real world may be much better than this worst scenario. Assume that each in-situ measurement composites of

$$S_i = S_{i,real} + S_{i,random}, \tag{1}$$

where $S_{i,real}$ is the real value of salinity and $S_{i,\,reandom}$ is the random error associated with each measurement. The global mean salinity is thus

$$\overline{S} = \frac{1}{N}\sum_{1}^{N} S_{i,real} + M\left(\overline{S_{i,random}}\right), \tag{2}$$

where N is the total number of in-situ observation, $M\left(\overline{S_{i,random}}\right)$ is the mathematical expectation of the random errors $S_{i,random}$.

In theory, the error of the global mean salinity can have a value within the range of [-0.001 0.001], depending on the nature of the errors involved in each in-situ measurement. However, if each measurement is a completely independent sampling, the corresponding error is also completely random. According to the theorem of large number, when the number of the independent measurement becomes very large, the mathematical expectation of the error approaches zero. Therefore, we can extract useful information from global mean salinity, assuming the global mean (or basin mean) salinity is meaningful to the accuracy of 10^{-4} or even higher. Of course, such estimates have to be calibrated with information from other means.

With the magnitude of salinity measurements and the signal associated with global sea level change in our mind, we are going to discuss the potential errors associated with some of the commonly used boundary conditions in Boussinesq models. Although some of such errors may seem quite small, they may not be negligible, in compared with the signals on the order of 10^{-4} or 10^{-5}.

Most currently used numerical models for the oceanic circulation are based on the Boussinesq approximations. The essential assumptions made in the Boussinesq approximations are as follows. First, mass conservation in the continuity equation is replaced by the volume conservation. Second, in the momentum equations, the variable density is replaced by a constant value and the effect of non-constant density is replaced by a buoyancy force term. Third, the mass advection of each tracer in tracer equations is replaced by a volumetric advection of each tracer.

Since sea water density varies over the range of 1,020- 1,060 (kg/m^3), the approximations made in such models may introduce non-negligible errors. However, many papers have been published in which people claimed that such models are quite accurate for simulating the oceanic circulation, and many ways of 'interpreting' data obtained from such models have been postulated. Nevertheless, there is a systematic bias of the salinity field, which may not be overcome within the framework of volume conservation approximation. In this note, several important issues related to salinity errors induced by volume conservation approximations will be explored.

2. Salinity errors associated with inaccurate upper boundary conditions

In this section we discuss simple models based on different boundary conditions for salinity. These models are highly simplified versions of the more elaborated models. The models consist of an upper layer box and a lower layer box. The upper layer box has a depth of *h*, and it is forced by precipitation (evaporation) from the upper surface, with a fixed rate of *dh* (m) per year. Through a relaxed condition, the upper layer box is linked to a lower layer box which is a water reservoir of salinity. This relaxation condition serves as a simplified mechanism mimicking the complex role of the basin-scale general circulation in maintaining the salinity in the upper layer box.

A. NBC (Natural Boundary Condition)

This boundary condition applies to a mass-conserved model only. The basic idea is that precipitation/evaporation consists of a air-sea fresh water flux only, and there is no salt flux across the air-sea interface. The basic equation is the salt conservation, i.e., the total amount of salt for each grid box is balanced, without any source/sink of salt associated with the air-sea interface

$$\partial_t\left[(h+\eta)\rho S\right]+\nabla_{3d}\cdot\left[(h+\eta)\vec{v}\rho S\right]+\nabla_{3d}\cdot\left(\kappa\nabla_{3d}\rho S\right)=0\,, \tag{3}$$

where h is the mean thickness of the upper layer box below the sea surface, η is the free surface, ρ is the water density, and S the salinity. The second and third terms represent contributions due to advection and diffusion. An accurate calculation of this equation requires running a complicated oceanic general circulation model, so it is beyond the scope of this note. Assuming that at the initial time, free surface elevation at this grid box is zero, and the precipitation rate is ω; after a time step Δt salinity in the upper box satisfies

$$\left(h\rho_0+dh\rho_f\right)S_1=h\rho_0 S_0-\left\{\nabla_{3d}\cdot\left[(h+\eta)\vec{v}\rho S\right]-\nabla_{3d}\cdot\left(\kappa\nabla_{3d}\rho S\right)\right\}\Delta t \tag{4}$$

where $dh=\omega\Delta t$ is the increment of upper layer thickness due to precipitation over one time step, S_0 is the salinity at the initial time step and S_1 is the salinity after one time step. We will parameterize the contribution due to advection and diffusion, represented by the terms in the curl bracket, in terms of a relaxation condition. Accordingly, Eq. (4) can be rewritten as follows

$$S_1 = \frac{h\rho_0 S_0}{h\rho_0 + dh\rho_f} + \frac{\Delta t}{T_r}\left(S_{ref} - S_1\right) = S_0 \frac{1}{1 + r_h r_\rho} + \frac{\Delta t}{T_r}\left(S_{ref} - S_1\right), \quad (5)$$

where a depth ratio is introduced

$$r_h = dh / h = \omega \Delta t / h = \Delta t / T_f \ll 1. \quad (6)$$

$T_f = h / \omega$ is the flushing time for the surface layer; $S_{ref} \simeq 35$ is the reference salinity. The last term in Eq. (5) represents a relaxation condition mimicking the role of the global thermohaline circulation in controlling the salinity of this box, where T_r is the relaxation time, which is on the order of year. In order to compare the effect of different boundary conditions, this term will have the same form and same relaxation constant for all models forced by different boundary conditions discussed in this note.

At the sea surface, when $T_0 = 5.2^o C$, freshwater has density of 999.943kg/m^3; thus, in our discussion here we will assume that temperature is kept constant at $T_0 = 5.2^o C$, and fresh water associated with precipitation and evaporation has a density $\rho_f = 1000$ (kg/m^3). Sea water density is assumed to be a linear function of salinity alone, i.e., $\rho = \rho_f (1 + \beta S)$ (kg/m^3), where $\beta \simeq 0.76 \times 10^{-3}$.

In Eq. (5), $\rho_0 = \rho_f \left(1 + \beta S_0\right)$ is the density of water in the upper layer, S_0 is the mean surface salinity, $r_\rho = \rho_f / \rho_0 \simeq 0.974$ is the ratio of freshwater density and seawater density. For an upper layer of 100 m thick and the typical precipitation rate on the order of 1 m/yr, the corresponding flushing time is about 100 yr, so that the time scale ratio $r_T = T_r / T_f \simeq 1/100 \ll 1$. In the steady state, $S_1 = S_0 = S$, so that

$$S = \frac{S}{1 + r_h r_\rho} + \frac{\Delta t}{T_r}\left(S_{ref} - S\right). \quad (7)$$

This relation is reduced to the salinity in the steady state

$$S_{NBC} = \frac{\left(1 + r_h r_\rho\right)\Delta t / T_r}{\left(1 + r_h r_\rho\right)\Delta t / T_r + r_h r_\rho} S_{ref}. \quad (8)$$

Since $r_h r_\rho \ll 1$, in a model forced by the natural boundary condition, salinity in the grid box is

$$S_{NBC} \simeq \frac{1}{1+r_T r_\rho} S_{ref} \,. \tag{9}$$

Thus, the salinity deviation from the global mean is

$$\Delta S_{NBC} = S_{NBC} - S_{ref} \simeq -\frac{r_T r_\rho}{1+r_T r_\rho} S_{ref} \simeq -r_T r_\rho \left(1 - r_T r_\rho\right) S_{ref} \,. \tag{10}$$

As will be discussed shortly, sea surface salinity deviation from the mean value can be calculated from the salinity climatology of the world oceans. Since the value of r_ρ is fixed, the time ratio of r_T for each grid point can be treated an independent parameter of the model; thus, instead of using parameters, such as ω, h, T_f, or T_r, for each surface grid point in the world ocean, a single parameter r_T can be inferred from the surface salinity distribution, and this can be used to estimate the potential error of surface salinity simulated from other models, as will be discussed shortly.

B. VNBC (Virtual Natural Boundary Condition)

As the second example, we discussed a boundary condition, which is most closely related to the natural boundary condition discussed above. The major difference is that the freshwater flux associated with precipitation (evaporation) is now used in a volume-conserved model. This boundary condition can be called VNBC, i.e., virtual natural boundary condition, or volumetric natural boundary condition. Since the model is a volume-conserving model, salinity in the model is a volumetric concentration of salt, which should be in unit of kg/m^3. This is quite different from the salinity unit used in a mass-conserving model, which is defined as the mass fraction and commonly accepted as a non-dimensional quantity, and it sometime called the practical salinity unit. Since salinity errors discussed in this note are related to volume-conserving model, the unit of kg/m^3 is used throughout the whole text, unless specified differently.

The salinity conservation equation, corresponding to Eq. (3) in the previous case, is now in the following form

$$\partial_t \left[\left(h+\eta\right) S \right] + \nabla_{3d} \cdot \left[\left(h+\eta\right) \vec{v} S \right] + \nabla_{3d} \cdot \left(\kappa \nabla_{3d} S \right) = 0 \,. \tag{11}$$

Note that although the term of natural boundary condition was first introduced in Huang (1993), strictly speaking, the model used by Huang is based on the Boussinesq approximations, so that the boundary condition discussed in that study belongs to this category. From Eq. (11), salinity of the upper box after mixing is

$$S_1 = \frac{hS_0}{h+dh} + \frac{\Delta t}{T_r}\left(S_{ref} - S_1\right) = S_0 \frac{1}{1+r_h} + \frac{\Delta t}{T_r}\left(S_{ref} - S_1\right). \tag{12}$$

At the equilibrium state, $S_1 = S_0 = S$; thus, the salinity in the final state is

$$S_{VNBC} = \frac{(1+r_h)}{(1+r_h)+T_r/T_f} S_{ref} \simeq \frac{1}{1+r_T} S_{ref}\,. \tag{13}$$

As a result, salinity deviation from the reference value is

$$\Delta S_{VNBC} = S_{VNBC} - S_{ref} = -\frac{r_T}{1+r_T} S_{ref} \simeq -r_T\left(1-r_T+r_T{}^2\right) S_{ref}\,. \tag{14}$$

Thus, the errors introduced by VNBC, as compared to the salinity produced by model under NBC, is

$$\Delta S_{VNBC} - \Delta S_{NBC} \simeq -r_T\left[1-r_T\left(1-r_T\right)-r_\rho\left(1-r_T r_\rho\right)\right] S_{ref} \tag{15}$$

C. VSFI (Virtual Salt Flux, In-situ)

In most numerical models based on volume conservation approximation, the role of freshwater flux associated with evaporation and precipitation has been converted into an equivalent salt flux through the air-sea interface, $S(E-P)$, where S is the in-situ (local) sea surface salinity. Therefore, salinity in a surface box obeys the following balance relation

$$\partial_t[hS] + \nabla_{3d}\cdot[h\vec{v}S] + \nabla_{3d}\cdot(\kappa\nabla_{3d}S) = -PS\,. \tag{16}$$

where P is the annual mean precipitation minus evaporation rate. With this notation, thus, over one time step the total amount of freshwater is $dh = P*\Delta t$. After mixing, salinity in the upper layer box is

$$S_1 = \frac{hS_0 - dhS_0}{h} + \frac{\Delta t}{T_r}\left(S_{ref}-S_1\right) = S_0 - \frac{\Delta t}{T_f}S + \frac{\Delta t}{T_r}\left(S_{ref}-S_1\right). \tag{17}$$

At the steady state, $S_1 = S_0 = S$; thus, salinity in this grid box is

$$S_{VSFI} = \frac{1}{1+r_T} S_{ref} \tag{18}$$

$$\Delta S_{VSFI} \simeq S_{VSFI} - S_{ref} = -\frac{r_T}{1+r_T} S_{ref} \simeq -r_T\left(1-r_T\right) S_{ref} \tag{19}$$

Accordingly, the errors introduced by VSFI, as compared to the salinity produced by model under NBC, is

$$\Delta S_{VSFI} - \Delta S_{NBC} \simeq -r_T\left[1-r_T\left(1-r_T\right)-r_\rho\left(1-r_T r_\rho\right)\right] S_{ref}\,. \tag{20}$$

Under our simple assumption, thus, surface salinity error induced by this boundary condition is the same as that induced by VNBC. One major concern about the VSFI is that this boundary condition can induce a systematic build up of salt in the model ocean. This problem is due to the fact that the global integration of the virtual salt flux defined in this boundary condition is not zero; thus, this boundary condition can induce a systematic increase of salt in the model ocean, which will be discussed shortly.

D. VSFM (Virtual Salt Flux, Mean)

To overcome the systematic increase of salt in the ocean induced by the VSFI formulation, the commonly used salinity boundary condition for volume-conserving models is a virtual salt flux condition based on a global mean salinity, instead of the local salinity used on the right-hand side of Eq. (16). Accordingly, the salinity balance in a grid box is as follows

$$\partial_t \left[hS\right] + \nabla_{3d} \cdot \left[h\vec{v}S\right] + \nabla_{3d} \cdot \left(\kappa \nabla_{3d} S\right) = -PS_{ref} \ . \tag{21}$$

Salinity after mixing in the upper layer box is

$$S_1 = \frac{hS_0 - dhS_{ref}}{h} + \frac{\Delta t}{T_r}\left(S_{ref} - S_1\right) = S_0 - r_h S_{ref} + \frac{\Delta t}{T_r}\left(S_{ref} - S_1\right). \tag{22}$$

In the steady state, the salinity is

$$S_{VSFM} = \left(1 - r_T\right) S_{ref} \ , \tag{23}$$

$$\Delta S_{VSFM} = S_{VSFM} - S_{ref} = -r_T S_{ref} \tag{24}$$

As a result, the errors introduced by VSFM, as compared to the salinity produced by model under NBC, is

$$\Delta S_{VSFM} - \Delta S_{NBC} = -r_T \left[1 - r_\rho \left(1 - r_T r_\rho\right)\right] S_{ref} \tag{25}$$

Although this boundary condition overcomes the problem of salt building up, it may introduce a systematic bias of salinity forcing. Whenever the local salinity is higher (lower) than the global mean surface salinity, the effect of local freshwater flux is exaggerated by a factor of $\left(S_{ss} / \overline{S}_{ss} - 1\right)$. As will be shown shortly, this boundary condition can lead to a surface salinity field which is lower than the observed field.

3. Application to the world oceans.

We first calculate the sea surface salinity deviation from the global mean surface salinity, as shown in Fig. 1. The global mean surface salinity is $\overline{S}_{ss} = 34.7832$, and the corresponding factor is $r_\rho = 0.974$. From this figure, we can use the local sea surface salinity to calculate the factor

$$\alpha = S_{ss}\left(\lambda, \theta\right) / \overline{S}_{ss} - 1 \tag{26}$$

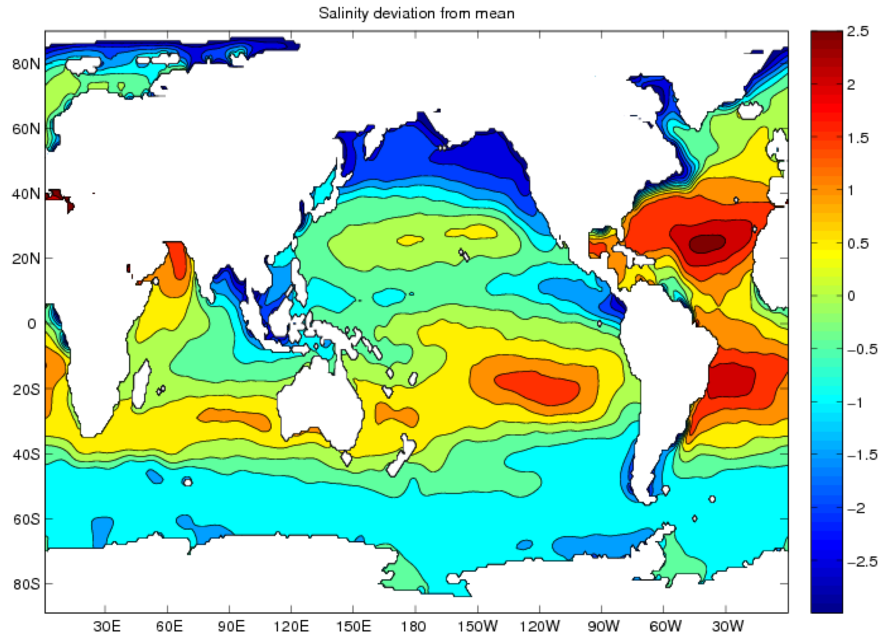

Fig. 1. Surface salinity distribution, deviation from the global mean, based on WOA01 dataset.

We will assume that a mass-conserving model can accurately simulate the surface salinity, thus, the sea surface salinity obtained from the natural boundary condition is represented by Eq. (9). We take the global mean (over the whole depth) salinity $\overline{S} = 34.7173$ as the reference salinity S_{ref} in these models. From Eq. (9), we can determine the factor r_T for each $1^o \times 1^o$ grid box, Fig. 2. For regimes with low surface salinity, such as the northern North Pacific Ocean and the Arctic Ocean, $r_T > 0$. On the other hand, for regimes associated with high-than-average the equivalent precipitation rate $\omega < 0$, and $r_T < 0$, indicating that evaporation excesses precipitation in these grid boxes.

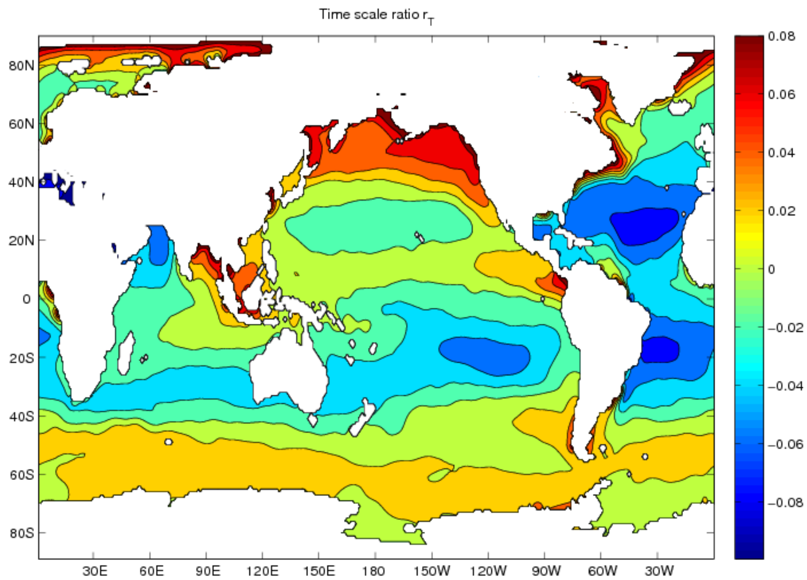

Fig. 2. Time scale ratio r_T diagnosed from surface salinity in the world oceans.

Using Eqs. (15), (20) and (25), we can make the theoretical estimate of salinity errors induced by the inaccurate boundary conditions. Thus, there is no need to select parameters T_r and T_f. Instead, using the observed surface salinity distribution, we can infer a single parameter, the time scale ration r_T, and use it to make estimate of the potential error bound for models under different upper boundary conditions for salinity.

These simple models provide the horizontal distribution of the estimated salinity errors. We emphasize that these estimates are not expected to be very accurate; however, they may serve as certain bound of salinity errors induced by inaccurate boundary conditions, such as the commonly used VNBC, VSFI and VSFM.

Surface salinity errors induced by the volume conservation approximation can be quite large in the oceans, Figs. 3 and 4. In particular, under the VNBC, the surface salinity error in the North Atlantic may be on the order of 0.20, Fig. 3. On the other hand, in the Northern North Pacific Ocean and the Arctic Ocean, a model under VNBC may induce a negative salinity bias on the order of 0.15 t 0.2, which is not negligible. Over the world oceans, the maximum salinity errors induced by VNBC are estimated at 0.286 (kg/m^3) in the Mediterranean Sea and minimum salinity errors are estimated at -0.223 (kg/m^3) in the Arctic Ocean, Fig. 2 and Table 1.

Type of boundary condition	Mean artificial source of salt ($kg/m^2/yr$)	Surface salinity error				Meridional Density difference in North Atlantic (kg/m^3)
		Global		North Atlantic		
		Maximum	Minimum	Maximum	Minimum	
VNBC	0	0.286	-0.223	0.091	-0.210	0.230

VSFM	0	0.006	-1.000	0.006	-0.940	0.719
VSFI	0.000135	0.286	-0.223	0.091	-0.210	0.230

Table 1. Errors induced in volume-conservation models, as compared with a mass-conserving model.

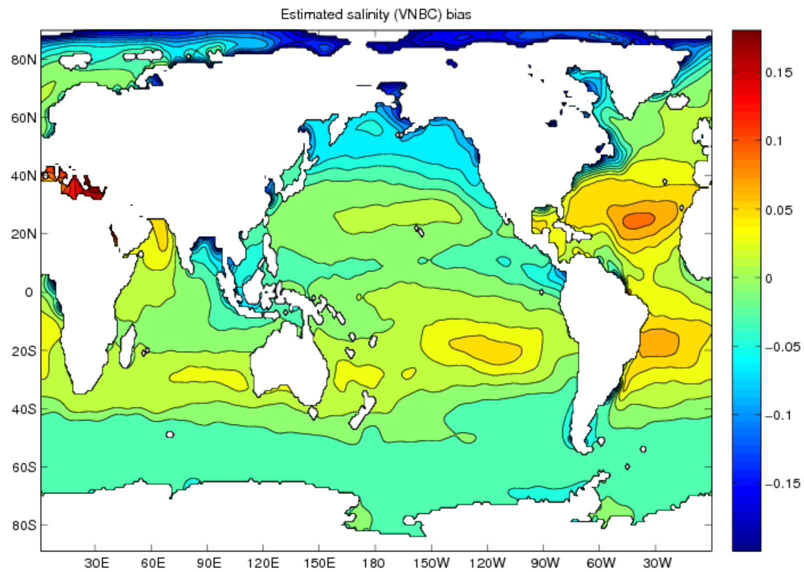

Fig. 3. Surface salinity errors induced in a volume-conserving model under VNBC.

As shown in Table 1, salinity errors induced by VNBC may induce a north-south salinity difference on the order of 0.30kg/m^3 in the North Atlantic Ocean, and this is equivalent to approximately a meridional density difference of 0.23 kg/m^3. Under the modern condition, the mean density difference in the North Atlantic Ocean is approximately 3.4 kg/m^3. Therefore, the error in salinity distribution induces a meridional density difference on the order of 7%. Such a density error is not negligible, in particular for studying the sensitivity of the meridional overturning circulation which is rather close to a possible bifurcation point.

As shown above, surface salinity errors induced by VSFI is approximately the same as that in VNBC. However, there is a systematic accumulation of salt in the ocean, which will be discussed in details in Section 4.

In order to avoid such an artificial salinity build-up in the model, the virtual salt flux should be defined by the global mean surface salinity, as discussed by Huang (1993). Although using the global mean surface salinity overcomes the problem of salt build-up, it also induces surface salinity deviation from that obtained from model under NBC, Fig. 4. It is interesting to note that surface salinity errors are mostly negative, i.e., surface salinity simulated by a volume-conserving model under VSFM, in general, is lower than that obtained from a model under NBC. The largest negative error appears in the northern North Pacific Ocean and Arctic Ocean. (Note that in order to show the solution in the North Atlantic Ocean, large negative salinity errors in the Arctic Ocean with errors lower than -0.2 were omitted in Fig. 4.)

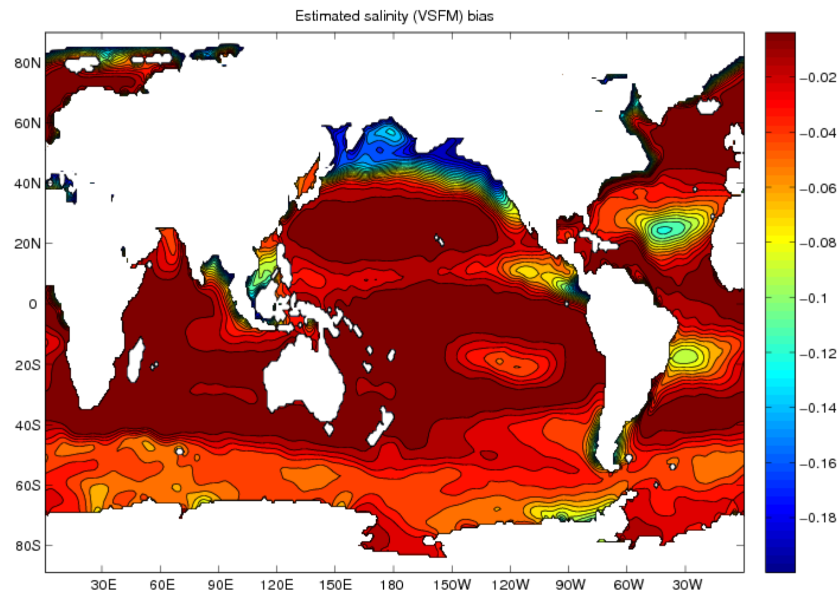

Fig. 4. Surface salinity errors induced in a volume-conserving model under VSFM.

An interesting phenomenon is the salinity errors in the middle of the subtropical gyres in the Atlantic Ocean are negative. In particular, the negative salinity errors in the middle of the subtropical North Atlantic Ocean are on the order of 0.15. In fact, in many regimes with evaporation dominating, surface salinity errors induced by model under VSFM are quite large and have a negative sign, Fig. 5.

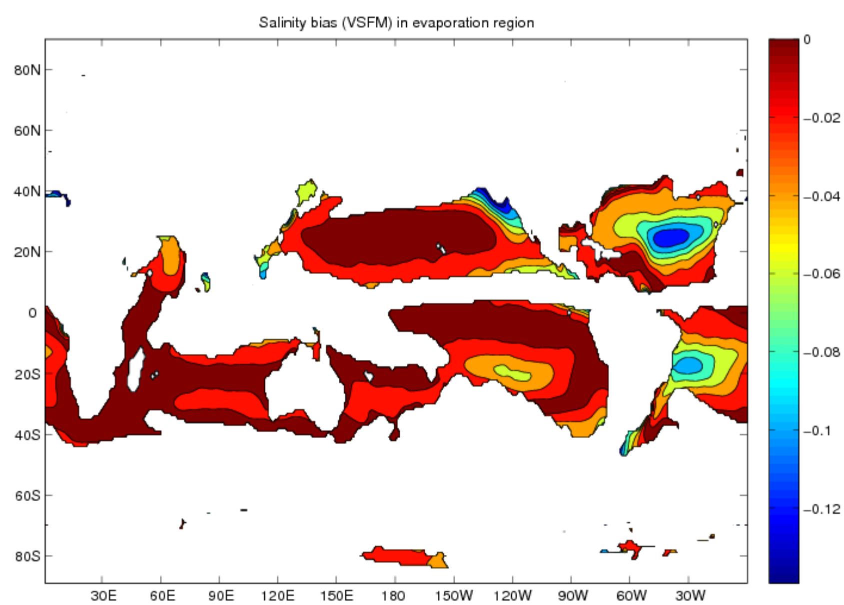

Fig. 5. Surface salinity errors induced under VSFM for the regime of net evaporation.

This phenomenon can be explained as follows. According to Eqs. (7) and (21), surface salinity deviation from the reference value in model under NBC and VSFM are $\Delta S_{VSFM} = -r_T S_{ref}$ and $\Delta S_{NBC} = -r_T r_\rho S_{ref} / (1 + r_T r_\rho)$ respectively. Recall that

$r_T < 0$ for regimes with evaporation overpowering precipitation. Thus, the ratio of surface salinity anomaly relative to the referred salinity anomaly is $\Delta S_{VSFM} / \Delta S_{NBC} = 1/r_\rho + r_T$. As a result, when $r_T > 1/r_\rho \simeq 1.027$, positive salinity anomaly over regime of evaporation obtained from the model under VSFM is lower than that obtained from the model under NBC. The regimes satisfying this criterion are shown in Fig. 6. The most outstanding regimes include: the subtropical gyres in the Atlantic Ocean, the South Pacific Ocean, and the Arabic Sea. This overall negative surface salinity anomaly induced by VSFM gives rise to a global mean surface salinity anomaly of 0.035 kg/m^3, which may not be negligible.

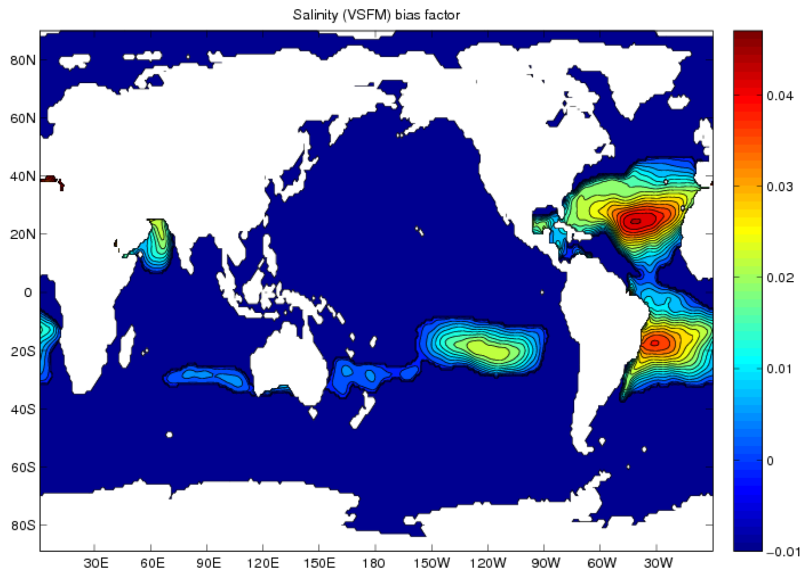

Fig. 6. Surface salinity bias factor under VSFM.

4. Artificial salinity increase associated with the virtual freshwater flux condition

Another major problem associated with VSFI discussed above is that the total amount of salt in the model ocean is gradually increased. This boundary condition can be simplified as

$$F_S = S(E-P). \tag{27}$$

There is an outstanding feature in the world oceans that high (low) salinity water on the sea surface is closely related to high evaporation (precipitation); thus, the global mean of the salt flux in Eq. (21) is not zero; instead, it has a positive value. Assume that the annual mean evaporation minus precipitation, plus river run-off, integrated over the world oceans is nearly balanced, the global mean of this salt flux is reduced to

$$\overline{F}_S = \overline{S(E-P)} = \overline{(S-\overline{S})(E-P)}. \tag{28}$$

Based on the Da Silver et al. (1994) dataset of monthly mean evaporation and precipitation, one can find the annual mean evaporation minus precipitation, as shown in Fig. 7.

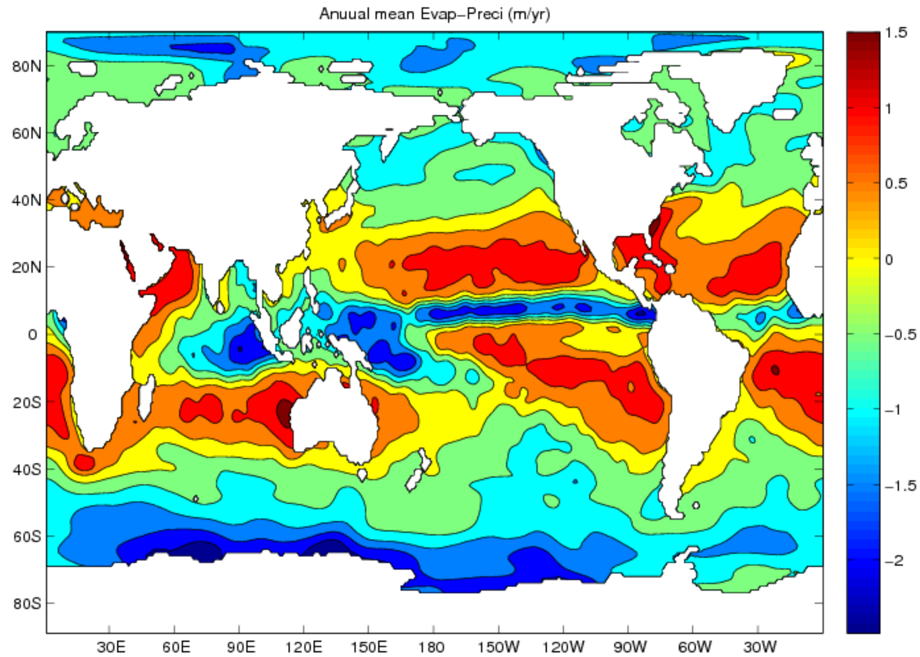

Fig. 7. Annual mean evaporation minus precipitation (m/yr).

Multiplying this field with the global salinity (deviation from the mean) as shown in Fig. 1, leads to a distribution of the artificial source of salt associated with this type of boundary condition. The largest artificial sources of salt appear in the subtropical gyres of the Atlantic Ocean and other places, Fig. 8. The global mean rate of this artificial source of salt is approximately 0.492 $kg/m^2/yr$. For an ocean with mean depth of 3668 m, this leads to a salinity increase of 0.000135$kg/m^3/yr$. As will be shown in Section 5, salinity build-up happens in many existing oceanic general circulation models, and such a problem may be a major deficit of these models for the study of global sea level changes.

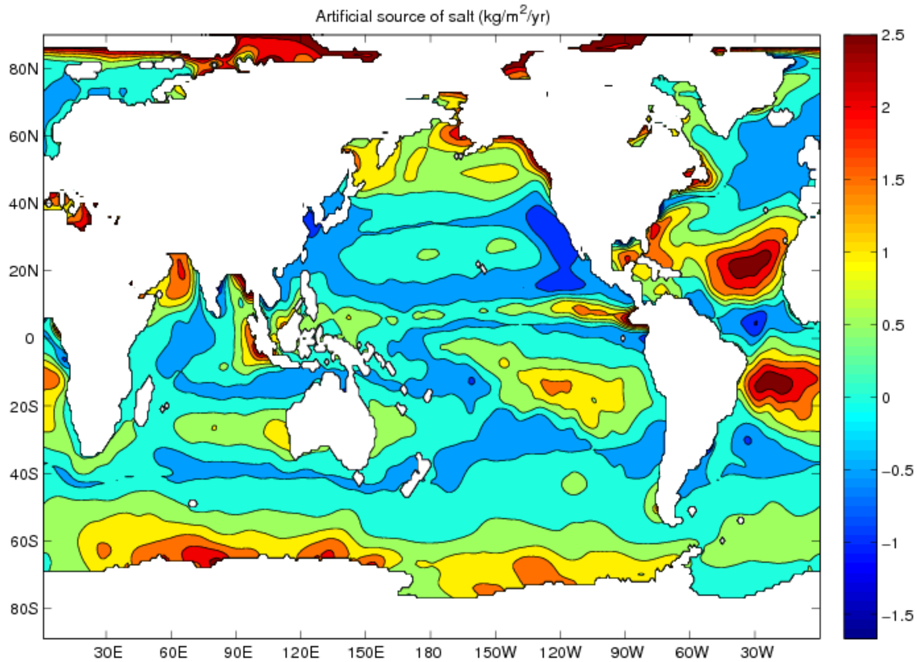

Fig. 8. Artificial source of salt associated with the virtual freshwater flux.

5. Salinity obtained from data assimilation based on a Boussinesq model may have a systematic negative bias

Due to the uncertainties of parameterizing sub-scale processes in the oceans, data assimilation has been used as a powerful tool in simulation and prediction of the oceanic environment. For a long time, there was technically a grand challenge in collecting routine salinity sampling in the world oceans. Thanks to the ARGO project, there is now an international observation net which collects salinity data from the upper 2km of the world ocean. Although the in-situ salinity measurements are great help for salinity simulation and prediction, there is a potentially major problem. Since ARGO floats can sample the salinity for the upper 2km only, assimilating such salinity data into a volume-conserving model may induce a systematic loss of salt in the model ocean, as will be discussed in the following section.

Assume there are two boxes with volume of $V_1 = V_2 = 1m^3$, and each of them has salinity of $S_1 = 35 + \delta S > S_2 = 35$. In the following discussion, we will assume that temperature is held at a fixed value $T_0 = 5.2^o C$; thus the total mass of these two box are

$$m_1 = V_1 \rho_1 = V_1 \rho_f (1 + \beta S_1); \tag{29}$$

$$m_2 = V_2 \rho_2 = V_2 \rho_f (1 + \beta S_2). \tag{30}$$

The total amount of salt in each box is

$$\mathsf{S}_1 = V_1 \rho_1 S_1 = V_1 \rho_f (1 + \beta S_1) S_1 \,; \tag{31}$$

$$\mathsf{S}_2 = V_2 \rho_2 S_2 = V_2 \rho_f (1 + \beta S_2) S_2 \,. \tag{32}$$

The new salinity after mixing these two boxes of water is

$$S = \frac{\mathsf{S}_1 + \mathsf{S}_2}{m_1 + m_2} = \frac{S_1 V_1 \rho_1 + S_2 V_2 \rho_2}{V_1 \rho_1 + V_2 \rho_2} \,. \tag{33}$$

On the other hand, salinity after mixing of these two boxes under the volume conservation assumption is

$$S_v = \frac{S_1 V_1 + S_2 V_2}{V_1 + V_2} \,. \tag{34}$$

Since we assume these two boxes have the same volume, it is easy to see that

$$S = \frac{S_1 \rho_1 + S_2 \rho_2}{\rho_1 + \rho_2} > \frac{S_1 + S_2}{2} = S_v \,;$$

thus, after mixing salinity in a mass conserving model is larger than that obtained from a volume conserving model because salty water has a slightly larger weight (density).

This point can be shown by the following example. There are two boxes of sea water, with salinity 30 and 35. If we use the volume-conserving model, the salinity after mixing is 32.5. On the other hand, salinity after mixing in the mass-conserving model is higher due to the density of water density:

$$\overline{S}_{m-model} = \frac{\rho_1 S_1 + \rho_2 S_2}{\rho_1 + \rho_2} = 32.5046 \,.$$

Since salinity data can be collected from parts of the world oceans only, primarily in the upper ocean, salinity in the un-sampled parts of the ocean is adjusted through mixing process. As shown above, however, there is a systematic bias of salinity loss due to the assumption of volume conservation instead of the mass and salt conservation. Although this bias may not be very large in the beginning of the data assimilation. After many time steps, the model ocean will gradually shift toward a state of freshness bias. This systematic bias can be overcome with a mass conserving model only.

The results of this mixing experiment are shown in Fig. 9. Assume that the volume of box 2 is assumed to be larger than volume of box 1, and their ratio is plotted as the vertical axis in Fig. 9. It is clear that for all cases, the salinity after mixing obtained from a mass conserving model is always larger than that obtained from a volume-conserving model.

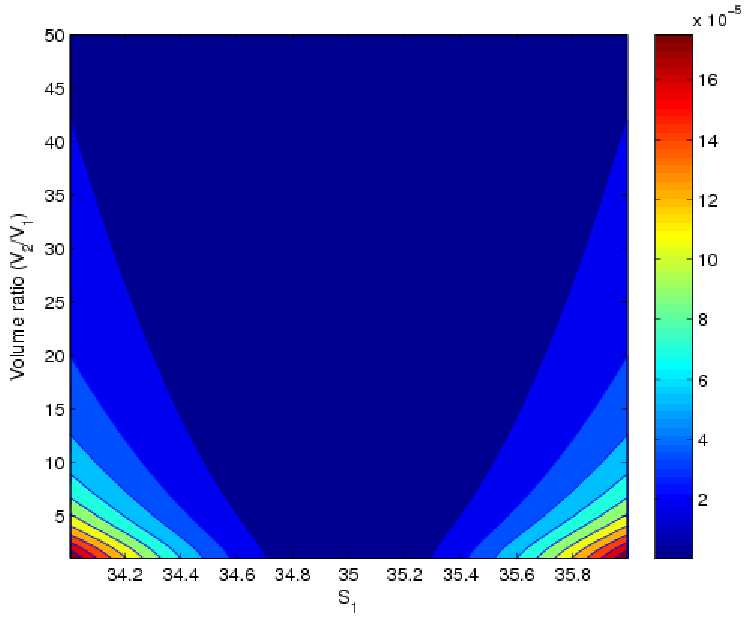

Fig. 9. The salinity difference (salinity of a mass-conserving model subtracting that a volume-conserving model).

To illustrate the problem associated with salinity data assimilation in the volume-conserving model, we use a two-by-two box model, Fig. 10. On the left-hand (right-hand) side, the model is forced by precipitation (evaporation) with a rate of dh per year. The upper layer boxes have a thickness of $h_u = 100m$, and the lower layer boxes have a thickness of $h_b = 1000m$.

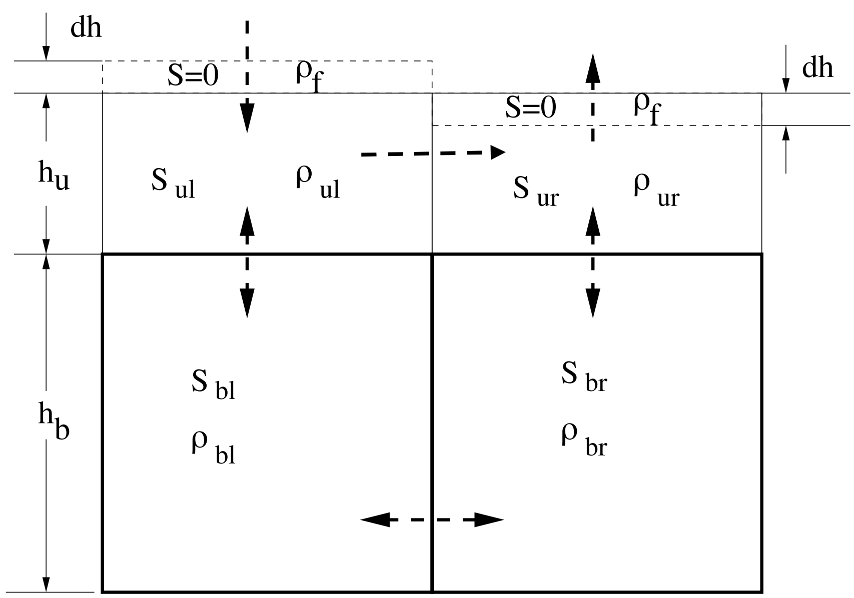

Fig. 10. Sketch of a 2x2 box model, illustrating the salinity balance in mass-conserving and volume-conserving models.

The fresh water from precipitation is assumed to be mixed uniformly with the salty water in the upper layer box. In a mass-conserving model (m-model hereafter), the salinity after mixing is

$$S_{ul,m,1} = \frac{h_u \rho_{ul,m,0} S_{ul,m,0}}{h_u \rho_{ul,m,0} + dh_{ul} \rho_f} \tag{35}$$

where $S_{ul,m,0}$ and $S_{ul,m,1}$ are the salinity in the upper left box of the m-model before and after mixing, $\rho_{ul,m,0} = \rho_f \left(1 + \beta S_{ul,m,0}\right)$ is the corresponding density before mixing, $dh_{ul} = \omega \Delta t$ is the freshwater flux due to precipitation. After mixing, part of the upper layer water is spread into other parts of the ocean during the adjustment process. As discussed by Huang and Jin (2002), most of the extra mass associated with precipitation is dispersed and there is a very week barotropic signal left behind, which will be ignored in the following discussion. Thus, we assume that the extra mass associated with precipitation is transported into the upper box of the right column which is exposed to the evaporation of the same strength, i.e., $dh_{ur} = -dh_{ul} = -\omega \Delta t$. Therefore, salinity of the water column after the second stage of mixing is

$$S_{bl,m,1} = \frac{h_u \rho_{ul,m,1} S_{ul,1} + h_b \rho_{bl,m,0} S_{bl,m,0}}{h_u \rho_{ul,m,1} + h_b \rho_{bl,m,0}} \tag{36}$$

where $S_{bl,m,0}$ and $S_{bl,m,1}$ are the salinity in the bottom left box of the m-model before and after mixing, $\rho_{bl,m,0} = \rho_f \left(1 + \beta S_{bl,m,0}\right)$ is the corresponding density before mixing.

At this point, we add on another box model which is a volume-conserving model (v-model hereafter) and has the same volumetric structure as the m-model. This model applies the data assimilation technical to predict the time evolution of salinity in the model ocean. Specifically, we assume that salinity obtained from the m-model in the upper layer is now taken as 'observation', and implemented into the v-model. There are many ways of assimilating the data into a model. In this study, we assume the simplest way of assimilation by forcing the upper layer box of the volume conserving model has the same salinity as that from the m-model.

Therefore, the salinity in the lower layer of the v-model after mixing is

$$S_{bl,v,1} = \frac{h_u \rho_{ul,1} S_{ul,1} + h_b S_{bl,m,0}}{h_u + h_b} \tag{37}$$

The essential difference between the m-model and the v-model is the principle of conservation of mass or volume, which is reflected into the equations discussed above.

The salinity balance on the right-hand side of the models is quite similar, except that this part of the model ocean is forced by a constant evaporation, with a rate of dh per year. The corresponding equations are quite similar to those for the left column, with the only difference in the sign of dh; thus, we will not include here.

Under the constant precipitation or evaporation, salinity in the left (right) column of the model constantly declines (increases). In order to maintain a nearly constant salinity in the model, we apply a mixing scheme between the left and right columns of each model, i.e.,

$$S_{l,m,1} = \frac{\alpha S_{bl,m,1}\rho_{bl,m,1} + (1-\alpha) S_{br,m,1}\rho_{br,m,1}}{\alpha\rho_{bl,m,1} + (1-\alpha)\rho_{br,m,1}} \tag{38}$$

$$S_{r,m,1} = \frac{\alpha S_{br,m,1}\rho_{br,m,1} + (1-\alpha) S_{bl,m,1}\rho_{bl,m,1}}{\alpha\rho_{br,m,1} + (1-\alpha)\rho_{bl,m,1}} \tag{39}$$

where $\alpha = 0.95$ is a coefficient, which can be selected but it is fixed for each model run. On the other hand, salinity after mixing between left and right column in the v-model is a simple weighted average of the salinity in each column

$$S_{l,v,1} = \alpha S_{bl,v,1} + (1-\alpha) S_{br,v,1} \tag{40}$$

$$S_{r,v,1} = \alpha S_{br,v,1} + (1-\alpha) S_{bl,v,1} \tag{41}$$

As time progresses, salinity in the left (right) column declines (increases), left panel of Fig. 11. Our major concern is the salinity errors induced by the volume conservation principle made used in the v-model. As shown in the right panel of Fig. 11, salinity deficit in the v-model grows with time, and over 50 yr, it reaches the order of 0.0005. It is clear such a salinity error is not acceptable.

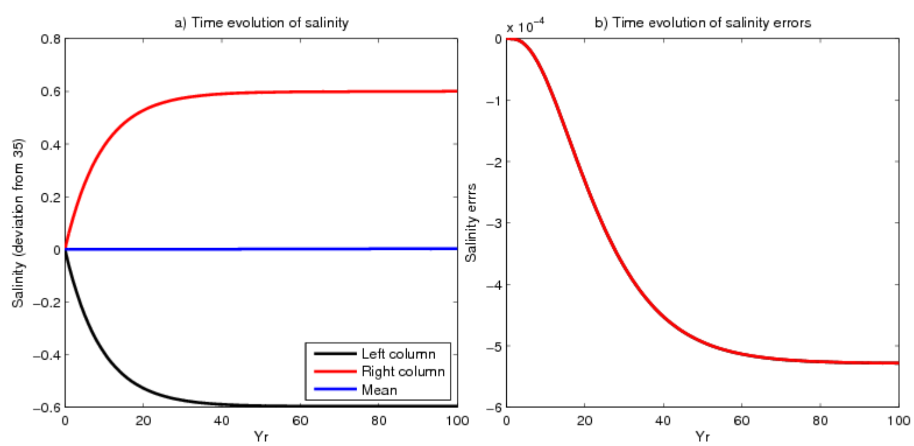

Fig. 11. Time evolution of salinity and the associated error in the v-model, with $\alpha = 0.95$.

The salinity error predicted by this simple model depends on the choice of α. At the limit of $\alpha = 1$, there is no communication between the deep boxes of the left and right columns, so that salinity in these two columns drift away, with the salinity in the

column exposed to precipitation (evaporation) goes down (increases upward) rapidly, left panel of Fig. 12. The corresponding salinity error is no rather small.
On the other hand, if the mixing ratio is reduced, the salinity error becomes large. At the range of $\alpha = 0.975$, the error is a maximal, with a value of approximately -0.0018 (kg/m^3). Such a large error is not acceptable.

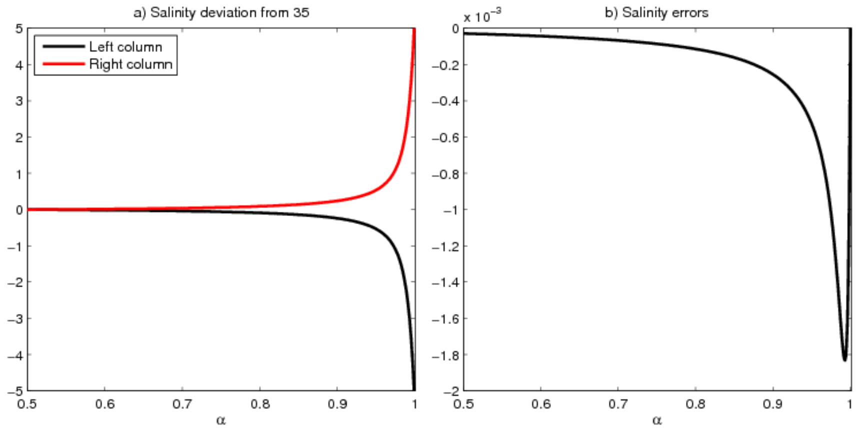

Fig. 12. Parameter sensitivity of salinity and its errors induced in the v-model.

The essential point in this model is that salinity observation is only available for the upper ocean, mostly through ARGO program or satellite salinity measurement in the future. Under such a constraint, a volume-conserving model applied to data assimilation will push a salt loss in the model ocean. Thus, unfortunately, mass-conserving model may be the only way to overcome these problems with salinity.

6. Examples taken from ECCO2 and SODA datasets

The ECCO model has been developed over the past decades, and there are many different products from the model. Our discussion here is focused on a specific case, ECCO2, which is forced by a new boundary condition since 2002, and over the past several years, the global mean salinity increased approximately 0.0025 (kg/m^3) based on (http://ecco2.jpl.gov/data1/cube/cube84/SALTanom/ , Ms. Ru Chen kindly provided salinity analysis based on the ECCO2 data). Over the 16 years of model simulation, the equivalent sea level drops nearly 35 cm. Such a large drop in sea level makes the model useless for the study of global sea level change. The exact way of model setting up in the ECCO2 remains unclear at this point, and will be explored. Nevertheless, our simple theoretical estimate is about half of the rate diagnosed from the ECCO data.

The most updated version of SODA comes the period of Jan. 1958- Dec. 2001. We define a mean salinity, S_{bar}, as the salinity averaged over the period from 1974 to 2001. The time evolution of the global mean salinity deviation from this mean salinity shows a gradual increase of the total amount of salt in the world ocean, Fig. 13. All data plotted here is kindly available from Dr. Jiang Hua.

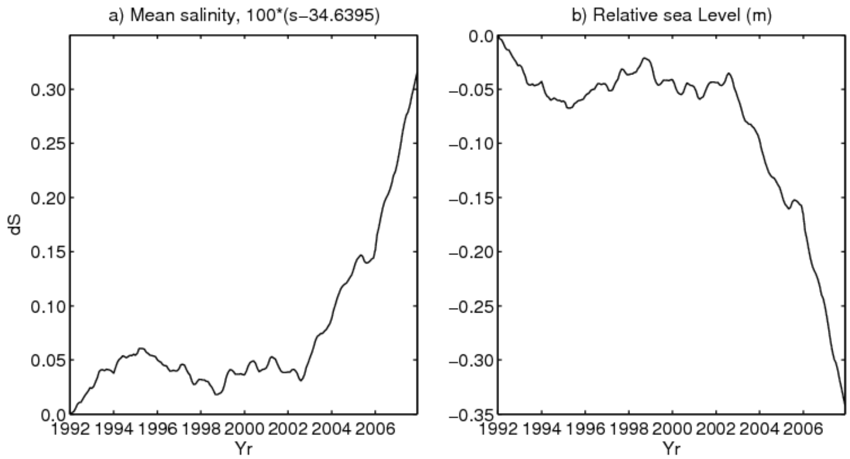

Fig. 13. Salinity and sea level diagnosed from ECCO2 output. Left panel: global mean salinity, right panel: change in the relative sea level (m).

To interpret this result, we have the following estimate. Assume that the global mean salinity perturbation is $\delta S = 10^{-4}(kg/m^3)$ (Note that salinity in volume conserving model is in unit of mass per volume); then the corresponding freshwater flux added to the ocean is

$$\delta h = -\frac{\delta S}{\overline{S}} H = -\frac{10^{-4}}{34.7} 3.67 \cdot 10^3 = -1.06 \cdot 10^{-2} \left(m\right)$$

This means an increase of 10^{-4} (kg/m^3) in global mean salinity is equivalent to taking out layer of 1 cm of freshwater from the world ocean.

Using this estimate, the mean salinity in SODA goes up for 180x10^{-4} (kg/m^3), so that the global mean sea level of SODA model should declines for 1.8 m over the period of the model integration, Fig. 14.

Apparently, the model continues to gain salt, Figs. 14 and 15. In particular, from year 1974 to 2001, the global mean salinity in the model seems to gain about 0.0004 (kg/m^3). This implies a decline of mean sea level on the order of 4cm over this time period. This seems to be opposite to the well-known trend of freshwater addition to the glacier melting, which is estimated to be on the order of 2 mm/yr, and equivalent to 5 cm over the period of 25 years.

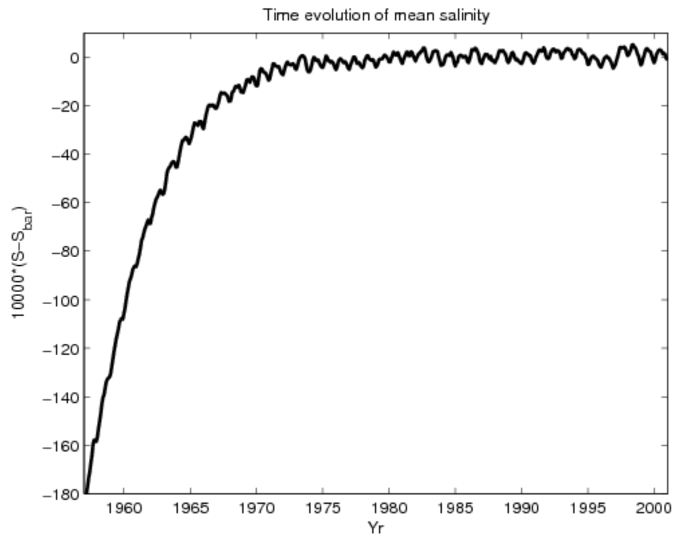

Fig. 14. Time evolution of the global mean salinity in SODA, started from year 1958.

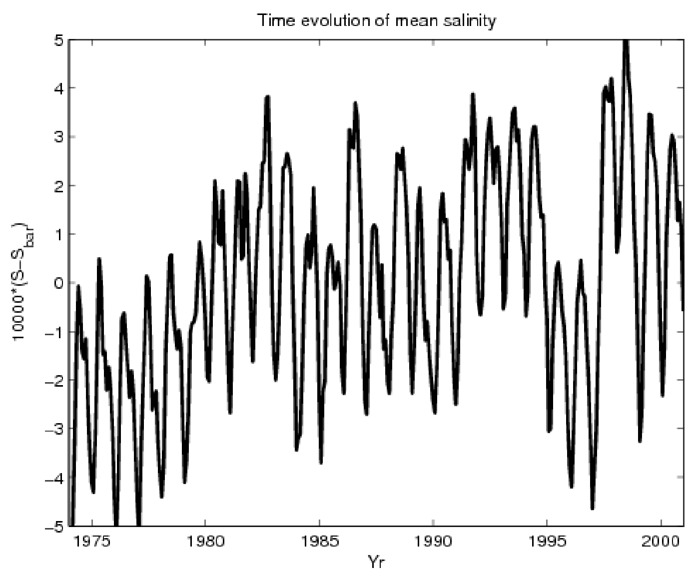

Fig. 15. A close-up view of the global mean salinity, diagnosed from SODA.

The mean annual cycle has the amplitude of 0.0004 (kg/m^3), Fig. 16. The annual cycle of salinity is equivalent to the amplitude of the annual freshwater flux of 4 cm/yr through the air-sea interface. This is about twice the amplitude of the annual mean sea level change (about 2 cm) as identified from satellite altimetry data.

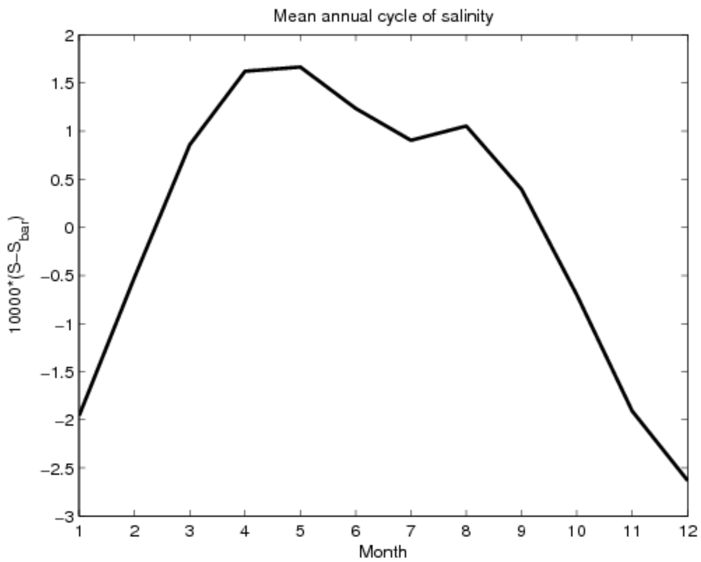

Fig. 16. The annual cycle of the global mean salinity for the period of 1974-2001, diagnosed from SODA.

7. Conclusion

In this note we explored the shortfalls of salinity simulation in models based on volume conservation, instead of mass conservation. We first set the salinity accuracy required for identify global sea level change as 1.0E-5 to 1.0E-4, and used this as a criterion to measure whether salinity errors produced in numerical models is tolerable.

Using simple box models, we showed that salinity errors induced by upper boundary conditions, such as VNBC, VSFI, and VSFM, for models based on volume conservation may not be acceptable. Furthermore, model under VSFI can induce a systematic build-up of salt in the model, and such a salt build-up is against the recent trend in freshening of the global ocean due to land-base glacier melting.

Although data assimilation including in-situ salinity measurements from ARGO and satellite salinometer may help to improve salinity simulation, a model based on volume conservation and salinity data from the upper ocean only cannot overcome the systematic bias of salinity after mixing.

These pitfalls of models based on volume conservation may not be easily overcome, and model based on mass conservation may be the only logic choice of overcome of such problems.

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