Simulation of underwater sound to understand processes is an indispensable tool in modern oceanography.

Lobsters, icebergs, and submarines have little in common except that they produce sound, like many other marine occupants. Noisy occupants include animals (from shrimp to whales), geophysical phenomena (from earthquakes to storms), and man's devices (from ships to energy turbines). Together, they create an underwater cacophony, now called the marine soundscape (Miksis-Olds et al., 2018). Interestingly, even silent things (such as a piece of muddy seabed or a parcel of warm water) may impact the soundscape because they affect the sound propagation.

Although sound provides a great deal of information about the underwater environment, unraveling and using the underwater clamor is not at all simple. In particular, one needs to understand the sound propagation. An excellent tool for understanding is computer modeling.

Computing simulated sound fields to understand sensed underwater sound is now a common practice in ocean science and engineering. The value for naval defense activities is perhaps obvious, but these simulations are finding a growing number of research applications. Applications include inversion and inference, study of marine fauna, system performance predictions, improved source localization, and improved navigation.

Accurate, detailed simulations of underwater sound can also motivate research expeditions by uncovering propagation behavior that may be difficult to tease out of sparse untargeted acoustic datasets but that are visible in computer simulations and then provable with targeted data collection. For our purposes, simulations mean computed predictions of spatial sound fields (amplitude and phase) from specified sources or sound pressure time series predictions for known emitted waveforms.

Sound Field Structure

Figure 1 shows an example computed field of harmonic 1,100-Hz sound refracting away from a layering anomaly in a shallow sea. The highly structured field that results from sound undergoing dispersion and refraction in this deceptively simple environment is of typical complexity for underwater sound. Underwater sound computation uses many approaches, partly because the complexity does not yield to any single approach (Jensen et al., 2011). In this article, we explore how the oceanographic community (as opposed to sonar system users) came to adopt computed sound simulation as a primary tool, what research it enables, and to what research it is indispensable.
Major reasons that sound fields in water are so frequently numerically computed are that the sound speed in the governing wave equation has a four-dimensionally varying nature with large spectral and dynamic ranges of variability, and the boundary conditions are applied at a shaped sea floor. The possible states of the sound field are thus endless. A few situations like water of uniform temperature over a flat sea floor can be solved analytically with fair accuracy, but to move forward, two-dimensional (2-D) and three-dimensional (3-D) numerical solutions have proven essential.

Simulation as a Tool
Faced with the vastness of the ocean and the richness of its features and physical processes, oceanographers use whatever tools or methodologies are available. These tools can be divided into three categories: observation, theory, and simulation. Many discoveries are made using a combination of these.

In ocean acoustics, the third of these, simulation, arguably began with the discovery in the 1940s of the ocean sound channel (sound fixing and ranging [SOFAR] channel) that allows sound to efficiently travel a long distance in nonpolar seas, away from the ocean surface and bottom. At that time, mathematical ray tracing, computed using some clever approximations, was used to predict sound propagation behavior. The ray trajectories undulate vertically in the channel, with sound speed gradients controlling the refraction. The predictions were rudimentary by today’s standards but were simulations nonetheless. Simulated propagation has changed in the last 40 years to be dominated by computational simulations that allow full-wave physics, moving on from the ray model of wave propagation (geometric model) dating back to Newton and transforming how oceanography involving acoustics is tackled.

Computations foster progress in at least two ways: stimulating discoveries and enabling better outcomes of data-based research. One example of the importance of simulation is that the Ocean Acoustics Library website (oalib-acoustics.org), which was conceived by the US Office of Naval Research in the 1990s and then developed and hosted for many years by Dr. Michael Porter, prominently features computational acoustic codes in its collection. Researchers from around the world use those codes, and others, with regularity.

Stimulation of Reason by Observation
or Simulation
The role of propagation simulation in wave-based remote sensing is clear to most. An early application of this was the use of seismic wave modeling to locate earthquakes from recorded ground motion. This advanced, eventually yielding joint solutions for earth structure, fault locations, shapes, and motions. Joint data/model analyses like this may form the majority of computational acoustics applications to ocean science, but something else can come from computer simulations: pure discovery. The stimulation of reason (theory) by observations is an ancient practice, often leading to discovery, and the interconnection between observational evidence and theory has been subjected to some critical thinking (Bogen, 2017). This includes considering the question of whether high-fidelity simulations of physical systems based on theories and rules constitute observations in their own right, although the predominant community answer is probably “no” at this time. (Bogen states: “…scientists continue to find ways to produce data that can’t be called observational without stretching the term to the point of vagueness.”) Nevertheless, we have found analysis of simulated ocean acoustic fields to stimulate many research directions as hypotheses; some are explored in the field.
Simulation Methods
There are numerous simulation methods in use, and each has strengths and weaknesses. Jensen et al. (2011) present the methods, explain the theory behind them, and provide application examples. Often, the weaknesses stem from the short wavelength of underwater sound with respect to ocean depth and width. For example, the 3-D solution for time-harmonic acoustics, available using the finite-element method for a volume with imposed absorbing or radiating boundaries, is unreasonable for the ocean because many grid points per wavelength are required in many scenarios, and the needed matrix solution methods are challenging for areas large enough to make a reasonable study. Facing this challenge, underwater acousticians have developed and/or refined alternatives. The already mentioned ray method is useful for many purposes. Normal mode and parabolic equation (PE) methods are other primary players for large-scale simulations. Each of the methods is elegantly based on the underlying theory of sound propagation and applied math methods.

Normal Mode Method
This is based on the standard differential equation math method of the separation of variables, where the vertical and horizontal structures of the sound field are given by different functions that are multiplied together to form the full solution. The vertical functions are the normal modes, which are trapped in the ocean waveguide bounded by the surface and the (usually partially absorbing) seabed.

Figure 2 shows mode shapes and how modes disperse, not all propagating with the same group speed. The modes propagate horizontally and can exchange energy (coupled-mode propagation) or not (adiabatic-mode propagation; mode-by-mode energy conservation). The adiabatic approximation gives the correct solution for a flat-bottomed ocean with a uniformly layered seabed, no waves, and a uniformly layered water but gives results with ever-decreasing accuracy as feature complexity is added to approach realistic conditions. The key to applying either technique is ensuring that the errors are acceptably small.

Figure 2. Shallow-water modal dispersion. Top left: an emitted sound waveform such as from a whale (central frequency of 50 Hz). Bottom left: received signals at three ranges (5, 15, and 30 km; \(r\)) after propagation in a shallow ocean of 100 m depth (about 3 wavelengths), wherein the sound interacts continually with the seabed and surface, effectively creating a waveguide. Use of a reference wave speed \((c_0)\) and reduced time \((t - r/c_0)\) places each signal initiation at zero. The interference of slightly upgoing and downgoing waves makes modes. Each mode is a standing wave in the vertical (top right) and propagates horizontally as a cylindrical wave but with a frequency-dependent group speed (bottom right). The scenario of variable group speed is called dispersion. As seen in the simulated waveforms, dispersion tends to lengthen the signal and to separate modes as range increases.
The application of the separation method to waves dates to nineteenth century studies of waves in layered media (e.g., seismic waves) and thence to quantum mechanics, where the modes are the energy states of atoms or molecules. Coupled-mode propagation, with energy exchange as sound moves away from a source, is analogous to time-dependent molecular energy states (think flames, mercury vapor lamps). Confusingly, mode propagation is sometimes treated with ray tracing (e.g. Heaney et al., 1991).

**Parabolic Equation Method**
This uses a trick to solve the Helmholtz equation. This equation results from imposing a single frequency (sine wave in time) while working with the wave equation. Making the further restriction that sound moves toward or away from point source in cylindrical coordinates, or in one direction along one axis for Cartesian coordinates, yields the parabolic wave equation. This has the troublesome square root operator, which requires another approximation before solving is possible. Various approximate forms of the operator are in use.

An interesting example of a discovery by simulation is the example of mode multipath from duct emission. In a study of propagation of sound between two ocean internal waves, which can trap sound between them, both adiabatic mode and 3-D PE simulations were made. Internal waves share physical properties with familiar surface waves, existing because stratified water (denser below) can oscillate around a flat-layered internal condition. Figure 3 shows that the normal modes, which disperse in two ways in the duct, can appear more than once at a distant receiver. In usual shallow-water ocean propagation (i.e., nearly continuous sound interaction with the seabed, low depth to wavelength ratio), the modes disperse in a regular fashion, each mode traveling at a characteristic group speed and appearing only once each. Before this numerical discovery, the double-mode arrivals had been seen and given many speculative explanations. Modal dispersion will come up in Signal-Processing Research and Simulations for Inversion.

**Acoustic Tomography and Thermometry**
Simulated propagation plays a key role in the acoustic sensing of ocean temperature and heat content. In this inverse technique, travel times for sound along known paths are used to estimate the average temperatures along the paths. The formula is \( t = \int_s \left( \frac{1}{c} \right) ds \), with the travel time equaling the along-path integral of the inverse sound speed (\( c \) is sound speed, and the differential \([ds]\) indicates integration along the continuous path from source \([S]\) to receiver \([R]\)). The sound speed is a known function of temperature, pressure and salinity, so this can be approximated as an integral involving temperature. Sensing over short ranges allows for a simple

![Figure 3. Simulation of sound field strength between two ocean internal waves in shallow water, with the waves (part of a three-wave packet) tapering to zero away from the source. The wavy surface marks the (smooth) boundary between warm water above and cold water below. Left: field at one frequency as per Figure 1. Right: arrival time series at a few locations for a pulse-style simulation. Mode one appears twice at bottom right, while it is absent in the frame to the left. From Lin et al., 2009.](image-url)
Ray tracing produces basic paths, but new computational methods give the so-called travel time-sensitivity kernel (Dzieciuch et al., 2013), which maps out locations where propagating sound is sensitive to the sound speed (mapping in a sense “where the sound goes,” although that is a simplified notion for sound propagation). Figure 4 shows a kernel example. The wave nature of sound means that it cannot be sensitive to only an infinitesimal ray but instead responds to a broader zone where the phase is somewhat coherent. A key point is that the way the sound propagates is critical to the inverse problem, and the more precise this can be computed, the better understood the tomographic inversion will be. For an ocean volume of arbitrary sound speed structure $c(x,y,z,t)$, the most trustworthy way to compute the kernel is with a full-wave computational simulation. This is because the kernel is a function of the geometry of the sound field itself (the structure of sound intensity and phase throughout the entire region). Here, the dueling particle and full-wave models of (sound) wave propagation, with Newton’s ray model treating waves as particles, meet again. In this situation, the ray model is a useful tool, but sometimes a better result can be obtained with the full model.

The kernel arguments apply to generalized sound governed by the wave equation. But situations of propagation with strong bottom interaction and distance many, many times the water depth can instead be analyzed using normal modes. In the 1960 Perth, Australia, to Bermuda propagation study, sound from an explosive source (no longer permitted) was recorded near Bermuda about 20,000 km away about 13,360 s later. The arrivals were explained using adiabatic modal propagation (Heaney et al., 1991). The computation of rays on earth for propagating modes was essential to show how the sound moved and sensed an average sound speed (temperature proxy) because without horizontal modal ray refraction from temperature gradients, no sound could pass both south of Africa and north of Brazil. A similar modal study was subsequently performed with a PE (Collins et al., 1995), with different numerical strengths.

**Signal-Processing Research**

Detecting signals of interest, localizing the source point of the signal, and tracking source position over time are tasks common to many wave-based remote-sensing and surveillance systems. Research into improving methods for these using underwater sound has leaned on computational methods.

Detection is the first-order operation. In Figure 1, white areas have low sound energy, and in the presence of noise, sound from the modeled source would not be detectable there. On the other hand, many locations have ample sound for detection. If currents are weak, sound propagation is reciprocal, so that one can see from an image like Figure 1 where sources would be detectable with a receiver at the modeled source position. Once detected, a source can be tracked over time if consistently received. The consistency of sound can be estimated and trackability evaluated by analyzing synthetic sound propagation patterns.

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**Figure 4.** The 2-D time-sensitivity kernels for two different sound frequencies. The source is at left and the receiver is at right and are connected by a sound channel-trapped ray path continuously curving toward the sound speed minimum. (Two paths connect the source and receiver, each corresponding to an arrival.) **Top:** kernel for 250 Hz; **bottom:** kernel for 75 Hz. The bandwidth is 18.75 Hz. The alternating bands of color signify delay or advance of sound with respect to a positive perturbation of sound speed at the location. From Dzieciuch et al., 2013.
Matched-field processing is a way to locate sound sources using receiver arrays. The structure of the sound phase and amplitude between the ocean surface and bottom is intricate across a receiver array and, to a large degree, is a unique function of the source and array locations (Jensen et al., 2011, Chap. 10). To locate the source, the received pattern is compared with patterns synthesized for all candidate source positions. Locations with good matches are considered likely source locations. The environmental conditions are usually complex enough that the patterns are best made with numerical propagation models, but this can be computationally expensive (more on this in Simulations for Inversion).

Physics-based signal-processing research and application also improves with better simulation. A simple dichotomy illustrates the choices one must make when processing signals. Again, considering array receivers, an option is to use classical plane-wave beamforming to analyze complex sound arrivals in the ocean (i.e., not plane waves), thus degrading performance because the signals do not match the plane-wave model. Another option is to use the physics (e.g., multipath/multimode, Doppler) to improve performance. Interesting results can be found when this is applied to small-aperture arrays and even single hydrophones. A popular current method is signal warping based on computed shallow-water waveguide modal dispersion. Figure 2 illustrates how modal dispersion causes a single pulse to morph into multiple pulses as the sound travels. Note that the various modes elongate uniquely. In signal warping, the time axis is adjusted (warped) to separate the modes in the frequency domain, which will work if the computed dispersion matches actual oceanic dispersion (Bonnel et al., 2019).

Simulations for Inversion

Because sound is very sensitive to environmental conditions, analyzing recorded sound can yield information about those conditions. This estimation of environmental properties is an example of data inversion (or the inverse problem) that pervades the earth sciences and other fields. To invert, one finds parameters of a natural state description that can be connected to a dataset with a modeled process. In ocean acoustic data inversion, the process is wave propagation influenced by the natural state, which must be understood. The understanding is called the forward problem and often takes the form of acoustic field prediction in a given environment. It can be solved using propagation models, our main topic.

In many inversion techniques, one must quantify the match between simulated data (called replicas) and experimental data. Replicas are computed for many sets of environmental parameters, and parameters are estimated by looking for the optimal fit between data and replicas. Inversion thus requires (1) powerful optimization algorithms to minimize a misfit function in a multidimensional space (the size of the space is the number of parameters to be estimated) and (2) effective propagation models that will be called on many times during the procedure. In Bayesian inversion, the trend is not only to estimate environmental parameters but also to infer the corresponding uncertainties. An approach to this uses Markov Chain Monte Carlo-like sampling methods (Dosso, 2002). It has been used to infer water column properties (Ballard and Becker, 2010), seabed properties (Bonnel et al., 2019), or both at the same time (Warner et al., 2015).

Interestingly, underwater acoustic inversion can have multiple aims. A clear goal is to learn geophysical/oceanographic information. We have already seen the tomography example where the objective is to sense ocean temperature structure. On the other hand, another goal for inversion is to infer parameters of a simplified ocean model, this model being physically inaccurate but acoustically equivalent to the true one. This specific application is particularly important for real-life users who need to run propagation models, from bioacousticians localizing whales to a navy assessing sonar performance.

Overall, today’s inversion research focuses more on the inverse methods or the inversion results than on the forward propagation models. This may indicate that propagation models are (thought to be) reliable enough. That being said, nonlinear inversion involves multiple calls of the forward models (sometimes millions), so fast models are beneficial, and using sophisticated models such as 3-D ones (Lin et al., 2013; Heaney et al., 2017) is impractical. For the same reasons, inversion usually does not allow the estimation of large parameter sets (like a range-dependent seabed). An approach to enabling larger efforts is using graphical processing units to run the forward propagation models (Belcourt et al., 2019).

Animal Studies

Locating vocalizing animals is an important step in marine mammal research. Not only is behavioral information provided when the animal is tracked over time, but passive acoustic localization is also important for accurate animal density assessment.
With multiple synchronous receivers, one can use arrival time-difference analysis to geometrically locate a source. Tiemann et al. (2004) used propagation modeling to locate whales with great success. Unfortunately, spread out synchronous systems are usually too costly for bioacoustic studies. Alternatives include the use of directional sensors or propagation models and inverse methods (see Simulations for Inversion). Here, the aim is only to infer source location; the unknown environmental parameters are seen as a nuisance, and the fact that the source signal is usually unknown is an extra difficulty. Nonetheless, when a synchronized vertical array is available, marine mammals can be localized with classical underwater acoustic inverse methods (e.g., Thode et al., 2000). Stepping down to a single sensor, a common arrangement for bioacoustic studies, causes further issues to arise. However, results are obtainable in this case when advanced signal-processing methods and propagation models are engaged. Toothed whales (high-frequency sources) can be localized by analyzing single-hydrophone ray arrivals (Tiemann et al., 2006). On the other hand, baleen whales (low-frequency sources) can be localized by analyzing the mode arrivals (e.g., Bonnel et al., 2014).

Marine mammals are not the only underwater dwellers producing and using sounds. Adequate propagation models are required to study all species. Considering humans, global noise models are needed to study the impact of anthropogenic activities on the entire marine ecosystem. Such noise models are discussed in Noise Modeling. On the other hand, many marine animals are studied in tanks and labs. Here, propagation models are required to (1) correctly understand the sound recorded in a reverberant tank and (2) predict sound properties in the sea from measurements performed in the lab. Of particular importance may be the use of propagation models for vector acoustics (water motion speed and direction; Heaney and Campbell, 2019) because fishes and crustaceans are highly sensitive to particle motion (Popper and Hawkins, 2018).

Simulations to Study Propagation Physics

As we have seen, ocean features control sound propagation in complex ways. Sound fields are affected by multiple interactions whose effects cascade nonlinearly along the propagation path. As detailed in Acoustic Tomography and Thermometry and Simulations for Inversion, the resultant fields can be used to infer environment variables, assisted by forward simulation. But the forward studies alone can be illuminating, particularly for nonlinearly chained events. The situation depicted in Figure 2 of a computation revealing a physics effect is not unique.

Some aspects to consider for studying chained propagation events is that the sound interactions with the environment are frequency dependent; higher frequencies are sensitive to smaller scale processes, whereas lower frequencies used over longer ranges will integrate the effects of more physical constraints. Ocean internal and surface features are dynamic and time variable, whereas the bottom shape and sediments below can be taken as static. Note, however, that the locations and angles of bottom interactions can change over time due to changing water column conditions, illustrating the chained nature.

Using these principles, sound simulation can be used as a tool to study and characterize complex sound/ocean-feature interaction (e.g., with internal waves, fronts, eddies and filaments, bottom vegetation, and coral reef roughness). After achieving that, a heady long-term goal would be the acoustic measurement of ocean phenomena with a synopticity that cannot be achieved by direct in situ measurements. The challenge is that sound signals arriving after traversing a volume are not each directly connected with a single ocean-state parameter but with many. This yields complex parameter-to-observation operators that require high-fidelity simulations to correlate the observed signal with the physical environment.

To illustrate, consider the acoustic effects of eddies, which have a huge range of parameters, and may control sound differently for varying source frequency and depth. Using an initial guess (ocean background), one can map the local features and identify the areas where alterations in sound speed (usually via altered temperature) are more likely to cause important sound field changes. The map in Figure 5, top right, shows an example with a lateral sound refraction metric based on the sound speed gradients that are based on US Navy Coastal Ocean Model (NCOM) ocean simulations. The maps in Figure 5, bottom, show simulations, from a 3-D ray/beam model (Porter, 2019), of the sound level estimate taken at two times for sound propagating outward from a point source. Sound rays beginning radially outward that strike eddy edges bend horizontally. At present, the quantification of the effects on sound fields of modeled eddies is
underway. Because the true ocean will differ, the background estimates could be used to plan a survey that would establish the full impact of the local dynamics on acoustic propagation.

This same area is known to support internal tides (tide-forced internal waves) with scales as short as a few kilometers. Because these features are small and changing rapidly, they are challenging to study with in situ sampling methods. However, because they affect sound speed, they have an acoustic impact. Figure 5 shows how a modeled sound level changes over a three-hour period. After 12 hours the simulated sound returns close to the initial conditions, confirming the strong modeled changes to be tidally related. The effects of the full spectrum of eddies and internal waves on sound remain to be established, with some aspects treated deterministically and some stochastically (Colosi, 2016; Duda, 2017). For example, a system using stochastic ocean simulations and acoustic simulations can infer the most likely environments (Coelho et al., 2015).

**Noise Modeling**

Computational modeling of underwater noise can be broken down into the component parts of determining the sources of noise and their parameters, tracking the sources and updating their parameters, propagating each sound, and then adding fields or field energies. Example sources would be ships, and parameters would be sound signatures as a function of speed, location, and speed.

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**Figure 5.** Sound modeling south of Long Island, NY, using conditions from an ocean model. Top left: surface current and temperature snapshot showing eddies. Top right: derived parameter (metric) thought to govern the horizontal refraction of sound, which could create areas of strong sound and shadow zones. Bottom: simulation results for sound propagating outward from a 1,500 Hz source at 10 m depth at 2 times, 3 hours apart. Energy reduction in the 10-m depth plane is shown in dB re source level. The up/down heaving of the sound speed layering from tide-driven internal waves changes the refraction and gives a strong reduction of sound energy near the surface over the time interval.
There are many motivations for modeling underwater noise. All human uses of active underwater sound (using generated sound signals) are subject to the signal-to-noise ratio at the receivers. If fully masked by noise, the sound is not usable. Additionally, marine mammals and other creatures are affected by underwater sound, both natural and anthropogenic. The levels of anthropogenic sound reaching marine mammals in their natural habitats can be estimated using verified noise models. One can easily imagine sound from multiple sources of known location, such as ships and breaking ocean waves from gales, each being modeled as outgoing in three dimensions, with the power from each summed everywhere to produce 3-D maps of noise. At the present time, noise models use ocean propagation conditions taken from ocean models (Figure 5), which, of course, do not fully match reality. This means that modeled noise fields will have uncertainty, to be evaluated most reliably with experiments.

Summary
We hope to have provided insight into the reasons why computed acoustic fields are so commonly incorporated into many types of oceanographic research. Both naturally occurring and man-made sounds can be used to learn about processes in the sea. Detailed knowledge of how the sound moves through the ocean, obtainable in many situations with computational methods, allows more of the information in the sound signals to be tapped for research purposes.

Acknowledgments
We thank the US Office of Naval Research for support. Timothy F. Duda acknowledges additional support from the Walter A. and Hope Noyes Smith Chair for Excellence in Oceanography award at WHOI (2019).

References


**BioSketches**

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